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Chiral Effects in gauges theories

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QCD has infinite vacuum states with the same energy
Topologically non equivalents

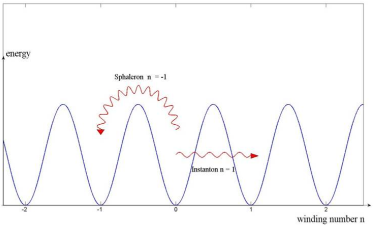
QCD axial anomaly

$$\Downarrow$$

$$N_L - N_R = 2nN_f, \;\; N_f = 6$$

$$\Downarrow$$

The axial charge conservation is violated



Instantons $\Leftrightarrow$ tunneling transitions

Sphalerons $\Leftrightarrow$ non tunneling transitions

QCD vacuum state $\rightarrow$ linear combination

$$|\theta> = \sum_n e^{in\theta} |n>$$

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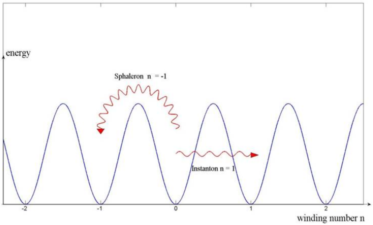

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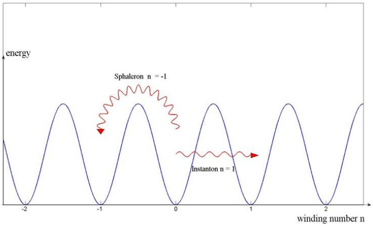

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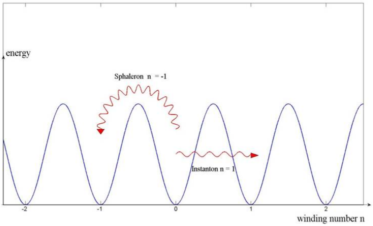

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QCD vacuum state $\rightarrow$ linear combination

$$|\theta >= \sum_n e^{in\theta}|n >$$

QCD special fields connect different topologic vacua $\leftrightarrow$ with different axial charges

$\Rightarrow$ conexion between the axial anomaly and QCD vacuum

$|\theta>$ can be reproduced if the term:

$$L_\theta = \frac{\alpha_s \theta}{8\pi} G^a_{\mu\nu} \tilde{G}^{\mu\nu}_a$$

is added to QCD Lagrangian in the Minkoswski space.

- $\alpha_s \rightarrow$ QCD coupling constant.
- $G^a_{\mu\nu} \rightarrow$ field tensor of gluonic force and $\tilde{G}^{\mu\nu}_a$ is its dual tensor.

!!!This term violates the P-and CP-invariance!!! $\Leftrightarrow$ Strong CP Problem

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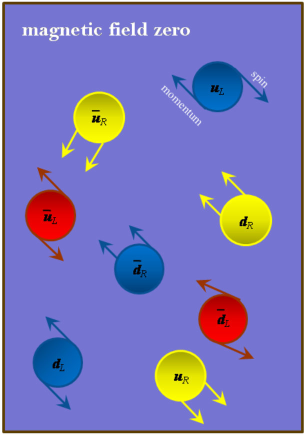
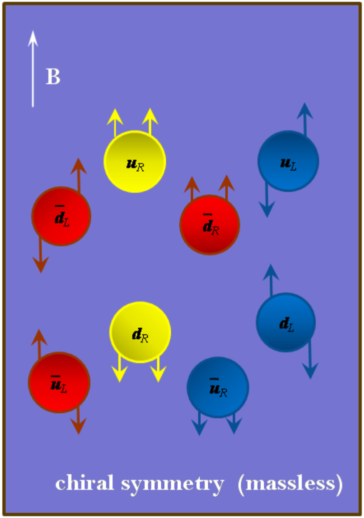
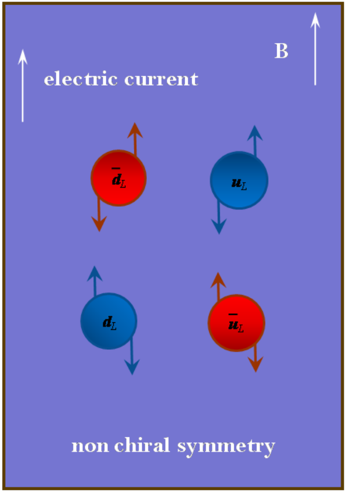


- No magnetic field $\Rightarrow$ No polarization.



- A field **B** will align the spins, depending on their electric charges
- R-helicity quark will has momentum opposite to a L-helicity one
- !!! A field **B** can distinguish between R and L!!!
- !!! An electric current along **B** is imposible



- What does a field **B** do with chirality?

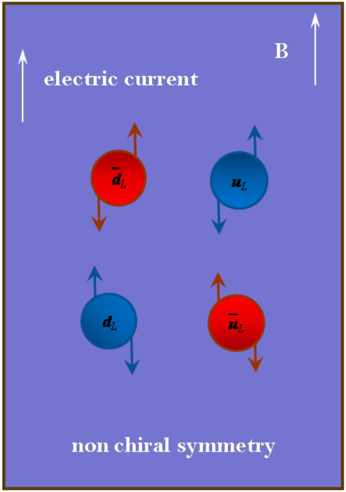
- e^+ move parallel to **B**
- e^- move antiparallel to **B**

- !!An electric current is created along **B** !!$\Leftrightarrow$ The Chiral Magnetic Effect in QCD

Electric current

$$\mathbf{j} = \frac{e^2}{2\pi^2}\mu_5 \mathbf{B}$$

!!!The chiral chemical potential μ_5 isn't well defined!!!



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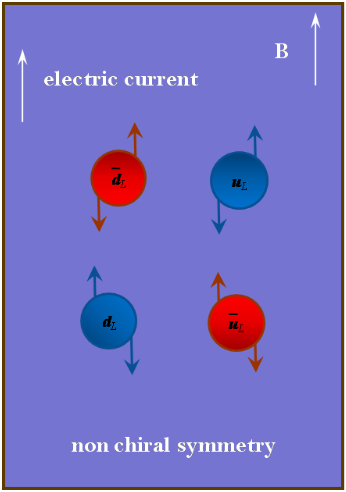

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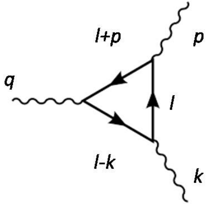

QED Axial anomaly

$$\partial_\mu j_A^\mu = -\frac{e^2}{16\pi^2}\tilde{F}^{\mu\nu}F_{\mu\nu} \neq 0$$

Axial anomaly in terms of the electric E and magnetic B fields

$$\partial_\mu j_A^\mu = \frac{e^2}{2\pi^2}\mathbf{E}\cdot\mathbf{B} \neq 0$$

Electromagnetic decay ($\pi^0 \to \gamma\gamma$)

$$\Gamma(\pi^0 \to 2\gamma) = \frac{\alpha_s^2}{32\pi^3}\frac{m_{\pi^0}^3}{f_\pi^2}$$

Motivation

The chiral effects have important applications in:

- Heavy-ion collisions experiments
- Dirac semimetals and graphene in magnetic fields
- Compact objets in astrophysics scenarios

The chiral effects are related with:

- Axial anomalies in quantum field theory
- QCD vacuum structure
- The Strong CP Problem
- The topological mass generation in Chern-Simons theories

Overview

- The results are based on the QFT formalism at finite temperature and density (massive fermions).
- We obtain a chiral current generation[1] in QED by longitudinal photons in a magnetized medium (Chiral magnetic effect).
- We introduce only an electromagnetic chemical potential-μ and not a μ_5.
- An anomaly relation for the axial current in a magnetized medium is found, $\rightarrow$ analogy to the Adler-Bell-Jackiw anomaly.
- We obtain an useful expression associated to the chiral asymmetry due to pair creation in a magnetized medium.
- In the static limit, an electric pseudovector current is obtained.
- We'll discuss the μ_5-term in the frame of electroweak theory.

1- J.L. Acosta Avalo, H. Pérez Rojas, Nucl Phys. B 909 (2016) 230-242.

We consider an electron-positron plasma in an external field **B**

Electrons and positrons move in bound states characterized by energy levels:

$$\varepsilon_{n_l,p_3} = \sqrt{p_3^2 + m^2 + |e|B(2n_l + 1 - sgn(e)s_3)}$$

- $s_3 = \pm 1 \rightarrow$ spin eigenvalues along x_3
- $n_l = 0, 1, ...$ are the Landau quantum numbers
- for $n_l = 0$, $s_3 = -1$ for electrons and $s_3 = 1$ for positrons

If we will make here the fundamental assumption: $2eB \gg \mu^2, T^2$
$\Rightarrow$ only the ground state LLL dominant

In equilibrium at temperature T and chemical potential μ

Net density of charged particles in the LLL

$$N_0 = \frac{eB}{2\pi^2}[\int_{-\infty}^{0} dp_3(n_R^e - n_L^p) + \int_0^{\infty} dp_3(n_L^e - n_R^p)]$$

Magnetization in the LLL

$$\mathcal{M}_0 = \frac{e}{4\pi^2}[\int_{-\infty}^{0} \frac{p_3^2 dp_3}{\varepsilon_0}(n_R^e + n_L^p) + \int_0^{\infty} \frac{p_3^2 dp_3}{\varepsilon_0}(n_L^e + n_R^p)]$$

$n^{e,p} = [1 + e^{(\varepsilon_0 \mp \mu)/T}]^{-1}$ are the densities(T in energy units)

- Exchange $p_3 \leftrightarrow -p_3 \Rightarrow$!!! equal densities with L,R helicities in the state of equilibrium!!!

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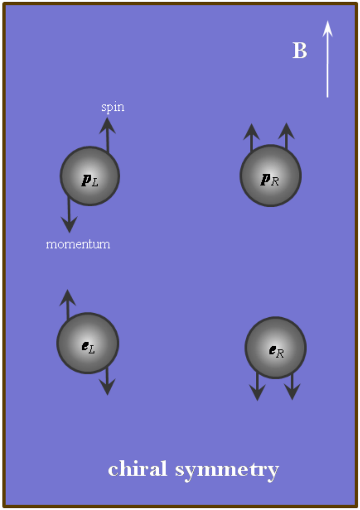

- $\Rightarrow$ there is not electric charges separation
- The magnetic moments are aligned along **B**, $\rightarrow$ paramagnetic behavior
- To higher Landau quantum numbers contribute $\rightarrow$ paramagnetic and diamagnetic terms

The equation of Schwinger-Dyson for the photon:

$$[k^2 g_{\mu\nu} - \Pi_{\mu\nu}(k|A_\mu^{ext})]A^\nu(k) = 0$$

- $A_\mu \rightarrow$ (*radiation field*) is a small perturbation added to A_μ^{ext} (*external field*)
- $A_\mu^{ext} + A_\mu \rightarrow$ the total external electromagnetic field

The quantum corrections are given for the tensor of polarization $\Pi_{\mu\nu}$

In a magnetized medium, for propagation along **B**, they are:

- Two transverse modes dependent of the C-symmetry-(Quantum Faraday Effect)
- A longitudinal mode independent of the C-symmetry-(Chiral Magnetic Effect)

For each mode it's obtained a dispersion law

$$k^2 = \eta_i(k_3, k_\perp, \omega, B), \qquad \Pi_{\mu\nu} b^{\nu(i)} = \eta_i b_\mu^{(i)}$$

$k^2 = k_3^2 + k_\perp^2 - \omega^2$, k_3 and $k_\perp$ are respectively the components of the photon four-momentum in directions $\|$ and $\perp$ to **B**, and ω its energy

The longitudinal mode is a pseudovector given by:

$$b_\mu^{(2)}(k) = Ac_\mu^{(2)}(k)$$

- $c_\mu^{(2)} = R_2(\tilde{F}k)_\mu \rightarrow$ pseudovector
- $R_2 = 1/B\sqrt{z_1}, z_1 = k_3^2 - \omega^2$ (normalization constant)
- $A \rightarrow$ parameter (in potential vector units)
- $\tilde{F}_{\mu\nu} \rightarrow$ the dual of the electromagnetic field tensor $F_{\mu\nu}$

The electric pseudovector associated to $b_\mu^{(2)}$ is:

$$\mathbf{E}_B = \mathbf{E}^{(2)}\mathbf{e}_B = A(k_3^2 - \omega^2)^{\frac{1}{2}}\mathbf{e}_B$$

!!! The longitudinal photon is not on the light cone, that is $k_3^2 - \omega^2 \neq 0$!!!

The electromagnetic current as a function of $A_\mu^{ext} + A_\mu$ depends on the two relativistic invariants:

$$\mathfrak{F} = \frac{1}{4} F^{\mu\nu} F_{\mu\nu} = \frac{1}{2}(B^2 - E^2) \simeq \frac{1}{2} B^2, \quad E \ll B$$

$$\mathfrak{G} = \frac{1}{4} \tilde{F}^{\mu\nu} F_{\mu\nu} = \mathbf{E} \cdot \mathbf{B}$$

The pseudoscalar $\mathfrak{G} \neq 0$ only for the longitudinal mode

An expansion in functional series gives:

$$j_\mu(A_\mu^{ext} + A_\mu) = j_\mu(A_\mu^{ext}) + (\delta j_\mu / \delta A_\nu^{ext}) A_\nu + \ldots$$

Its linear term in A_ν is:

$$j_i = \Pi_{i\nu} A_\nu = Y_{ij} E_j, \quad \nu = 1,2,3,4, \quad i,j = 1,2,3$$

- $E_j = i(\omega A_j - k_j A_0)$ is the electric field, with $A_4 = iA_0$ and $k_4 = i\omega$
- $j_\mu(A_\mu^{ext}) = N_0 \delta_{\mu 4}$

The complex conductivity tensor or admittivity is:

$$Y_{ij} = \Pi_{ij}/i\omega$$

$A_\mu \rightarrow$ is a linear combination of the eigenmodes $b_\mu^{(i)}$

If we will consider $A_\mu = b_\mu^{(2)}$ $(\mathbf{E} \parallel \mathbf{B}) \Rightarrow$ problem in $(1+1)$ dimensions

This is strictly valid if we consider only the LLL

We will use the two-dimensional identity of Dirac matrices

$$\gamma^\mu \gamma^5 = -\epsilon^{\mu\nu} \gamma_\nu$$

We can study the properties of the axial vector current by using results already derived for the vector current

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We must observe that from the linear approximation of j_i, and the eigenvalue equation one gets also:

$$j_i = \Pi_{i\nu} A^\nu = s b_i^{(2)}$$

- $s = c_\nu^{(2)} \Pi^\nu_\rho c^{\rho(2)} \rightarrow$ is the eigenvalue of $\Pi_{\mu\nu}$ corresponding to the longitudinal mode
- !!!! $b_\nu^{(2)}$ is a pseudovector !!!! $\Rightarrow$!!! the current j_ν is also a pseudovector!!!
- This is necessary for the breaking of chiral symmetry

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Using the two-dimensional transversality condition $\boxed{\Pi_{\mu\nu}k_{\nu} = 0}$, we obtain:

The non-conservation of the two-dimensional axial current:

$$k_{\mu}j^{\mu}_{A} = \frac{z_1}{k_4} j_3 \neq 0$$

Electric pseudovector breaks the chiral symmetry in both the C-symmetric and non-symmetric cases

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A chiral magnetic effect is produced in the frame of QED

We are interested only in the region of real frequency $k_3^2 > z_1$ and momentum

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The current density can be written in the general form as:

$$j_i = \sigma^0_{ij} E_j + (E \times S)_i$$

$$\sigma^0_{ij} = Im[\Pi^s_{ij}]/\omega, \quad S_i = \frac{1}{2}\epsilon^{ijk}\sigma^H_{jk}$$

$$\sigma^H_{jk} = Im[\Pi^A_{ij}]/\omega$$

- $\epsilon^{ijk} \rightarrow$ is the third rank antisymmetric unit tensor.
- $\Pi^s_{ij}, \Pi^A_{ij} \rightarrow$ are the symmetric and antisymmetric parts of $\Pi_{\mu\nu}$.

The first term of the current density corresponds to the Ohm current and the second is the Hall current.

Here we'll only work with the current associated to the longitudinal mode.

The current density associated to the longitudinal mode can be expressed in the form:

$$j_3 = \sigma^0_{33} E_3$$

$$\sigma^0_{33} = Im[\Pi_{33}]/\omega = -\omega Im[s]/z_1$$

!!! σ^0_{33} is the chiral conductivity!!!

Scalar s in the one-loop approximation is given by:

$$s = -\frac{e^3 B}{\pi^2} \sum_{n=0}^{\infty} \int_{-\infty}^{\infty} \frac{dp_3}{\varepsilon_q} \left[\frac{\alpha_n \varepsilon_{n,0}^2 (2p_3 k_3 + z_1)}{4z_1 p_3^2 + 4p_3 k_3 z_1 + z_1^2 - 4\omega^2 \varepsilon_{n,0}^2}\right]$$
$$\cdot [n^p(\varepsilon_q) + n^e(\varepsilon_q) - 1]$$

$$\varepsilon_{n,0} = \sqrt{m^2 + 2eBn}, \qquad n = n_l + 1/2 + s_3/2$$
$$\alpha_n = 2 - \delta_{n,0}, \qquad q = (n, p_3)$$

The imaginary part of the scalar s is given by:

$$Im[s] = -\frac{e^3 B}{2\pi} \sum_{n=0}^{\infty} \alpha_n \varepsilon_{n,0}^2 S_n$$

The chiral conductivity at finite temperature T and density μ is given by:

$$\sigma_{33}^{0} = \frac{e^3 B\omega}{2\pi z_1} \sum_{n=0}^{\infty} \alpha_n \varepsilon_{n,0}^2 S_n$$

$$S_n = \{\theta(z_1)\Delta N + \theta(-4\varepsilon_{n,0}^2 - z_1)\Delta H\}/\Lambda$$

$$\Lambda = \sqrt{z_1(z_1 + 4\varepsilon_{n,0}^2)}$$

!!! Scattering or pair creation contribute $\Rightarrow$ chiral conductivity !!!

The chiral conductivity contains both diamagnetic and paramagnetic terms

$$\Delta N = [N(\varepsilon_r) - N(\varepsilon_r + \omega)]$$

ΔN accounts for the excitation of particles $[\varepsilon(p_3, n) \longrightarrow \varepsilon(p_3 + k_3, n)]$ by increasing their momentum along $\mathbf{B}$ (only for $z_1 > 0$).

$$\Delta H = [H(-\varepsilon_s) + H(\omega + \varepsilon_s) - 2]$$

ΔH accounts for the pair creation (only in the region $z_1 < -4\varepsilon^2_{n,0}$).

$$N = n^e(\varepsilon_r) + n^p(\varepsilon_r), \quad H = n^e(\varepsilon_s) + n^p(\omega - \varepsilon_s)$$

$$\varepsilon_s = (\omega z_1 + |k_3|\Lambda)/2z_1, \quad \varepsilon_r = (-\omega z_1 + |k_3|\Lambda)/2z_1, \quad r, s = (n, \omega, k_3)$$

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For magnetic fields very strong, such that: $2eB \gg \mu^2, T^2 \Rightarrow$ the LLL dominant.

Chiral conductivity in the LLL

$$\sigma_{33}^0 = \frac{e^3 B\omega}{2\pi z_1} m^2 S_0$$

$$S_0 = \frac{\theta_1 \Delta N_R + \theta_2 \Delta H_R}{\sqrt{z_1(z_1 + 4m^2)}}$$

$$\theta_1 = \theta(z_1), \qquad \theta_2 = \theta(-4\varepsilon_{n,0}^2 - z_1)$$

In the low frequency limit $\omega \to 0 \Rightarrow |k_3| \gg \omega$

We have conditions very close to the static electric field case.

Chiral current is given by:

$$j_3 = \frac{e^3}{8\pi}\frac{m^2}{|k_3|\lambda}\frac{1}{T}\left[\frac{1}{1+\cosh(\frac{\lambda-\mu}{T})}+\frac{1}{1+\cosh(\frac{\lambda+\mu}{T})}\right](\mathbf{E}^{(2)}\cdot\mathbf{B})$$

- $\mathbf{E}^{(2)} = A|k_3|\mathbf{e}_3$
- $\lambda = (\sqrt{k_3^2+4m^2})/2$

!!!Notice that $j_3 \neq 0$ for $\mu = 0$!!!

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Taking into account the expression for the four-divergence of j_A^μ and the equation for σ_{33}^0 it is obtained:

Anomaly relation for j_A^μ in a medium of massive particles

$$k_\mu j_A^\mu = \beta[m\mathbb{A}(m) + \mathbb{C}(m)]\frac{e^2}{2\pi^2}\mathbf{E}\cdot\mathbf{B}$$

$$\mathbb{A}(m) = (2\pi m/e)\sum_{n=0}^{\infty}\alpha_n S_n$$

$$\mathbb{C}(m) = 8\pi B\sum_{n=1}^{\infty} n S_n$$

If we consider $z_1 < -4m^2$, only the LLL.

Pair creation contribution to the non conservation of j_A^μ

$$k_\mu j_A^\mu = \beta \frac{m^2}{\sqrt{z_1(z_1+4m^2)}} \; \Delta H_R \; (\mathbf{E}^{(2)} \cdot \mathbf{B}).$$

!!!The pair creation can generate a chiral asymmetry in a magnetized medium!!!

We consider the Weinberg-Salam model including leptons and quarks

The electric and weak neutral charges are:

$$Q^e = e[P^i_\nu \epsilon^{i3k} W^k_\nu + \frac{i}{2} P_{\psi_L}(\tau^3 - I)\psi_L - iP_{e_R} e_R + \frac{i}{2} P_{Q_L}(\tau^3 + \frac{I}{3})Q_L + \frac{2i}{3}$$

$$P_{u_R} u_R - \tfrac{i}{3} P_{d_R} d_R - iP_{\phi^\dagger}[\tfrac{1}{2}(\tau^3 + I)\phi^\dagger] + i[\tfrac{1}{2}(\tau^3 + I)\phi]P_\phi]$$

$$Q^N = \frac{1}{\sqrt{g^2 + g'^2}}[-g^2 P^i_\nu \epsilon^{ij3} W^j_\nu - \frac{i}{2} P_{\psi_L}(g^2\tau^3 + g'^2 I)\psi_L - ig'^2 P_{e_R}$$

$$e_R - \tfrac{i}{2} P_{Q_L}(g^2\tau^3 - g'^2 \tfrac{I}{3})Q_L + \tfrac{2i}{3} g'^2 P_{u_R} u_R - \tfrac{i}{3} g'^2 P_{d_R} d_R + iP_{\phi^\dagger}[\tfrac{1}{2}(g^2\tau^3$$

$$-g'^2 I)\phi^\dagger] - i[\tfrac{1}{2}(g^2\tau^3 - g'^2 I)\phi]P_\phi]$$

Where the conjugated momenta are:

$$P^i_\mu = iG^i_{4\mu}, \quad P^B_\mu = iF^i_{4\mu}, \quad P_{\psi_L} = i\bar{\psi}_L\gamma_0, \quad P_{e_R} = i\bar{e}_R\gamma_0,$$

$P_{Q_L} = i\bar{Q}_L\gamma_0, \quad P_{u_R} = i\bar{Q}_R\gamma_0, \quad P_{d_R} = i\bar{d}_R\gamma_0, \quad P_\phi = i(\partial_4 + i\frac{g}{2}\tau^i$
$W^i_4 + i\frac{g'}{2}B_4)\phi^\dagger, \quad P_{\phi^\dagger} = i(\partial_4 - i\frac{g}{2}\tau^i W^i_4 - i\frac{g'}{2}B_4)\phi, \quad P_{\bar{\psi}_L} = 0$
$P_{\bar{Q}_L} = 0, \quad P_{\bar{d}_R} = 0, \quad P_{\bar{e}_R} = 0, \quad P_{\bar{u}_R} = 0$

The field tensor of the $SU(2)$ non-abelian field

$$G^i_{\mu\nu} = \partial_\mu W^i_\nu - \partial_\nu W^i_\mu + g\epsilon^{ijk} W^i_\mu W^i_\nu$$

The abelian gauge field tensor

$$F_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$$

Starting from density matrix, we may get

The partition funtional

$$Z = N(\beta)\int \prod_{l,m} DP_B^m DP_{\bar\psi}^l DP_\psi^l DB^m D\bar\psi^l D\psi^l \prod_n \delta(C^n)\times \prod_{j=0}^{3}\delta(G_j)\times$$

$$\times(Det\mathbb{M})\times e^{\int_0^\beta dx_4 \int d^3x[iP^{B^l}\dot{B}^l + iP_\psi^m \dot\psi^m - \mathbb{H} + \sum \mu_i N_i]}$$

- $N(\beta)$ → temperature dependent constant
- B^l → bosons
- ψ^l → fermions
- $\delta(C^n)$ → delta functions for the constraints (momenta)
- $\delta(G_j)$ → delta functions for the gauge conditions
- $Det\mathbb{M}$ → Faddev-Popov determinant
- $\mathbb{H}$ → Hamiltonian

where

- $N_1 = Q^e/e \rightarrow$ charged particles density
- $N_2 = N^l = -i(P_{\psi_L}\psi_L + P_{e_R}e_R) \rightarrow$ lepton number density
- $N_3 = Q^N/cot2\theta \rightarrow$ neutral charge density, $\quad tan\theta = g'/g$
- $N_4 = N^Q = -i(P_{Q_L}Q_L + P_{u_R}u_R + P_{d_R}d_R) \rightarrow$ quark density

Chemical equilibrium equations

$$\mu_{e_R} - \mu_{e_L} = \mu_3\Lambda(g,g'), \quad \mu_{u_R} - \mu_{u_L} = \mu_3\Lambda(g,g')$$

$$\mu_{d_R} - \mu_{d_L} = \mu_3\Lambda(g,g'), \quad \Lambda(g,g') = \left(\frac{g^2+g'^2}{g^2-g'^2}\right)$$

chemical potential associated to the Higgs scalar

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- If $\mu_3 = 0$ for $T < T_c$ (symmetry restoration temperature) $\rightarrow$ gauge invariance plus symmetry breaking would lead to the validity of the Higgs mechanism
- If $\mu_3 \neq 0 \rightarrow$ Higgs field σ and the Goldstone boson h^3 would be weakly-neutrally charged, but h^3 can be eliminated from the theory by taking the unitary gauge

Scalar field $\rightarrow \phi = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ \xi \end{pmatrix} + \begin{pmatrix} ih^1 + h^2 \\ \sigma + ih^3 \end{pmatrix},$

$\xi \neq 0$ is the symmetry breakdown parameter

$\Rightarrow$ a non-conservation of the weak neutral charge due to Higgs mechanism !! contradiction !!

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$$\mu_5 = \mu_R - \mu_L$$

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- In an Electroweak plasma $\rightarrow$ there is a relation between the neutral chemical potential-μ_3 associated to the weak-neutral charge-Q^N, μ_σ and μ_5

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  - non-conservation of the weak-neutral charge due to Higgs mechanism for $T < T_c$
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Conclusions:

- We conclude, that as a consequence of:
  - The breaking of spacial symmetry due to **B**
  - The breaking of chiral symmetry due to **B** and **E**

Chiral effects are generated as:

- Quantum Faraday Effect
- Chiral Magnetic Effect

The main results are:

- We obtained the chiral current generation in QED by longitudinal photons in a magnetized medium

!!!Chiral Magnetic Effect in QED!!!

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As difference of Chiral Magnetic Effect discussed in the literature:

- We didn't need to introduce a μ_5
  - In QCD: The imbalanced chirality is associated to $\rightarrow$ winding number $\rightarrow$ Instantons-Sphalerons
  - In our case: The imbalanced chirality is associated to $\rightarrow$ longitudinal pseudovector photons
- An electromagnetic chemical potential μ was introduced
  - But the electric current reported is C-symmetry
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Conclusions:

- The chiral conductivity was calculated at finite temperature and density

  It is due to:

  - Electrons and positrons scattered by longitudinal photons(inside the light cone)
  - The pair creation due to longitudinal photons (out of light cone)

  For fixed values $(k_3, \omega) \rightarrow$ only one of these process contribute

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- In the QFT formalism at finite temperature and density.
  - We obtained an anomaly relation for the axial current in a magnetized medium Analogy to the Adler-Bell-Jackiw relation
- This means the possibility of longitudinal photon splitting in two transverse ones in a magnetized medium
- An expression for the chiral current in terms of photon self-energy tensor was given. A similar expression could be found for a quark-antiquark gas, if we consider QCD coupled to electromagnetism
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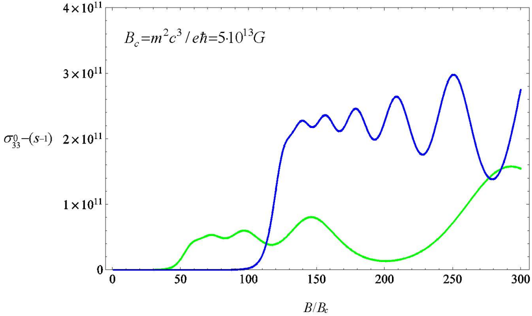
.......Thanks.......



- $\hbar c k_3 = 1\ MeV$
- $\hbar\omega = 10\ eV$
- Blue curve: $\mu = 25\ MeV$ $(n = 10)$
- Green curve: $\mu = 12\ MeV$ $(n = 5)$

If we consider:

- The C-non symmetric case.
- Low temperatures (positron contribution is negligible).
- $mc \gg p_3$, con $\mu \gtrsim mc^2$

Chiral current in the non relativistic limit

$$\mathbf{j}_3 = \frac{\alpha e}{16\pi} \frac{mc^2}{|p_3|} \frac{e^{-|\frac{P_3^2}{2m} - \mu_0|/T}}{T} (\mathbf{E}^{(2)} \cdot \mathbf{B}) \mathbf{e}_3$$

- About the relation

$$\langle j_A^\mu \rangle = -\epsilon^{\mu\nu} \langle j_\nu^V \rangle \quad \Leftrightarrow \quad \gamma^\mu \gamma^5 = -\epsilon^{\mu\nu} \gamma_\nu$$

- We can use the above relation due to:

1. Electric field associated to longitudinal mode is $\parallel$ **B**.
2. In the LLL $\rightarrow n = 0 \Leftrightarrow p_\perp \equiv 0$.
3. Condition $eB \gg T^2, \mu^2 \Rightarrow$ confine the system to the LLL.

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- About the 2-dimension nature of LLL.

Dirac equation

$$[\gamma^\nu(\partial_\nu + ieA_\nu^{ext}) + m]G(x, x'|A^{ext}) = \delta(x - x')$$

Analytic prolongation $(p_0 \to -ip_4 + \mu)$ of the time Fourier transformation of $G(x, x'|A^{ext})$

$$G(p_4, \mathbf{x}, \mathbf{x}'|A^{ext}) = -\frac{1}{2\pi^2}\sum_{p_4}\int dp_2 dp_3[(p_4 + i\mu)^2 + \varepsilon_q^2]^{-1}$$

$$\times M(p_3, p_4, n, \zeta, \zeta')e^{i[p_2(x_2 - x_2') + p_3(x_3 - x_3')]}$$

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Where the matrix $M(p_3, p_4, n, \zeta, \zeta')$ is:

$$\begin{pmatrix} H_{n-1,n-1}(x) & 0 & -D_{n-1,n-1} & -E_{n-1,n} \\ 0 & H_{n,n}(x) & -E_{n-1,n} & D_{n,n} \\ D_{n-1,n-1} & -E_{n-1,n} & H_{n-1,n-1}(-x) & 0 \\ -E_{n-1,n} & -D_{n,n} & 0 & H_{n,n}(-x) \end{pmatrix}$$

$$E_{k,k'} = \mp i(2neB)^{\frac{1}{2}}\phi_k(\zeta)\phi_{k'}(\zeta')$$

$$D_{k,k'} = \pm p_3\phi_k(\zeta)\phi_{k'}(\zeta')$$

$$H_{k,k'}(x) = H_{k,k'}(ip_4 - \mu) = (m + ip_4 - \mu)\phi_k(\zeta)\phi_{k'}(\zeta')$$

Hermite functions multiply by $(eB)^{1/4}$

$$\phi_n(\zeta) = \frac{(eB)^{\frac{1}{4}}}{\pi^{\frac{1}{4}}2^{\frac{n}{2}}(n!)^{\frac{1}{2}}}e^{\frac{\zeta^2}{2}}H_n(\zeta), \quad \zeta = \sqrt{eB}(x_1 + x_0)$$

In the LLL, the matrix $M(p_3, p_4, 0, \zeta) =$

$$\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & (m - ip_4 + \mu) & 0 & \pm p_3 \\ 0 & 0 & 0 & 0 \\ 0 & \mp p_3 & 0 & (m + ip_4 - \mu) \end{pmatrix}$$

$$M(p_3, p_4, 0, \zeta)] \quad \Leftrightarrow \quad D_{2\times 2}$$

Con $\mu = 0$

$$det[D] = 0 \quad \Leftrightarrow \quad m^2 + p_3^2 = p_0^2$$

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- Helicity and the direction $\vec{p}$.

Dirac hamiltonian H_D and spin operator $\vec{S}$

$$H_D = \vec{\alpha}\cdot\vec{p} + \beta m, \quad \beta = \gamma^0,\ \vec{\alpha} = \gamma^0\vec{\gamma}$$

$$\vec{S} = \frac{i}{4}\vec{\gamma}\times\vec{\gamma}$$

Commutator between H_D and spin projection in the direction $\vec{n}$

$$[H_D, \vec{S}\cdot\vec{n}] = i(\vec{p}\times\vec{n})\cdot\vec{\alpha}$$

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- $[H_D, \vec{S}\cdot\vec{n}] \neq 0$,otherwise.

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- $[H_D, \vec{S}\cdot\vec{n}] = 0$,if $\vec{p} \parallel \vec{n} \Rightarrow$ Helicity $\Rightarrow$ movement constant .
- $[H_D, \vec{S}\cdot\vec{n}] \neq 0$,otherwise.

- Helicity and the direction $\vec{p}$.

Dirac hamiltonian H_D and spin operator $\vec{S}$

$$H_D = \vec{\alpha}\cdot\vec{p} + \beta m, \quad \beta = \gamma^0,\ \vec{\alpha} = \gamma^0\vec{\gamma}$$

$$\vec{S} = \frac{i}{4}\vec{\gamma}\times\vec{\gamma}$$

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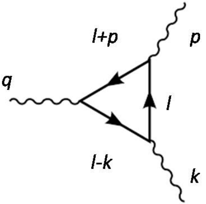
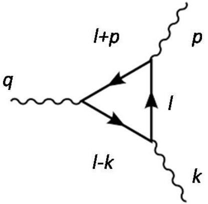


axial anomaly

$$\partial_\mu j^\mu_A = -\frac{e^2}{16\pi^2}\tilde{F}^{\mu\nu}F_{\mu\nu} \neq 0$$

$$\partial_\mu j^\mu_A = \frac{e}{\pi}E \neq 0, \quad \rightarrow \ (1+1) \ \ dimensions$$

$$\partial_\mu j^\mu_A = \frac{e^2}{2\pi^2}\mathbf{E}\cdot\mathbf{B} \neq 0, \quad \rightarrow \ (3+1) \ \ dimensions$$



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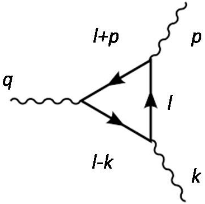

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- Anomaly relation in the limits $\mu \to 0$ and $T \to 0$.

Anomaly relation for j_A^μ in a magnetized medium of massive particles

$$k_\mu j_A^\mu = \beta[m\mathbb{A}(m) + \mathbb{C}(m)]\frac{e^2}{2\pi^2}\mathbf{E}\cdot\mathbf{B}$$

$$\mathbb{A}(m) = (2\pi m/e)\sum_{n=0}^{\infty}\alpha_n S_n, \quad \mathbb{C}(m) = 8\pi B\sum_{n=1}^{\infty} n S_n$$

Dependence with μ and T

$$S_n(\mu,T) = \frac{\{\theta(z_1)\Delta N + \theta(-4\varepsilon_{n,0}^2 - z_1)\Delta H\}}{\Lambda}$$

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