Cross-tests of CMB features in the primordial spectra

Spyros Sypsas



STARS/SMFNS 2017

Spyros Sypsas dli, fcfm, UChile Cross-tests of CMB features in the primordial spectra

Based on 1512.08977 (PLB), 1612.09253 (PRD), 1702.08756 (JCAP)

in collaboration with:

Gonzalo Palma, Jinn-Ouk Gong, Stephen Appleby, Dhiraj Kumar Hazra, Arman Shafieloo, Walter Riquelme and Bastián Pradenas

Outline

- 1 Intro
 - CMB spectra
 - Scope
- 2 Inversion Methods
- 3 Bispectrum-Power Spectrum correlation
- 4 Mixed Bispectrum Templates
- 5 Tensor Power Spectrum
- 6 Concluding Remarks

イロト イポト イヨト イヨト

Inversion Methods Bispectrum-Power Spectrum correlation Mixed Bispectrum Templates Tensor Power Spectrum Concluding Remarks

CMB spectra Scope



E

Inversion Methods Bispectrum-Power Spectrum correlation Mixed Bispectrum Templates Tensor Power Spectrum Concluding Remarks

CMB spectra Scope

ISOTROPY OF THE COSMIC MICROWAVE BACKGROUND

MAP990004

Spyros Sypsas dfi, fcfm, UChile Cross-tests of CMB features in the primordial spectra

イロト イポト イヨト イヨト

Ξ

Inversion Methods Bispectrum-Power Spectrum correlation Mixed Bispectrum Templates Tensor Power Spectrum Concluding Remarks

CMB spectra Scope

Scalar Perturbations in single field inflation



flat gauge:
$$g_{ij} = e^{2\rho}(\delta_{ij} + \gamma_{ij})$$
 and $\phi = \phi_0(t) + \delta\phi(x, t)$
unitary gauge: $g_{ij} = e^{2\rho + \zeta}(\delta_{ij} + \gamma_{ij})$ and $\phi = \phi_0(t)$
 $\zeta \sim H\delta\phi$

 ζ causes tiny $\mathcal{O}(10^{-5})$ temperature anisotropies in the CMBL , and $\mathcal{O}(10^{-5})$

Spyros Sypsas dli, fclm, UChile Cross-tests of CMB features in the primordial spectra

Inversion Methods Bispectrum-Power Spectrum correlation Mixed Bispectrum Templates Tensor Power Spectrum Concluding Remarks

CMB spectra Scope



Spyros Sypsas dfi, fcfm, UChile Cross-tests of CMB features in the primordial spectra

< □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □

Ξ

Inversion Methods Bispectrum-Power Spectrum correlation Mixed Bispectrum Templates Tensor Power Spectrum Concluding Remarks

CMB spectra Scope

Measuring the distribution of these small anisotropies in the CMB has been our main window into the very early Universe

 $\exists \rightarrow$

Inversion Methods Bispectrum-Power Spectrum correlation Mixed Bispectrum Templates Tensor Power Spectrum Concluding Remarks

CMB spectra Scope

Measuring the distribution of these small anisotropies in the CMB has been our main window into the very early Universe

• $\langle \zeta \zeta \rangle \sim \frac{H^2}{M_{\rm Pl}^2 \epsilon}$

Ξ

CMB spectra Scope

Measuring the distribution of these small anisotropies in the CMB has been our main window into the very early Universe

- $\langle \zeta \zeta \rangle \sim \frac{H^2}{M_{\rm Pl}^2 \epsilon}$
- They follow nearly gaussian statistics

CMB spectra Scope

Measuring the distribution of these small anisotropies in the CMB has been our main window into the very early Universe

•
$$\langle \zeta \zeta \rangle \sim \frac{H^2}{M_{\rm Pl}^2 \epsilon}$$

- They follow nearly gaussian statistics
- But is that all? What about $\langle \zeta(k_1)\zeta(k_2)\zeta(k_3)\rangle \sim f_{NL}S(k_1,k_2,k_3)$

CMB spectra Scope

Measuring the distribution of these small anisotropies in the CMB has been our main window into the very early Universe

•
$$\langle \zeta \zeta \rangle \sim \frac{H^2}{M_{\rm Pl}^2 \epsilon}$$

- They follow nearly gaussian statistics
- But is that all? What about $\langle \zeta(k_1)\zeta(k_2)\zeta(k_3)\rangle \sim f_{NL}S(k_1,k_2,k_3)$
- the shape function $S(k_1, k_2, k_3)$ and the overall amplitude f_{NL} are model dependent

CMB spectra Scope

Measuring the distribution of these small anisotropies in the CMB has been our main window into the very early Universe

•
$$\langle \zeta \zeta \rangle \sim \frac{H^2}{M_{\rm Pl}^2 \epsilon}$$

- They follow nearly gaussian statistics
- But is that all? What about $\langle \zeta(k_1)\zeta(k_2)\zeta(k_3)\rangle \sim f_{NL}S(k_1,k_2,k_3)$
- the shape function $S(k_1, k_2, k_3)$ and the overall amplitude f_{NL} are model dependent
- for canonical single field inflation we have $f_{NL} \sim \mathcal{O}(\epsilon)$ and a local shape

イロト イポト イヨト イヨト

Inversion Methods Bispectrum-Power Spectrum correlation Mixed Bispectrum Templates Tensor Power Spectrum Concluding Remarks

CMB spectra Scope

• What if there is some short period of sudden change in the slow roll dynamics? (e.g. inflationary potential with a step, turning trajectory, etc.)

I □ ► I □ ►

-

Inversion Methods Bispectrum-Power Spectrum correlation Mixed Bispectrum Templates Tensor Power Spectrum Concluding Remarks

CMB spectra Scope

- What if there is some short period of sudden change in the slow roll dynamics? (e.g. inflationary potential with a step, turning trajectory, etc.)
- In that case the amplitude can be larger than usual slow roll and the shape can be quite different too

Inversion Methods Bispectrum-Power Spectrum correlation Mixed Bispectrum Templates Tensor Power Spectrum Concluding Remarks

CMB spectra Scope

- What if there is some short period of sudden change in the slow roll dynamics? (e.g. inflationary potential with a step, turning trajectory, etc.)
- In that case the amplitude can be larger than usual slow roll and the shape can be quite different too
- Does the 2-pt function allow for such speculation?

- 4 同下 - 4 同下 - 4 同下

Inversion Methods Bispectrum-Power Spectrum correlation Mixed Bispectrum Templates Tensor Power Spectrum Concluding Remarks

CMB spectra Scope



Spyros Sypsas dfi, fcfm, UChile

Cross-tests of CMB features in the primordial spectra

Inversion Methods Bispectrum-Power Spectrum correlation Mixed Bispectrum Templates Tensor Power Spectrum Concluding Remarks

CMB spectra Scope

We have a pretty dense set of data for the power spectrum allowing for a semi bottom-up approach. But for the bispectrum the data set is quite sparse.

We need to predict shapes so that observational surveys can verify them or rule them out.

That is, PLANCK can only tell us if a given shape has a non-empty overlap with the existing data.

CMB spectra Scope

Non gaussian estimator: define an inner product between two shape functions as

$$S_1 \star S_2 \equiv \int_{x,y} (xy)^4 S_1(x,y) S_2(x,y)$$

. Now what PLANCK measures is

$$f_{nl}^{eq} = rac{S_{data} \star S_{eq}}{S_{eq} \star S_{eq}}$$

For example, if a model predicts equilateral non gaussianity with amplitude $f_{nl}^{eq-model} = 1/c_s^2$ and shape function S_{model} , then

$$f_{nl}^{eq} = rac{S_{model} \star S_{eq}}{S_{eq} \star S_{eq}} rac{1}{c_s^2}$$

Spyros Sypsas dfi, fcfm, UChile Cross-tests of CMB features in the primordial spectra

イロト イポト イヨト イヨト

Inversion Methods Bispectrum-Power Spectrum correlation Mixed Bispectrum Templates Tensor Power Spectrum Concluding Remarks

CMB spectra Scope

The aim is to...

• Relate the bispectrum to the power spectrum

200

Inversion Methods Bispectrum-Power Spectrum correlation Mixed Bispectrum Templates Tensor Power Spectrum Concluding Remarks

CMB spectra Scope

The aim is to...

- Relate the bispectrum to the power spectrum
- Relate the tensor spectrum to the scalar one



-

Inversion Methods Bispectrum-Power Spectrum correlation Mixed Bispectrum Templates Tensor Power Spectrum Concluding Remarks

CMB spectra Scope

The aim is to...

- Relate the bispectrum to the power spectrum
- Relate the tensor spectrum to the scalar one



General Idea

$$P = \int A(t)$$
$$B(k_1, k_2, k_3) = f_{NL}(A)S(k_1, k_2, k_3)$$
$$\boxed{P(A) \to A(P) \to B(P)}$$

Spyros Sypsas dfi, fcfm, UChile Cross-tests of CMB features in the primordial spectra

イロト イヨト イヨト イヨト

Ξ

Generalised Slow Roll / Fourier Transform

$$S_{2} = m_{\rm Pl}^{2} \int d^{3}x d\tau a^{2} \epsilon \left(\frac{\dot{\mathcal{R}}^{2}}{c_{s}^{2} = 1} - (\nabla \mathcal{R})^{2} \right)$$
$$\eta = -\int_{0}^{\infty} \frac{dk}{k} m (-k\tau) \frac{d \log \mathcal{P}_{\mathcal{R}}}{d \log k}$$

But η sources the relevant 3-pt interactions:

$$S_3 \supset \int d^4x a^3 \epsilon m_{
m Pl}^2 \left[-\eta \dot{\mathcal{R}}^2 \mathcal{R} + rac{\eta}{a^2} \mathcal{R} (
abla \mathcal{R})^2
ight]$$

Computing the bispectrum using in-in formalism and plugging this result into the final formula we get the desired

$$B_S(P_S)$$

The bispectrum template finally reads Appleby/Gong/Hazra/Shafieloo/SS '15, Palma '14

$$B_{\mathcal{R}}(k_1, k_2, k_3) = \frac{(2\pi)^4 \mathcal{P}_{\mathcal{R}}^2}{(k_1 k_2 k_3)^3} \left\{ \left[\left(k_1^2 + k_2^2 + k_3^2 \right) \frac{k_1 k_2 + k_2 k_3 + k_3 k_1}{16k} + \frac{(k_1 k_2)^2 + (k_2 k_3)^2 + (k_3 k_1)^2}{8k} - \frac{k_1 k_2 k_3}{8} \right] (1 - n_{\mathcal{R}}) + \frac{k_1 k_2 k_3}{8} \alpha_{\mathcal{R}} \right\},$$

We may test this formula using numerical computation of the bispectrum for a known model with a feature in the potential: $V(\phi) = \frac{1}{2}m^2\phi^2 \left[1 + \alpha \tanh\left(\frac{\phi - \phi_0}{\Delta\phi}\right)\right]$



Spyros Sypsas dfi, fcfm, UChile

Cross-tests of CMB features in the primordial spectra

Prediction of the bispectrum shape



Figure: $f_{\rm NL}$ in the (left) squeezed and (right) equilateral limit. The dark (light) band encloses 68% (95%) of the reconstructed $\mathcal{P}_{\mathcal{R}}$. The plot covers the entire range considered in this work, $k = (10^{-3}, 0.12) \text{ Mpc}^{-1}$. The inset plots exhibit certain k-bands of interest.

Spyros Sypsas dfi, fcfm, UChile Cross-tests of CMB features in the primordial spectra

・ロト ・日 ・ ・ ヨ ・ ・

. ⊒ . ⊳

Prediction of the bispectrum shape



Figure: Heat maps of $f_{\rm NL}^{+2\sigma} - f_{\rm NL}^{\rm fid}$ (top) and $f_{\rm NL}^{-2\sigma} - f_{\rm NL}^{\rm fid}$ (bottom) as a function of k_3/k_1 and k_2/k_1 , with $k_1 = 0.06 \,{\rm Mpc}^{-1}$. Regions of interest are $f_{\rm NL}^{+2\sigma} - f_{\rm NL}^{\rm fid} < 0$ and $f_{\rm NL}^{-2\sigma} - f_{\rm NL}^{\rm fid} > 0$, red (blue) contours in the top (bottom) panel, indicating areas where the featureless expectation value lies outside the 95% contours.

Spyros Sypsas dfi, fcfm, UChile Cross-tests of CMB features in the primordial spectra

Using these methods we can also produce templates for a generic situation where there are features in both the potential and kinetic terms of the scalar perturbations

$$S_3 \supset \int d^4 x a^3 \epsilon m_{
m Pl}^2 \left[c_1 \dot{\mathcal{R}}^2 \mathcal{R} + rac{c_2}{a^2} \mathcal{R} (
abla \mathcal{R})^2
ight]$$

After computing with in-in and inverting with Fourier we get

$$\int_{-\infty}^{\infty} dk e^{-i(1+x+y)k\tau} \frac{S_{\mathcal{R}}(k,x,y)}{(2\pi)^4} k \frac{8}{2\pi i} = \frac{(1+x^2+y^2)}{2(xy)^2(1+x+y)^4} (c_2 \tau)^{\prime\prime\prime} - \frac{(x+y+xy)}{(xy)^2(1+x+y)^4} (c_1 \tau)^{\prime\prime\prime}$$

Main idea:

we may now fix 2 triangle configurations, solve the algebraic system for c_1, c_2 , and plug them back to the bispectrum Gong/Palma/SS '16

Spyros Sypsas dfi, fcfm, UChile Cross-tests of CMB features in the primordial spectra

Ξ

3 proposed concistency relations

$$S_{\mathcal{R}}(k, x, y) = \frac{6(x + y + xy) - 3(1 + x^{2} + y^{2})}{(1 + x + y)^{2}} S_{\mathcal{R}}\left(\frac{1 + x + y}{3}k, 1, 1\right) - 4\frac{x + y + xy - (1 + x^{2} + y^{2})}{(1 + x + y)^{2}} S_{\mathcal{R}}\left(\frac{1 + x + y}{2}k, 1, 0\right).$$
(1)

$$S_{\mathcal{R}}(k, x, y) = \frac{18(x + y + xy) - 15(1 + x^2 + y^2)}{(1 + x + y)^2} S_{\mathcal{R}}\left(\frac{1 + x + y}{3}k, 1, 1\right) \\ - 16\frac{x + y + xy - (1 + x^2 + y^2)}{(1 + x + y)^2} S_{\mathcal{R}}\left(\frac{1 + x + y}{2}k, 1/2, 1/2\right).$$
(2)

$$S_{\mathcal{R}}(k, x, y) = -\frac{6(x + y + xy) - 5(1 + x^{2} + y^{2})}{(1 + x + y)^{2}} S_{\mathcal{R}}^{loc} \left(\frac{1 + x + y}{2}k, 1, 0\right) \\ + \frac{8(x + y + xy) - 4(1 + x^{2} + y^{2})}{(1 + x + y)^{2}} S_{\mathcal{R}}^{fold} \left(\frac{1 + x + y}{2}k, 1/2, 1/2\right).$$
(3)

Spyros Sypsas dfi, fcfm, UChile Cross-tests of CMB features in the primordial spectra

Ξ

We can play the same game for the tensor power spectrum:

$$k^{3}\frac{\Delta \mathcal{P}_{T}}{\mathcal{P}_{0}}(k) = -\frac{1}{4}\int_{-\infty}^{+\infty} d\tau \left[\frac{\delta_{T}}{2\tau^{2}} - \frac{\delta_{T}}{\tau^{4}}\right] e^{-2ik\tau}$$

$$k^{3}\frac{\Delta \mathcal{P}_{S}}{\mathcal{P}_{0}}(k) = -\frac{1}{4}\int_{-\infty}^{+\infty} d\tau \left[\frac{\delta_{S}''}{2\tau^{2}} - \frac{\delta_{S}}{\tau^{4}}\right] e^{-2ik\tau}$$

For the tensor modes we have $\delta_T \simeq \epsilon$, while for scalars $\delta_S \simeq \tau^2 \epsilon'' \simeq \tau^2 \delta''_T$

イロト イポト イヨト イヨト

Main idea:

$$\frac{\Delta \mathcal{P}_{S}}{\mathcal{P}_{0}} = \mathcal{F}(\delta_{S}) \rightarrow \delta_{S}(\delta_{T}) \rightarrow \frac{\Delta \mathcal{P}_{S}}{\mathcal{P}_{0}} = \mathcal{F}(\delta_{T}) \rightarrow \frac{\Delta \mathcal{P}_{S}}{\mathcal{P}_{0}} = \mathcal{F}(\frac{\Delta \mathcal{P}_{T}}{\mathcal{P}_{0}}) \text{ invert! } \frac{\Delta \mathcal{P}_{T}}{\mathcal{P}_{0}} = \mathcal{F}(\frac{\Delta \mathcal{P}_{S}}{\mathcal{P}_{0}})$$
Palma/Pradenas/Riquelme/SS '16

$$\frac{\Delta \mathcal{P}_T}{\mathcal{P}_0} = -6 \iint d \ln k \ \epsilon \frac{\Delta \mathcal{P}_S}{\mathcal{P}_0}$$

Ξ

Conclusions

• We have used inversion methods to produce templates for the primordial spectra in cases of sharp features (which are supported at 2σ by PLANCK PPS data)

イロト イポト イヨト イヨト

Conclusions

- We have used inversion methods to produce templates for the primordial spectra in cases of sharp features (which are supported at 2σ by PLANCK PPS data)
- background expansion rate with a fixed sound speed (potential feature)

イロト イポト イヨト イヨト

Conclusions

- We have used inversion methods to produce templates for the primordial spectra in cases of sharp features (which are supported at 2σ by PLANCK PPS data)
- background expansion rate with a fixed sound speed (potential feature)
- generic models (potential/kinetic features)

- 4 同下 4 戸下 4 戸下

Conclusions

- We have used inversion methods to produce templates for the primordial spectra in cases of sharp features (which are supported at 2σ by PLANCK PPS data)
- background expansion rate with a fixed sound speed (potential feature)
- generic models (potential/kinetic features)
- as well as the power spectrum of tensor modes

イロト イポト イヨト イヨト

Future directions

This is a tool to produce multiple templates for any n-point function of scalar-tensor perturbations. Features in late-time observables? (matter power spectrum)

Thank you !

Spyros Sypsas dfi, fcfm, UChile

Cross-tests of CMB features in the primordial spectra

< □ > < □ > < □ > < □ > < □ > < □ >