Gravitational and electromagnetic signatures of accretion into a black hole

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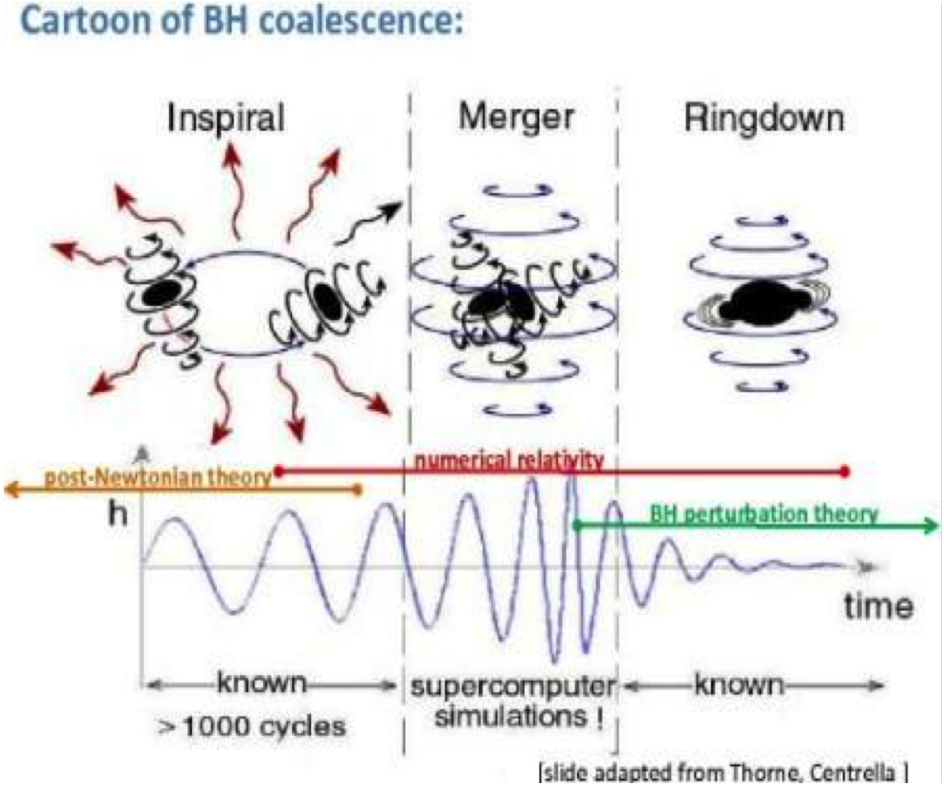
Outline

Gravitational waves phases



We are working in the ringdown phase, we use one black hole and particles falling into the black hole.
Cardoso et. al. (2016) analyze inspiral and ringdown using extremal black hole.

Motivation

- Our goal is to describe the gravitational waves generated by the motion of disks of matter in the background of a black hole using perturbation theory (Teukolsky equation).
- For simplicity we focused on the emission of gravitational waves when a black hole is perturbed by a surrounding pressure-less fluid matter.
- Specifically, we work with the curvature perturbations within the null tetrad formulations developed by Newman and Penrose.
- We analyze Schwarzschild and Reissner-Nordstrom space-time.

Schwarszchild space-time

We obtained the so called Teukolsky equation for vacuum type D spacetimes (Teukolsky '73), and then focused on the Schwarzschild spacetime written in Kerr-Schild coordinates.

$$ds^2 = -\left(1 - \frac{2M}{r}\right) dt^2 + \frac{4M}{r}\, dt\, dr + \left(1 + \frac{2M}{r}\right) dr^2 + r^2\, d\Omega^2,$$

the tetrad components as

$$l^\mu = \left(\frac{1}{2} + \frac{M}{r}, \frac{1}{2} - \frac{M}{r}, 0, 0\right) \ , \quad k^\mu = (1, -1, 0, 0) \ , \quad m^\mu = \frac{1}{r\sqrt{2}}(0, 0, 1, i \csc\theta).$$

The normalization is such that $l^\mu\, k_\mu = -m^\mu\, m^*{}_\mu = 1$. The directional derivatives are $\mathbf{D} = l^\mu\, \partial_\mu$, $\mathbf{\Delta} = \kappa^\mu\, \partial_\mu$ and $\delta = m^\mu\, \partial_\mu$.
From twelve spin coefficients the non-zero for this metric are

$$\mu_s = \frac{1}{r} \ , \quad \rho_s = \frac{r - 2\,M}{2\,r^2} \ , \quad \epsilon_s = -\frac{M}{2\,r^2} \ , \quad \alpha_s = \frac{\cot\theta}{2\sqrt{2}\,r} \ , \quad \beta_s = -\alpha_s \ .$$

The non-vanishing Weyl component $\Psi_2 = \frac{M}{r^3}$, $\quad \Psi_0 = \Psi_1 = \Psi_3 = \Psi_4 = 0$.

Bianchi identities

In order to obtain the perturbed equations, one starts from the Bianchi identities and the definition of the Riemann tensor (Chandrasekhar '83)

$$R_{\mu\nu\lambda\tau\,;\sigma} + R_{\mu\nu\sigma\lambda\,;\tau} + R_{\mu\nu\tau\sigma\,;\lambda} = 0\ , \qquad R_{\sigma\mu\nu\lambda}\, Z_a{}^{\sigma} = Z_{a\,\mu;\nu\lambda} - Z_{a\,\mu;\lambda\nu}.$$

These equations are projected on a null tetrad.
We obtain a master equation for the perturbed scalar (outgoing waves)

$$\Psi_4 \quad = \quad -C_{\mu\nu\lambda\tau}\, k^{\mu}\, m^{*\nu}\, k^{\lambda}\, m^{*\tau},$$

the Teukolsky equation

$$\begin{aligned}&[(\mathbf{\Delta} - 4\,\mu - \mu^* - 3\,\gamma + \gamma^*)\ (\mathbf{D} + \rho - 4\,\epsilon)\\ &- (\delta^* - (3\,\alpha + \beta^* + 4\,\pi - \tau^*))\ (\delta - 4\,\beta + \tau) + 3\Psi_2]\ \Psi_4{}^{(1)} = -4\pi\, T_4,\end{aligned}$$

the source

$$T_4 = \hat{\mathcal{T}}^{k\,k}\, T_{k\,k} + \hat{\mathcal{T}}^{k\,m^*}\, T_{k\,m^*} + \hat{\mathcal{T}}^{m^*\,m^*}\, T_{m^*\,m^*},$$

$T_{kk} = T_{\mu\nu} k^{\mu} k^{\nu}$, $T_{km^*} = T_{\mu\nu} k^{\mu} m^{*\nu}$, $T_{m^*m^*} = T_{\mu\nu} m^{*\mu} m^{*\nu}$.

Maxwell equations

For the electromagnetic case, we use the the Faraday's tensor $F_{\mu\nu}$:

$$\phi_1 \equiv \frac{1}{2} F_{\mu\nu}(l^{*\mu}k^{\nu} + m^{*\mu}k^{\nu}), \qquad \phi_2 \equiv F_{\mu\nu}m^{*\mu}k^{\nu}.$$

$\phi_0 = \phi_1 = 0$ in the Schwarzschild case. Projecting the Maxwell's equations $F^{\mu\nu}{}_{;\nu} = 4\,\pi\,J^{\mu}$ into the null tetrad,

$$\begin{aligned}&[(-\,\mathbf{\Delta} + 2\,\mu + \mu^* + \gamma - \gamma^*)\,(-\,\mathbf{D} - \rho + 2\,\epsilon) - \\ &(-\,\delta^* + \alpha + \beta^* + 2\,\pi - \tau^*)\,(-\,\delta + 2\,\beta - \tau)]\,\phi_2 = 4\,\pi\,J_2\ ,\end{aligned}$$

where

$$J_2 \;=\; (-\mathbf{\Delta} + 2\,\mu + \mu^* + \gamma - \gamma^*)\;J_{m^*} - (-\delta^* + \alpha + \beta^* + 2\,\pi - \tau^*)\;J_k.$$

$$J_k = J_{\mu}k^{\mu}, \qquad J_m = J_{\mu}m^{\mu}$$

are the projections of the current vector J_{μ}.

Matter content

We consider the matter source to be described by a pressure-less fluid

$$T_{\mu\nu} = \rho\, u_\mu\, u_\nu \ ,$$

where ρ is the rest mass density and u_μ is the four velocity of the dust. Furthermore, we consider that the fluid is in-falling radially in the black hole and the four velocity has only temporal and radial components,

$$u^\mu = \left(u^0, u^1, 0, 0\right) \ .$$

The evolution of the fluid is described by the continuity equation for the current vector,

$$J_\mu = \rho\, u_\mu,$$

and the conservation equation for the stress energy tensor:

$$J^\mu{}_{;\mu} = 0 \quad \text{and} \quad T^{\mu\nu}{}_{;\mu} = 0 \ .$$

Spherical harmonics decomposition

With the radial in-falling matter, plus a decomposition of the density in terms of the spherical harmonics with zero weight

$$\rho = \sum_{lm} \rho_{l,m}(t,r)\, {Y_0}^{l,m}(\theta,\phi) \ .$$

it is possible to separate the angular and radial part of the sources. Furthermore, expanding the perturbation of Ψ_4, Ψ_3 as

$$\Psi_4 = \sum_{lm} R^G_{l,m}(t,r)\, {Y_{-2}}^{l,m}(\theta,\phi) \ ,$$
$$\Psi_3 = \sum_{lm} R^E_{l,m}(t,r)\, {Y_{-1}}^{l,m}(\theta,\phi) \ ,$$

we can get a radial-time equation. Here $l = 0, 1, 2$ and $-m < l < m$.
Peeling theorem states that in the asymptotic region, the Weyl scalar behave as $\Psi_i \approx \frac{1}{r^{5-i}}$.
Spin weight $s = 0$ escalar, $s = \pm 1$ electromagnetic, $s = \pm 2$ gravitational.

Schwarzschild space time

Gravitational equation

$$\left[\Box^{\Psi}_{tr} + \frac{1}{r^2}\Box^{-2}_{\theta\,\varphi}\right] r\Psi_4^{(1)} = 16\pi\, r\, T_4 \ ,$$

where the radial-temporal operator is

$$\Box^{\Psi}_{tr} = -\left(1+\frac{2M}{r}\right)\frac{\partial^2}{\partial t^2} + \left(1-\frac{2\,M}{r}\right)\frac{\partial^2}{\partial r^2} + \frac{4M}{r}\frac{\partial^2}{\partial t\partial r} + 2\left(\frac{2}{r}+\frac{M}{r^2}\right)\frac{\partial}{\partial t} + 2\left(\frac{2}{r}-\frac{M}{r^2}\right)\frac{\partial}{\partial r} + 2\frac{M}{r^3},$$

$$\Box^{-2}_{\theta\varphi} \quad = \quad \frac{\partial^2}{\partial\theta^2} + \frac{1}{\sin^2\theta}\frac{\partial^2}{\partial\varphi^2} + \cot\theta\,\frac{\partial}{\partial\theta} - 4\,i\,\frac{\cos\theta}{\sin^2\theta}\frac{\partial}{\partial\varphi} - 2\,\frac{1+\cos^2\theta}{\sin^2\theta}\ .$$

Electromagnetic case

$$\left[\Box^{\phi}_{tr} + \frac{1}{r^2}\Box^{-1}_{\theta\,\varphi}\right] r\phi_2 = -8\pi r J_2,$$

where the radial-temporal operator

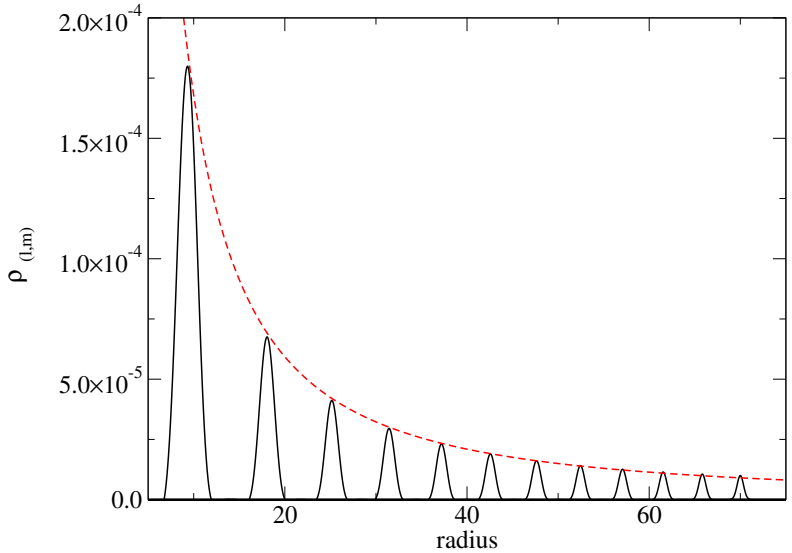
$$\Box^{\phi}_{tr} = -\left(1+\frac{2\,M}{r}\right)\frac{\partial^2}{\partial t^2} + \left(1-\frac{2M}{r}\right)\frac{\partial^2}{\partial r^2} + \frac{4\,M}{r}\frac{\partial^2}{\partial t\partial r} + \frac{2}{r}\frac{\partial}{\partial t} + \frac{2}{r}\frac{\partial}{\partial r}\ ,$$

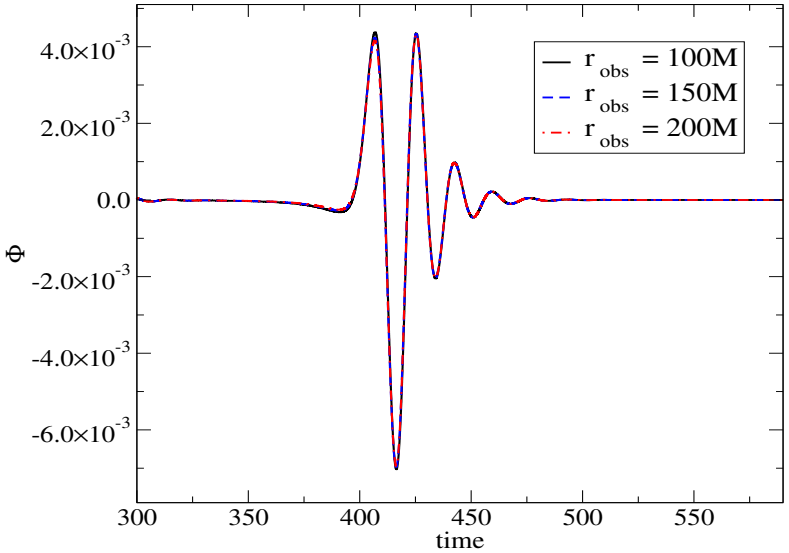
$$\Box^{-1}_{\theta\varphi} \quad = \quad \frac{\partial^2}{\partial\theta^2} + \frac{1}{\sin^2\theta}\frac{\partial^2}{\partial\varphi^2} + \cot\theta\,\frac{\partial}{\partial\theta} - 2\,i\,\frac{\cos\theta}{\sin^2\theta}\frac{\partial}{\partial\varphi} - \frac{1}{\sin^2\theta}.$$

Numerical implementation

The resulting second order equation can be reduced to a first order system. We parametrize the shells of matter using a Gaussian pulse

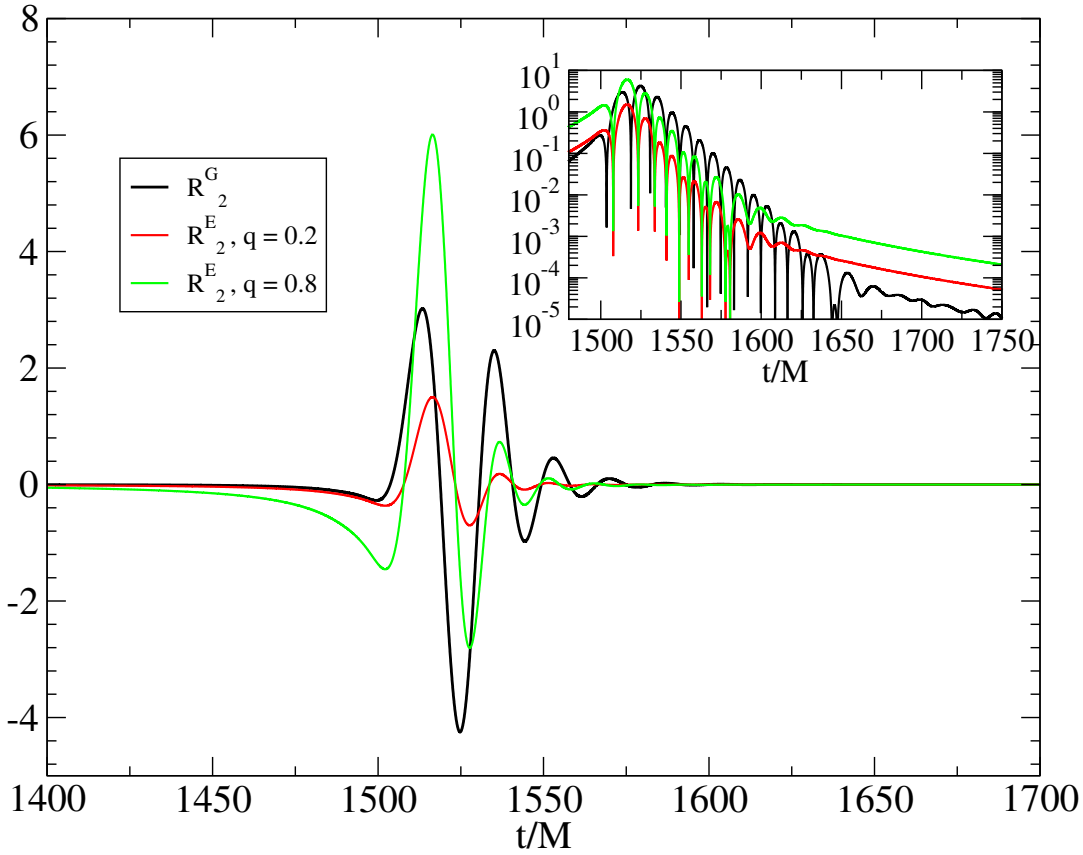
$$\rho_{l,m}(r, t = 0) = A_0\, e^{-(r-r_0)^2/2\sigma^2},$$





Schwarszchild space-time

We obtained the frequencies using a fourier transform and the results from the evolution match the results in the frequency domain



Reissner-Nordstrom space-time

Reissner-Nordstrom spacetime written in Kerr-Schild coordinates

$$ds^2 = -\left(1 - \frac{2\,M}{r} + \frac{Q^2}{r^2}\right) dt^2 + 2\left(\frac{2\,M}{r} - \frac{Q^2}{r^2}\right) dtdr + \left(1 + \frac{2\,M}{r} - \frac{Q^2}{r^2}\right) dr^2 + r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right),$$

and the tetrad

$$l^\mu = \frac{1}{2}\left(1 + \frac{2\,M}{r} - \frac{Q^2}{r^2}, 1 - \frac{2\,M}{r} + \frac{Q^2}{r^2}, 0, 0\right), \qquad k^\mu = (1, -1, 0, 0), \qquad m^\mu = \frac{1}{\sqrt{2}\,r}\left(0, 0, 1, i\,\csc\theta\right). \tag{1}$$

whereas the non-zero spin coefficients are

$$\mu_s = \frac{1}{r}, \quad \rho_s = \frac{r^2 - 2Mr + Q^2}{2r^3}, \quad \epsilon_s = -\frac{1}{2}\left(\frac{M}{r^2} - \frac{Q^2}{r^3}\right), \quad \beta_s = -\alpha_s = -\frac{1}{2\sqrt{2}}\frac{\cot\theta}{r}.$$

The directional derivatives are $\mathbf{D} = l^\mu\,\partial_\mu$, $\mathbf{\Delta} = \kappa^\mu\,\partial_\mu$ and $\delta = m^\mu\,\partial_\mu$.
Maxwell component $\Phi_{11} = \varphi_1\varphi_1 = \frac{Q^2}{2r^4}$. Weyl component $\Psi_2 = \frac{M}{r^3} - \frac{Q^2}{r^4}$.

RN system equations

Maxwell equation

$$
\begin{aligned}
&[(D-(\rho_s-3\epsilon_s-\overline{\epsilon}_s))(\Delta+(2\mu_s+\overline{\mu}_s+2\gamma_s))\\
&-(\delta+(3\beta_s-\overline{\alpha}_s-\tau_s))(\overline{\delta}+(2\alpha_s+2\pi_s-\tau_s))-3\Psi_2]\varphi_2^{(1)}\\
&=2\varphi_1[(D+(2\rho_s-\overline{\rho}_s+3\epsilon_s+\overline{\epsilon}_s))\nu_s{}^{(1)}-(\delta-(\overline{\alpha}_s-3\beta_s-\overline{\pi}_s-2\tau_s))\lambda_s{}^{(1)}-2\Psi_3^{(1)}]+2\pi J_2^{(1)}.
\end{aligned}
$$

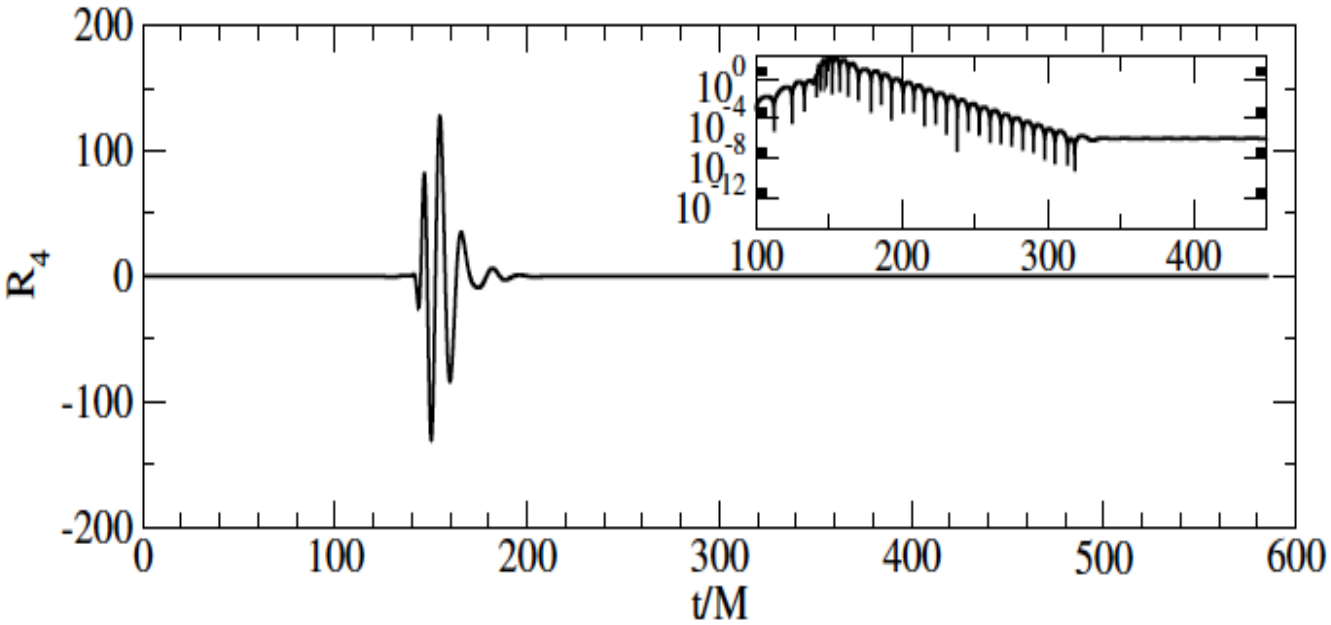
Einstein equations

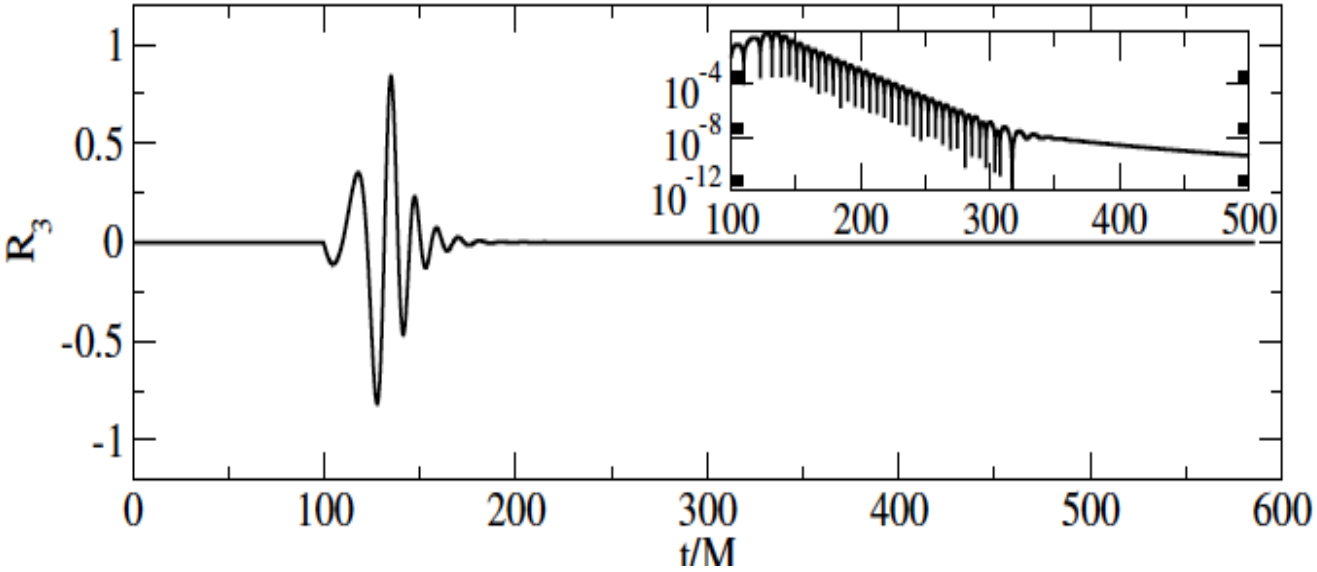
$$
\begin{aligned}
&\left\{\left[\Delta+\chi\,\mu_s\left(5-\frac{4\,\Phi_{11}\,(14\,\Phi_{11}-9\,\Psi_2)}{(2\Phi_{11}-3\,\Psi_2)^2}\right)\right](D+\rho_s-4\epsilon_s)-\chi\,\mathcal{O}_{3b}\mathcal{O}_{2a}-2\,\Phi_{11}+3\,\Psi_2\right\}\Psi_4^{(1)}\\
&-\frac{4\,\Phi_{11}}{2\Phi_{11}+3\,\Psi_2}\left(\Delta-\frac{14\,\Phi_{11}-9\,\Psi_2}{2\Phi_{11}-3\,\Psi_2}\,\mu_s\right)\mathcal{O}_{1b}\,\Psi_3^{(1)}=\\
&-\left[\frac{6\,\Psi_2}{2\Phi_{11}+3\,\Psi_2}\,\Delta+4\,\chi\left(2-\frac{\Phi_{11}\,\mu_s\,(14\,\Phi_{11}-9\,\Psi_2)}{(2\Phi_{11}-3\,\Psi_2)^2}\right)\right]\left(\overline{\delta}+2\,\beta_s\right)T_{21}^{(1)}\\
&+\left[\Delta+\chi\,\mu_s\left(5-\frac{4\,\Phi_{11}\,(14\,\Phi_{11}-9\,\Psi_2)}{(2\Phi_{11}-3\,\Psi_2)^2}\right)\right](\Delta-\mu_s)\,T_{20}^{(1)}-\chi\,[\mathcal{O}_{3b}\mathcal{T}_{2a}]\,T_{22}^{(1)},
\end{aligned}
\tag{2}
$$

$$
\begin{aligned}
&\left[\left(D+2\,\rho_s-4\,\epsilon_s-4\,\rho_s\,\frac{\Phi_{11}}{3\,\Psi_2+2\,\Phi_{11}}\right)\mathcal{O}_{2b}+\frac{1}{\chi}\mathcal{O}_{5b}\mathcal{O}_{1b}+6\,\Psi_2+4\,\Phi_{11}\right]\Psi_3^{(1)}\\
&-4\,\frac{\Phi_{11}}{3\,\Psi_2-2\,\Phi_{11}}\left[D+\rho_s-4\,\epsilon_s+6\,\rho_s\,\frac{\Psi_2}{3\,\Psi_2+2\,\Phi_{11}}\right]\mathcal{O}_{2a}\,\Psi_4^{(1)}=\\
&4\,\pi\left(D+\rho_s-4\,\epsilon_s-4\,\rho_s\,\frac{\Phi_{11}}{3\,\Psi_2+2\,\Phi_{11}}\right)\mathcal{T}_{2a}\,T_{22}{}^{(1)}+\frac{\pi}{\varphi_1}\,(3\,\Psi_2+2\,\Phi_{11})\,J_2^{(1)}.
\end{aligned}
\tag{3}
$$

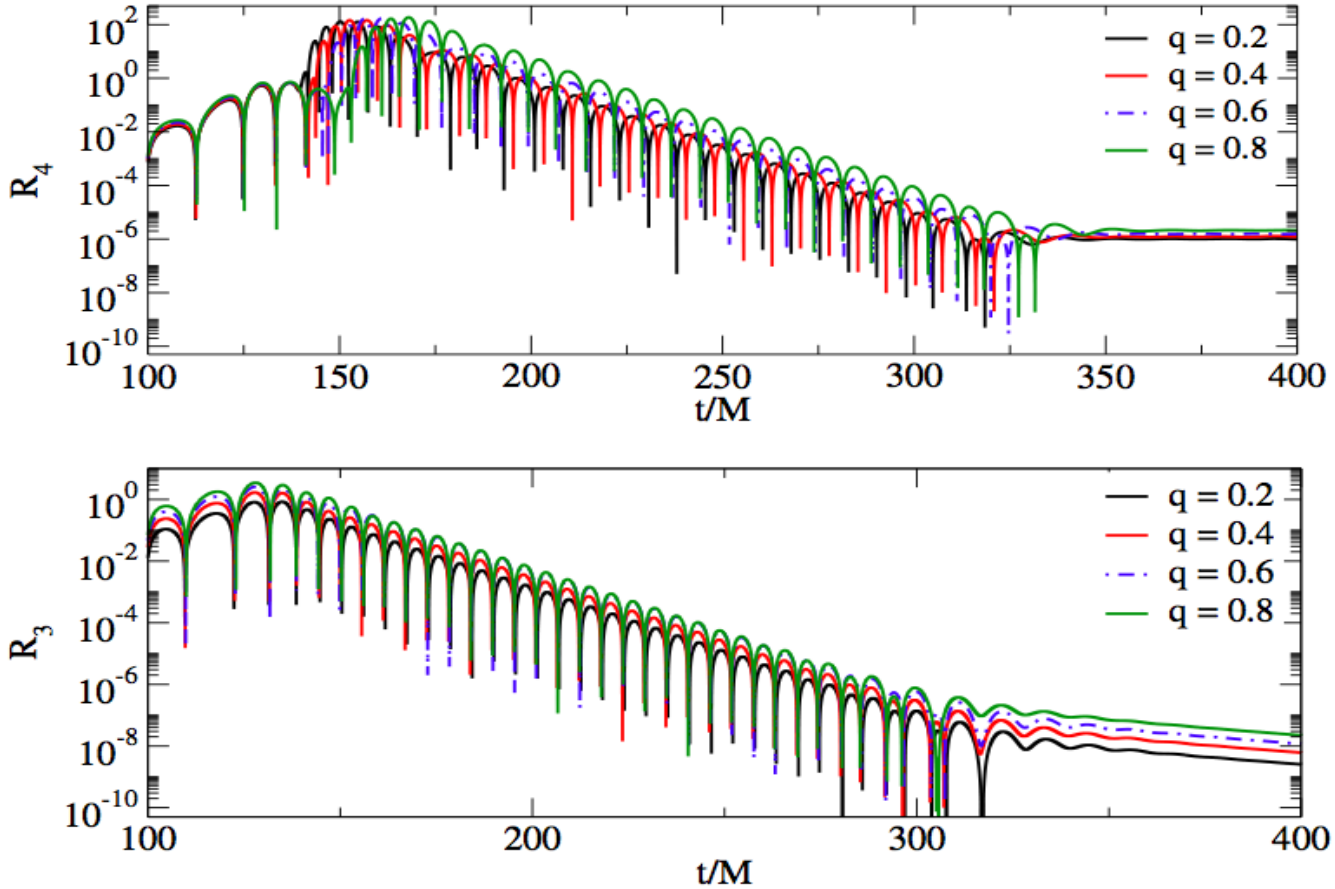
$\Psi_3^{(1)}$ electromagnetic wave for RN black hole. $\chi:=\frac{2\Phi_{11}-3\,\Psi_2}{2\Phi_{11}+3\,\Psi_2}$, $\mathcal{O}nn$ are differential operators.

Reissner-Nordstrom space-time





Quasinormal modes RN



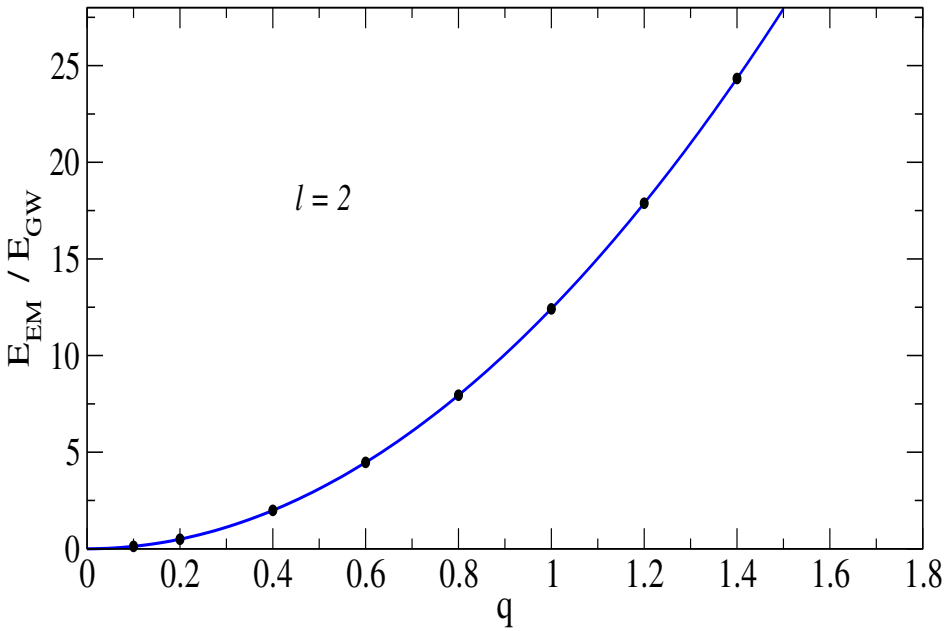
The difference in phase in R_4 is due to the difference in the infalling time of the charge particles. There is not change in the frequency because this depend of the parameters of the black hole.

Energy

We estimate the energy radiated as

$$\frac{d}{dt}E_{GW} = \lim_{r\to\infty}\frac{1}{16\pi}\sum_{\ell,m}|\int_{-\infty}^{t}dt'\,R_\ell^G(t')|^2, \quad \frac{d}{dt}E_{EM} = \lim_{r\to\infty}\frac{1}{4\pi}\sum_{\ell,m}|R_\ell^E(t)|^2\ .$$

We found a quadratic dependence between the electromagnetic and gravitational energy emitted, of the form $E_{EM}/E_{GW} = a\,q^2$, with $a = 12.417$, for $\ell = 2$ respectively.



Conclusions

- The compactness of the shells affects the gravitational and electromagnetic emission.
- Pressure-less matter induces electric and gravitational quasi-normal modes on both signals. However there is no direct mixing between frequencies.
- The electromagnetic energy emitted after the falling of matter is related with the energy carried by the gravitational waves via q^2.
- We are working in the Kerr-Newman case.

Comercial

In México we organize the Tematic Network of Gravitational Waves and Black Holes, we invite students, professors (master, Ph.D, postdoctoral, hue sped professor) to join us, you can fin information in the web page:

www.redtematicaanyog.mx

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