An emergent Van der Waals-like description of black hole entropy

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- Classical General Relativity (CGR) is a very complete theory
- ...but with some important problems
- The gravitational field is treated in a purely classical manner
- There are singularities
- The issue with singularities: either CGR breaks down at them or is it incomplete (what comes out of a singularity?)

- Strings (extra dimensions)
- Ashtekar variables (canonical cuantization)

$$\{\tilde{E}^a_i(x), A^j_b(y)\} = 8\pi G\beta\delta^a_b\delta^j_i\delta^3(x-y) \tag{1}$$

- Euclidean path integral

$$Z = \int \mathcal{D}[g]\exp\left(-I_E[g]\right) \tag{2}$$

$$\begin{aligned} I_E &= -\frac{1}{16\pi G}\int_{\mathcal{M}} d^4x\sqrt{g}\,(R-2\lambda) \\ &- \frac{1}{8\pi G}\int_{\partial\mathcal{M}} d^3x\sqrt{h}K - I_{\rm matter} + I_{\rm count} \end{aligned} \tag{3}$$

If you can heat it, it has an internal structrure (Boltzmann)

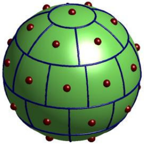
Strategy

- *Associate an entropy with null surfaces. Demand maximisation of S of all null surfaces to get the dynamics.*
- *Holographic equipartition*

$$E = \int_{^3\mathrm{vol}} d^3x\sqrt{h}\rho_{\mathrm{Komar}} = \frac{1}{2}\int_{\partial(^3\mathrm{vol})} d^2x \frac{\sqrt{\sigma}}{l_p^2} T_{\mathrm{loc}} \qquad (4)$$

- *Spacetime atoms*

$$\begin{aligned} E &= \frac{1}{2} N T_{loc} \\ N &= \frac{\mathrm{Area}\ \partial(^3\mathrm{vol})}{l_p^2} \end{aligned} \qquad (5)$$



Spacetime atoms: N *plaquettes* of area l_p^2 tesellating the horizon with no holes between them

What about the volume term in the Gauss theorem expressing equipartition?

Particularize to Spherically symmetric Petrov D-type solutions

$$E = \frac{1}{2} N T_{loc} = \left(\frac{R}{12} - \Psi_2 \right) r^3 \quad (6)$$

- Gravitational *monopole* of the source: second Weyl scalar

$$\Psi_2 := C_{\alpha\beta\gamma\delta} l^\alpha m^\beta \bar{m}^\gamma n^\delta \tag{7}$$

- Possible cosmological constant: scalar curvature ($R = 4\lambda$ in CGR)
- Choice of an appropriate null surface: event horizon: $g^{00}(r_+) = 0$

$$ds^2 = -\left(1 - \frac{2M}{r} + \frac{\lambda}{3}r^2\right)dt^2 + \left(1 - \frac{2M}{r} + \frac{\lambda}{3}r^2\right)^{-1} dr^2 + r^2 d\Omega^2 \quad (8)$$

Define

- Mass $M = \frac{r_+}{2}\left(1 + \frac{\lambda}{3}r^2\right)$
- Pressure $P = \frac{\lambda}{8\pi}$
- Areal volume $V = \frac{4}{3}\pi r_+^3$ (is the relevent quantity when we consider the action of pressure on a surface area)
- $N = A \; (l_p^2 = 1)$

Smarr law for SAdS black holes

$$M = 2TS - 2PV \tag{9}$$

- T is the Hawking temperature
- $S = \frac{A}{4}$ is the Bekenstein–Hawking entropy

In this case, the equation of state turns to be

$$P = \frac{\bar{N}T}{V} - \frac{1}{2\pi}\frac{\bar{N}^2}{V^2} \tag{10}$$

This VdW equation is a completely equivalent way of writing the Smarr law for SAdS if

$$\bar{N} = \frac{N}{6} \tag{11}$$

Hawking temperature under control

$$T = \frac{PV}{\bar{N}} + \frac{1}{2\pi}\frac{\bar{N}}{V} = 2Pr_+ + \frac{1}{2\pi r_+} \tag{12}$$

Microscopic derivation of the Bekenstein–Hawking entropy?

Our strategy

- Write a canonical partition function for this formal gas living inside V to obtain the Smarr (VdW) law
- Compute the entropy to obtain $S = \frac{A}{4} = \frac{N}{4} = \frac{3\bar{N}}{2}$

Assumptions

1. Boltzmann gas of massive indistinguishable particles (spacetime atoms)
2. Horizon configuration energy
3. $(\bar{N}, P, T)$ ensemble
4. Degeneracy threshold

Justification

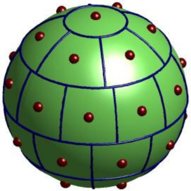
1. Loop quantum gravity: the number of Chern–Simons states on the punctured horizon implies $\frac{A}{4}$. Heuristic picture: indistinguishable gas of Boltzmann punctures reproduces the calculation (ref Pérez) is employed.
2. Not necessary but nice. We already have the equation of state.
3. Euclidean quantum gravity: (Gibbs energy) $\beta G = I_E$.
4. Subtle point.

$$
\begin{aligned}
Z_{\mathrm{can}} &= Z_{\mathrm{free}} \times Z_{\mathrm{int}} \\
Z_{\mathrm{can}} &= \frac{1}{\bar{N}!}\left(\frac{V}{\Lambda^3}\right)^{\bar{N}} e^{-\beta\frac{\bar{N}^2}{2\pi V}} \\
U_{\mathrm{int}} &= \bar{N}\int d\Omega^2 \int_0^{r_+} r^2 \rho U_0(r) dr \\
U_0 &= -\frac{1}{2\pi}\frac{\delta(r-r_+)}{4\pi r^2} \\
\Delta &= \int_0^V Z_{\mathrm{can}}\, e^{-\beta P V'} dV'
\end{aligned}
\tag{13}
$$

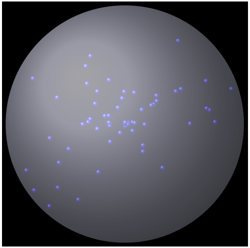
1. Equation of state: $P = \frac{\bar{N}T}{V} - \frac{1}{2\pi}\frac{\bar{N}^2}{V^2}$ (Smarr law)
2. Entropy : $S = \frac{N}{4} - \frac{N}{6}\ln\left(\rho\Lambda^3\right)$
3. Assume degeneracy threshold for the *gas* $\rightarrow S = \frac{N}{4}$

Do we have a physical picture for the emergence of a SAdS event horizon?

Padmanabhan's view: horizon = N plaquettes of area l_p^2



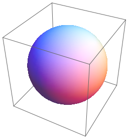
Our view: $\bar{N}$ particles dancing at Hawking's temperature with a pressure given by λ



What is the meaning of $\bar{N}\Lambda^3 = V$ in our model?

The degeneracy condition can be expressed as

$$\Lambda^3 = 2r_+ l_p^2 \tag{14}$$



1. There is an atom in each box of volume Λ^3
2. Fill the volume with these Λ–boxes
3. Then the plaquette representation should emerge at the boundary

1. We have proved (not shown here) that this approach also works for a Reissner–Nördstrom–AdS black hole *under the same degeneracy condition* (at the corresponding Hawking temperature)
2. Therefore, in these cases, semiclassical general relativity is recovered when $\rho\Lambda^3 = 1$
3. What is the meaning of $\rho\Lambda^3 \neq 1$?. Beyond semiclassical general relativity?

Thank you for your attention!