Hartle's slow rotation effects in magnetized white dwarfs

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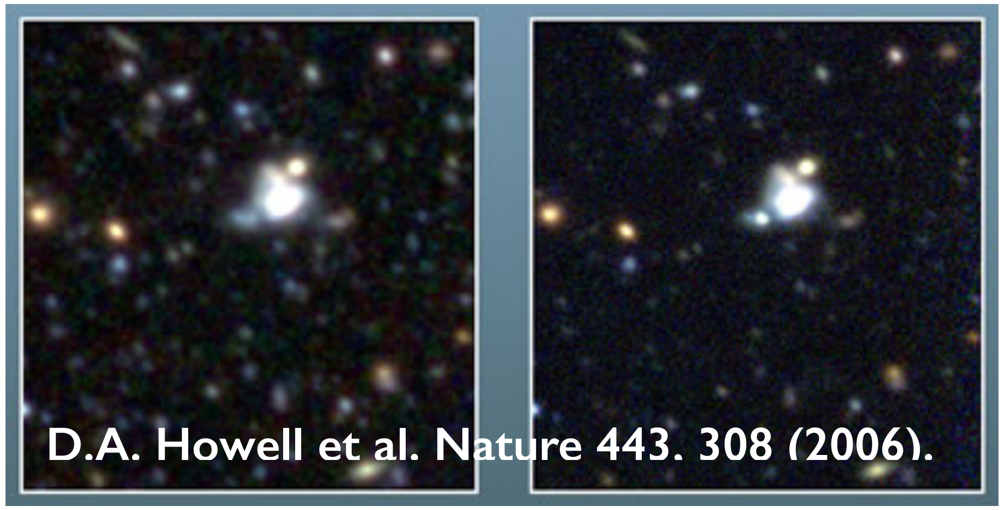
Outline

- Motivation
- Brief description of Hartle's formalism for slow rotation
- Results
- Conclusions
- Perspectives

Motivation

- Observations of SNe Ia with high luminosity and low kinetic energy with **Super-Chandrasekhar-mass WDs** as most probable progenitors.



- **How to explain these Super-Chandrasekhar-mass WDs?**
  - Presence of magnetic fields.
  - Rotation.
  - Coalescence, …

Motivation

- **MAGNETIC FIELDS** brake rotational symmetry leading to an anisotropic energy-momentum tensor.

$$T_{\mu\nu} = \mathrm{diag}(E, P_\perp, P_\perp, P_\parallel)$$

$$E = TS + \mu N + \Omega,$$
$$P_\perp = -\Omega - B\mathcal{M},$$
$$P_\parallel = -\Omega,$$

- **Spherical symmetry not suitable any more.**
- In cylindrical coordinates:

$$ds^2 = -e^{2\Phi(r)}dt^2 + e^{2\Lambda(r)}dr^2 + r^2 d\phi^2 + e^{2\Psi(r)}dz^2$$

$$\begin{aligned}
P'_\perp &= -\Phi'(E + P_\perp) - \Psi'(P_\perp - P_\parallel), \\
4\pi e^{2\Lambda}(E + P_\parallel + 2P_\perp) &= \Phi'' + \Phi'(\Psi' + \Phi' - \Lambda') + \frac{\Phi'}{r}, \\
4\pi e^{2\Lambda}(E + P_\parallel - 2P_\perp) &= -\Psi'' - \Psi'(\Psi' + \Phi' - \Lambda') - \frac{\Psi'}{r}, \\
4\pi e^{2\Lambda}(P_\parallel - E) &= \frac{1}{r}(\Psi' + \Phi' - \Lambda').
\end{aligned}$$

$$\frac{M_T}{R_\parallel} = 4\pi \int_0^{R_\perp} r e^{\Phi(r)+\Psi(r)+\Lambda(r)}(E - 2P_\perp - P_\parallel)dr$$

Astron.Nachr./AN 336 , No. 8/ 9, 851 – 855 (2015)

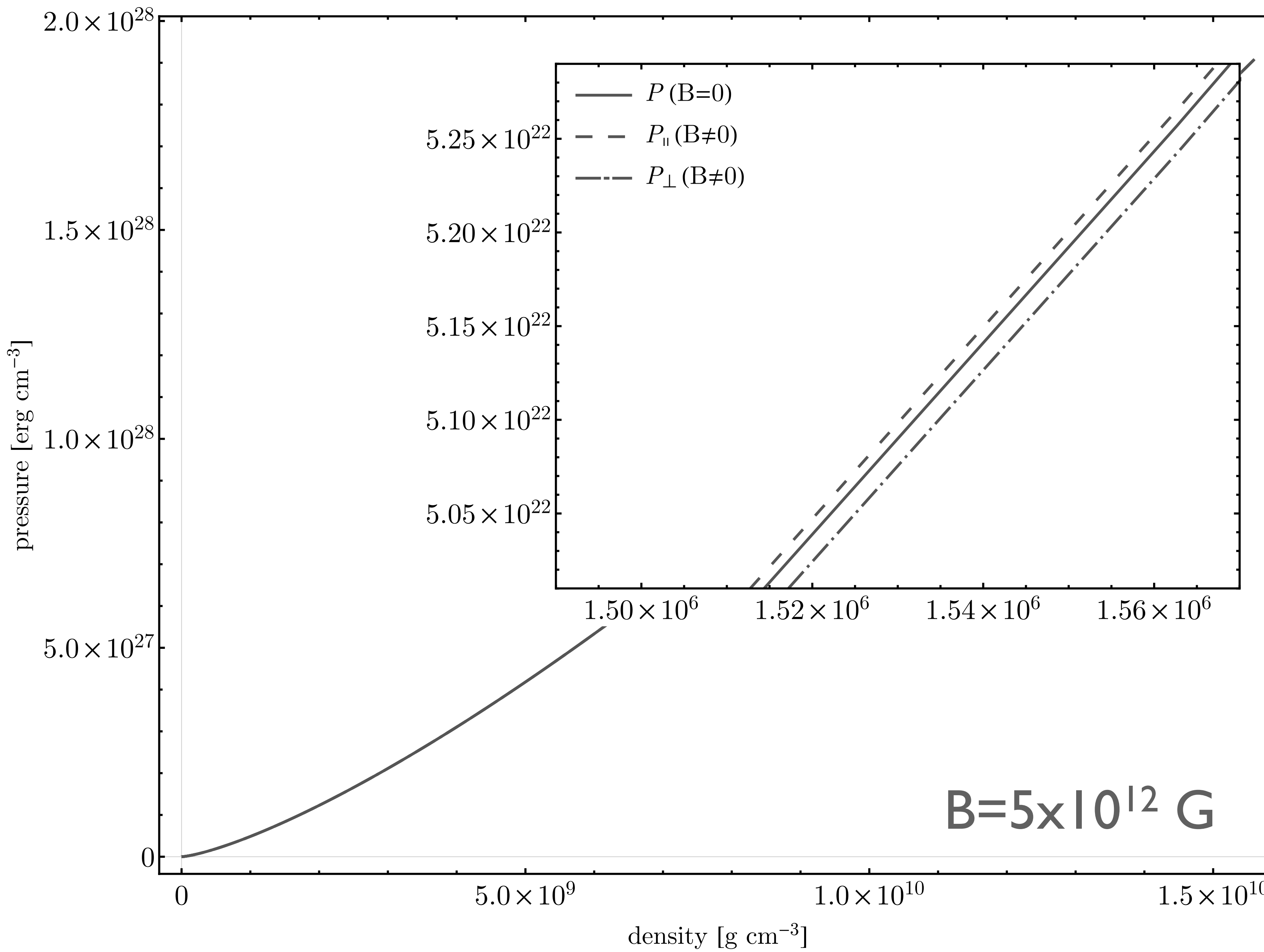
Motivation

EOS obtained considering a degenerate electron gas in an external magnetic field with fixed value lower than the Schwinger critical value (**$B_c = m^2/e = 4.41\times10^{13}$ G).**

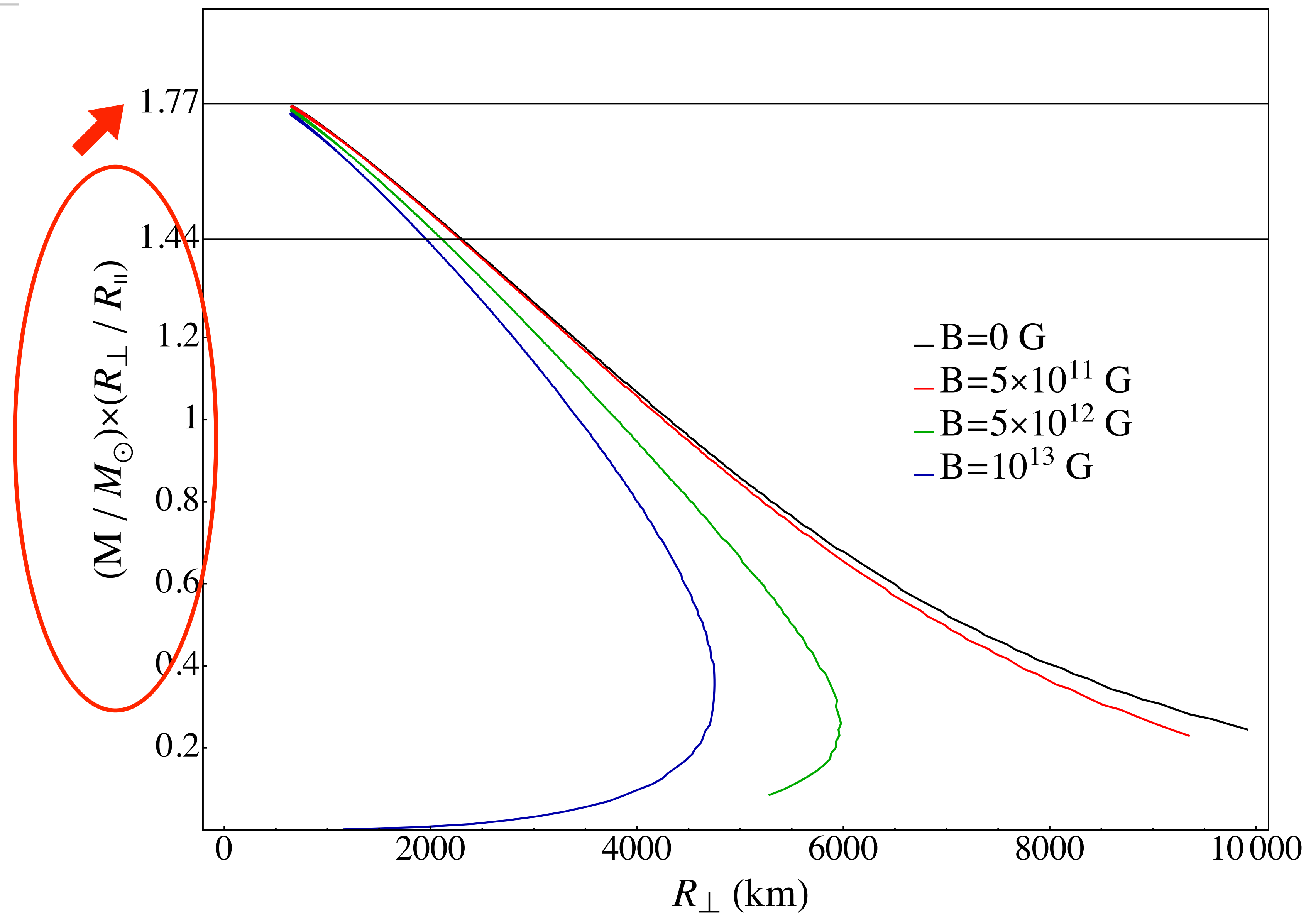
$$E(x,0,B) = E(x,0,0) + f(x,B)$$
$$P_\perp(x,0,B) = P(x,0,0) - g(x,B)$$
$$P_\parallel(x,0,B) = P(x,0,0) + g(x,B)$$

$$f, g \sim \left[\frac{B}{B_c}\right]^2$$

$$x = p_F/m_e$$
$$p_F = \sqrt{\mu^2 - m_e^2}$$



Motivation



$$\frac{M_T}{R_\|} = 4\pi \int_0^{R_\perp} r e^{\Phi(r)+\Psi(r)+\Lambda(r)} (E - 2P_\perp - P_\|) dr$$

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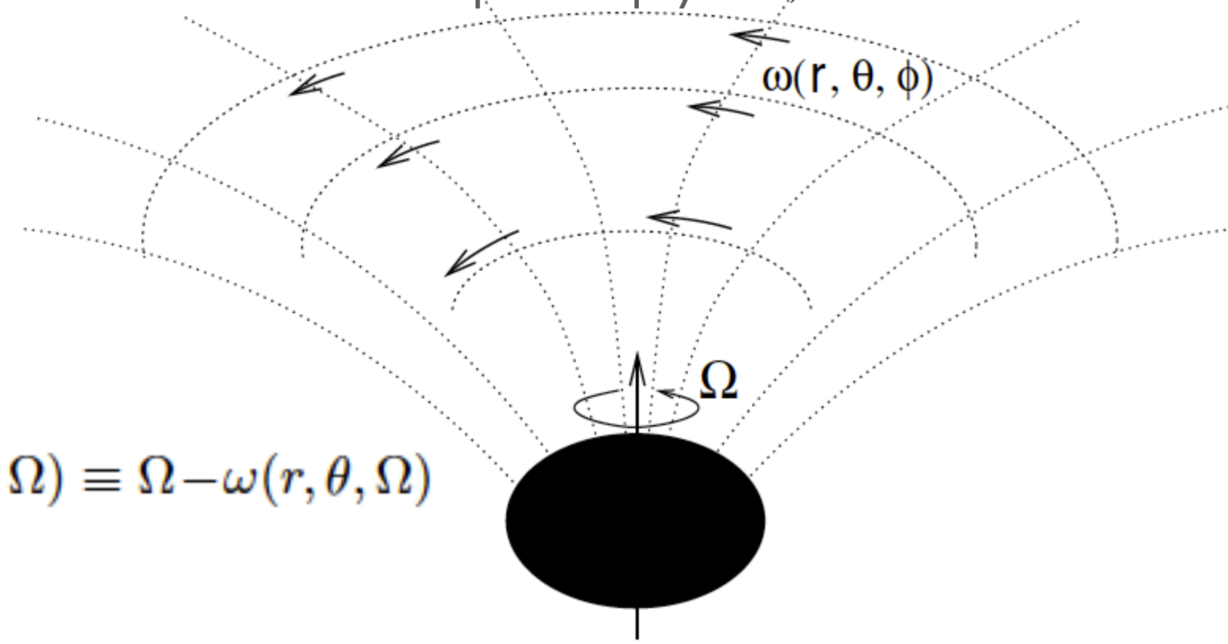
Motivation

F. Weber, ed., "Pulsars as astrophysical laboratories for nuclear and particle physics", 1999.



$$\bar{\omega}(r,\theta,\Omega) \equiv \Omega - \omega(r,\theta,\Omega)$$

- **ROTATION:**
  - Deforms the star producing a flattening at the poles and a blowup in the equatorial direction. Then, the metrics depends also on the polar coordinate θ.
  - Stable configurations may support higher masses than corresponding static ones, therefore changing the space-time geometry.
  - Introduces the general relativistic effect of dragging of inertial frames, thus appearing a non-diagonal term in the metric tensor.
    - This imposes a self-consistency condition into the stellar structure equations, since star's properties like mass and rotation frequency determine how much the inertial frames are dragged in the direction of rotation.

Hartle's formalism for slow rotation

J. B. Hartle, ApJ, vol. 150, p. 1005, Dec. 1967.
J. B. Hartle and K. S. Thorne, ApJ, vol. 153, p. 807, Sept. 1968.

- Rotation velocity small in comparison with the *relativistic* Keplerian frequency, so that the solutions are stable and below the mass shedding limit.

$$\Omega_K^J \approx 0.7\sqrt{\frac{M}{R^3}}$$

- **To do so:**
  - Consider equilibrium rotating configurations as a linear perturbation of the known static counterpart.
  - Expand field equations up to second order in the angular velocity of the star.

$$\begin{aligned} ds^2 = \; & e^{\nu(r)}\{1+2[h_0(r,\Omega)+h_2(r,\Omega)P_2(\cos\theta)]\}dt^2 \\ & + \left[1-\frac{2M(r)}{r}\right]^{-1}\left\{1+2\frac{m_0(r,\Omega)+m_2(r,\Omega)P_2(\cos\theta)}{r-2M(r)}\right\}dr^2 \\ & + r^2[1+2k_2(r,\Omega)P_2(\cos\theta)]\left\{d\theta^2+\sin^2\theta\,[d\phi-\omega(r,\theta,\Omega)dt]^2\right\}+O(\Omega^3). \end{aligned}$$

Axisymmetric uniform rotation

$$T_{\alpha\beta} \equiv T^0_{\alpha\beta}+\Delta T_{\alpha\beta}, \qquad \Delta P = (E+P)\left[p_0+p_2P_2(\cos\theta)\right],$$

$$\Delta T_{\alpha\beta} \equiv (\Delta E+\Delta P)u_\alpha u_\beta+\Delta P g_{\alpha\beta}. \qquad \Delta E = \Delta P\frac{dE}{dP},$$

Hartle's formalism for slow rotation

- Structure equations

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J. B. Hartle and K. S. Thorne, ApJ, vol. 153, p. 807, Sept. 1968.

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$$\frac{dP}{dr} = -\frac{(E+P)(M+4\pi r^3 P)}{r(r-2M)}$$
$$\frac{dM}{dr} = 4\pi r^2 E,$$
$$\frac{d\nu}{dr} = -\frac{2}{E+P}\frac{dP}{dr}$$

1st order perturbations

$$\frac{d\bar\omega}{dr} = \kappa,$$
$$\frac{d\kappa}{dr} = \frac{4\pi r(E+P)(r\kappa + 4\,\bar\omega)}{r-2M} - 4\frac{\kappa}{r}$$

2nd order perturbations, *l*=0

$$\frac{dm_0}{dr} = 4\pi r^2 (E+P)\frac{dE}{dP}p_0 + \frac{r^3 e^{-\nu}}{3}(r-2M)\left[\frac{\kappa^2}{4} + \frac{8\pi r(E+P)\,\bar\omega^2}{r-2M}\right]$$
$$\frac{dp_0}{dr} = -\frac{m_0(1+8\pi r^2 P)}{(r-2M)^2} - \frac{4\pi r^2(E+P)}{r-2M}p_0 + \frac{r^3 e^{-\nu}}{3}\left[\frac{\kappa^2}{4} + \frac{\bar\omega^2}{r}\left(\frac{2}{r} - \frac{d\nu}{dr}\right) + \frac{2\,\kappa\,\bar\omega}{r}\right]$$

2nd order perturbations, *l*=2

$$\frac{dv_2}{dr} = -h_2\frac{d\nu}{dr} + \frac{r^3 e^{-\nu}}{3}(r-2M)\left[\frac{2}{r} + \frac{d\nu}{dr}\right]\left[\frac{\kappa^2}{4} + \frac{4\pi r(E+P)\,\bar\omega^2}{r-2M}\right],$$
$$\frac{dh_2}{dr} = h_2\left[-\frac{d\nu}{dr} + \frac{8\pi r^3(E+P) - 4M}{r^2(r-2M)}\left(\frac{d\nu}{dr}\right)^{-1}\right] - \frac{4v_2}{r(r-2M)}\left(\frac{d\nu}{dr}\right)^{-1}$$
$$+ \frac{r^3 e^{-\nu}}{3}\left(\frac{d\nu}{dr}\right)^{-1}\left\{\frac{\kappa^2}{4}\left[(r-2M)\left(\frac{d\nu}{dr}\right)^2 - \frac{2}{r}\right] + \frac{4\pi r(E+P)\,\bar\omega^2}{r-2M}\left[(r-2M)\left(\frac{d\nu}{dr}\right)^2 + \frac{2}{r}\right]\right\}$$

Hartle's formalism for slow rotation

- Structure equations

$$\frac{dP}{dr} = -\frac{(E+P)(M+4\pi r^3 P)}{r(r-2M)}$$
$$\frac{dM}{dr} = 4\pi r^2 E,$$
$$\frac{d\nu}{dr} = -\frac{2}{E+P}\frac{dP}{dr}$$

$$\frac{d\bar{\omega}}{dr} = \kappa,$$
$$\frac{d\kappa}{dr} = \frac{4\pi r(E+P)(r\kappa+4\bar{\omega})}{r-2M} - 4\frac{\kappa}{r}$$

$$\frac{dm_0}{dr} = 4\pi r^2(E+P)\frac{dE}{dP}p_0 + \frac{r^3 e^{-\nu}}{3}(r-2M)\left[\frac{\kappa^2}{4} + \frac{8\cdots}{\cdots}\right.$$
$$\frac{dp_0}{dr} = -\frac{m_0(1+8\pi r^2 P)}{(r-2M)^2} - \frac{4\pi r^2(E+P)}{r-2M}p_0 + \frac{r^3 e^{-\nu}}{3}\left[\frac{\kappa\cdots}{4}\right.$$

$$\frac{dv_2}{dr} = -h_2\frac{d\nu}{dr} + \frac{r^3 e^{-\nu}}{3}(r-2M)\left[\frac{2}{r}+\frac{d\nu}{dr}\right]\left[\frac{\kappa^2}{4} + \frac{4\pi\cdots}{\cdots}\right.$$
$$\frac{dh_2}{dr} = h_2\left[-\frac{d\nu}{dr} + \frac{8\pi r^3(E+P)-4M}{r^2(r-2M)}\left(\frac{d\nu}{dr}\right)^{-1}\right] - \frac{\cdots}{r(r\cdots}$$
$$+\frac{r^3 e^{-\nu}}{3}\left(\frac{d\nu}{dr}\right)^{-1}\left\{\frac{\kappa^2}{4}\left[(r-2M)\left(\frac{d\nu}{dr}\right)^2 - \frac{2}{r}\right] + \cdots\right.$$

- Total angular momentum: $J = R^4\kappa(R)/6$
- Angular velocity of the star: $\Omega = \bar{\omega}(R) + 2J/R^3$
- Moment of inertia: $I = J/\Omega$
- Total mass: $M_T = M(R) + m_0(R) + J^2/R^3$
- Quadrupole moment:

$$Q = \frac{8}{5}M^3\frac{h_2 + v_2 - J^2/(MR^3)}{\frac{2M\,Q_2^1\left(\frac{R}{M}-1\right)}{\sqrt{R(R-2M)}} + Q_2^2\left(\frac{R}{M}-1\right)} + \frac{J^2}{M}$$

- Polar and equatorial radius:

$$R_p = r(R,0) = R + \xi_0(R) + \xi_2(R),$$
$$R_{eq} = r(R,\tfrac{\pi}{2}) = R + \xi_0(R) - \frac{\xi_2(R)}{2}$$

with

$$r(R,\theta) = R + \xi_0(R) + \xi_2(R)P_2(\cos\theta),$$
$$\xi_0 = -p_0^*(E+P)\left(\frac{dP}{dr}\right)^{-1}$$
$$\xi_2 = -p_2^*(E+P)\left(\frac{dP}{dr}\right)^{-1}$$

- Eccentricity:

$$\varepsilon = \sqrt{1-\left(\frac{R_p}{R_{eq}}\right)^2}$$

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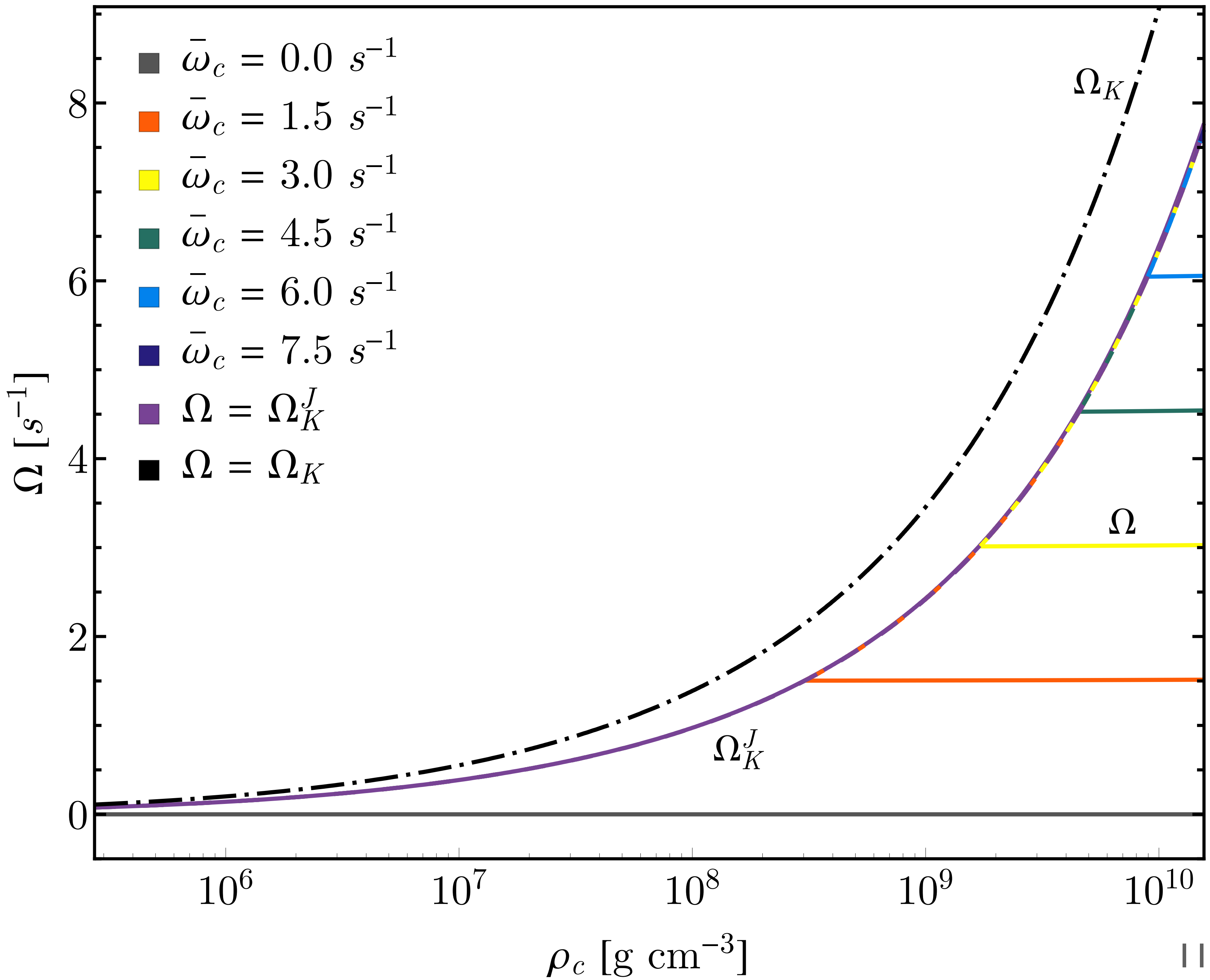
Results: rotation effects

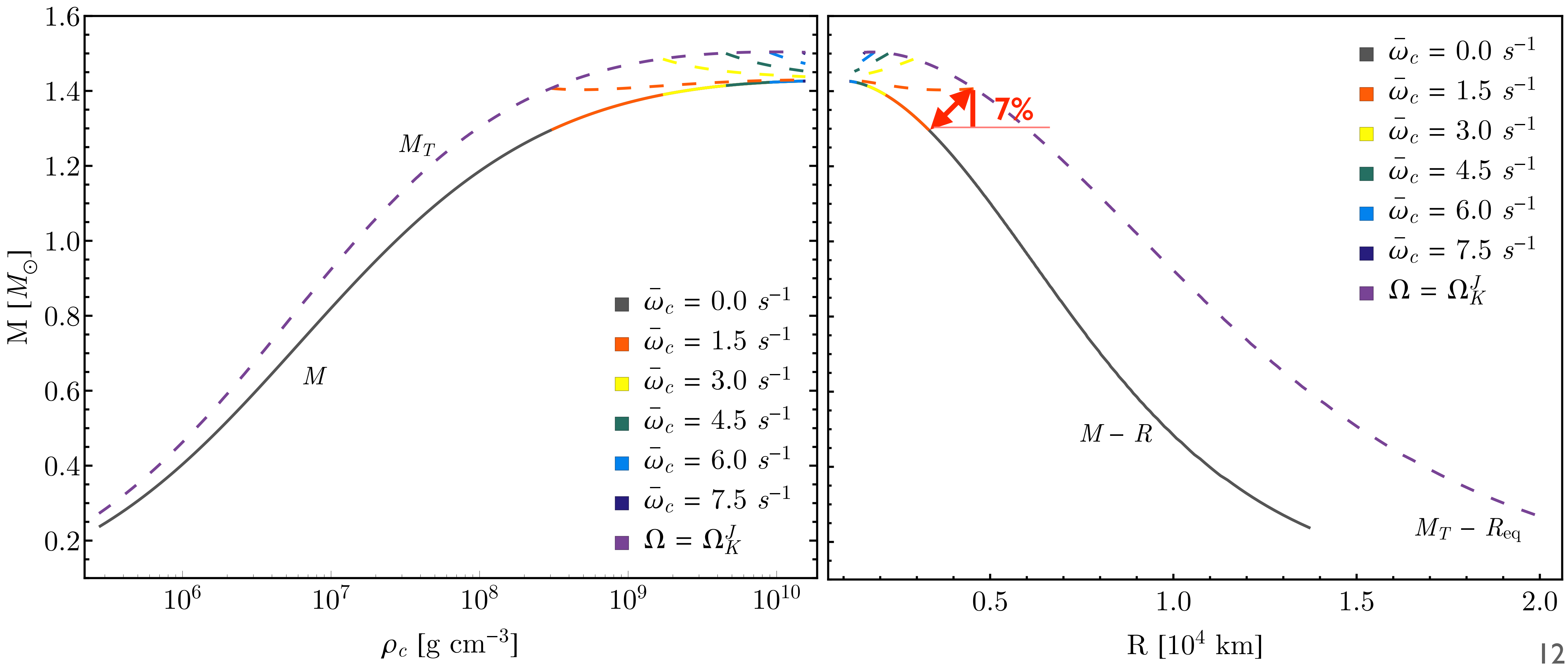


For higher angular velocities less central densities are *accepted.*

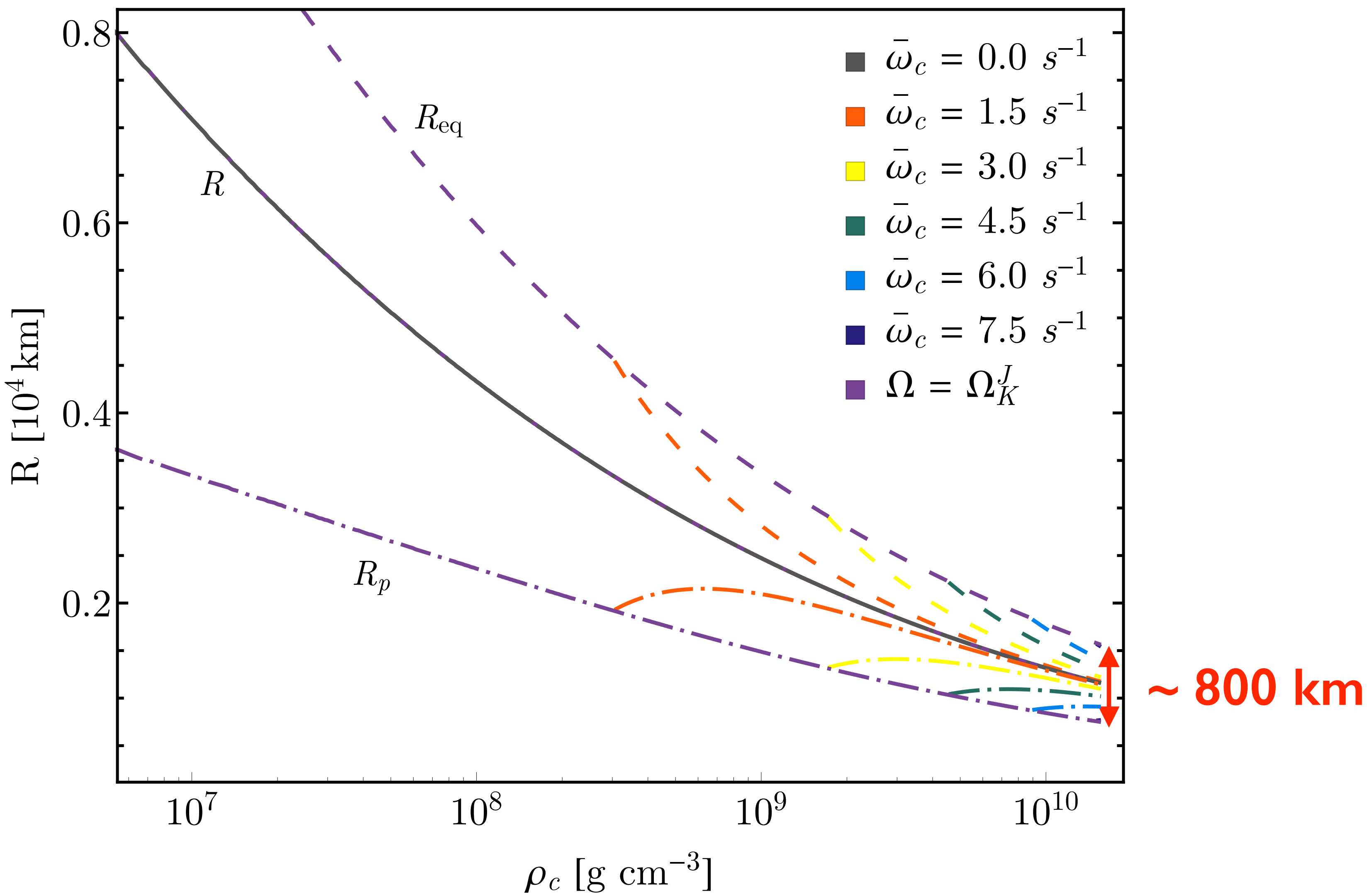
Results: rotation effects

Mass increases up to ~ 1.5 M$_\odot$.

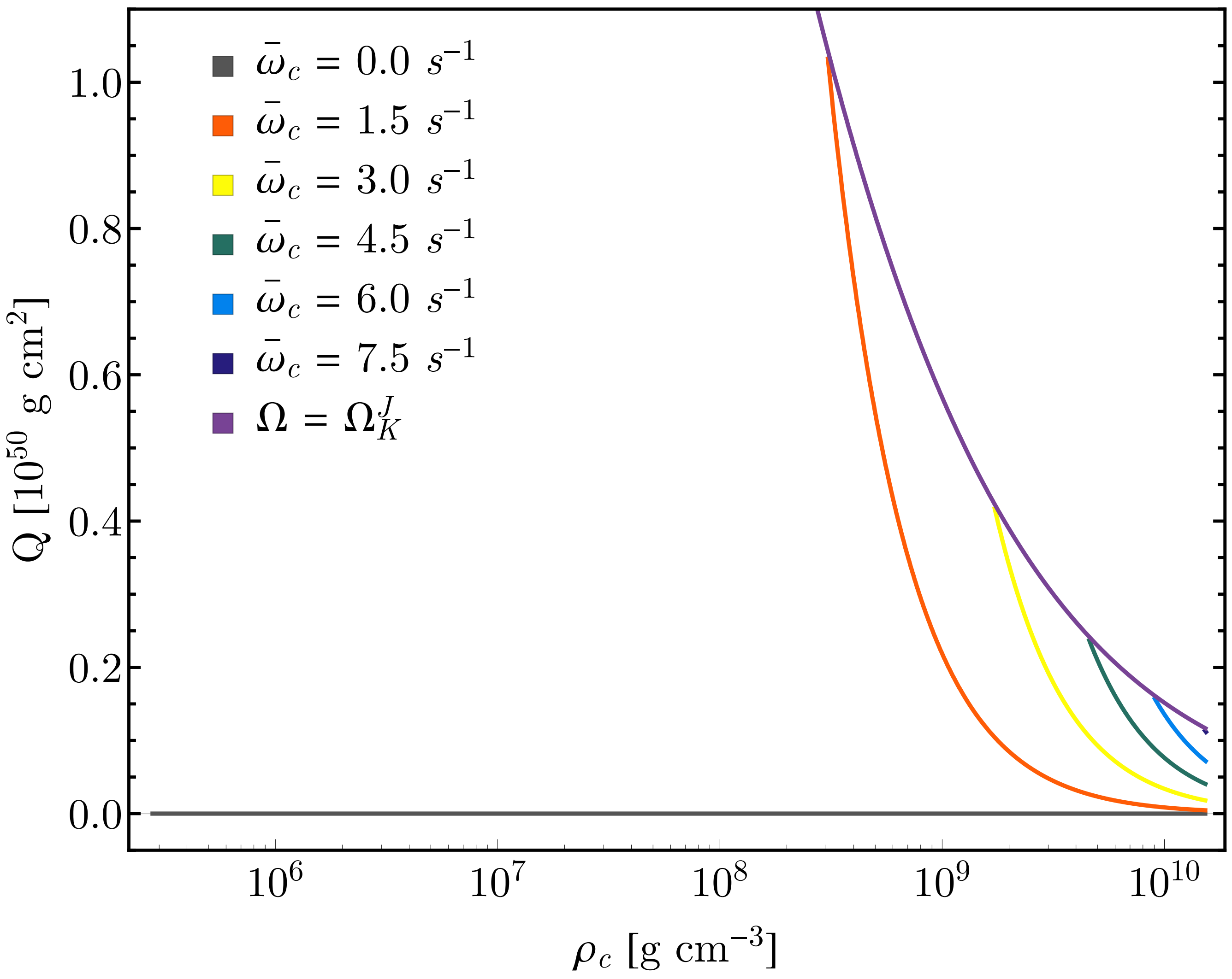
Compatible with 1.45 M$_\odot$ solutions for the Keplerian sequence in (B. Franzon & S. Schramm, 2015)



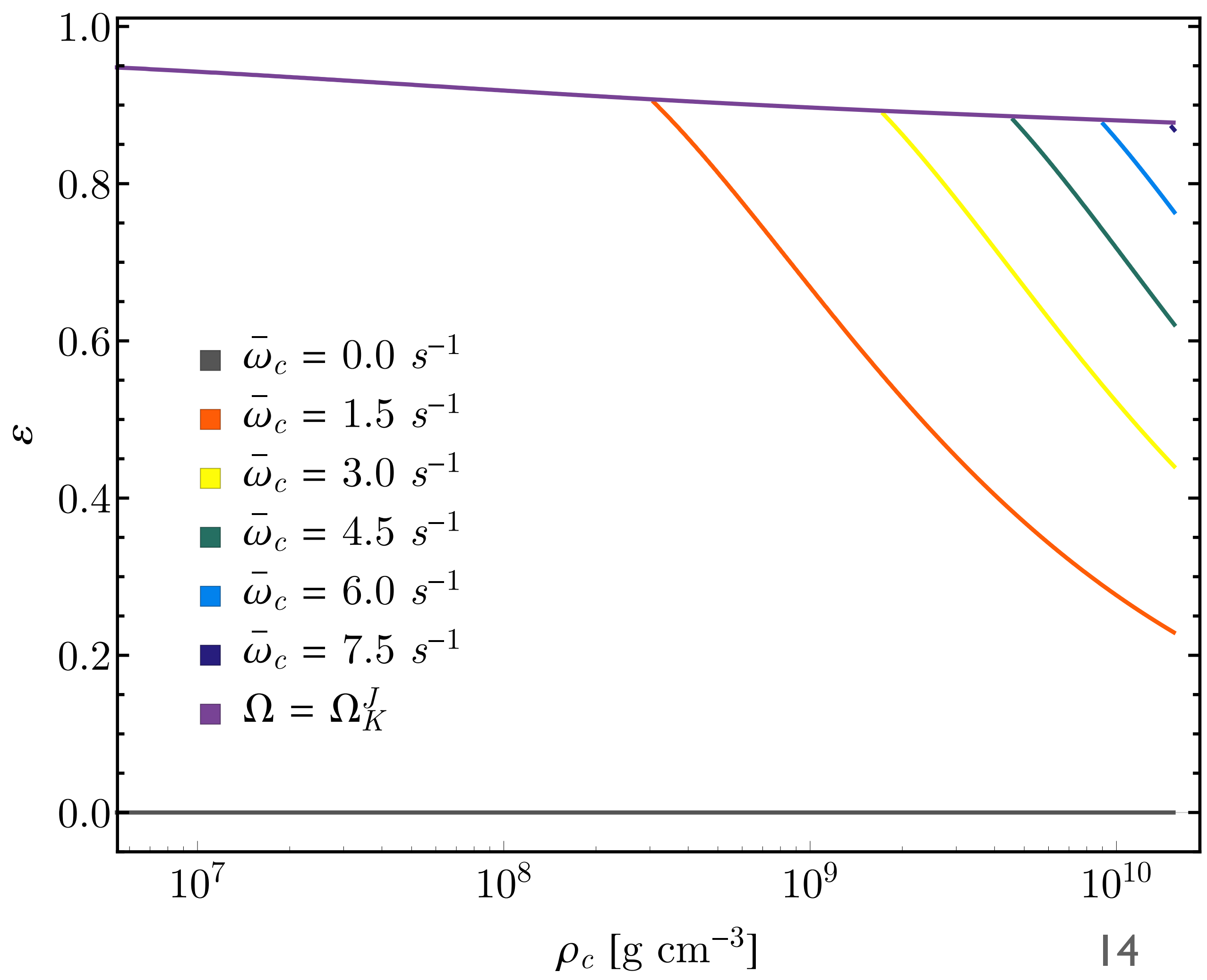
Results: rotation effects



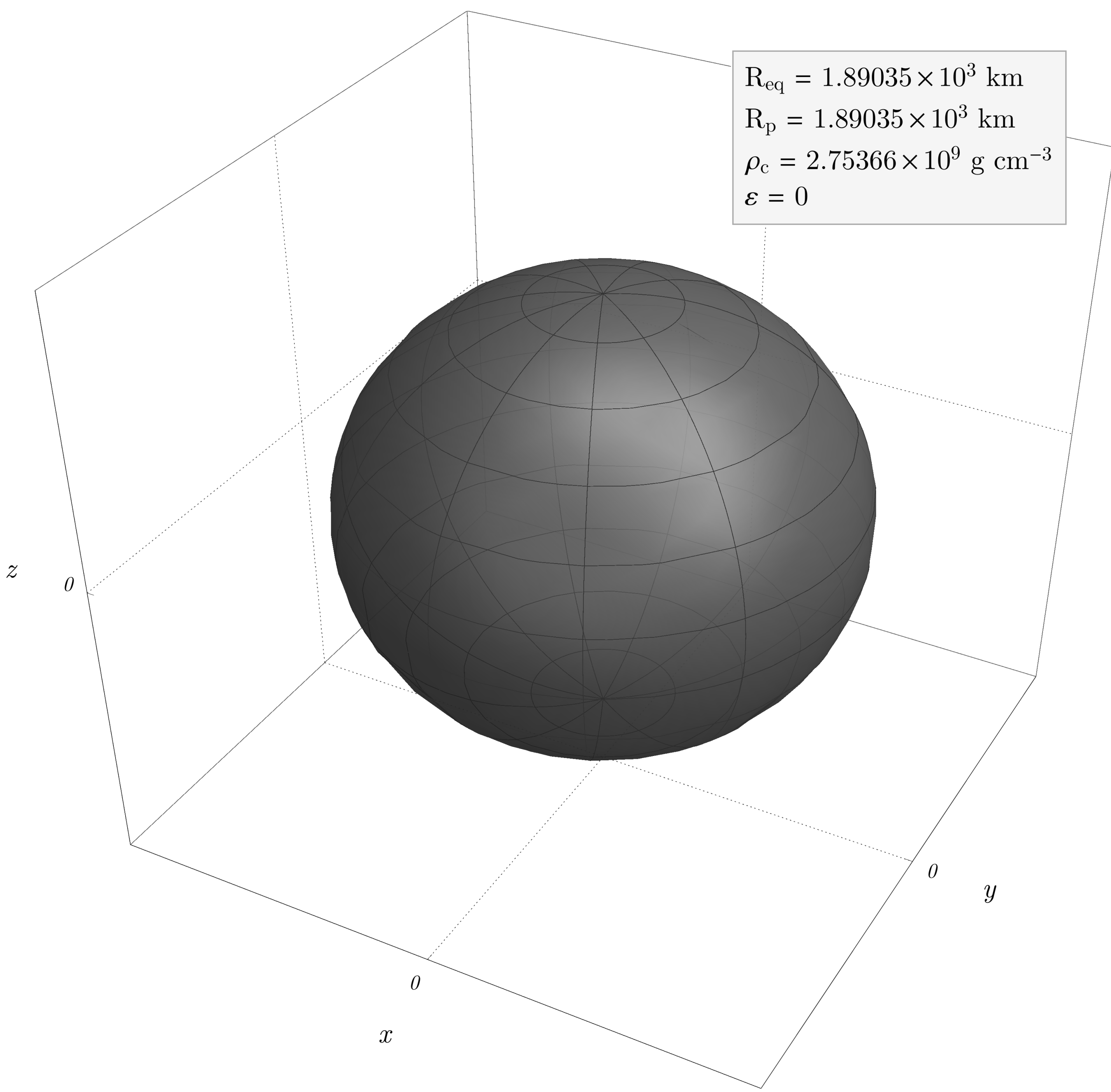
Results: rotation effects

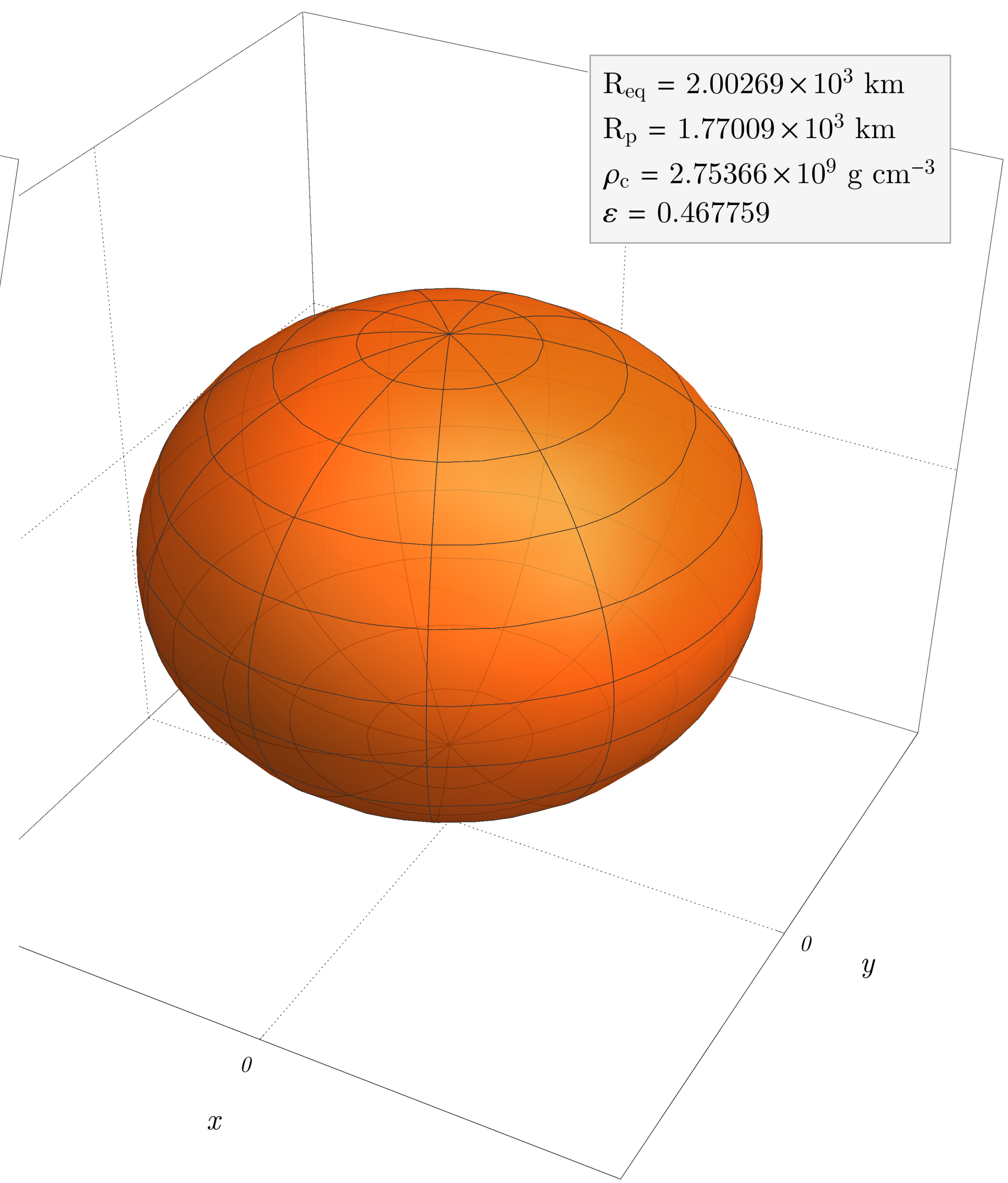


The higher the density the less the star is deformed.



Results: rotation effects

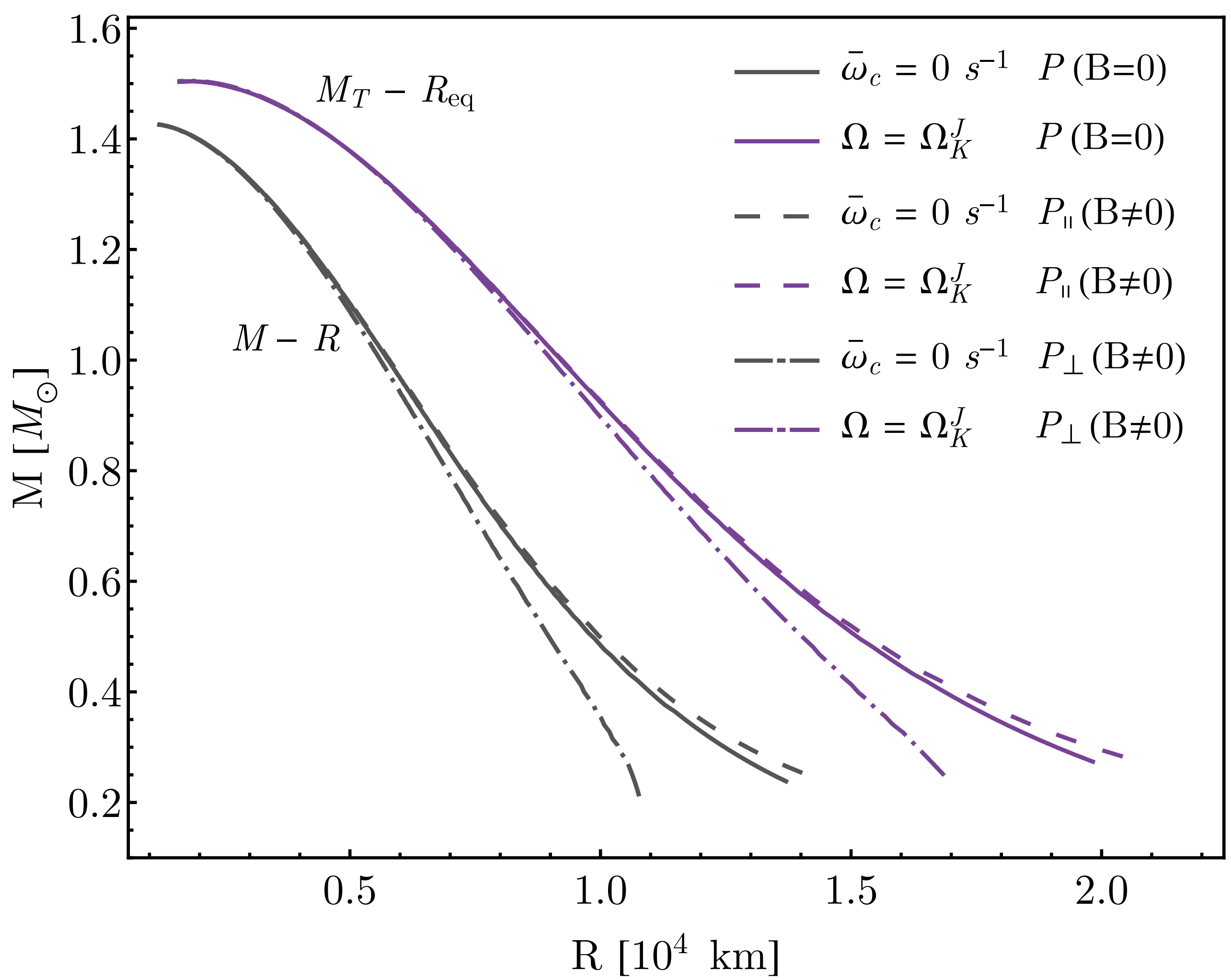




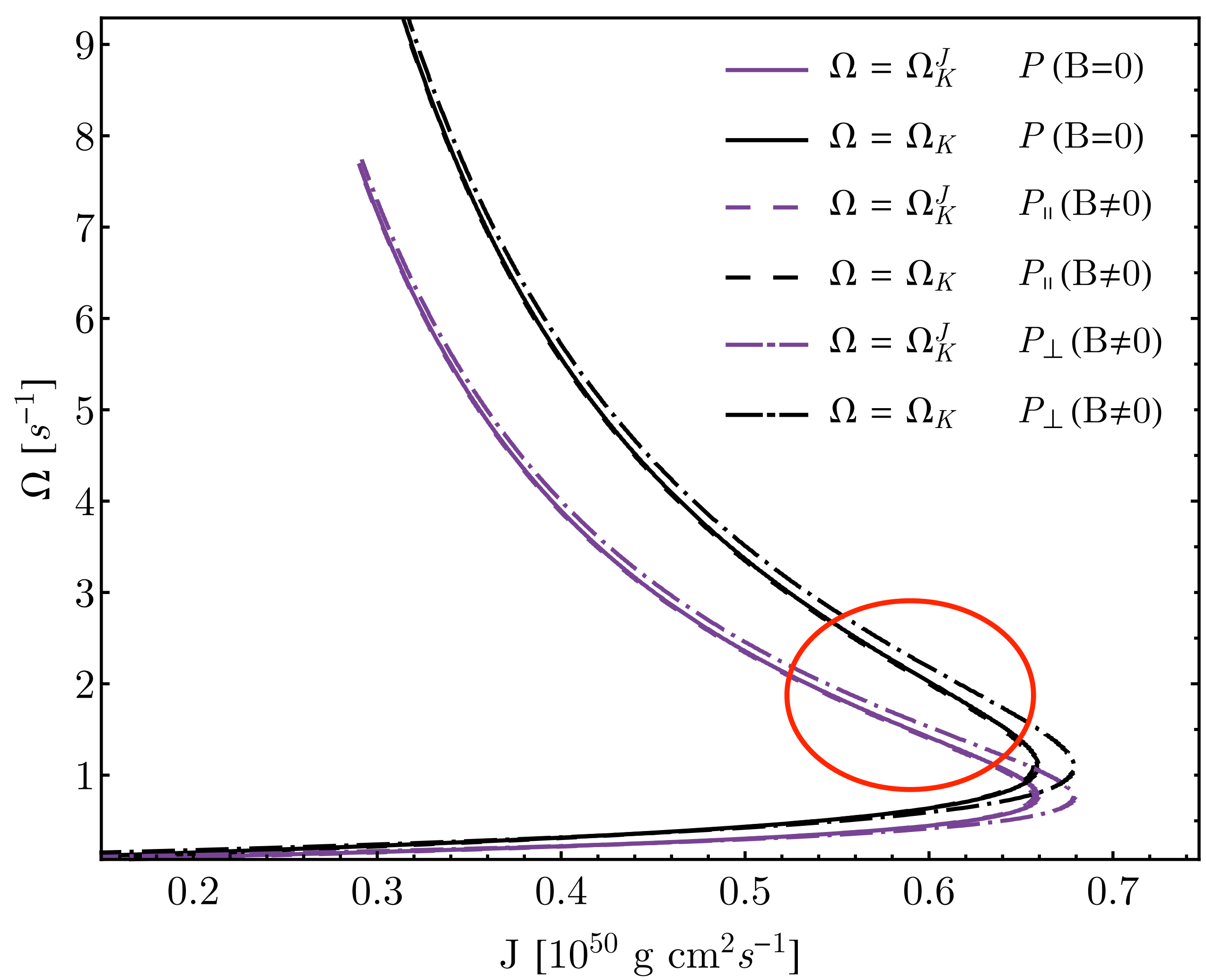
Results: rotation & magnetic field effects

B=5x10^{12} G

Perpendicular pressure solutions have lower static mass and radius.



Slightly higher angular velocities and angular momentum can be reached.

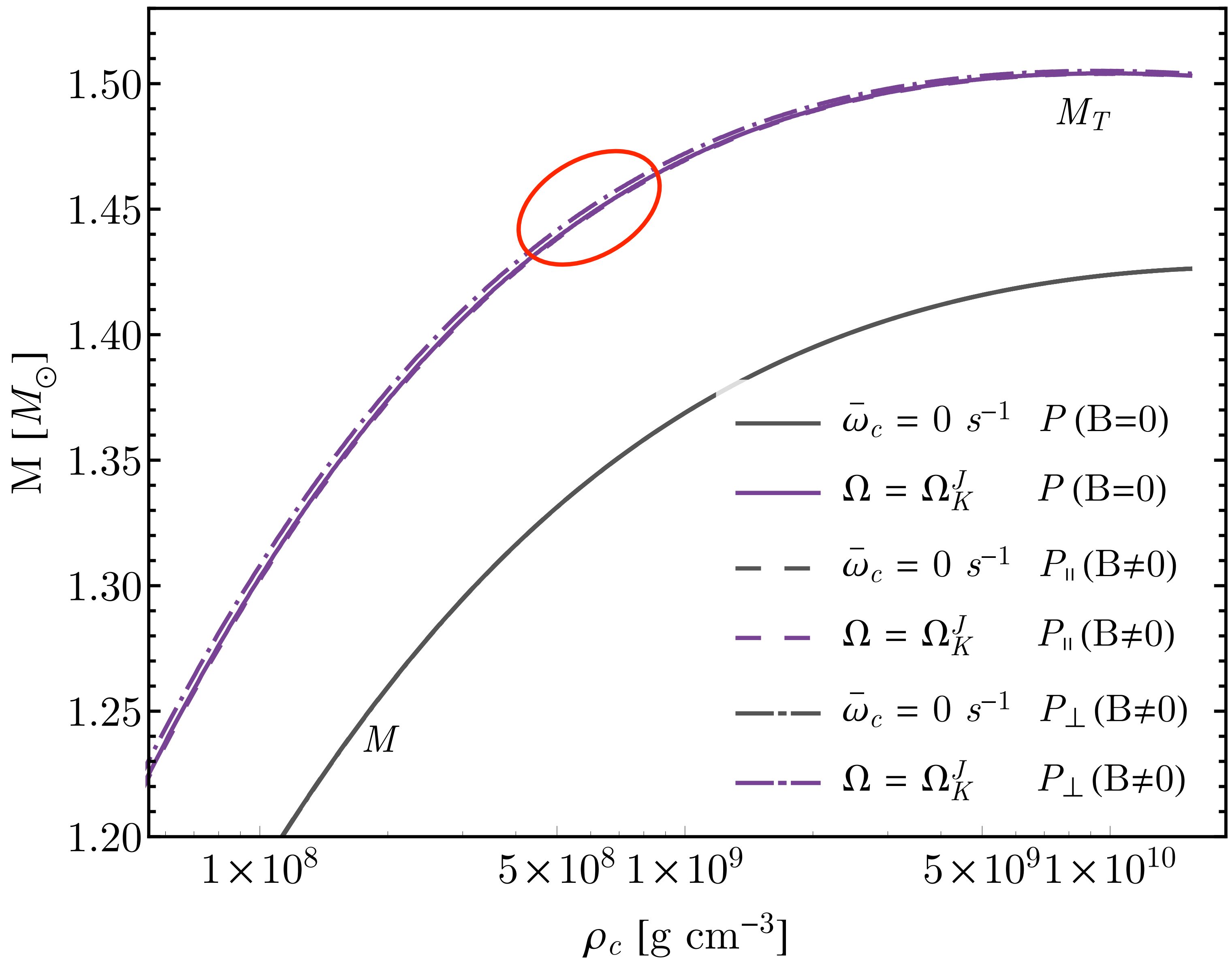


Results: rotation & magnetic field effects

B=5x10[12] G

≈1% increase in the total mass for the configurations with perpendicular pressure (softer EOS).

Rotation effects are dominant since magnetic field is included only in the EOS.



Conclusions

- Rotational effects in non-magnetic and magnetic white dwarfs studied within Hartle's formalism.
- To first order in the angular velocity the only variation with respect to the static counterpart is the appearance of the angular momentum, the angular velocity and the moment of inertia.
- To second order in the angular velocity, rotational effects decrease as the density increases and mass augments in ~7%.
- The slower the velocity the better the approximation.
- Magnetic field effects are much smaller than rotation effects. This follows from the assumption that the magnetic field only influences the EOS.
- For the perpendicular pressure EOS (softer) we obtain smaller sizes, so that slightly higher angular velocities and angular momenta are reached, thus obtaining ~1% higher masses.

Next Steps

- Other EOS.
- Include both pressures simultaneously.
- Consider magnetic field effects not only on EOS.
- Other configurations for the magnetic field.
- Magnetohydrodynamics.
- …

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Thank you for your attention.