$(2+1)$ Scale dependent gravity coupled to a nonlinear electrodynamic source

Ernesto Contreras

Departamento de Física
Universidad de Los Andes
Bogotá-Colombia

Scale dependent three-dimensional charged black holes in linear and non-linear electrodynamics

Ángel Rincón [a,1], Ernesto Contreras [b,2,c], Pedro Bargueño [2], Benjamin Koch [1], Grigorios Panotopoulos [3], Alejandro Hernández-Arboleda [2]

[1]Instituto de Física, Pontificia Universidad Católica de Chile,
Av. Vicuña Mackenna 4860, Santiago, Chile
[2]Departamento de Física, Universidad de los Andes,
Apartado Aéreo 4976, Bogotá, Distrito Capital, Colombia
[3]CENTRA, Instituto Superior Técnico, Universidade de Lisboa,
Av. Rovisco País 1, Lisboa, Portugal
Received: date / Accepted: date

Abstract In the present work we study the scale dependence at the level of the effective action of charged black holes in Einstein-Maxwell as well as in Einstein-power-Maxwell theories in (2+1)-dimensional spacetimes without a cosmological constant. We allow for scale dependence of the gravitational and electromagnetic couplings, and we solve the corresponding generalized field equations imposing the "null energy condition". Certain properties, such as horizon structure and thermodynamics, are discussed in detail.

Keywords Black holes; Scale dependence; 2+1 gravity.

PACS PACS code1 · PACS code2 · more

particular, non-linear electromagnetic models are introduced in order to describe situations in which this field is strong enough to invalidate the predictions provided by the linear theory. Originally the Born-Infeld non-linear electrodynamics was introduced in the 30's in order to obtain a finite self-energy of point-like charges [4]. During the last decades this type of action reappears in the open sector of superstring theories [5] as it describes the dynamics of D-branes [6]. Also, these kind of electrodynamics have been coupled to gravity in order to obtain, for example, regular black holes solutions [8–10], semiclassical corrections to the black hole entropy [11] and novel exact solutions with a cosmological constant acting as an effective Born–Infeld cut–off [12]. A particularly interesting class of NLED theories is the so called power-Maxwell theory described by a Lagrangian den-

Outline

- Overview
- $(2+1)$ Gravity $+$ NLED source
- $(2+1)$ Scale dependent Gravity $+$ NLED source
- Summary

Overview

Why scale dependent gravity?

$$G_0 \rightarrow G_k$$
$$\Lambda \rightarrow \Lambda_k$$

$k :=$ momentum scale

$$G_0 \rightarrow G(r)$$
$$\Lambda \rightarrow \Lambda(r)$$

$$S_0 = \int d^3x\sqrt{-g}\left[\frac{R-\Lambda}{16\pi G_0}\right] \rightarrow S = \int d^3x\sqrt{-g}\left[\frac{R-\Lambda(r)}{16\pi G(r)}\right]$$

Overview

IOP PUBLISHING — CLASSICAL AND QUANTUM GRAVITY

Class. Quantum Grav. **30** (2013) 175009 (17pp) — doi:10.1088/0264-9381/30/17/175009

Black hole solution for scale-dependent gravitational couplings and the corresponding coupling flow

Carlos Contreras[1], **Benjamin Koch**[2] **and Paola Rioseco**[2]

[1] Departamento de Física, Universidad Técnica Federico Santa María, Casilla 110-V, Valparaíso, Chile
[2] Pontificia Universidad Católica de Chile, Av. Vicuña Mackenna 4860, Santiago, Chile

E-mail: bkoch@fis.puc.cl

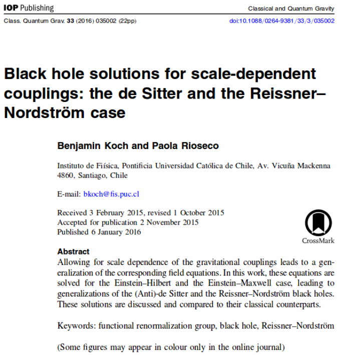
Received 28 March 2013, in final form 26 June 2013
Published 8 August 2013
Online at stacks.iop.org/CQG/30/175009

Abstract
We study a particular solution for the generalized Einstein Hilbert action with scale-dependent couplings $G(r)$ and $\Lambda(r)$. The form of the couplings is not imposed, but rather deduced from the existence of a non-trivial symmetrical solution. A classical-like choice of the integration constants is found. Finally, the induced flow of the couplings is derived and compared to the flow that is obtained in the context of the exact renormalization group approach.

PACS numbers: 04.60., 04.70.

(Some figures may appear in colour only in the online journal)

Overview



IOP Publishing
Classical and Quantum Gravity
Class. Quantum Grav. **33** (2016) 035002 (22pp)
doi:10.1088/0264-9381/33/3/035002

Black hole solutions for scale-dependent couplings: the de Sitter and the Reissner–Nordström case

Benjamin Koch and Paola Rioseco

Instituto de Física, Pontificia Universidad Católica de Chile, Av. Vicuña Mackenna 4860, Santiago, Chile

E-mail: bkoch@fis.puc.cl

Received 3 February 2015, revised 1 October 2015
Accepted for publication 2 November 2015
Published 6 January 2016

CrossMark

Abstract
Allowing for scale dependence of the gravitational couplings leads to a generalization of the corresponding field equations. In this work, these equations are solved for the Einstein–Hilbert and the Einstein–Maxwell case, leading to generalizations of the (Anti)-de Sitter and the Reissner–Nordström black holes. These solutions are discussed and compared to their classical counterparts.

Keywords: functional renormalization group, black hole, Reissner–Nordström

(Some figures may appear in colour only in the online journal)

$$e_0 \to e(r)$$

$$S_0 = \int d^3x \sqrt{-g}\left[\frac{R}{16\pi G_0} - \frac{\mathcal{L}}{e_0^2}\right] \to S = \int d^3x \sqrt{-g}\left[\frac{R}{16\pi G(r)} - \frac{\mathcal{L}}{e(r)^2}\right]$$

Overview

Why nonlinear electrodynamics?

$$\mathcal{L} \quad \to \quad \mathcal{L}(F)$$

The Born-Infeld Lagrangian

$$\mathcal{L} = -b^2 \left(\sqrt{1 - \frac{1}{b^2}(E^2 - B^2)} - 1 \right)$$

Overview

VOLUME 80, NUMBER 23 PHYSICAL REVIEW LETTERS 8 JUNE 1998

Regular Black Hole in General Relativity Coupled to Nonlinear Electrodynamics

Eloy Ayón-Beato* and Alberto García

Departamento de Física, Centro de Investigación y Estudios Avanzados del IPN, Apdo. Postal 14-740, 07000 México DF, Mexico

(Received 14 November 1997)

The first regular *exact* black hole solution in general relativity is presented. The source is a nonlinear electrodynamic field satisfying the weak energy condition, which in the limit of weak field becomes the Maxwell field. The solution corresponds to a charged black hole with $|q| \leq 2 s_c m \approx 0.6m$, having the metric, the curvature invariants, and the electric field regular everywhere. [S0031-9007(98)06332-7]

PACS numbers: 04.20.Jb, 04.70.Bw

In general relativity the existence of singularities appears to be a property inherent to most of the physically relevant solutions of Einstein equations, in particular, to all known up-to-date black hole *exact* solutions [1]. The Penrose cosmic censorship conjecture states that these singularities must be dressed by event horizons; no causal connection could exist between the interior of a black hole with the exterior fields, thus pathologies occurring at the singular region would have no influence on the exterior region, and the physics outside would be well behaved (cf. [2] for a review on the recent status of this conjecture).

To avoid the black hole singularity problem, some

$$\boldsymbol{g} = -\left(1 - \frac{2mr^2}{(r^2+q^2)^{3/2}} + \frac{q^2r^2}{(r^2+q^2)^2}\right)d\boldsymbol{t}^2 + \left(1 - \frac{2mr^2}{(r^2+q^2)^{3/2}} + \frac{q^2r^2}{(r^2+q^2)^2}\right)^{-1} d\boldsymbol{r}^2 + r^2 d\boldsymbol{\Omega}^2, \tag{1}$$

while the associated electric field E is given by

$$E = qr^4\left(\frac{r^2-5q^2}{(r^2+q^2)^4} + \frac{15}{2}\frac{m}{(r^2+q^2)^{7/2}}\right). \tag{2}$$

Notice that this solution asymptotically behaves as the Reissner-Nordström solution, i.e.,

Overview

3 June 1999

ELSEVIER

PHYSICS LETTERS B

Physics Letters B 456 (1999) 28–33

Three dimensional black hole coupled to the Born-Infeld electrodynamics

Mauricio Cataldo [1, 2], Alberto García [3, 4]

Departamento de Física, Facultad de Ciencia, Universidad de Santiago de Chile, Avda. Ecuador 3493, Casilla 307, Santiago, Chile

Received 9 December 1998; received in revised form 23 March 1999
Editor: M. Cvetič

Abstract

A nonlinear charged version of the $(2+1)$-anti de Sitter black hole solution is derived. The source to the Einstein equations is a Born-Infeld electromagnetic field, which in the weak field limit becomes the $(2+1)$-Maxwell field. The obtained Einstein-Born-Infeld solution for certain range of the parameters (mass, charge, cosmological and the Born-Infeld constants) represent a static circularly symmetric black hole. Although the covariant metric components and the electric field do not exhibit a singular behavior at $r=0$ the curvature invariants are singular at that point.

PACS: 04.20.Jb
Keywords: 2 + 1 dimensions; Born-Infeld black hole

Overview

PHYSICAL REVIEW D, VOLUME 61, 084003

Regular (2+1)-dimensional black holes within nonlinear electrodynamics

Mauricio Cataldo*
Departamento de Física, Facultad de Ciencias, Universidad del Bío-Bío, Avda. Collao 1202, Casilla 5-C, Concepción, Chile and Departamento de Física, Facultad de Ciencia, Universidad de Santiago de Chile, Avda. Ecuador 3493, Casilla 307, Santiago, Chile

Alberto García†
Departamento de Física, Centro de Investigación y de Estudios Avanzados del IPN, Apartado Postal 14–740, C.P. 07000, México, D.F. Mexico and Departamento de Física, Facultad de Ciencia, Universidad de Santiago de Chile, Avda. Ecuador 3493, Casilla 307, Santiago, Chile
(Received 22 April 1999; published 15 March 2000)

(2+1)-regular static black hole solutions with a nonlinear electric field are derived. The source to the Einstein equations is an energy momentum tensor of nonlinear electrodynamics, which satisfies the weak energy conditions and in the weak field limit becomes the (2+1)-Maxwell field tensor. The derived class of solutions is regular: the metric, curvature invariants, and electric field are regular everywhere. The metric becomes, for a vanishing parameter, the (2+1)-static charged BTZ solution. A general procedure to derive solutions for the static BTZ (2+1)-spacetime for any nonlinear Lagrangian depending on the electric field is formulated; for relevant electric fields one requires the fulfillment of the weak energy conditions.

PACS number(s): 04.20.Jb, 97.60.Lf

Overview

In this talk...

$$\mathcal{L}(F) = C^{\beta}|F|^{\beta}$$
$$F := \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

Power-Maxwell source...

...as a source of

- Classical $(2+1)$ gravity
- Scale dependent $(2+1)$ gravity

$$G_0 \rightarrow G(r)$$
$$e_0 \rightarrow e(r)$$
$$\mathcal{L} \rightarrow \mathcal{L}(F)$$

$(2+1)$ Gravity + NLED source

$$S = \int d^3x \sqrt{-g}\left[\frac{R}{16\pi G_0} - \frac{\mathcal{L}(F)}{e_0^{2\beta}}\right],$$

- metric signature $(-,+,+)$
- natural units $c = \hbar = k_B = 1$

$(2+1)$ Gravity + NLED source

Variations respect to the metric field lead to

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi \frac{G_0}{e_0^{2\beta}} T_{\mu\nu},$$

where

$$T_{\mu\nu} := \mathcal{L}_F g_{\mu\nu} - \mathcal{L}(F) F_{\mu\gamma} F_{\nu}{}^{\gamma}$$

Variations respect to the electromagnetic four–potential A_μ

$$D_\mu \left(F^{\mu}{}_{\nu} \mathcal{L}_F \right) = 0.$$

(2 + 1) Gravity + NLED source

ELSEVIER

29 June 2000

Physics Letters B 484 (2000) 154–159

PHYSICS LETTERS B

www.elsevier.nl/locate/npe

(2 + 1)-dimensional black hole with Coulomb-like field

Mauricio Cataldo [a], Norman Cruz [b], Sergio del Campo [c], Alberto García [d]

[a] *Departamento de Física, Facultad de Ciencias, Universidad del Bío-Bío, Avda. Collao 1202, Casilla 5-C, Concepcíon, Chile*
[b] *Departamento de Física, Facultad de Ciencia, Universidad de Santiago de Chile, Avda. Ecuador 3493, Casilla 307, Santiago, Chile*
[c] *Instituto de Física, Facultad de Ciencias Básicas y Matemáticas, Universidad Católica de Valparaíso, Avda. Brasil 2950, Valparaíso, Chile*
[d] *Departamento de Física, CINVESTAV-IPN, Apartado Postal 14-740, C.P., 07000 México D.F., Mexico*

Received 17 April 2000; accepted 16 May 2000
Editor: M. Cvetič

Abstract

A (2 + 1)-static black hole solution with a nonlinear electric field is derived. The source to the Einstein equations is a nonlinear electrodynamics, satisfying the weak energy conditions, and it is such that the energy momentum tensor is traceless. The obtained solution is singular at the origin of coordinates. The derived electric field $E(r)$ is given by $E(r) = q/r^2$, thus it has the Coulomb form of a point charge in the Minkowski spacetime. This solution describes charged (anti)-de Sitter spaces. An interesting asymptotically flat solution arises for $\Lambda = 0$.

PACS: 04.20.Jb
Keywords: 2 + 1 dimensions; Non-linear black hole

$(2+1)$ Gravity + NLED source

Consideration: Static and circularly symmetric spacetime and

$$ds^2 = -f(r)dt^2 + f(r)^{-1}dr^2 + r^2 d\Omega^2$$
$$F_{\mu\nu} = (\delta^t_\mu \delta^r_\nu - \delta^r_\mu \delta^t_\nu)E(r)$$

For $\beta = 3/4$ and $C := 2^{7/3}3^{-4/3}e_0^2 Q_0^{2/3}$

$$f(r) = \frac{4G_0 Q_0^2}{3r} - G_0 M_0,$$
$$E(r) = \frac{Q_0}{r^2}$$

$$T^\mu{}_\mu = 0!!$$

$(2+1)$ Scale dependent Gravity + NLED source

$$
\begin{aligned}
G_0 &\rightarrow G(r) \\
e_0 &\rightarrow e(r)
\end{aligned}
$$

$$S = \int d^3x\sqrt{-g}\left[\frac{R}{16\pi G(r)} - \frac{\mathcal{L}(F)}{e(r)^{2\beta}}\right]$$

Variations respect to the metric field lead to

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = 8\pi\frac{G(r)}{e^{2\beta}(r)}T_{\mu\nu} - \Delta t_{\mu\nu},$$

where

$$\Delta t_{\mu\nu} = G(r)(g_{\mu\nu}\Box - \nabla_\mu\nabla_\nu)\frac{1}{G(r)}.$$

$(2+1)$ Scale dependent Gravity + NLED source

Variations respect to the electromagnetic four–potential A_μ

$$D_\mu\left(\frac{F^\mu{}_\nu\mathcal{L}_F}{e(r)^{2\beta}}\right)=0.$$

For static and circularly symmetric solutions we consider

$$\begin{aligned} ds^2 &= -f(r)dt^2+f(r)^{-1}dr^2+r^2d\Omega^2 \\ F_{\mu\nu} &= (\delta^t_\mu\delta^r_\nu-\delta^r_\mu\delta^t_\nu)E(r) \\ \beta &= \frac{3}{4} \end{aligned}$$

from where

$$E(r)=\frac{Q_0}{r^2}\left(\frac{e(r)}{e_0}\right)^3$$

We must solve a set of differential equations for $\{f(r),G(r),e(r)\}$!!!

$(2+1)$ Scale dependent Gravity + NLED source

The null energy condition

$$T_{\mu\nu}\ell^{\mu}\ell^{\nu} \geq 0.$$

where $\ell^{\mu} = \{\frac{1}{\sqrt{f}}, \sqrt{f}, 0\}$.

Now,

$$\cancelto{0}{\ell^{\mu}\ell^{\nu}R_{\mu\nu}} - \frac{1}{2}\cancelto{0}{\ell^{\mu}\ell^{\nu}g_{\mu\nu}R} = 8\pi\frac{G(r)}{e^{2\beta}(r)}\cancelto{0}{\ell^{\mu}\ell^{\nu}T_{\mu\nu}} - \ell^{\mu}\ell^{\nu}\Delta t_{\mu\nu},$$

Then,

$$\Delta t_{\mu\nu}\ell^{\mu}\ell^{\nu} = 0,$$

from where

$$G(r)\frac{d^2G(r)}{dr^2} - 2\left(\frac{dG(r)}{dr}\right)^2 = 0,$$

which solution reads

$$G(r) = \frac{G_0}{1+\epsilon r}$$

$\epsilon \to$ running parameter!

$(2+1)$ Scale dependent Gravity $+$ NLED source

Solutions

$$f(r) = \frac{4G_0 Q_0^2}{3r(r\epsilon+1)^3} - \frac{M_0 G_0 \left(r^3\epsilon^2 + 3r^2\epsilon + 3r\right)}{3r(r\epsilon+1)^3},$$

$$e(r)^3 = e_0^3 \left[\frac{(2r\epsilon(3r\epsilon+2)+1)}{(r\epsilon+1)^4} - \frac{M_0 r^3 \epsilon^2 (r\epsilon+4)}{4Q_0^2(r\epsilon+1)^4}\right].$$

In the limit $\epsilon \to 0$ we recover the classical results

$$\lim_{\epsilon\to 0} G(r) = G_0,$$

$$\lim_{\epsilon\to 0} E(r) = \frac{Q_0}{r^2},$$

$$\lim_{\epsilon\to 0} f(r) = \frac{4G_0 Q_0^2}{3r} - G_0 M_0,$$

$$\lim_{\epsilon\to 0} e(r)^3 = e_0^3.$$

...as expected!!

$(2+1)$ Scale dependent Gravity + NLED source

Ricci scalar

$$R \approx -4G_0\epsilon\left[\frac{M_0 + 4Q_0^2\epsilon}{r}\right] + \mathcal{O}(r).$$

Kretschmann scalar

$$\mathcal{K} \approx \frac{32G_0^2Q_0^4}{3r^6}\left[1 - \left(\frac{M_0}{Q_0^2}\epsilon + 4\epsilon^2\right)r^2 + \mathcal{O}(r^{-3})\ \epsilon\right],$$

....the singularity persist at $r = 0$!!

(2 + 1) Scale dependent Gravity + NLED source

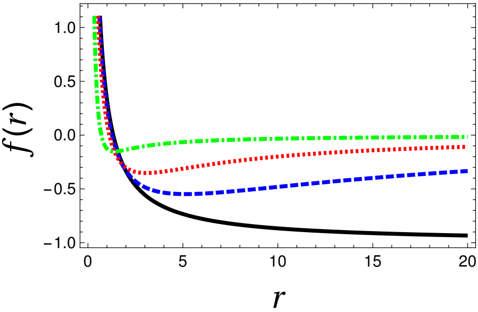


Figure: Lapse function $f(r)$ for $\epsilon = 0.00$ (black solid line), $\epsilon = 0.04$ (blue dashed line), $\epsilon = 0.15$ (dotted red line) and $\epsilon = 1.00$ (dotted dashed green line). The values of the rest of the parameters have been taken as unity.

$(2+1)$ Scale dependent Gravity + NLED source

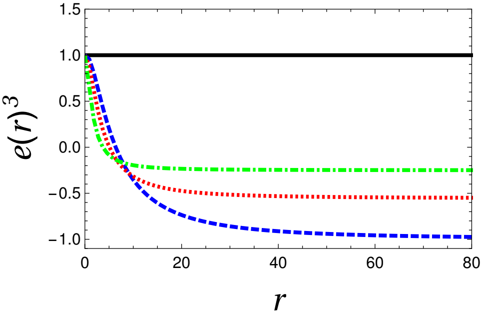


Figure: Electromagnetic coupling $e(r)^3$ for $\epsilon = 0.00$ (black solid line), $\epsilon = 0.25$ (dashed blue line), $\epsilon = 0.45$ (dotted red line) and $\epsilon = 1.00$ (dotted dashed green line). The values of the rest of the parameters have been taken as unity.

$$\lim_{r\to\infty} e(r)^3 \propto -\frac{e_0^3}{\epsilon}$$

(2 + 1) Scale dependent Gravity + NLED source

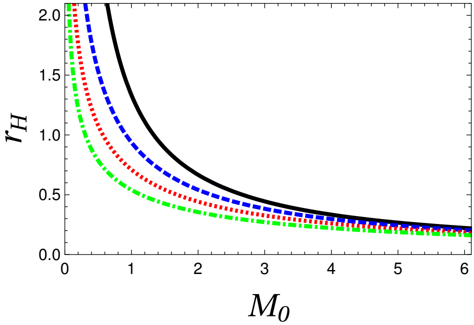


Figure: Black hole horizons r_H as a function of the mass M_0 for $\epsilon = 0.00$ (black solid line), $\epsilon = 0.40$ (blue dashed line), $\epsilon = 1.00$ (dotted red line) and $\epsilon = 2.00$ (dotted dashed green line). The values of the rest of the parameters have been taken as unity.

$(2+1)$ Scale dependent Gravity + NLED source

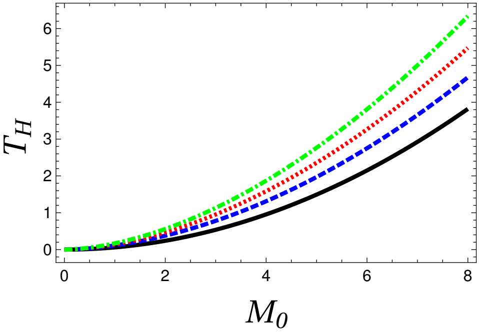


Figure: Hawking temperature T_H as a function of the classical mass M_0 for $\epsilon = 0.00$ (black solid line), $\epsilon = 20.00$ (blue dashed line), $\epsilon = 50.00$ (dotted red line) and $\epsilon = 100.00$ (dotted dashed green line). The values of the rest of the parameters have been taken as unity.

$(2+1)$ Scale dependent Gravity + NLED source

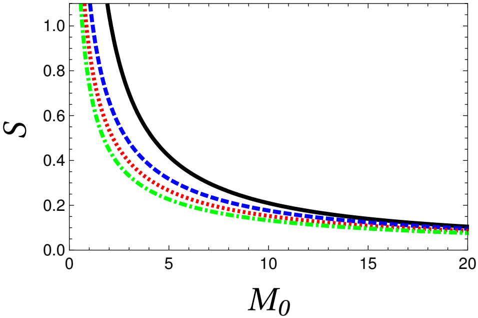


Figure: The Bekenstein-Hawking entropy S as a function of the classical mass M_0 for $\epsilon = 0.00$ (black solid line), $\epsilon = 20.00$ (blue dashed line), $\epsilon = 50.00$ (dotted red line) and $\epsilon = 100.00$ (dotted dashed green line). The other values have been taken as unity.

$(2+1)$ Scale dependent Gravity + NLED source

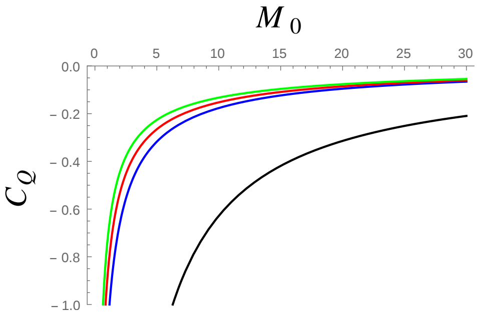


Figure: The Heat Capacity as a function of the classical mass M_0 for $\epsilon = 0.00$ (black solid line), $\epsilon = 20.00$ (blue line), $\epsilon = 50.00$ (red line) and $\epsilon = 100.00$ (green line). The other values have been taken as unity.

Summary

- An exact solution for scale dependent gravity coupled to a power Maxwell source was obtained.
- Scale dependent gravitational couplings induce non trivial deviations from classical Black Holes solutions.
- The singularity at $r = 0$ and the non-stability of the solutions persists.

...Gracias!