Hadron Resonance Gas and lattice QCD Equation of State

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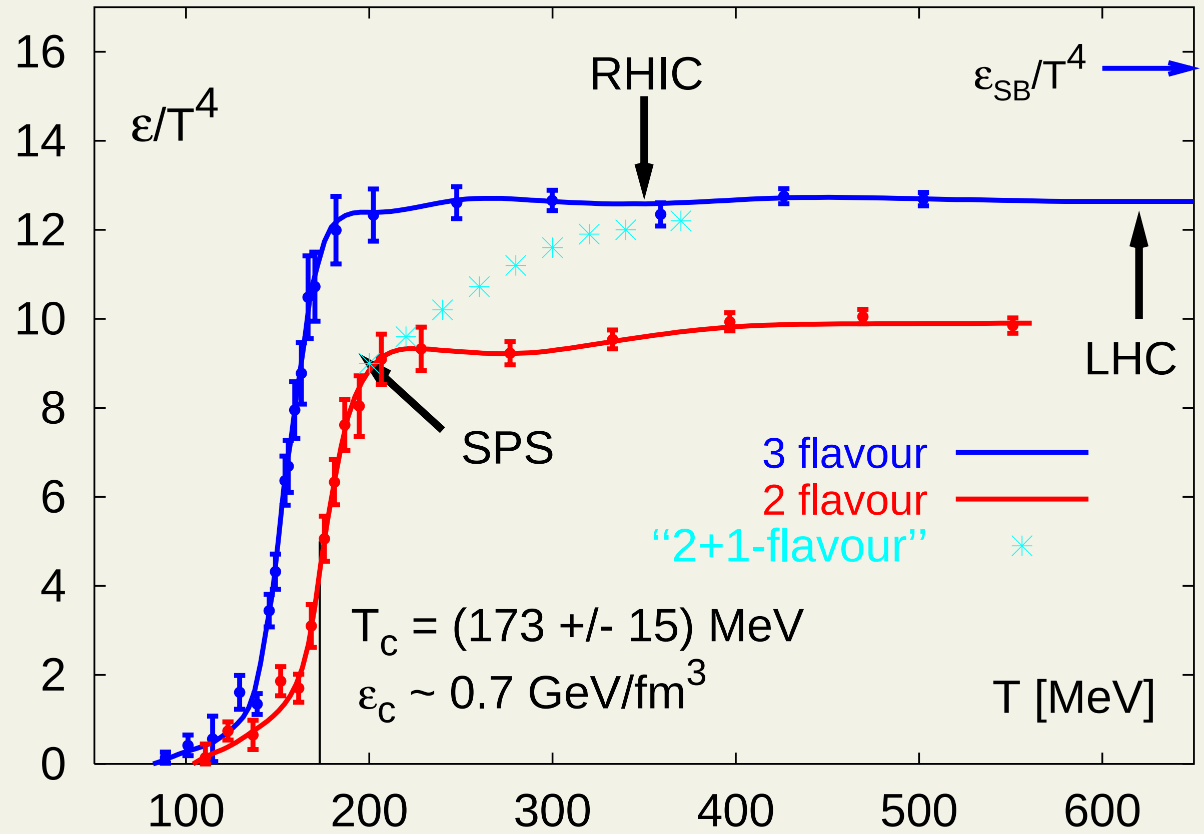
QCD equation of state

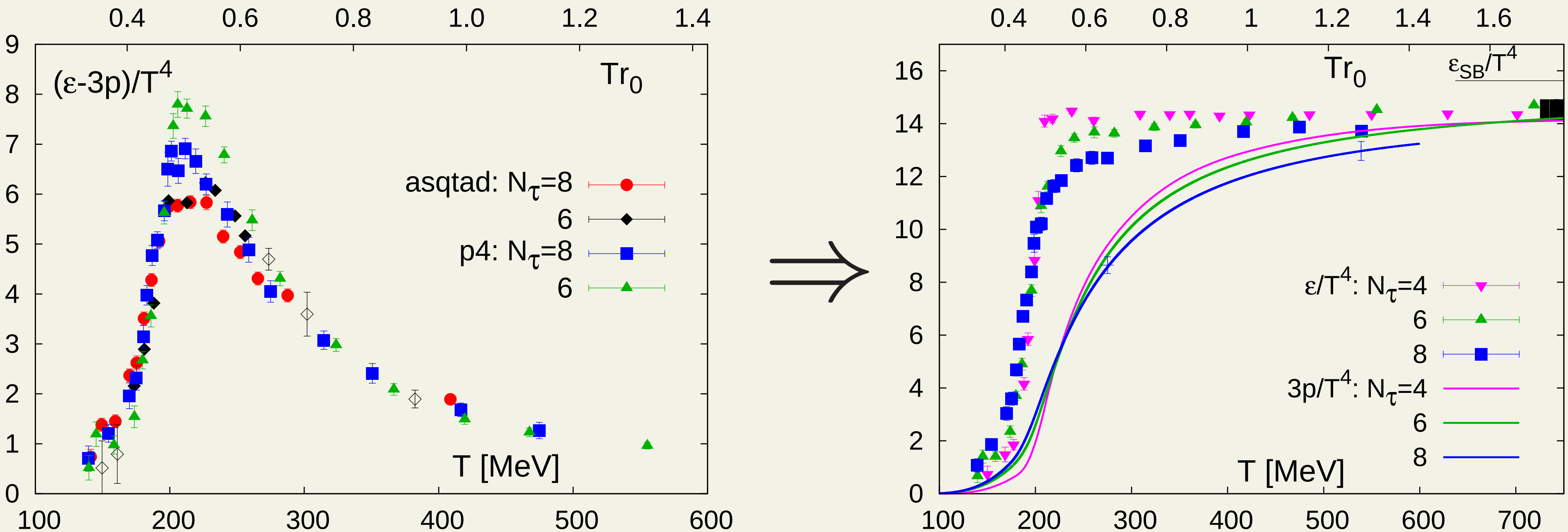
lattice QCD (Karsch & Laermann, hep-lat/0305025):



- **EoS from first principles**
- **seldom used in hydro calculations**

EoS by hotQCD collaboration

Bazavov *et al.* arXiv:0903.4379 [hep-lat]



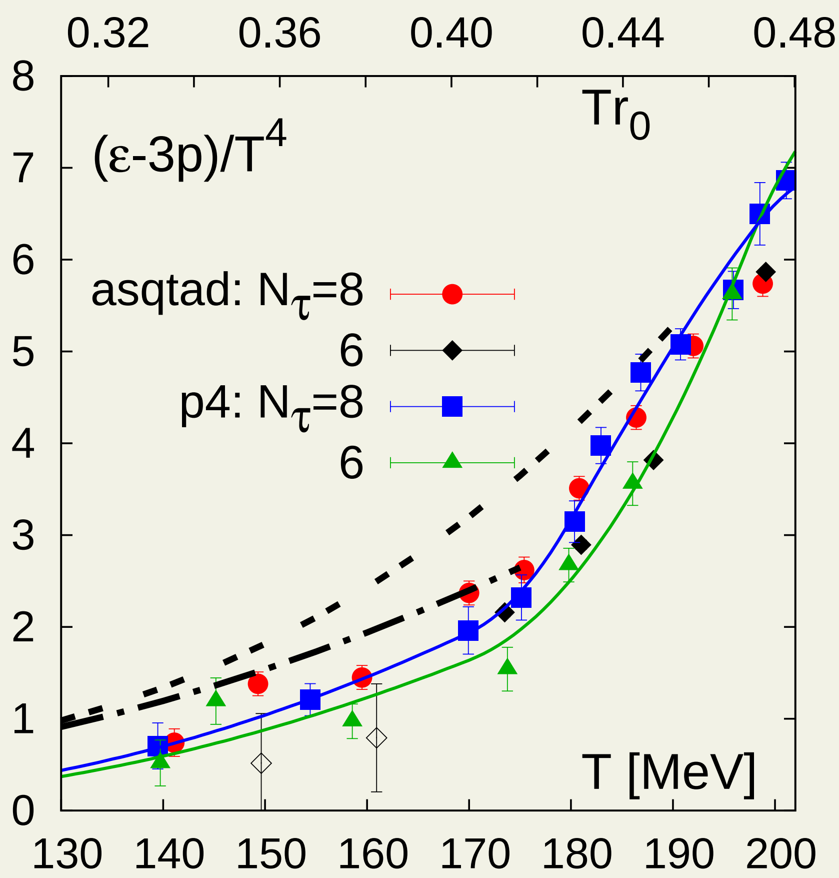
- **evaluate interaction measure** $(\epsilon - 3P)/T^4$
- **obtain pressure via**

$$\frac{P}{T^4} - \frac{P_0}{T_0^4} = \int_{T_0}^{T} \mathrm{d}T' \, \frac{\epsilon - 3P}{T'^5}$$

- **What is** $P(T_0)$**?**
- **What is** $(\epsilon(T_0) - 3P(T_0))/T_0^4$**?**
- **How good is lattice below** T_c**?**

Interaction measure below T_c

Bazavov *et al* arXiv:0903.4379



- **Lattice EoS $\neq$ Hadron Resonance Gas EoS**

Hadron Resonance Gas model

- EoS of interacting hadron gas well approximated by non-interacting gas of hadrons and resonances

$$P(T) = \sum_i \int \mathrm{d}^3p \, \frac{p^2}{3E} \, f(p,T)$$

- valid when
- interactions mediated by resonances
- resonances have zero width

- Prakash & Venugopalan, NPA546, 718 (1992): experimental phase shifts
- Gerber & Leutwyler, NPB321, 387 (1989): chiral perturbation theory

$\Rightarrow$ HRG good approximation at low temperatures

$\rightarrow$ lattice should reproduce HRG at $T \leq 120 - 140$ MeV

- practical problem: how to convert fluid to particles?
- energy conservation iff EoS is the same before and after freeze-out

Hadrons on lattice

- **Hadron masses depend on lattice cutoff**

$\Rightarrow$ i.e. on temperature:

$$\begin{aligned} m_i(a) &= m_i(a_0) + B_i(a^2 - a_0^2) \\ a &= \frac{T_0}{N_\tau T} \end{aligned}$$

where $T_0 = 197$ **MeV and** $a_0 = 0.125$

+ 16 pseudoscalar mesons on lattice

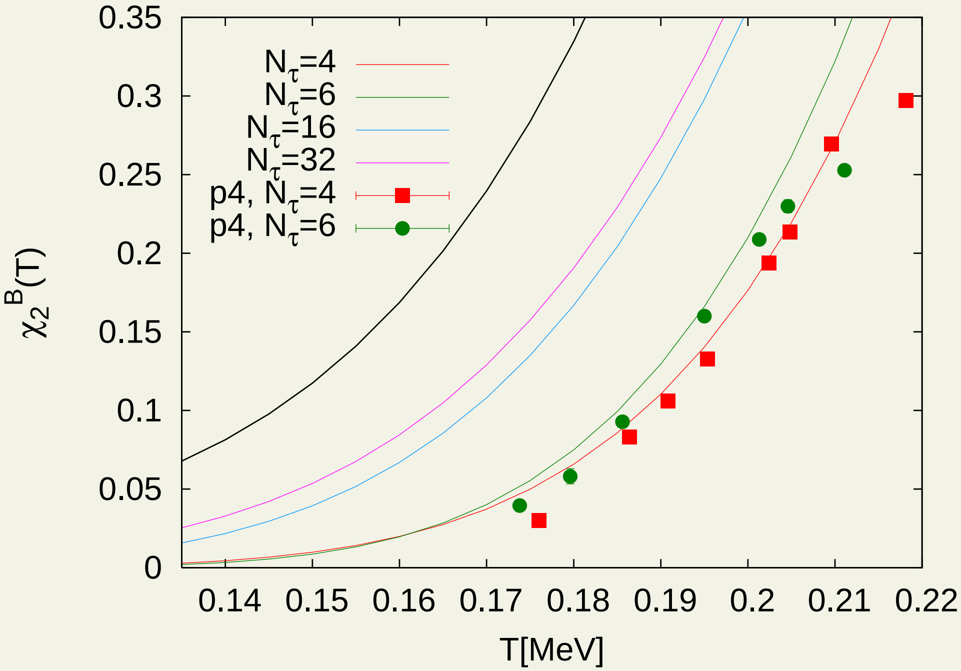
- **HRG with lattice mass spectrum?**

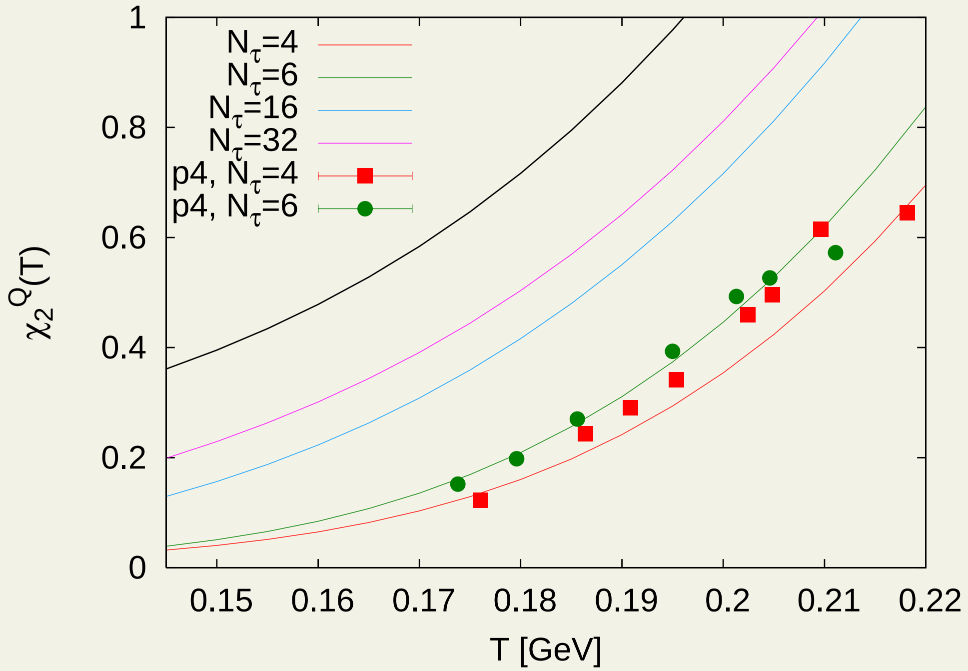
Hadronic fluctuations

i.e. baryon number, strangeness and charge susceptibilities

$$\chi_2^x = \frac{1}{VT^3}\frac{\partial^2 \ln Z}{\partial(\mu_x/T)^2} = \frac{1}{T^2}\frac{\partial^2 P}{\partial \mu_x^2},$$

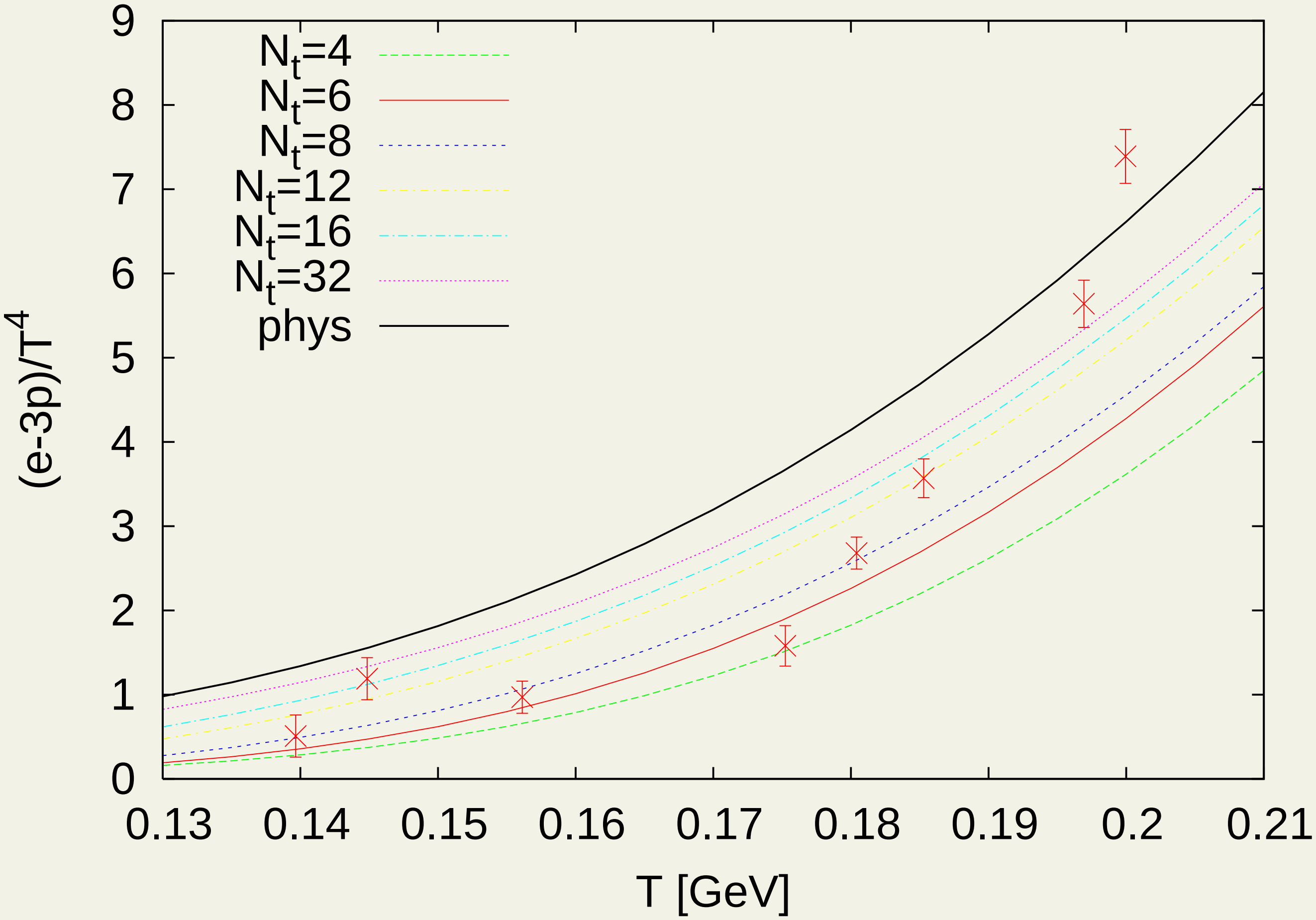
where $\mu_x = \mu_B, \mu_S$ **or** μ_Q





- **Lattice masses → fluctuations in resonance gas and lattice similar**

Interaction measure

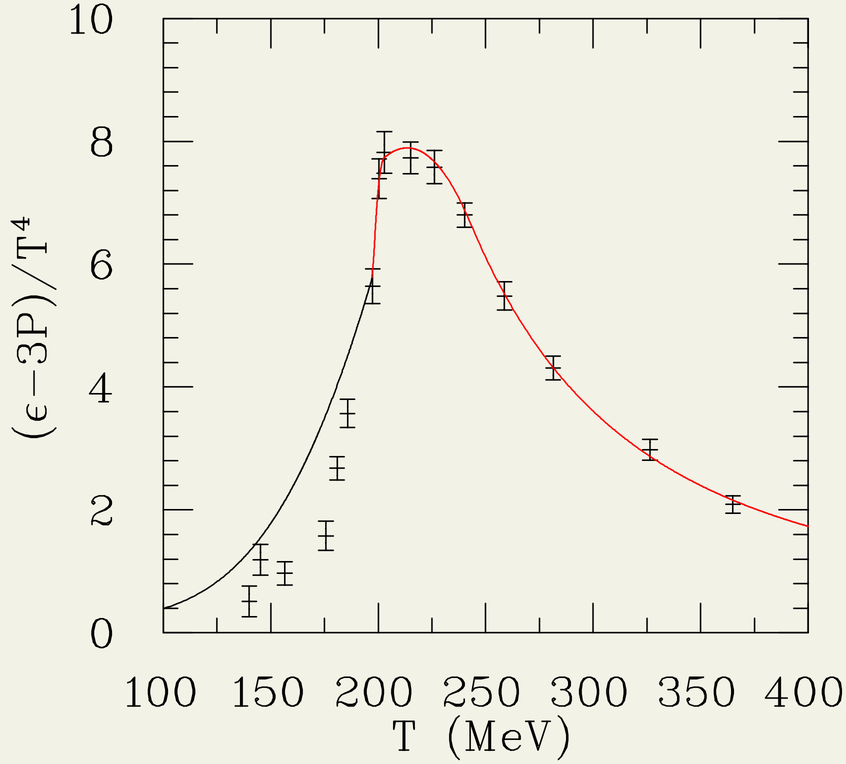


- **very little room for modifications in hadron gas**
- **BUT, what is physical mass spectrum?**
- **conservative estimate: free particle masses**

Phenomenological EoS

- $T < T_{sw}$: **HRG interaction measure (black)**
- $T > T_{sw}$: **Lattice interaction measure (red)**

- **lattice $N_\tau = 6$ data, Cheng *et al.* Phys. Rev. D 77, 014511 (2008)**



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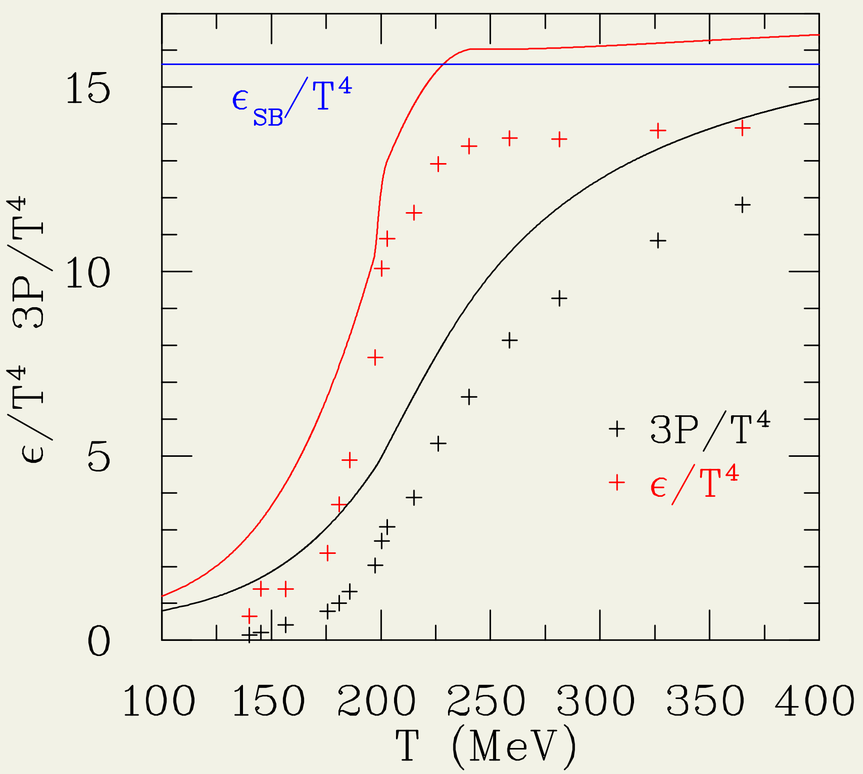


- ϵ **and** P **overshoot Stefan-Boltzmann limit!**
- **Interaction measure too large, but where?**

Procedure for EoS

- **HRG below** $T \approx 180 - 190$ **MeV**
- **Parametrize lattice using:**

$$\frac{\epsilon - 3P}{T^4} = \frac{d_2}{T^2} + \frac{d_4}{T^4} + \frac{c_1}{T^{n_1}} + \frac{c_2}{T^{n_2}}$$

- **Require that:**

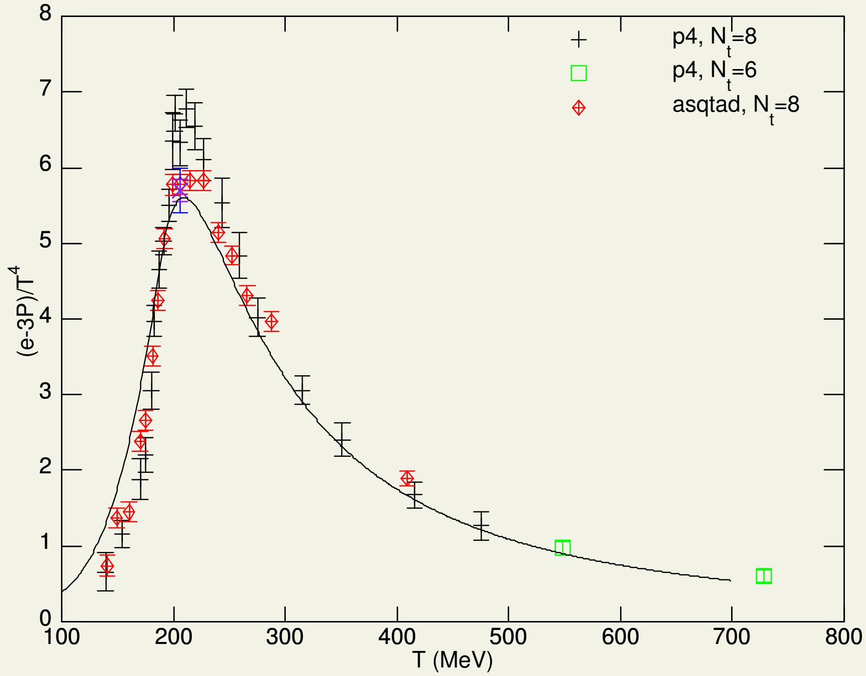
$$\left.\frac{\epsilon - 3P}{T^4}\right|_{T_0}, \quad \left.\frac{\mathrm{d}}{\mathrm{d}T}\frac{\epsilon - 3P}{T^4}\right|_{T_0}, \quad \left.\frac{\mathrm{d}^2}{\mathrm{d}T^2}\frac{\epsilon - 3P}{T^4}\right|_{T_0} \quad \text{are continuous} \tag{1}$$

$$\left.\frac{P}{T^4}\right|_{T=800\mathrm{MeV}} = 0.95\,\frac{s_{\mathrm{SB}}}{T^4} \tag{2}$$

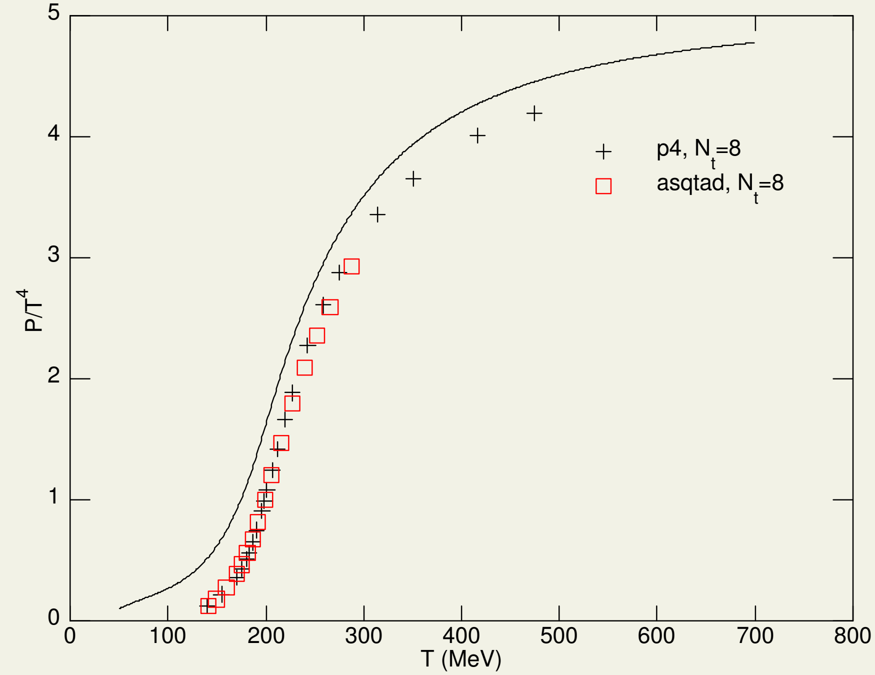
$\Longrightarrow T_0$, d_4, c_1, c_2 **fixed**

- χ^2 **fit to lattice above** $T = 250$ **MeV + one point at** $T = 206$ **MeV**
- **We get** $T_0 = 184.2$ **MeV**, $d_2 = 0.2588$, $d_4 = 3.483 \cdot 10^{-3}$,
  $c_1 = -1.634 \cdot 10^{-6}$, $c_2 = 2.560 \cdot 10^{-28}$, $n_1 = 9$, $n_2 = 37$

Phenomenological EoS



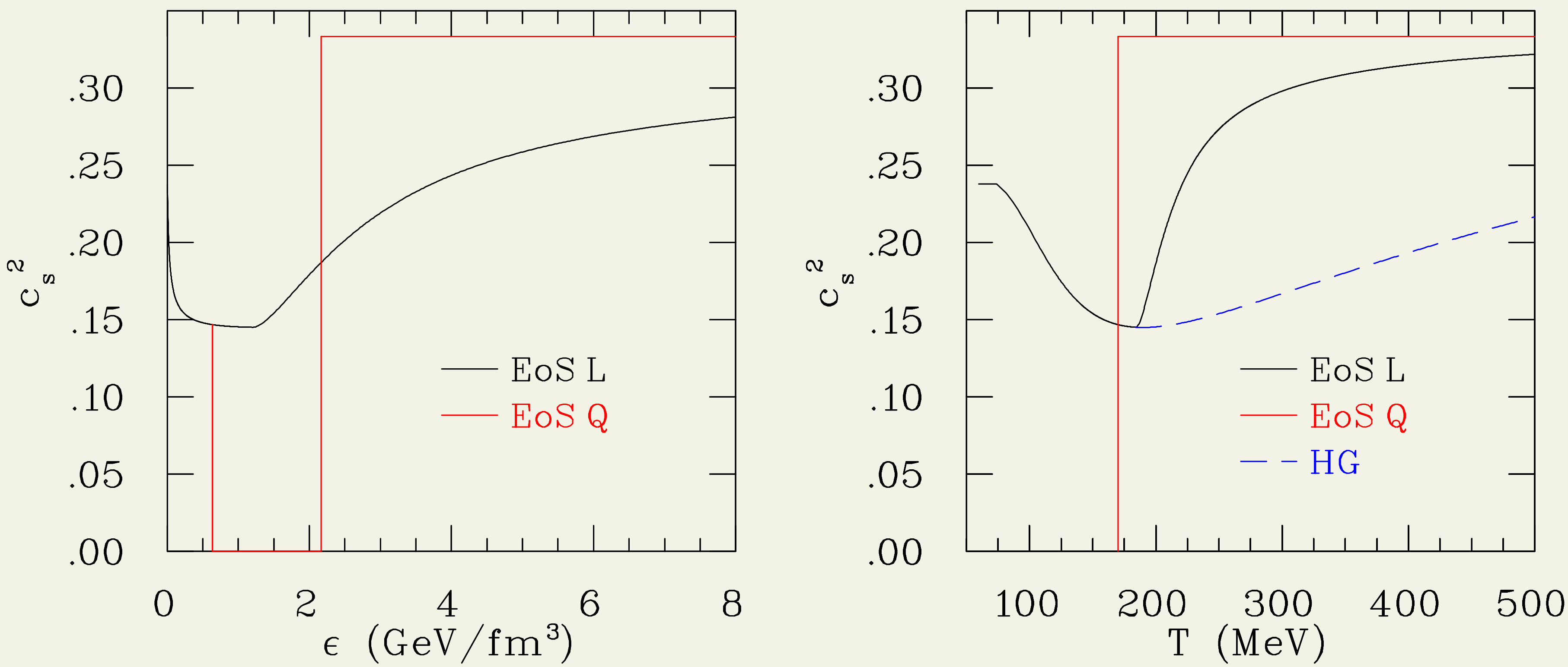
$\Rightarrow$



- **interpolated interaction measure** $(\epsilon - 3P)/T^4$
- **obtain pressure via**

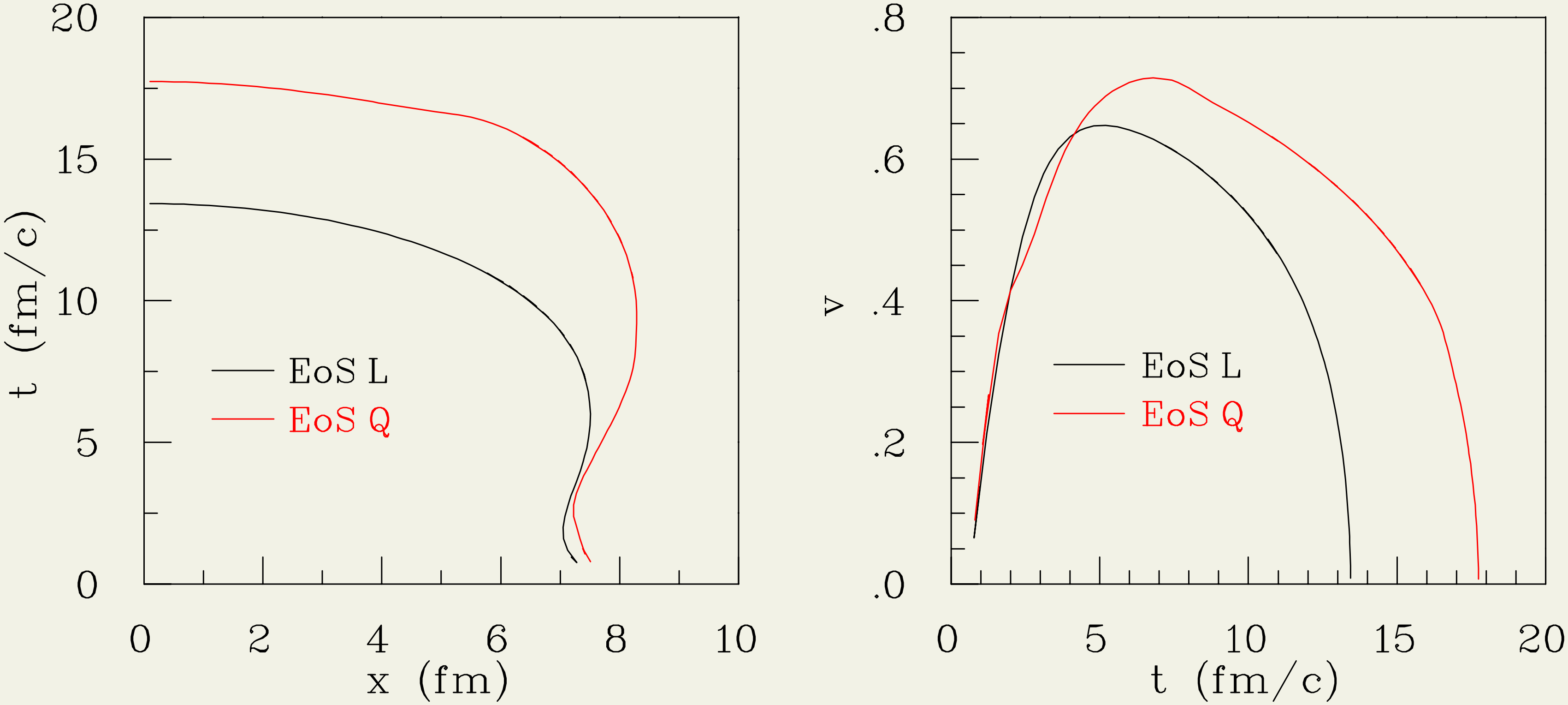
$$\frac{P}{T^4} - \frac{P_0}{T_0^4} = \int_{T_0}^{T} \mathrm{d}T' \, \frac{\epsilon - 3P}{T'^5}$$

Speed of sound



- **no softening below the HRG!**

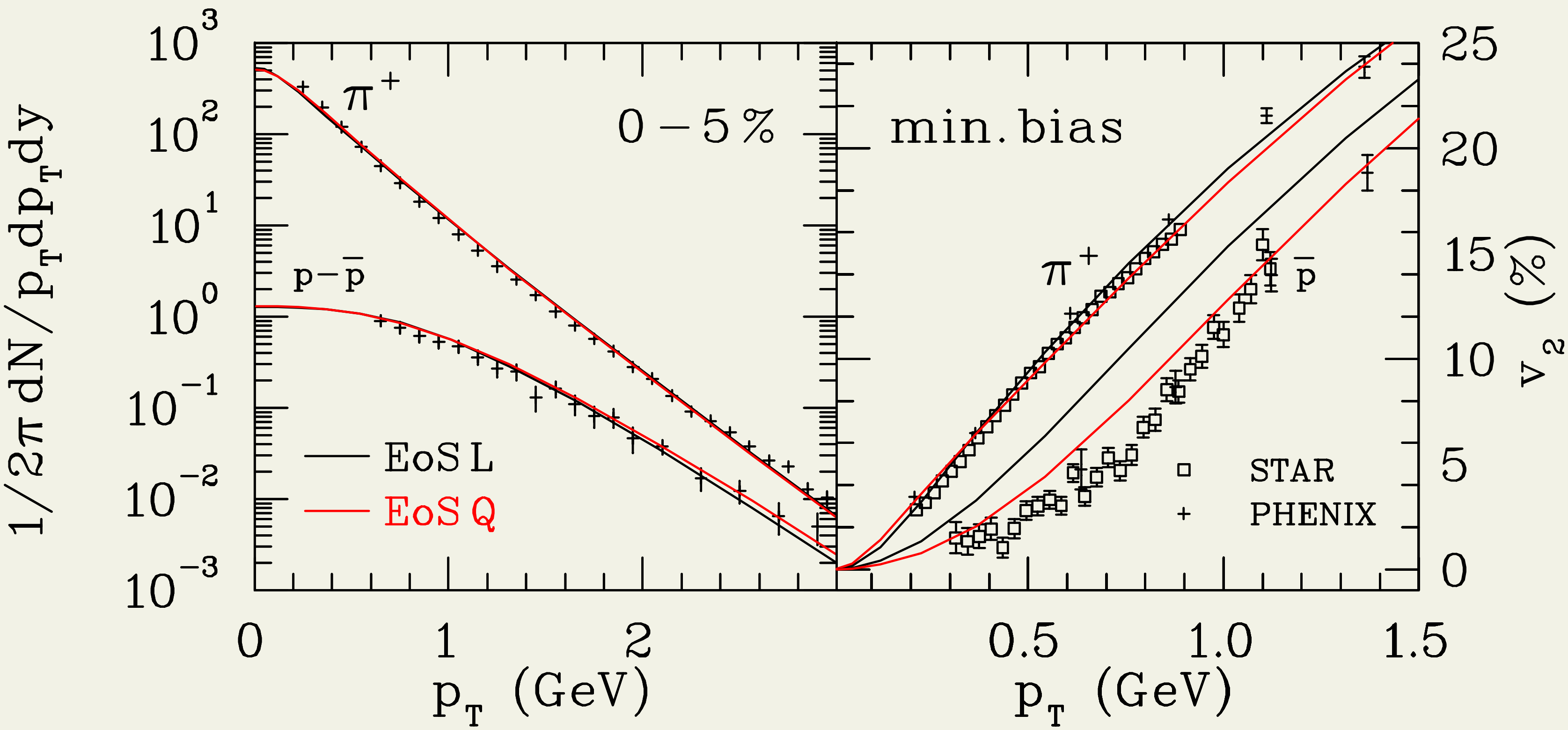
Effect on expansion



- **Harder EoS** $\Longrightarrow$ **Faster decoupling** BUT **smaller flow velocity**

Effect on flow I

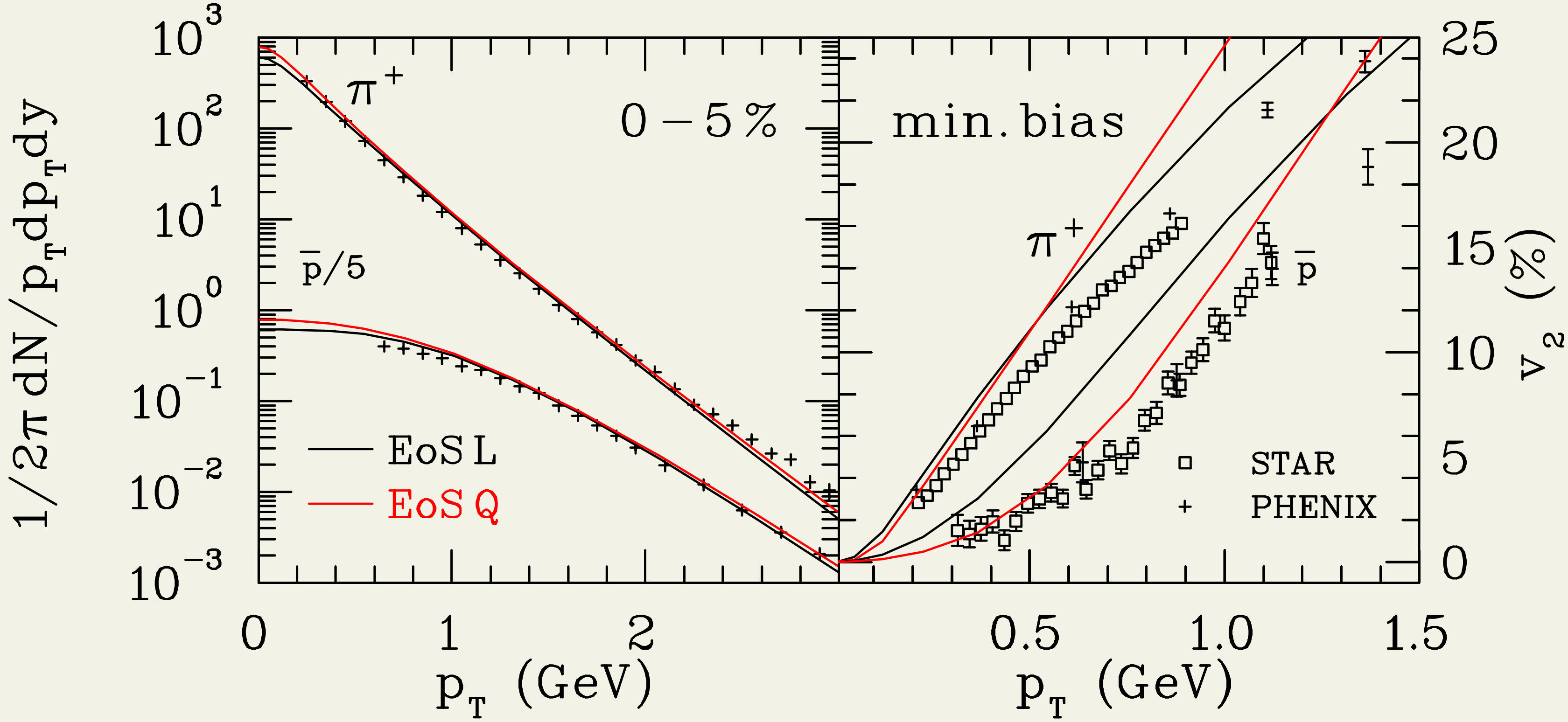
- ideal hydro, Au+Au at $\sqrt{s_{NN}} = 200$ GeV
- chemical equilibrium



- EoS Q: first order phase transition at $T_c = 170$ MeV, $T_{dec} = 125$ MeV
- EoS L: interpolation from HRG to lattice, $T_{dec} = 140$ MeV

Effect on flow II

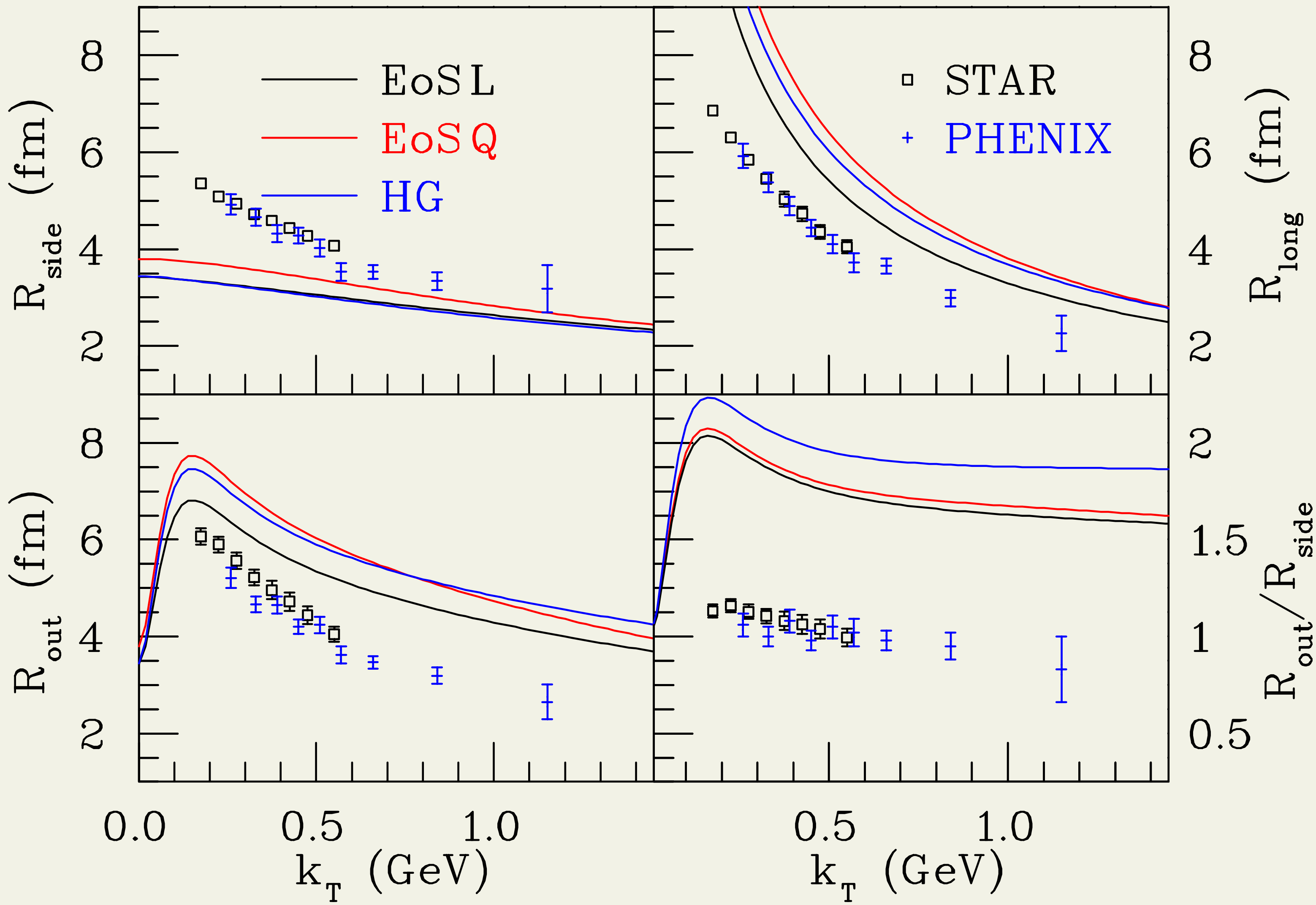
- **ideal hydro, Au+Au at $\sqrt{s_{NN}} = 200$ GeV**
- $T_{\rm chem} = 150$ **MeV**



- **EoS Q:** $T_{dec} = 120$ **MeV**, $s_{\rm ini} \propto N_{bin}$ $\tau_0 = 0.2$ **fm/**c
- **EoS L:** $T_{dec} = 120$ **MeV**, $\epsilon_{\rm ini} \propto N_{bin}$, $\tau_0 = 0.6$ **fm/**c

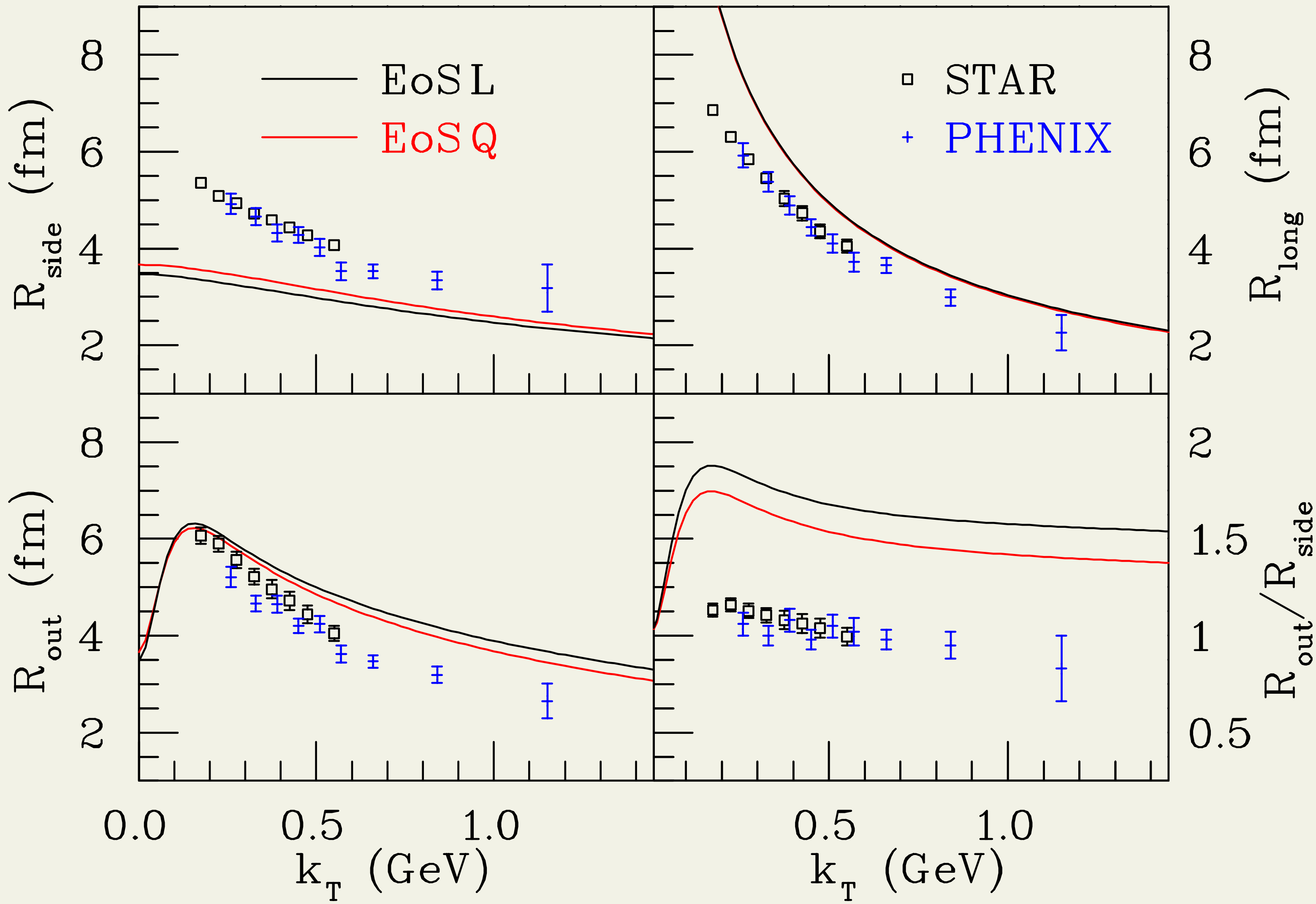
Two particle correlations I

- **Not true HBT radii but variances**



- R_{out}/R_{side} **is largest when there is no phase transition!**

Two particle correlations II



Conclusions

- below T_c lattice and HRG differ because of hadron mass spectrum

$\Rightarrow$ HRG good description below T_c

- proton $v_2(p_T)$ sensitive to EoS
- lattice EoS improves the fit to HBT radii
- first order phase transition largest R_{out}/R_{side}
- can we measure η/s and EoS simultaneously?