Canonical Transformation Path to Gauge Theories of Gravity

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Overview

1. Aim: Derive the theory of gravity from first principles along the line of non-Abelian gauge theories.

2. Method: Use the canonical transformation theory in order to ensure that the action principle is maintained. Here, we restrict the calculations to scalar and vector boson fields.

3. Key results:

    - the connection coefficients are the gauge fields of gravity
    - the coupling terms of fields and gauge fields are uniquely defined
    - the Hamiltonian for spacetime dynamics must be at least quadratic in the canonical momenta of the gauge fields
    - spin fields contribute with additional source terms to the spacetime dynamics

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Minimal Set of Basic Principles

1. Action Principle: The system dynamics follows from the variation of its action S, namely $\delta S \stackrel{!}{=} 0$.

2. Special Principle of Relativity: The action integrand must be form-invariant (symmetric) under (global) Lorentz transformations.

3. General Principle of Relativity: The action integrand must be form-invariant (symmetric) under local Lorentz transformations.

4. Gauge Principle: Promoting a global symmetry of a given system to a local symmetry by adding appropriate gauge fields yields a physically meaningful theory.

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Hamiltonian action principle for **static** spacetime

Action principle for a system of real scalar and vector fields

$$S = \int_R \left(\pi^\alpha \frac{\partial \phi}{\partial x^\alpha} + p^{\beta\alpha} \frac{\partial a_\beta}{\partial x^\alpha} - \mathcal{H}(\pi^\mu, \phi, p^{\nu\mu}, a_\nu, x^\mu) \right) \mathrm{d}^4 x$$

with

$$\delta S \stackrel{!}{=} 0, \quad \delta\phi\big|_{\partial R} = \delta a_\mu\big|_{\partial R} \stackrel{!}{=} 0.$$

Calculus of variations: $\delta S = 0$ holds exactly for the solutions of the

Covariant canonical field equations

$$\frac{\mathrm{d}q_i}{\mathrm{d}t} = \frac{\partial H}{\partial p^i} \quad \rightarrow \quad \frac{\partial \phi}{\partial x^\mu} = \frac{\partial \mathcal{H}}{\partial \pi^\mu}, \qquad \frac{\partial a_\nu}{\partial x^\mu} = \frac{\partial \mathcal{H}}{\partial p^{\nu\mu}}$$

$$\frac{\mathrm{d}p^i}{\mathrm{d}t} = -\frac{\partial H}{\partial q_i} \quad \rightarrow \quad \frac{\partial \pi^\alpha}{\partial x^\alpha} = -\frac{\partial \mathcal{H}}{\partial \phi}, \qquad \frac{\partial p^{\nu\alpha}}{\partial x^\alpha} = -\frac{\partial \mathcal{H}}{\partial a_\nu}$$

$$\frac{\mathrm{d}e}{\mathrm{d}t} = \frac{\partial H}{\partial t}\bigg|_{\mathrm{expl}} \quad \rightarrow \quad \frac{\partial T_\mu{}^\alpha}{\partial x^\alpha} = \frac{\partial \mathcal{H}}{\partial x^\mu}\bigg|_{\mathrm{expl}}, \qquad T_\mu{}^\alpha = \text{E-M tensor}$$

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with $\sqrt{-g}\,\mathrm{d}^4 x$ the invariant volume form and the tensor densities

$$\tilde{\pi}^\mu = \pi^\mu \sqrt{-g}, \quad \tilde{p}^{\mu\nu} = p^{\mu\nu}\sqrt{-g}, \quad \tilde{k}^{\mu\lambda\nu} = k^{\mu\lambda\nu}\sqrt{-g}, \quad \tilde{\mathcal{H}} = \mathcal{H}\sqrt{-g}.$$

$g_{\alpha\lambda}(x)$ denotes the system's metric and g the metric's determinant.

Covariant canonical field equations

$$\frac{\partial \phi}{\partial x^\mu} = \frac{\partial \tilde{\mathcal{H}}}{\partial \tilde{\pi}^\mu}, \qquad \frac{\partial a_\nu}{\partial x^\mu} = \frac{\partial \tilde{\mathcal{H}}}{\partial \tilde{p}^{\nu\mu}}, \qquad \frac{\partial g_{\nu\lambda}}{\partial x^\mu} = \frac{\partial \tilde{\mathcal{H}}}{\partial \tilde{k}^{\nu\lambda\mu}}$$

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Maxwell Hamiltonian in arbitrary spacetime

Example

For the Maxwell Hamiltonian in a Riemannian space with metric $g_{\nu\mu}(x)$

$$\tilde{\mathcal{H}}_{\mathrm{M}} = -\tfrac{1}{4}\tilde{p}^{\alpha\beta}\tilde{p}^{\xi\eta}\,\frac{g_{\alpha\xi}g_{\beta\eta}}{\sqrt{-g}} + j_{\alpha}(x)\,a_{\beta}\,g^{\alpha\beta}\sqrt{-g}, \qquad \tilde{p}^{\mu\nu} = -\tilde{p}^{\nu\mu},$$

the field equations emerge for a torsion-free spacetime and $g_{\nu\mu;\xi} = 0$ as

$$\frac{\partial a_{\nu}}{\partial x^{\mu}} = \frac{\partial\tilde{\mathcal{H}}_{\mathrm{M}}}{\partial\tilde{p}^{\nu\mu}} = -\tfrac{1}{2}p_{\nu\mu} \qquad \Rightarrow \qquad p_{\nu\mu} = \frac{\partial a_{\mu}}{\partial x^{\nu}} - \frac{\partial a_{\nu}}{\partial x^{\mu}}$$

$$\frac{\partial\tilde{p}^{\nu\alpha}}{\partial x^{\alpha}} = -\frac{\partial\tilde{\mathcal{H}}_{\mathrm{M}}}{\partial a_{\nu}} = -j^{\nu}(x)\sqrt{-g} \qquad \Rightarrow \qquad p^{\nu\alpha}{}_{;\alpha} = -j^{\nu}(x)$$

$$T^{\mu\nu} = \frac{2}{\sqrt{-g}}\frac{\partial\tilde{\mathcal{H}}_{\mathrm{M}}}{\partial g_{\nu\mu}} = -p^{\mu}{}_{\alpha}\,p^{\nu\alpha} - j^{\nu}a^{\mu} - j^{\mu}a^{\nu} + g^{\mu\nu}\left(\tfrac{1}{4}p^{\alpha\beta}p_{\alpha\beta} + j^{\alpha}a_{\alpha}\right)$$

Requirement of form-invariance for the action principle

For a gauge theory that includes a general mapping of spacetime $x \mapsto X$, we need the connection coefficients $\gamma^{\eta}{}_{\alpha\xi}$ as additional dynamic quantities

Condition for canonical transformations under a dynamical spacetime

$$S = \int_R \left(\tilde{\pi}^{\beta} \frac{\partial \phi}{\partial x^{\beta}} + \tilde{p}^{\alpha\beta} \frac{\partial a_{\alpha}}{\partial x^{\beta}} + \tilde{k}^{\alpha\lambda\beta} \frac{\partial g_{\alpha\lambda}}{\partial x^{\beta}} + \tilde{q}_{\eta}{}^{\alpha\xi\beta} \frac{\partial \gamma^{\eta}{}_{\alpha\xi}}{\partial x^{\beta}} - \tilde{\mathcal{H}} + \frac{\partial \tilde{\mathcal{F}}_2^{\beta}}{\partial x^{\beta}} \right) \mathrm{d}^4 x$$

$$= \int_{R'} \left(\tilde{\Pi}^{\beta} \frac{\partial \Phi}{\partial X^{\beta}} + \tilde{P}^{\alpha\beta} \frac{\partial A_{\alpha}}{\partial X^{\beta}} + \tilde{K}^{\alpha\lambda\beta} \frac{\partial G_{\alpha\lambda}}{\partial X^{\beta}} + \tilde{Q}_{\eta}{}^{\alpha\xi\beta} \frac{\partial \Gamma^{\eta}{}_{\alpha\xi}}{\partial x^{\beta}} - \tilde{\mathcal{H}}' \right) \mathrm{d}^4 X.$$

- The integrands must be world scalar densities in order to be form-invariant under general spacetime transformations.
- $\rightsquigarrow$ The partial derivatives must be promoted to covariant derivatives.
- $\rightsquigarrow$ The connection coefficients γ, Γ must be the gauge quantities.
- $\tilde{\mathcal{F}}_2^{\beta}$ is the generating function of the canonical transformation $x \mapsto X$.

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General CT rules under dynamic spacetime

General rules for a generating function of type $\tilde{\mathcal{F}}_2^{\mu}(\tilde{\Pi}, \phi, \tilde{P}, a, \tilde{K}, g, \tilde{Q}, \gamma, x)$

$$\tilde{\pi}^{\mu} = \frac{\partial \tilde{\mathcal{F}}_2^{\mu}}{\partial \phi} \qquad \delta^{\mu}_{\nu} \Phi = \frac{\partial \tilde{\mathcal{F}}_2^{\kappa}}{\partial \tilde{\Pi}^{\nu}} \frac{\partial X^{\mu}}{\partial x^{\kappa}} \left| \frac{\partial x}{\partial X} \right|$$

$$\tilde{p}^{\nu\mu} = \frac{\partial \tilde{\mathcal{F}}_2^{\mu}}{\partial a_{\nu}} \qquad \delta^{\mu}_{\beta} A_{\alpha} = \frac{\partial \tilde{\mathcal{F}}_2^{\kappa}}{\partial \tilde{P}^{\alpha\beta}} \frac{\partial X^{\mu}}{\partial x^{\kappa}} \left| \frac{\partial x}{\partial X} \right|$$

$$\tilde{k}^{\xi\zeta\mu} = \frac{\partial \tilde{\mathcal{F}}_2^{\mu}}{\partial g_{\xi\zeta}} \qquad \delta^{\mu}_{\beta} G_{\alpha\lambda} = \frac{\partial \tilde{\mathcal{F}}_2^{\kappa}}{\partial \tilde{K}^{\alpha\lambda\beta}} \frac{\partial X^{\mu}}{\partial x^{\kappa}} \left| \frac{\partial x}{\partial X} \right|$$

$$\tilde{q}_k{}^{ij\mu} = \frac{\partial \tilde{\mathcal{F}}_2^{\mu}}{\partial \gamma^k{}_{ij}} \qquad \delta^{\mu}_{\nu} \Gamma^{\eta}{}_{\alpha\xi} = \frac{\partial \tilde{\mathcal{F}}_2^{\kappa}}{\partial \tilde{Q}_{\eta}{}^{\alpha\xi\nu}} \frac{\partial X^{\mu}}{\partial x^{\kappa}} \left| \frac{\partial x}{\partial X} \right|$$

$$\left| \frac{\partial x}{\partial X} \right| := \frac{\partial \left(x^0, \ldots, x^3\right)}{\partial \left(X^0, \ldots, X^3\right)} \qquad \tilde{\mathcal{H}}' = \left(\tilde{\mathcal{H}} + \left. \frac{\partial \tilde{\mathcal{F}}_2^{\alpha}}{\partial x^{\alpha}} \right|_{\text{expl}} \right) \left| \frac{\partial x}{\partial X} \right|$$

Gauge Hamiltonian

The generating function $\tilde{\mathcal{F}}_2^{\mu}$ is devised to define the required mappings

$$\Phi(X) = \phi(x), \qquad A_\mu(X) = a_\alpha(x) \frac{\partial x^\alpha}{\partial X^\mu}, \qquad G_{\nu\mu}(X) = g_{\alpha\lambda}(x) \frac{\partial x^\alpha}{\partial X^\nu} \frac{\partial x^\lambda}{\partial X^\mu}$$

and

$$\Gamma^{\kappa}{}_{\alpha\beta}(X) = \gamma^{\xi}{}_{\eta\tau}(x) \frac{\partial x^\eta}{\partial X^\alpha} \frac{\partial x^\tau}{\partial X^\beta} \frac{\partial X^\kappa}{\partial x^\xi} + \frac{\partial^2 x^\xi}{\partial X^\alpha \partial X^\beta} \frac{\partial X^\kappa}{\partial x^\xi}.$$

$\rightsquigarrow$ $\tilde{\mathcal{F}}_2^{\beta}$ simultaneously defines the rules for the conjugate fields $\tilde{\pi}$, $\tilde{p}$, $\tilde{k}$, $\tilde{q}$ and for the Hamiltonians.

We thus encounter the "gauge" Hamiltonian $\tilde{\mathcal{H}}_{\mathrm{G}}$ (after "some algebra"!)

$$\tilde{\mathcal{H}}_{\mathrm{G}} = \tilde{\mathcal{H}} + \left(\tilde{p}^{\alpha\beta} a_\xi + \tilde{k}^{\alpha\lambda\beta} g_{\xi\lambda} + \tilde{k}^{\lambda\alpha\beta} g_{\lambda\xi}\right) \gamma^{\xi}{}_{\alpha\beta} + \tfrac{1}{2} \tilde{q}_{\eta}{}^{\alpha\xi\beta} \left(\frac{\partial \gamma^{\eta}{}_{\alpha\xi}}{\partial x^\beta} + \frac{\partial \gamma^{\eta}{}_{\alpha\beta}}{\partial x^\xi} + \gamma^{\tau}{}_{\alpha\beta} \gamma^{\eta}{}_{\tau\xi} - \gamma^{\tau}{}_{\alpha\xi} \gamma^{\eta}{}_{\tau\beta} \right)$$

The Hamiltonian $\tilde{\mathcal{H}}'_{\mathrm{G}}$ has the same form in the transformed fields.

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$$+ \tfrac{1}{2}\tilde{q}_{\eta}{}^{\alpha\xi\beta}\left(\frac{\partial \gamma^{\eta}{}_{\alpha\xi}}{\partial x^\beta} + \frac{\partial \gamma^{\eta}{}_{\alpha\beta}}{\partial x^\xi} + \gamma^{\tau}{}_{\alpha\beta}\gamma^{\eta}{}_{\tau\xi} - \gamma^{\tau}{}_{\alpha\xi}\gamma^{\eta}{}_{\tau\beta}\right)$$

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and

$$\Gamma^{\kappa}{}_{\alpha\beta}(X) = \gamma^{\xi}{}_{\eta\tau}(x)\frac{\partial x^\eta}{\partial X^\alpha}\frac{\partial x^\tau}{\partial X^\beta}\frac{\partial X^\kappa}{\partial x^\xi} + \frac{\partial^2 x^\xi}{\partial X^\alpha \partial X^\beta}\frac{\partial X^\kappa}{\partial x^\xi}.$$

$\rightsquigarrow$ $\tilde{\mathcal{F}}_2^{\beta}$ simultaneously defines the rules for the conjugate fields $\tilde{\pi}$, $\tilde{p}$, $\tilde{k}$, $\tilde{q}$ and for the Hamiltonians.

We thus encounter the "gauge" Hamiltonian $\tilde{\mathcal{H}}_{\mathrm{G}}$ (after "some algebra"!)

$$\tilde{\mathcal{H}}_{\mathrm{G}} = \tilde{\mathcal{H}} + \left(\tilde{p}^{\alpha\beta}a_\xi + \tilde{k}^{\alpha\lambda\beta}g_{\xi\lambda} + \tilde{k}^{\lambda\alpha\beta}g_{\lambda\xi}\right)\gamma^{\xi}{}_{\alpha\beta} + \tfrac{1}{2}\tilde{q}_\eta{}^{\alpha\xi\beta}\left(\frac{\partial\gamma^{\eta}{}_{\alpha\xi}}{\partial x^\beta} + \frac{\partial\gamma^{\eta}{}_{\alpha\beta}}{\partial x^\xi} + \gamma^{\tau}{}_{\alpha\beta}\gamma^{\eta}{}_{\tau\xi} - \gamma^{\tau}{}_{\alpha\xi}\gamma^{\eta}{}_{\tau\beta}\right)$$

The Hamiltonian $\tilde{\mathcal{H}}'_{\mathrm{G}}$ has the same form in the transformed fields.

Generally invariant action principle

Inserting the Hamiltonians $\tilde{\mathcal{H}}_{\mathrm{G}}$, $\tilde{\mathcal{H}}'_{\mathrm{G}}$ into the above action functionals yields

Gauged action

$$S = \int_R \left(\tilde{\pi}^{\beta} \phi_{;\beta} + \tilde{p}^{\alpha\beta}\, a_{\alpha;\beta} + \tilde{k}^{\alpha\lambda\beta}\, g_{\alpha\lambda;\beta} - \tfrac{1}{2} \tilde{q}_{\eta}{}^{\alpha\xi\beta} r^{\eta}{}_{\alpha\xi\beta} - \tilde{\mathcal{H}} \right) \mathrm{d}^4 x$$
$$= \int_{R'} \left(\tilde{\Pi}^{\beta} \Phi_{;\beta} + \tilde{P}^{\alpha\beta}\, A_{\alpha;\beta} + \tilde{K}^{\alpha\lambda\beta}\, G_{\alpha\lambda;\beta} - \tfrac{1}{2} \tilde{Q}_{\eta}{}^{\alpha\xi\beta} R^{\eta}{}_{\alpha\xi\beta} - \tilde{\mathcal{H}}' \right) \mathrm{d}^4 X.$$

- The partial derivatives of the fields ϕ, a_{μ}, and $g_{\nu\mu}$ in the original action functional are indeed converted into covariant derivatives.
- In contrast, the partial derivatives of the non-tensorial quantities $\gamma^{\eta}{}_{\alpha\xi}$ cannot be converted into covariant derivatives.
- Miraculously, the terms of the calculated gauge Hamiltonians $\tilde{\mathcal{H}}_{\mathrm{G}}$, $\tilde{\mathcal{H}}'_{\mathrm{G}}$ complement these derivatives to the Riemann curvature tensors r, R.
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The "free" gauge field Hamiltonian $\tilde{\mathcal{H}}_{\mathrm{Dyn}}$

As common to all gauge theories,

- the gauge formalism yields the coupling terms of the fields of the given system to the gauge fields,
- the gauge formalism does not provide the Hamiltonian describing the dynamics of the "free" gauge fields,
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Final action functional

$$S = \int_R \left(\tilde{\pi}^{\beta} \phi_{;\beta} + \tilde{p}^{\alpha\beta} a_{\alpha;\beta} + \tilde{k}^{\alpha\lambda\beta} g_{\alpha\lambda;\beta} - \tfrac{1}{2} \tilde{q}_{\eta}{}^{\alpha\xi\beta} r^{\eta}{}_{\alpha\xi\beta} - \tilde{\mathcal{H}} - \tilde{\mathcal{H}}_{\mathrm{Dyn}} \right) \mathrm{d}^4 x$$

with a form-invariant Hamiltonian $\tilde{\mathcal{H}}_{\mathrm{Dyn}}(g, \tilde{k}, \tilde{q}) = \tilde{\mathcal{H}}_{\mathrm{Dyn}}(G, \tilde{K}, \tilde{Q})$.

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Canonical field equations for given $\tilde{\mathcal{H}}$ and $\tilde{\mathcal{H}}_{\mathrm{Dyn}}$

$$\phi_{;\,\mu} = \frac{\partial\tilde{\mathcal{H}}}{\partial\tilde{\pi}^{\mu}}, \qquad \tilde{\pi}^{\,\beta}{}_{;\,\beta} = -\frac{\partial\tilde{\mathcal{H}}}{\partial\phi} + 2\tilde{\pi}^{\,\beta} s^{\,\alpha}{}_{\beta\alpha}$$

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- Throughout tensor equations $\rightsquigarrow$ form-invariant in any reference frame.
- $\tilde{\mathcal{H}}_{\mathrm{Dyn}}$ must be postulated for the set of equations to be closed.
- The torsion ($s^{\,\lambda}{}_{\beta\alpha} \neq 0$) and metricity ($g_{\xi\lambda;\,\mu} \neq 0$) may be non-zero.
- Spin-1 fields yield additional source terms beyond the Einstein gravity.

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Discussion of $\tilde{\mathcal{H}}_{\mathrm{Dyn}}$

- For U(1) and SU(N) (Yang-Mills) gauge theories, $\mathcal{H}_{\mathrm{Dyn}}$ is uniquely given by a purely quadratic momentum term (Maxwell Hamiltonian!).
- For geometrodynamics, $\tilde{\mathcal{H}}_{\mathrm{Dyn}}$ must be at least quadratic in $\tilde{q}_{\eta}{}^{\xi\beta\alpha}$ to get a well-defined and non-trivial equation for the Riemann tensor r.
- The simplest choice is to postulate $\tilde{\mathcal{H}}_{\mathrm{Dyn,post}}$ as purely quadratic in $\tilde{q}$ and not depending on $\tilde{k}$, which yields the metricity condition $g_{\alpha\beta;\lambda} = 0$

$$\tilde{\mathcal{H}}_{\mathrm{Dyn,post}} = \tfrac{1}{4} g_1\, \tilde{q}_{\eta}{}^{\alpha\xi\beta}\, \tilde{q}_{\alpha}{}^{\eta\tau\lambda}\, g_{\xi\tau} g_{\beta\lambda} \frac{1}{\sqrt{-g}}, \qquad g_1 : \text{ coupling constant}$$

For the classical vacuum ($\tilde{\mathcal{H}} \equiv 0$), the set of canonical equations reduces to

$$r_{\alpha\beta\tau\eta}\, r^{\alpha\beta\tau\xi} - \tfrac{1}{4}\delta^{\xi}_{\eta}\, r_{\alpha\beta\tau\sigma}\, r^{\alpha\beta\tau\sigma} - \left(r_{\eta}{}^{\xi\beta\alpha} s^{\lambda}{}_{\beta\alpha} - 2 r_{\eta}{}^{\xi\beta\lambda} s^{\alpha}{}_{\beta\alpha} \right)_{;\lambda} = 0.$$

⇝ This is a set of homogeneous second-order equations for $\gamma^{\lambda}{}_{\beta\alpha}$.

⇝ The solutions can then be inserted into the first-order equations for $g_{\alpha\beta}$.

Discussion of $\tilde{\mathcal{H}}_{\rm Dyn}$

- For U(1) and SU(N) (Yang-Mills) gauge theories, $\mathcal{H}_{\rm Dyn}$ is uniquely given by a purely quadratic momentum term (Maxwell Hamiltonian!).
- For geometrodynamics, $\tilde{\mathcal{H}}_{\rm Dyn}$ must be at least quadratic in $\tilde{q}_{\eta}{}^{\xi\beta\alpha}$ to get a well-defined and non-trivial equation for the Riemann tensor r.
- The simplest choice is to postulate $\tilde{\mathcal{H}}_{\rm Dyn,post}$ as purely quadratic in $\tilde{q}$ and not depending on $\tilde{k}$, which yields the metricity condition $g_{\alpha\beta;\lambda} = 0$

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$$g_{\mu\nu} = \begin{pmatrix} -e^{2\kappa(r)} & 0 & 0 & 0 \\ 0 & e^{2\lambda(r)} & 0 & 0 \\ 0 & 0 & r^2 & 0 \\ 0 & 0 & 0 & r^2 \sin^2\theta \end{pmatrix},$$

we find with the integration constants m and Λ

$$g_{00} = -\left(1 - \frac{2m}{r} - \tfrac{1}{3}\Lambda r^2\right), \qquad g_{11} = \left(1 - \frac{2m}{r} - \tfrac{1}{3}\Lambda r^2\right)^{-1}.$$

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Outlook

1. Key results:
   - the connection coefficients γ are the gauge fields of gravity
   - the coupling terms of fields and gauge fields are uniquely defined
   - the gauge procedure truncates with the introduction of γ — no infinite hierarchy of gauge fields and pertaining transformation rules occurs
   - the Hamiltonian $\tilde{\mathcal{H}}_{\mathrm{Dyn}}$ for the "free gauge fields" must be at least quadratic in the canonical momenta $\tilde{q}$ of the gauge fields γ
   - spin fields contribute with additional source terms to the spacetime dynamics, which is beyond the Einstein theory.
2. Actual work:
   - Discussion of other options for $\tilde{\mathcal{H}}_{\mathrm{Dyn}}$: adding a term linear in $\tilde{q}$
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Conclusions

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- The connection coefficients γ are the gauge fields of gravity.
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