High energy scattering in QCD at small Bjorken x:

from ultra-high energy neutrinos and cosmic rays to high energy heavy ion collisions

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OUTLINE

Perturbative QCD

asymptotic freedom

particle production at high $\mathbf{p_t}$

collinear factorization

QCD at high energy (CGC)

high energy => small x

large number of gluons

Applications: particle production

DIS, *pp, pA, AA*

Quantum ChromoDynamics (QCD)

theory of interactions between quarks and gluons $\mathbf{SU(N_c)}$ with $N_c = 3$

$$\mathcal{L} = -\frac{1}{4} G^a_{\mu\nu} G^{\mu\nu}_a + \sum^{f} \bar{\Psi}^\alpha_i \left[i \not{D} - m_f \right]^{ij}_{\alpha\beta} \Psi^\beta_j$$

$$G^a_{\mu\nu}(x) \equiv \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - g f^{abc} A^b_\mu A^c_\nu$$

color index:
$$\begin{aligned} a, b, c &= 1, \cdots, 8 \\ \alpha, \beta &= 1, 2, 3 \end{aligned}$$

f^{abc} *group structure constant*

Lorentz index: $\mu, \nu = 0, 1, 2, 3$

$\not{D} \equiv D_\mu \gamma^\mu$ *with* $\{\gamma^\mu, \gamma^\nu\} = 2 g^{\mu\nu}$

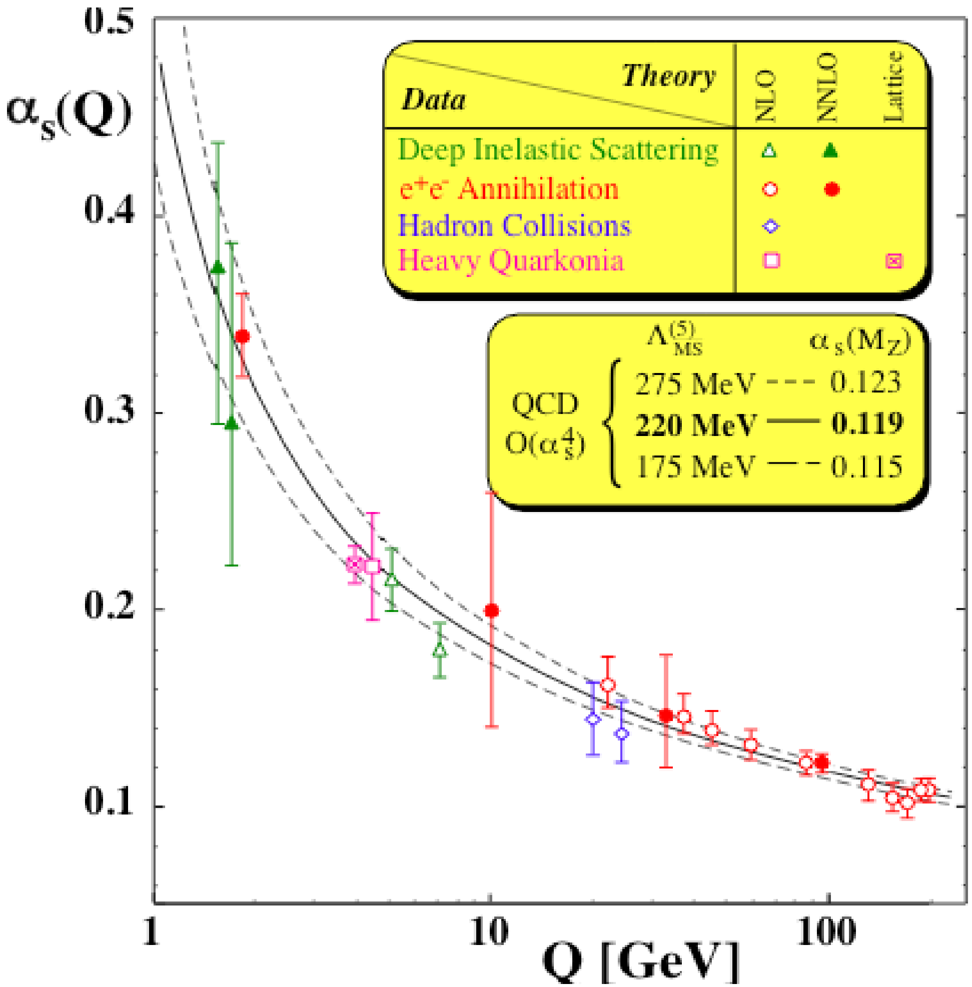
spinor index: $i, j = 1, 2, 3, 4$

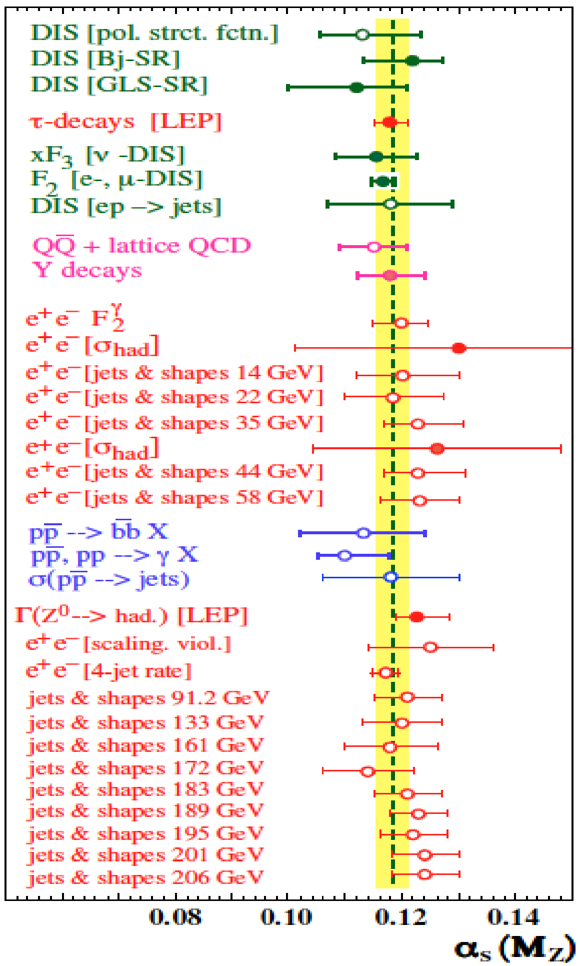
$D_\mu \equiv \partial_\mu + i g A_\mu$ *covariant derivative*

Quarks:
Fermions, spin 1/2
4x1 spinor, come in N_c colors
6 flavors (up, down, ..., top)
carry electric charge

Gluons:
Bosons, spin 1
come in $N_c^2 - 1$ colors
flavor blind
have no electric charge

Perturbative QCD



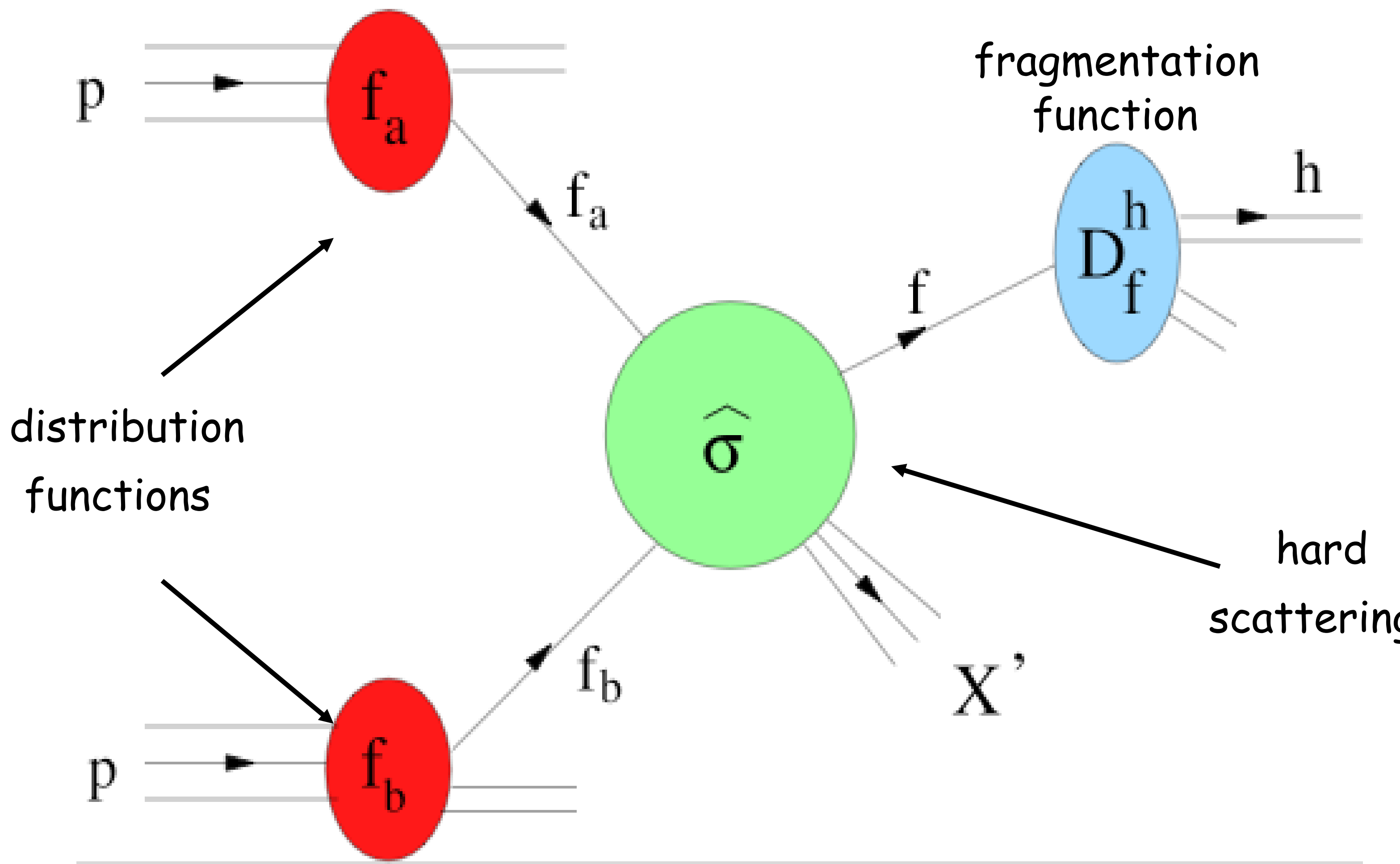


running of the coupling constant

expansion in powers of the coupling $\alpha_s \ll 1$

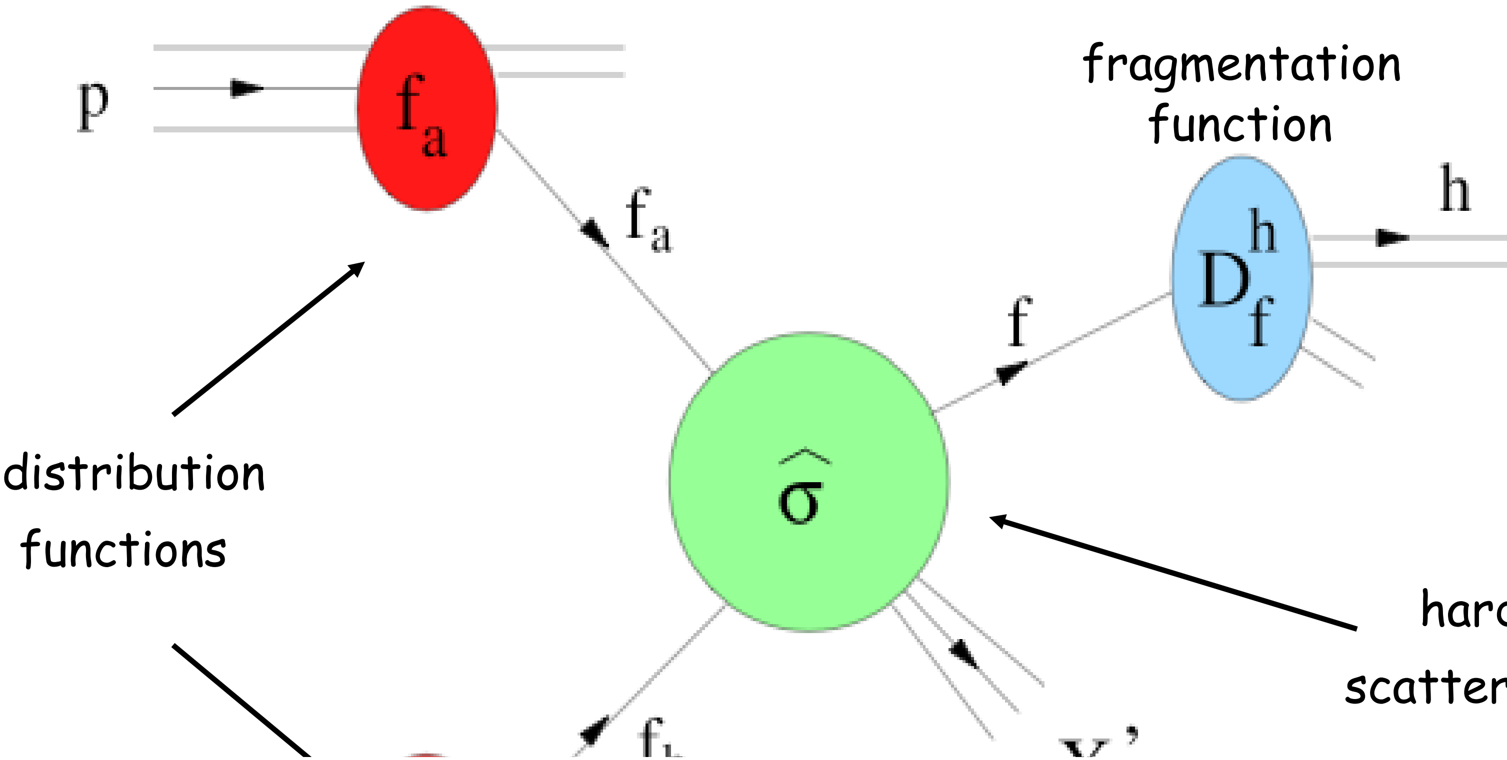
pQCD in pp Collisions

Collinear factorization :separation of long and short distances



pQCD in pp Collisions

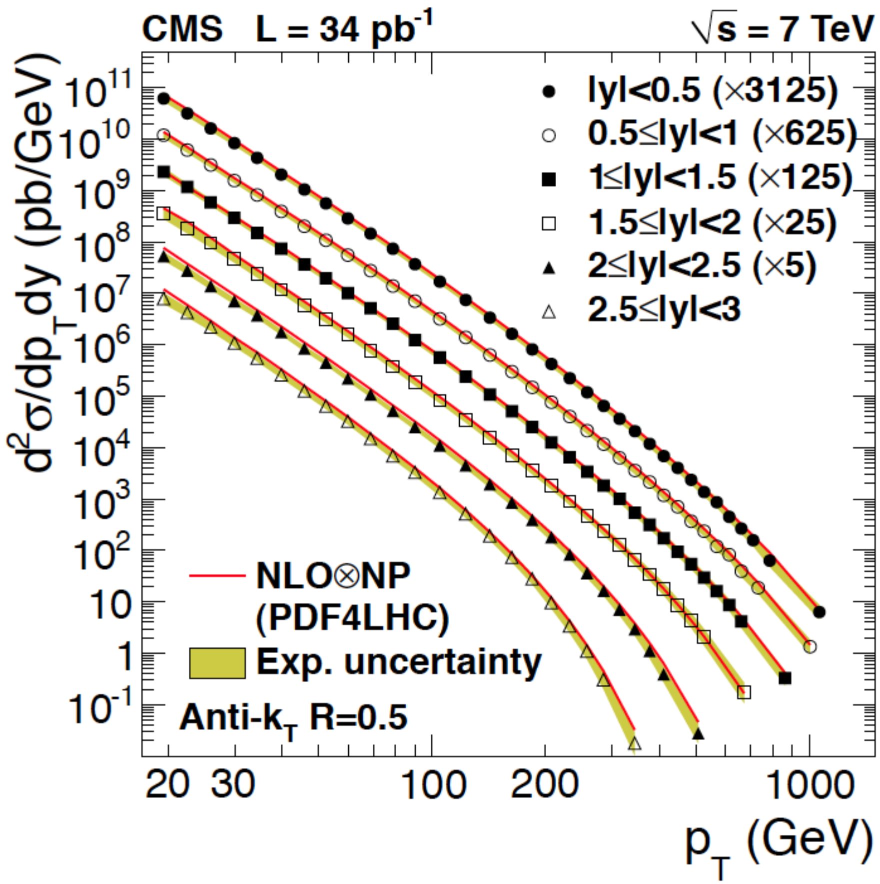
Collinear factorization :separation of long and short distances



$$d\sigma = \int dx_1\, dx_2\, dz\, f_a^{H1}(x_1, M^2)\, f_b^{H2}(x_2, M^2)\, D_c^h(z, M^2)$$

$$\otimes \quad d\hat{\sigma}_{ab}^c(x_1 P_{H1}, x_2 P_{H2}, P_h/z, M^2)$$

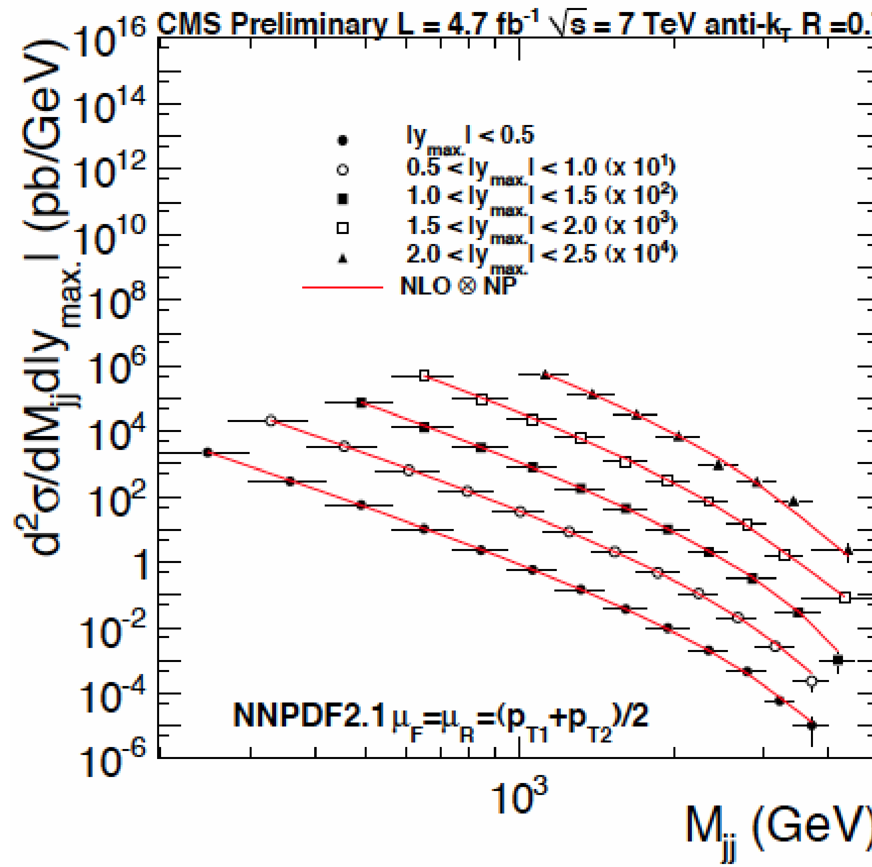
pQCD: *a success story*



two recent examples from the LHC:

1-jet and di-jet cross sections

many other final-states available



results are starting to be used in global fits to constrain PDFs

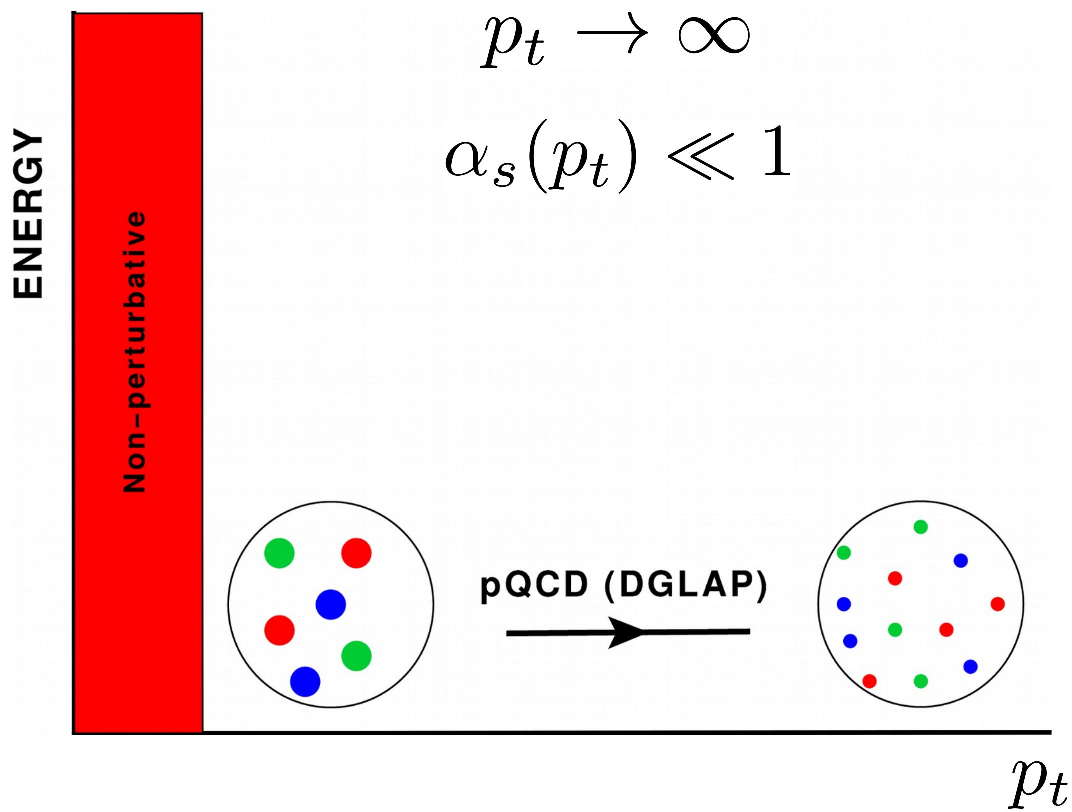
particularly sensitive to gluons

$$gg \to gg \qquad gq \to gq$$

QCD: the standard paradigm

$$p_t \to \infty$$

$$\alpha_s(p_t) \ll 1$$



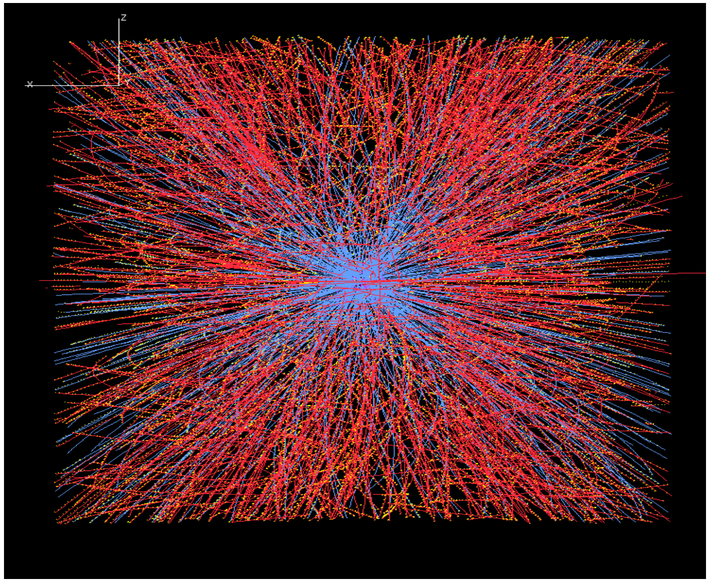
pQCD tools:
twist expansion,
collinear factorization

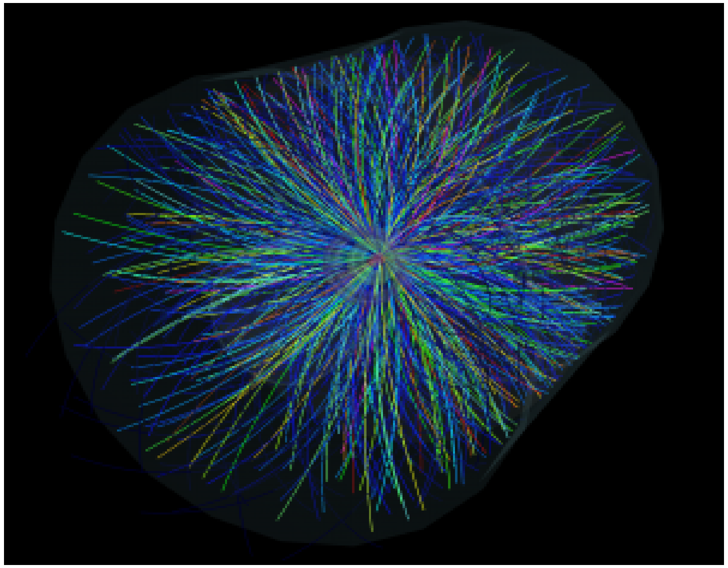
$$\sqrt{\mathbf{s}} \to \infty$$

but bulk of QCD phenomena happens at low p_t

Nucleus-Nucleus (AA) Collisions:

Quark-Gluon Plasma





$$\sqrt{S} \sim 200\,\mathrm{GeV}$$

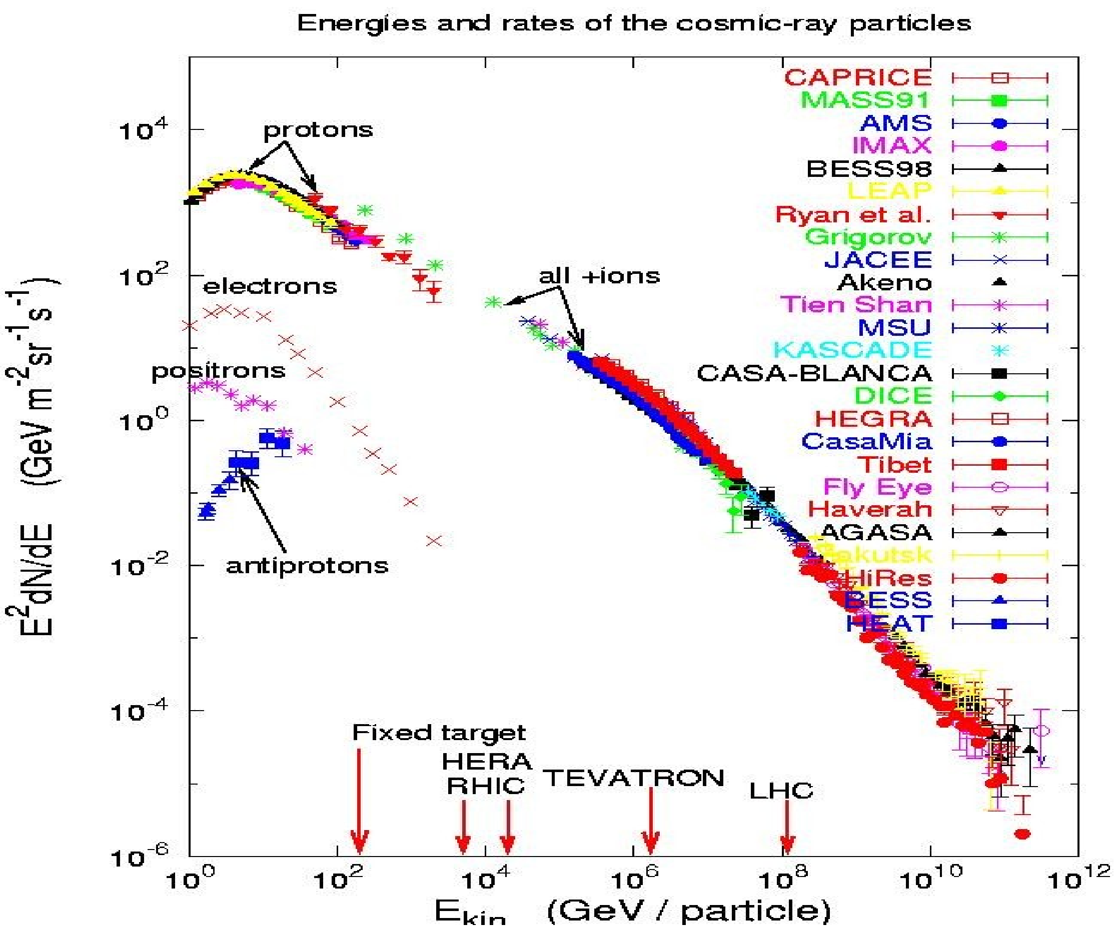
$$\sqrt{S} \sim 5\,\mathrm{TeV}$$

$$\mathrm{RHIC}: \ \frac{dN_{ch}}{d\eta} \sim 700$$

$$\mathrm{LHC}: \ \frac{dN_{ch}}{d\eta} \sim 1600$$

$$x \sim \frac{p_t}{\sqrt{S}}\, e^{-y} \rightarrow 0$$

High Energy Cosmic Rays



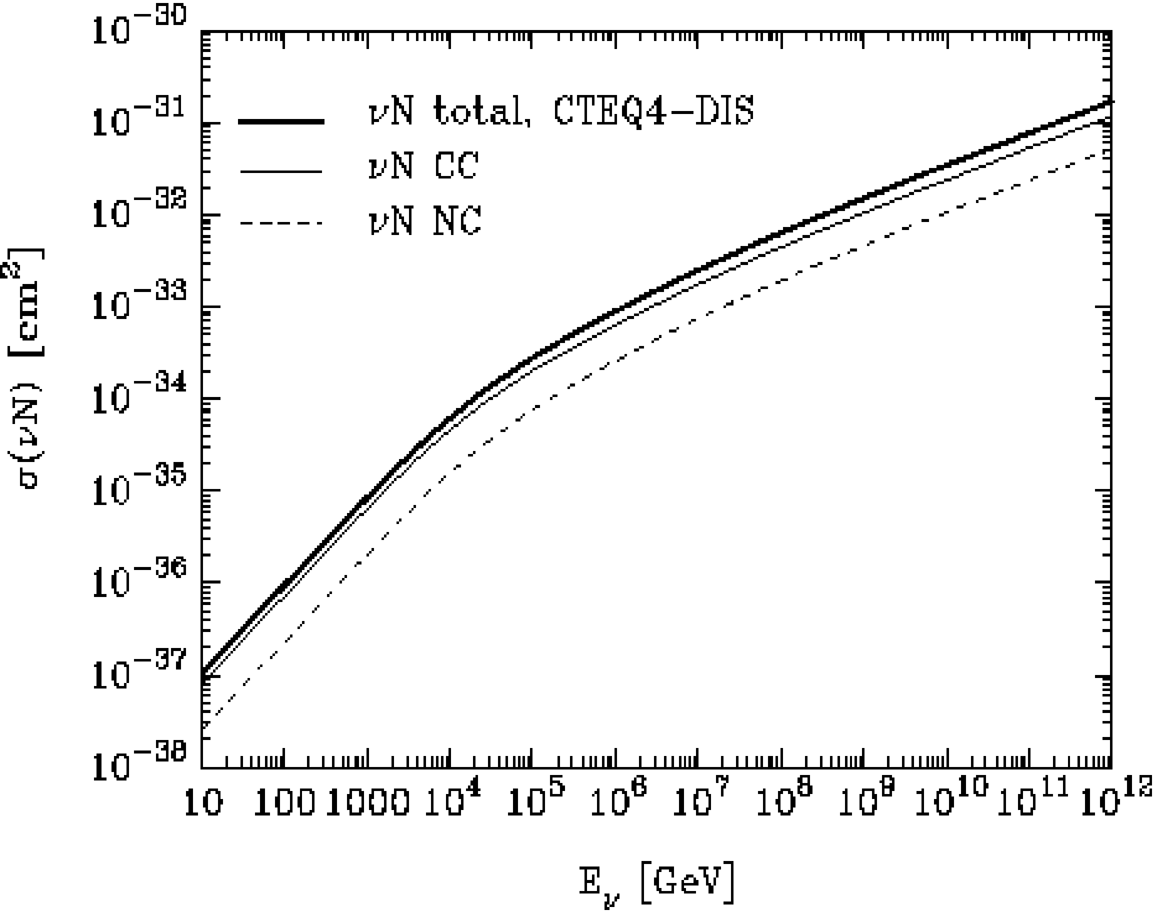
$$\mathbf{p\,A \to X}$$

$$\sqrt{\mathbf{S}} \sim \mathbf{10^{2-3} TeV}$$

most particles/energy are in the <u>forward rapidity</u> region and have low p_t

$$\mathbf{x \sim \frac{p_t}{\sqrt{S}} e^{-y} \to 0}$$

Ultra-High Energy Neutrinos



$$\nu\,\mathbf{N} \to \nu\,\mathbf{X}$$

$$\sqrt{\mathbf{S}} \sim \mathbf{10^{2-3}TeV}$$

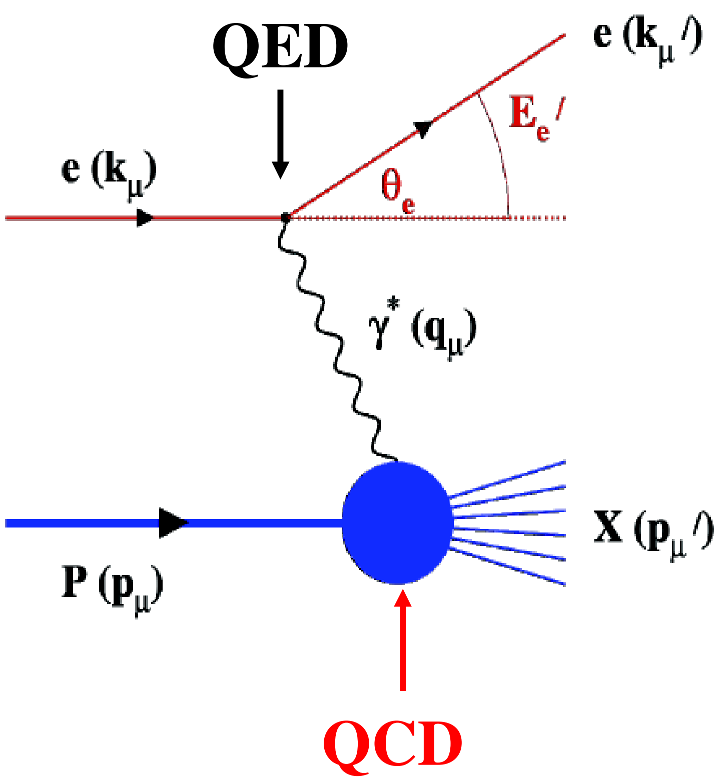
$$\mathbf{x} \sim \frac{\mathbf{M_z}}{\sqrt{\mathbf{S}}} \to \mathbf{0}$$

total cross section dominated by Q ~ M_Z

need to understand structure of hadrons at *very small x*

Deeply Inelastic Scattering (DIS)

probing hadron structure



(F_1, F_2 structure functions)

Kinematic Invariants

$$Q^2 = -q^2 = -(k_\mu - k_\mu')^2$$

$$Q^2 = 4E_e E_e' \sin^2\left(\frac{\theta_e'}{2}\right)$$

Measure of resolution power

$$y = \frac{pq}{pk} = 1 - \frac{E_e'}{E_e}\cos^2\left(\frac{\theta_e'}{2}\right)$$

Measure of inelasticity

$$x = \frac{Q^2}{2pq} = \frac{Q^2}{sy}$$

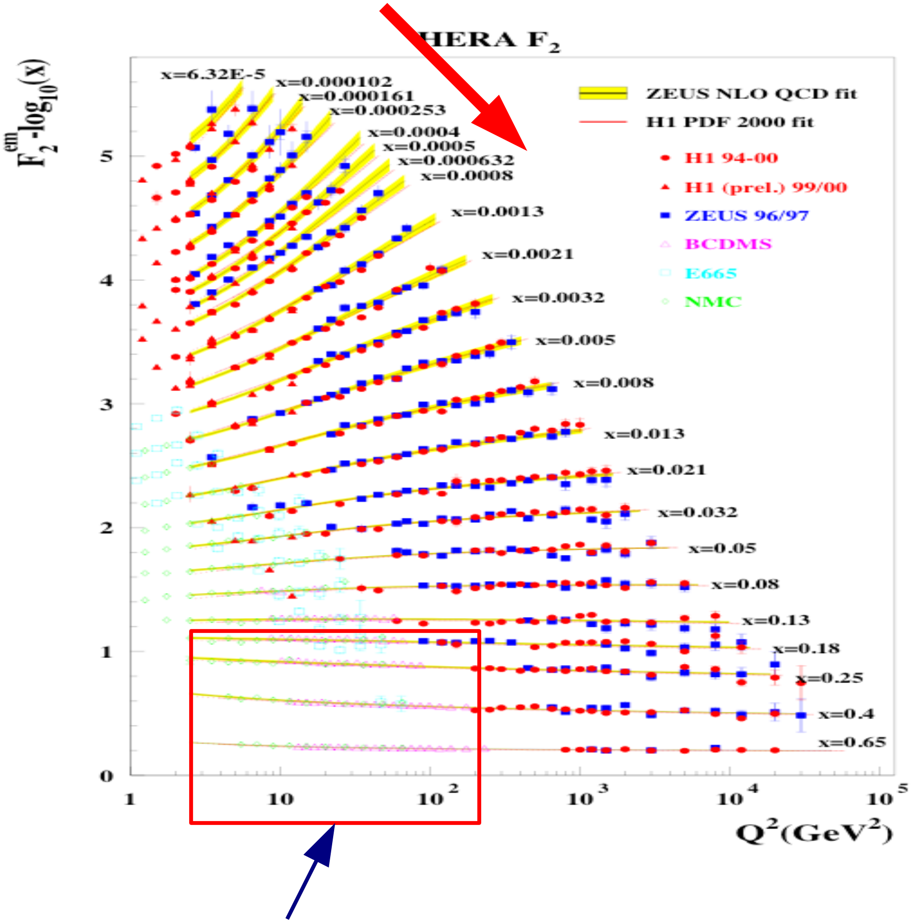
Measure of momentum fraction of struck quark

$$\mathbf{s} \equiv (\mathbf{p} + \mathbf{k})^2$$

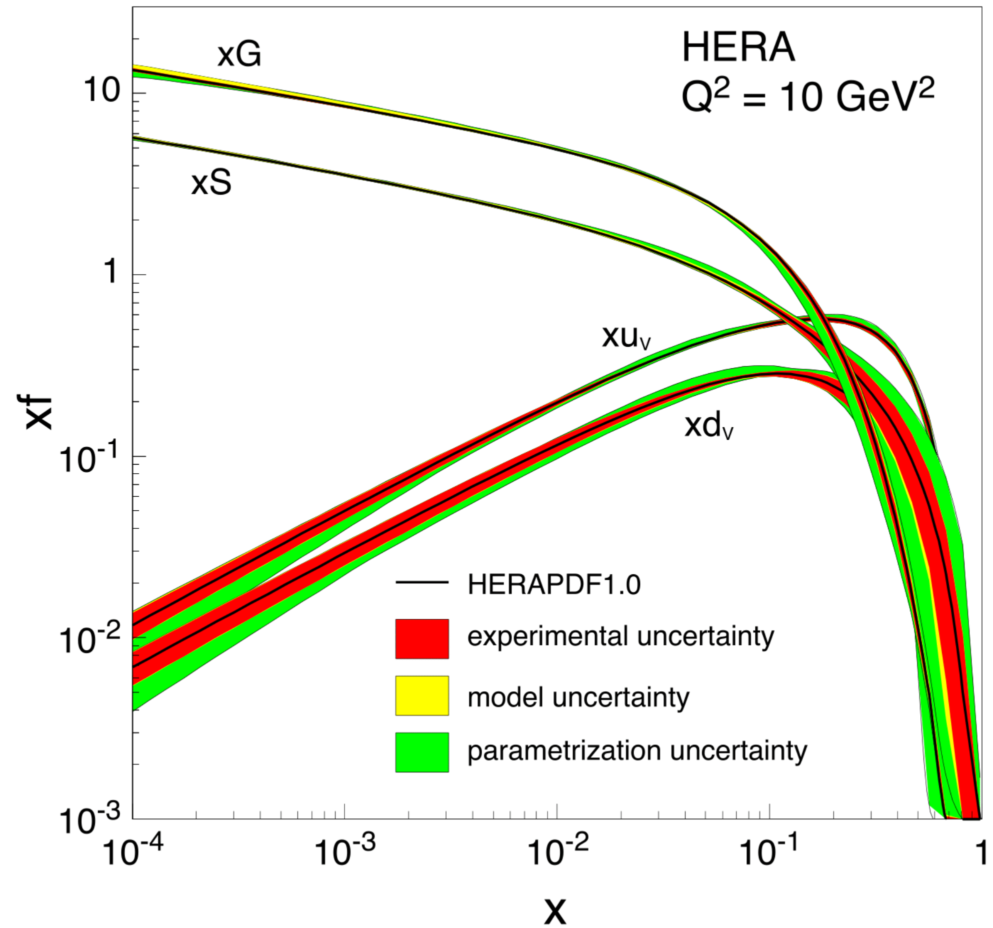
Deep Inelastic Scattering

$$F_2 \equiv \sum_{f=q,\bar{q}} e_f^2 \, xq(x, Q^2)$$

QCD: <u>scaling violations</u>



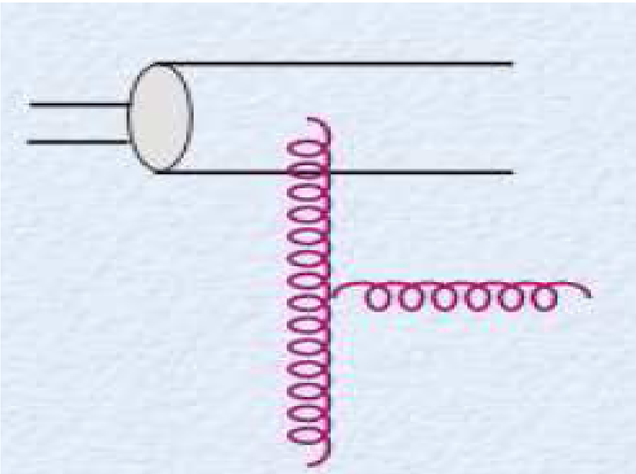
early experiments (SLAC,...):
scale invariance of hadron structure



large number of gluons at small x

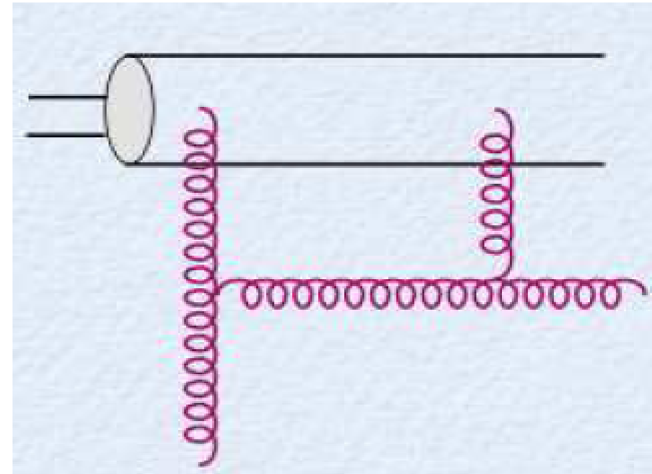
Perturbative QCD breaks down at small x

"attractive" bremsstrahlung *vs.* *"repulsive" recombination*



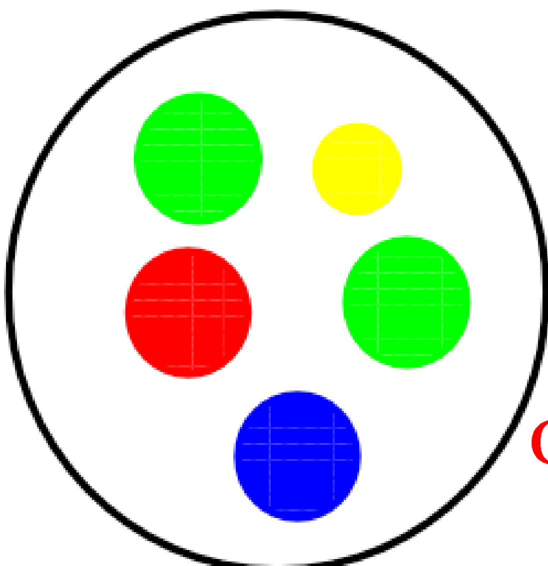
$$S \to \infty, \;\; Q^2 \;\; fixed$$

$$x_{Bj} \equiv \frac{Q^2}{S} \to 0$$



included in pQCD

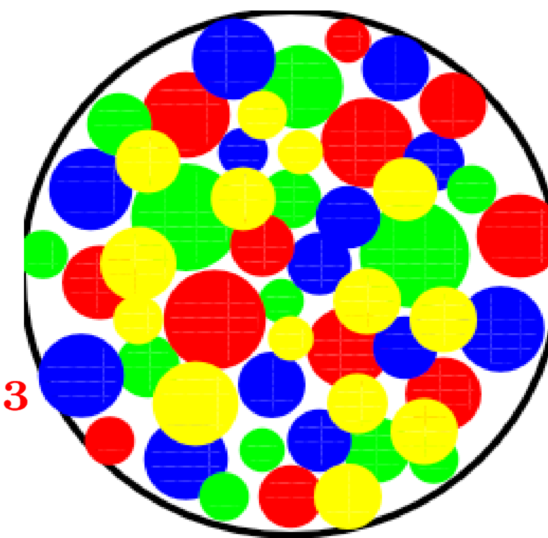
not included in pQCD



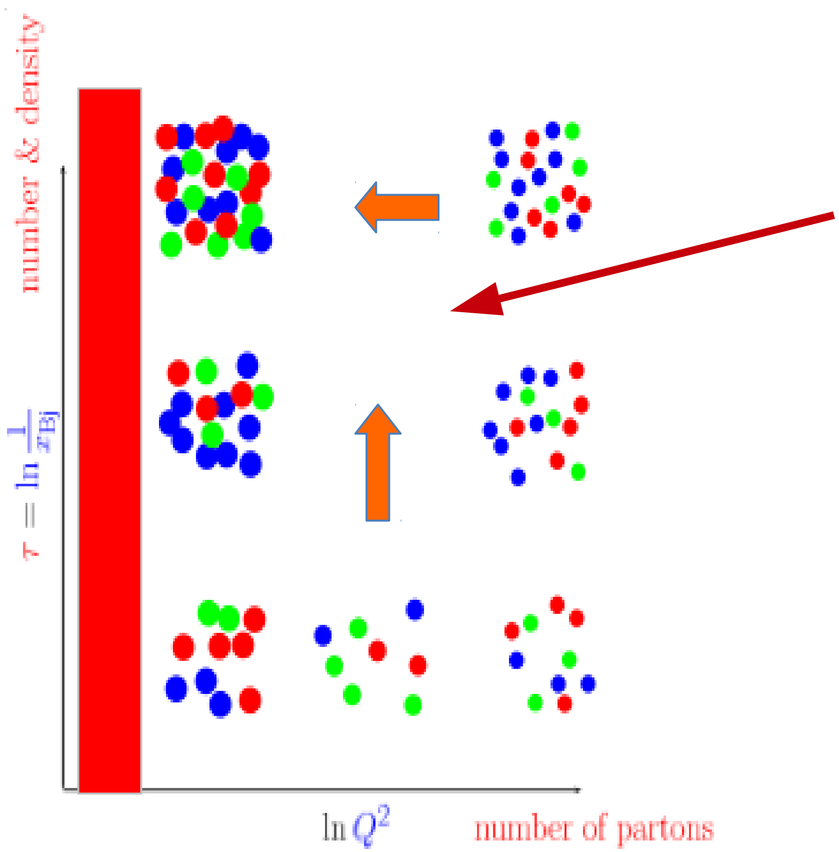
energy ~ 1/x

$$\frac{\alpha_s}{Q^2} \, \frac{xG(x, Q^2)}{\pi r^2} \sim 1$$

$$Q_s^2(x, b_t, A) \sim A^{1/3} \left(\frac{1}{x}\right)^{0.3}$$



Many-body dynamics of universal gluonic matter



How does this happen ?

How do correlation functions of these evolve ?

Is there a universal fixed point for the RG evolution of d.o.f ?

Are there scaling laws ?

Probing saturation in high energy collisions

"nucleus-nucleus" (dense-dense)

need quite a bit of modeling

"proton-nucleus" (dilute-dense)

much less modeling

<u>DIS</u>

structure functions (diffraction)
<u>NLO</u> di-hadron/jet correlations
<u>3-hadron/jet angular correlations</u>

signatures in production spectra

multiple scattering via Wilson lines:

p_t broadening

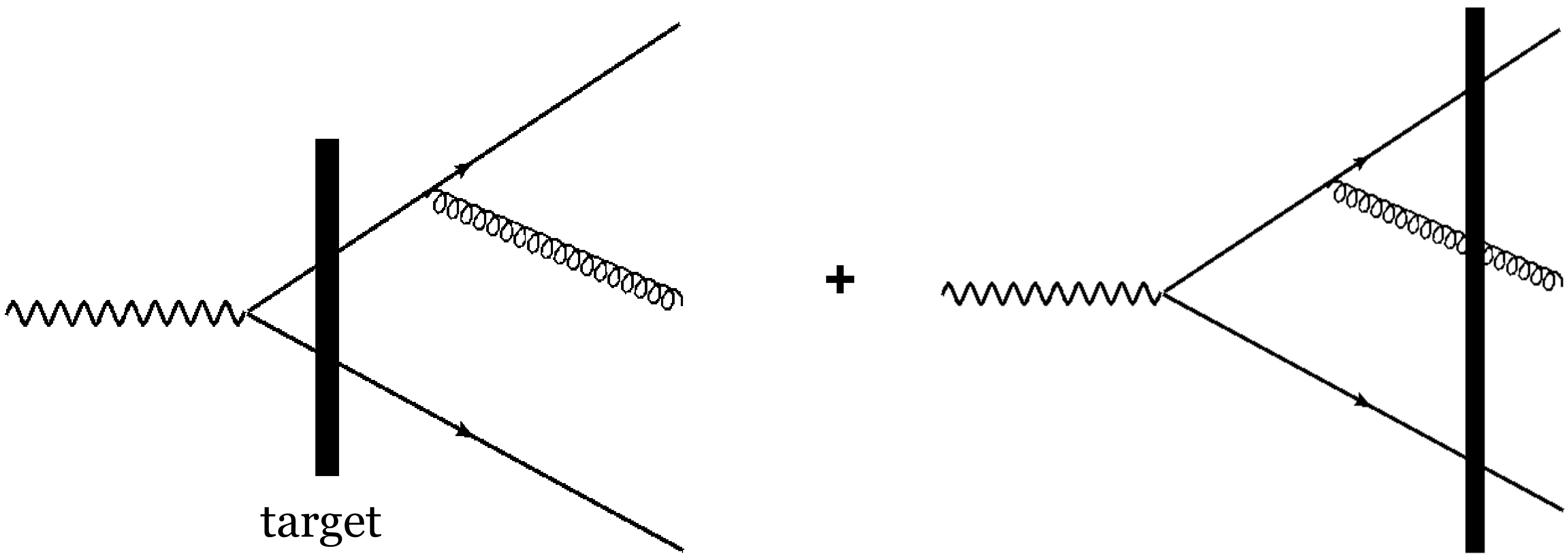
X-evolution via JIMWLK:

suppression of spectra/away side peaks

probing saturation with angular correlations

3-parton production in DIS

$$\gamma^{\star}\mathbf{T} \rightarrow \mathbf{q}\,\bar{\mathbf{q}}\,\mathbf{g}\,\mathbf{X}$$

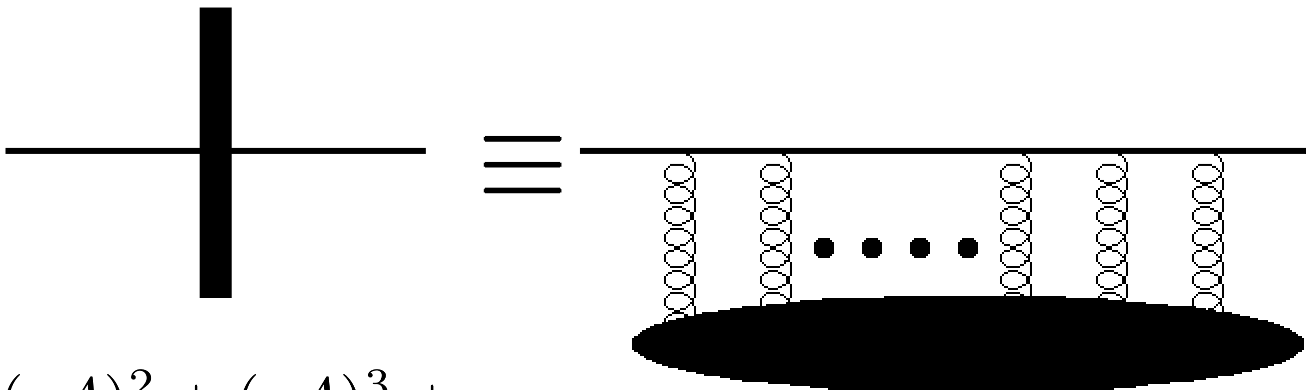


+ radiation from anti-quark

Ayala, Hentschinski , Jalilian-Marian, Tejeda-Yeomans; PLB761 (2016) 229

scattering of a quark from the target

target (proton, nucleus) as a classical color field
quark propagator in the background color field: Wilson line V



$$\sim gA + (gA)^2 + (gA)^3 + \cdots$$

$$\mathrm{S}_F(q,p) \equiv (2\pi)^4 \underbrace{\delta^4(p-q)\, S_F^0(p)}_{\text{no interaction}} + S_F^0(q)\, \underbrace{\tau_f(q,p)}_{\text{interaction}}\, S_F^0(p) \qquad \text{with} \qquad S_F^0(p) = \frac{i}{\not p + i\epsilon}$$

$$\tau_f(q,p) \quad \equiv \quad (2\pi)\delta(p^+ - q^+)\,\gamma^+ \int d^2x_t\, e^{i(q_t - p_t)\cdot x_t}$$
$$\left\{\theta(p^+)[V(x_t) - 1] - \theta(-p^+)[V^\dagger(x_t) - 1]\right\}$$

$$\mathrm{V}(\mathrm{x}_t) = \hat{p}\, e^{ig\int dz^+\, A^-(z^+, x_t)}$$

similar for gluon propagator

spinor helicity methods

Review:
L. Dixon, hep-ph/9601359

massless quarks: helicity eigenstates

$$u_\pm(k) \equiv \frac{1}{2}(1 \pm \gamma_5)\, u(k) \qquad \overline{u_\pm(k)} \equiv \overline{u(k)}\,\frac{1}{2}(1 \mp \gamma_5)$$

$$v_\mp(k) \equiv \frac{1}{2}(1 \pm \gamma_5)\, v(k) \qquad \overline{v_\mp(k)} \equiv \overline{v(k)}\,\frac{1}{2}(1 \mp \gamma_5)$$

helicity operator

$$\mathrm{h} \equiv \vec{\Sigma}\cdot\hat{p} = \begin{pmatrix} \vec{\sigma}\cdot\hat{p} & 0 \\ 0 & \vec{\sigma}\cdot\hat{p} \end{pmatrix}$$

$$\vec{\Sigma}\cdot\hat{p}\, u_\pm(p) = \pm u_\pm(p)$$
$$-\vec{\Sigma}\cdot\hat{p}\, v_\pm(p) = \pm v_\pm(p)$$

$$u_+(k) = v_-(k) = \frac{1}{2^{1/4}}\begin{bmatrix} \sqrt{k^+} \\ \sqrt{k^-}e^{i\phi_k} \\ \sqrt{k^+} \\ \sqrt{k^-}e^{i\phi_k} \end{bmatrix} \qquad u_-(k) = v_+(k) = \frac{1}{2^{1/4}}\begin{bmatrix} \sqrt{k^-}e^{-i\phi_k} \\ -\sqrt{k^+} \\ -\sqrt{k^-}e^{-i\phi_k} \\ \sqrt{k^+} \end{bmatrix}$$

with $e^{\pm i\phi_k} \equiv \dfrac{k_x \pm i k_y}{\sqrt{2k^+\,k^-}} = \sqrt{2}\,\dfrac{k_t\cdot\epsilon_\pm}{k_t}$

$$n^\mu = (n^+ = 0, n^- = 1, n_\perp = 0)$$
$$\bar{n}^\mu = (\bar{n}^+ = 1, \bar{n}^- = 0, \bar{n}_\perp = 0)$$

and

$$k^\pm = \frac{E \pm k_z}{\sqrt{2}}$$

$$\epsilon_\pm = \frac{1}{\sqrt{2}}(1, \pm i)$$

spinor helicity methods

notation:

$$|i^\pm > \equiv |k_i^\pm > \equiv u_\pm(k_i) = v_\mp(k_i) \qquad < i^\pm| \equiv < k_i^\pm| \equiv \overline{u}_\pm(k_i) = \overline{v}_\mp(k_i)$$

basic spinor products:

$$< i\, j > \;\equiv\; < i^-|j^+ > = \overline{u}_-(k_i)\, u_+(k_j) \;=\; \sqrt{|s_{ij}|}e^{i\phi_{ij}} \qquad \cos\phi_{ij} \;=\; \frac{k_i^x k_j^+ - k_j^x k_i^+}{\sqrt{|s_{ij}|k_i^+ k_j^+}}$$

$$[i\, j] \;\equiv\; < i^+|j^- > = \overline{u}_+(k_i)\, u_-(k_j) \;=\; -\sqrt{|s_{ij}|}e^{-i\phi_{ij}} \qquad \sin\phi_{ij} \;=\; \frac{k_i^y k_j^+ - k_j^y k_i^+}{\sqrt{|s_{ij}|k_i^+ k_j^+}}$$

with

$$\begin{aligned} s_{ij} &= (k_i + k_j)^2 = 2k_i \cdot k_j \\ &= - < ij > [ij] \end{aligned}$$

and

$$\begin{aligned} < ii > &= [ii] = 0 \\ < ij] &= [ij >= 0 \end{aligned}$$

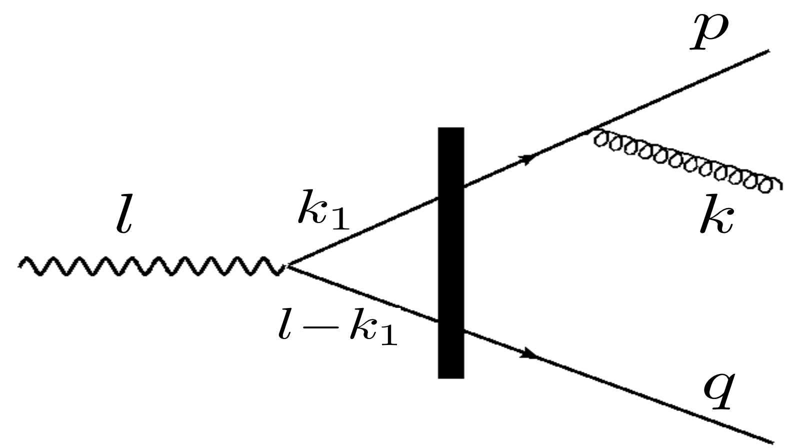
charge conjugation $\quad < i^+|\gamma^\mu|j^+ > = < j^-|\gamma^\mu|i^- >$

Fierz identity $\quad < i^+|\gamma^\mu|j^+ >< k^+|\gamma^\mu|l^+ > = 2[ik] \; < lj >$

any off-shell momentum $\quad k^\mu \equiv \bar{k}^\mu + \dfrac{k^2}{2k^+} n^\mu \quad$ where $\bar{k}^\mu$ is on-shell $\quad \bar{k}^2 = 0$

any on-shell momentum $\quad \not{p} = |p^+ >< p^+| + |p^- >< p^-|$

Diagram A1



Numerator: Dirac Algebra

$$a_1 \equiv \overline{u}(p)\,(k)\,(\not p + \not k)\ \not k_1\ (l)\,(\not k_1 - \not l)\ v(q)$$

longitudinal photons

quark anti-quark gluon helicity: + - +

$$\not l = l^+ \not\overline{n} - \frac{Q^2}{2l^+}\not n$$

$$a_1^{L;+-+} \;=\; -\frac{\sqrt{2}}{[n\,k]}\frac{Q}{l^+}\,[n\,p] < k\,p > [n\,p] < n\,\overline{k}_1 > \left[n\,\overline{k}_1\right] < n\,q >$$
$$(< n\,\overline{k}_1 > \left[n\,\overline{k}_1\right] - l^+ < n\,\overline{n} > [n\,\overline{n}])$$

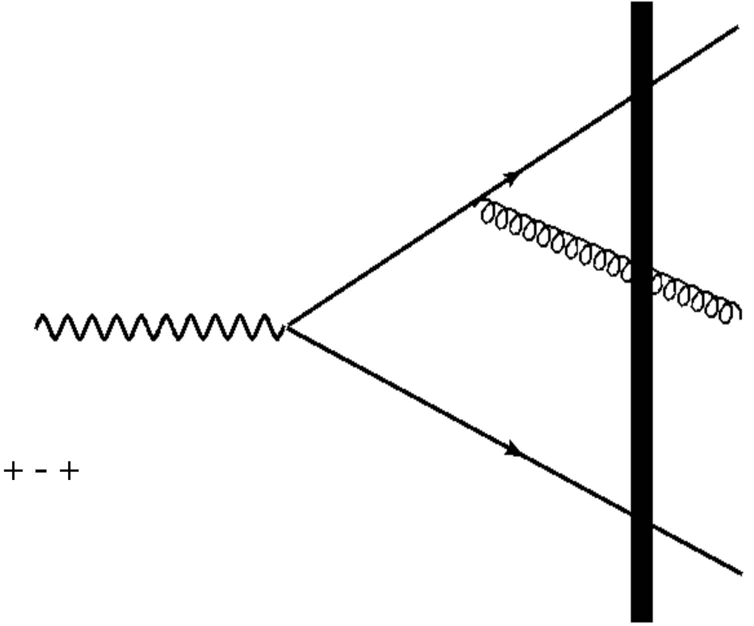
with

$$< np >= -[np] = \sqrt{2p^+}$$

transverse photons: +

$$a_1^{\perp=+;+-+} = -\frac{\sqrt{2}}{[nk]}[pn] < kp > [pn] < nk_1 > [k_1 n] < \bar{n}k_1 > [k_1 n] < nq >$$

Diagram A3



Numerator: Dirac Algebra

longitudinal photons

quark anti-quark gluon helicity: + - +

$$
\begin{aligned}
a_3^{L;+-+} &= \frac{\sqrt{2}Q}{l^+\,[n\bar{k}_2]}[pn]\Big(< n\bar{k}_1 > [\bar{k}_1 n] - < n\bar{k}_2 > [\bar{k}_2 n]\Big) < \bar{k}_2\bar{k}_1 > [\bar{k}_1 n] \\
&\quad \Big(< n\bar{k}_1 > [\bar{k}_1 n] - l^+ < n\bar{n} > [\bar{n}n]\Big) < nq > \\
&= -2^4 Q(l^+)^2 \frac{(z_1 z_2)^{3/2}}{z_3}\left[z_3\, k_{1\,t}\cdot\epsilon - (z_1+z_3)k_{2\,t}\cdot\epsilon\right]
\end{aligned}
$$

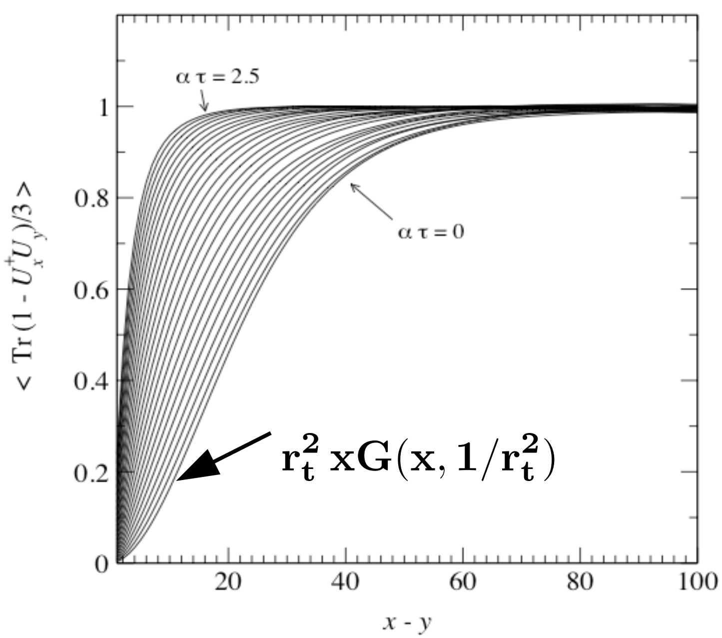
the rest is some standard integrals, <u>we know how to compute the numerators efficiently</u>

add up the amplitudes, add, square.. : **get (trace of) products of Wilson lines**

Dipoles at large N_c : BK eq.

$$\frac{d}{dy}\mathbf{T}(\mathbf{x_t}-\mathbf{y_t}) = \frac{\bar{\alpha}_s}{2\pi}\int d^2\mathbf{z_t}\,\frac{(\mathbf{x_t}-\mathbf{y_t})^2}{(\mathbf{x_t}-\mathbf{z_t})^2(\mathbf{y_t}-\mathbf{z_t})^2}\times$$

$$[\mathbf{T}(\mathbf{x_t}-\mathbf{z_t})+\mathbf{T}(\mathbf{z_t}-\mathbf{y_t})-\mathbf{T}(\mathbf{x_t}-\mathbf{y_t})-\mathbf{T}(\mathbf{x_t}-\mathbf{z_t})\mathbf{T}(\mathbf{z_t}-\mathbf{y_t})]$$



$\tilde{\mathbf{T}}(\mathbf{p_t}) \to \log\left[\frac{\mathbf{Q_s^2}}{\mathbf{p_t^2}}\right]$ **saturation region**

$\tilde{\mathbf{T}}(\mathbf{p_t}) \to \frac{1}{\mathbf{p_t^2}}\left[\frac{\mathbf{Q_s^2}}{\mathbf{p_t^2}}\right]^{\gamma}$ $R_{pA} \sim A^{\frac{-\gamma}{3}}$

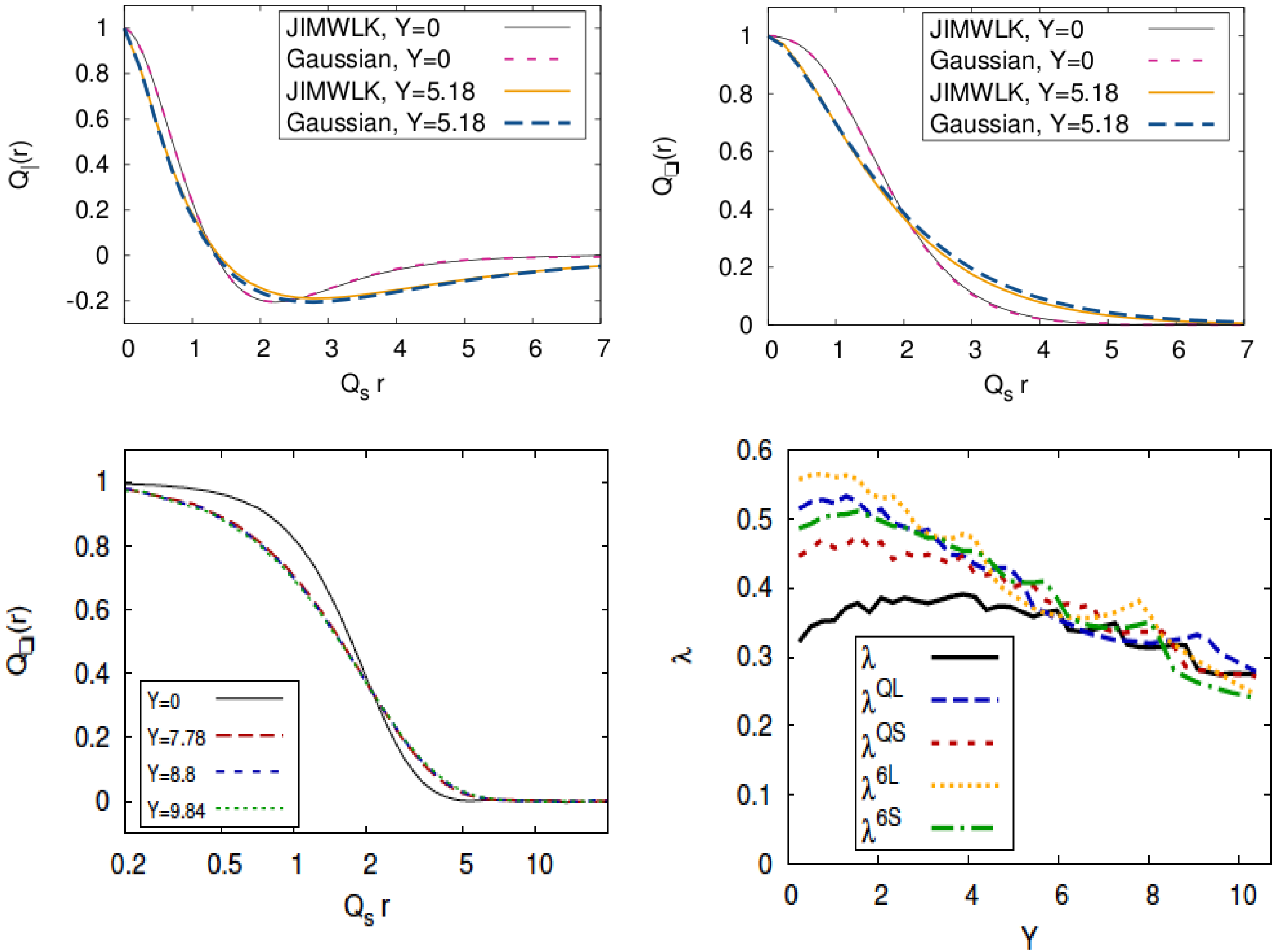
$\gamma < 1$

$\tilde{\mathbf{T}}(\mathbf{p_t}) \to \frac{1}{\mathbf{p_t^2}}\left[\frac{\mathbf{Q_s^2}}{\mathbf{p_t^2}}\right]$ **pQCD region**

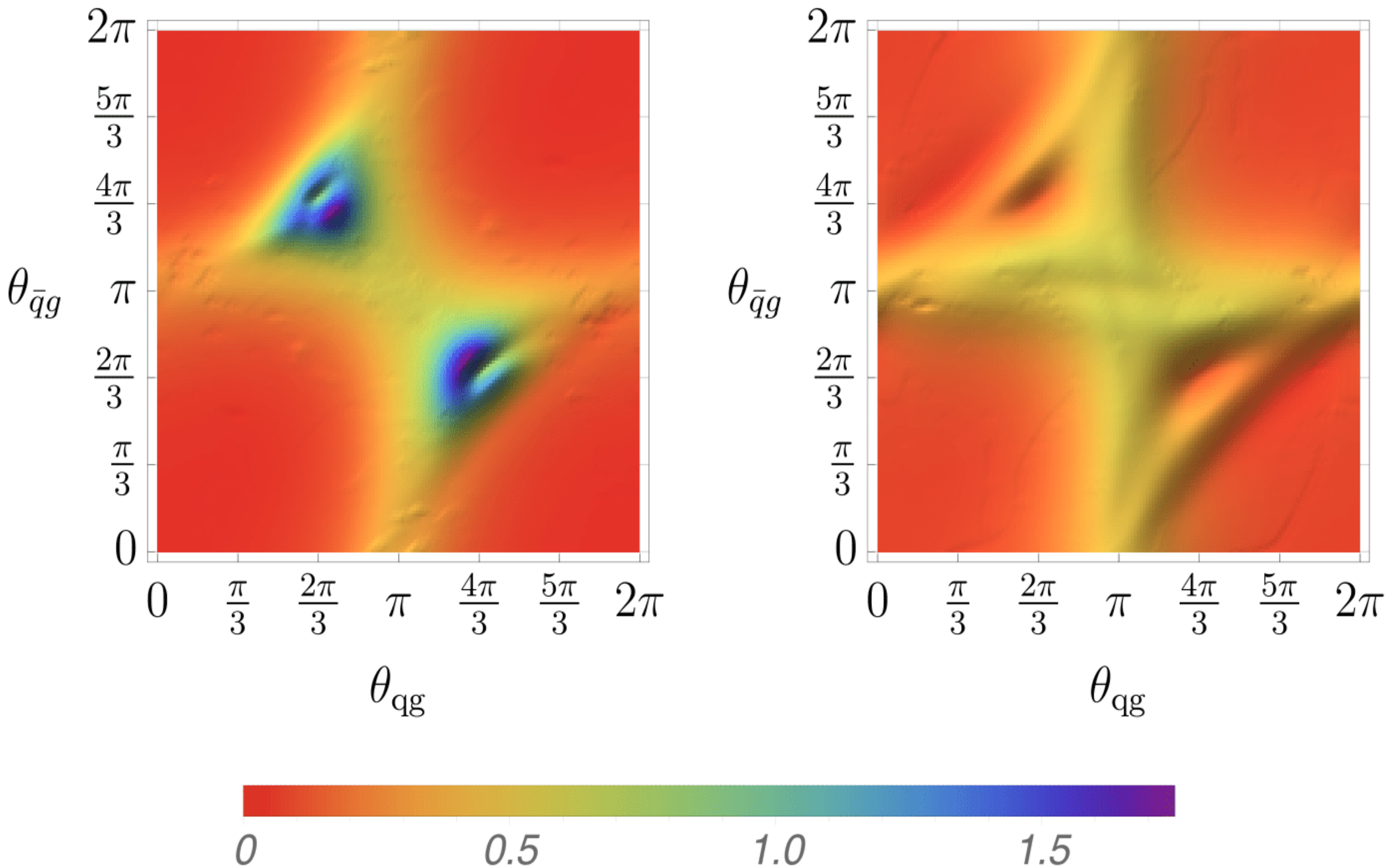
Rummukainen-Weigert, NPA739 (2004) 183
NLO: Balitsky-Kovchegov-Weigert-Gardi-Chirilli (2007-2008)

Quadrupole: $Q(r,\bar{r},\bar{s},s) \equiv \frac{1}{N_c} < Tr\, V(r)\, V^\dagger(\bar{r})\, V(\bar{s})\, V^\dagger(s) >$

Dumitru-Jalilian-Marian-Lappi-Schenke-Venugopalan:PLB706 (2011) 219



3-parton azimuthal angular correlations



multiple scattering:

***broadening** of the peak*

x-evolution:

***reduction** of magnitude*

SUMMARY

CGC is a systematic approach to high energy collisions

high gluon density: re-sum multiple soft scatterings

high energy: re-sum large logs of energy (rapidity or log 1/x)

Leading Log CGC works (too) well

it has been used to fit a wealth of data; ep, eA, pp, pA, AA

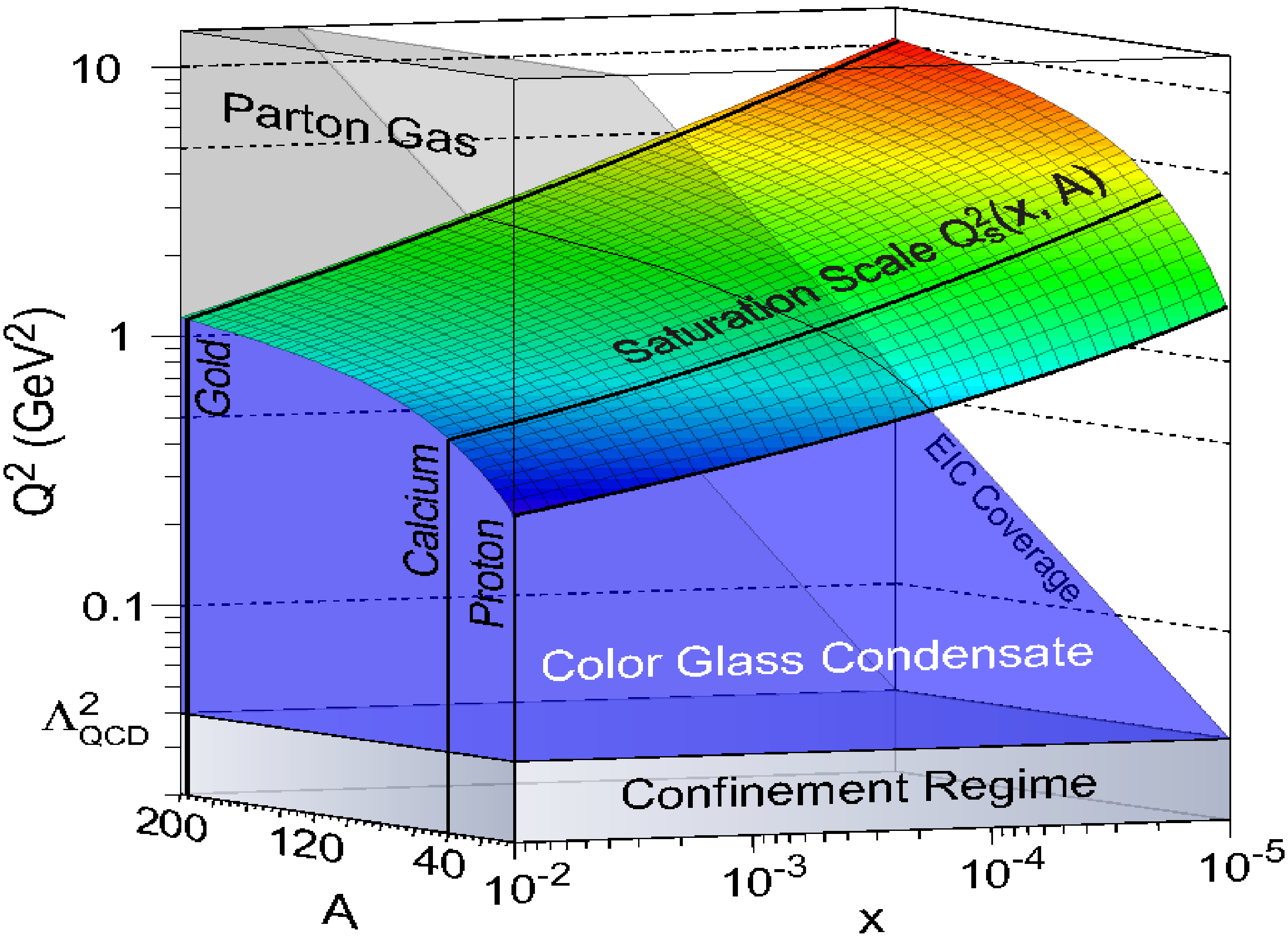
Precision (NLO) studies are needed

available for DIS, single inclusive forward production in pp, pA

Azimuthal angular correlations offer a unique probe of CGC

<u>3-hadron/jet correlations</u> should be even more discriminatory

The Saturation Scale Q_s



x 9/4 for gluons