FPGA acceleration of Model Predictive Control for ITER Plasma current and shape control

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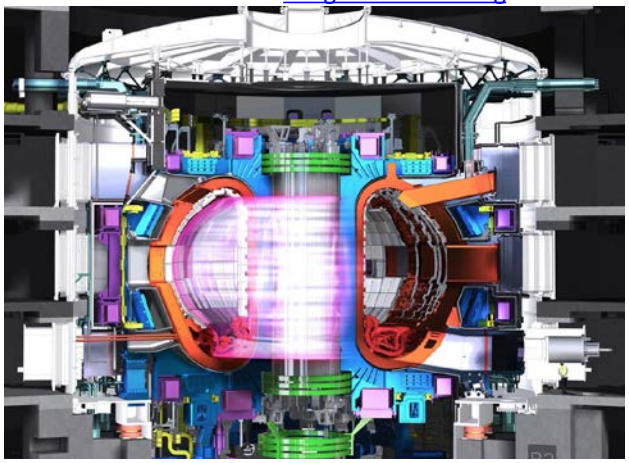
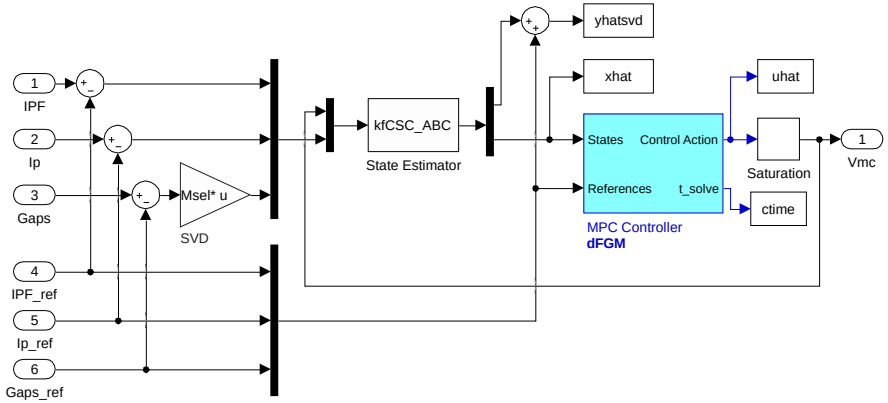
Overview

- Plasma Current and Shape Controller (PCSC) for ITER
- Singular Value Decomposition (SVD) -based Model Predictive Control (MPC) using
- a dual Fast Gradient Method (dFGM) quadratic programming (QP) solver
- FPGA acceleration with a High-Level Synthesis (HLS) approach

Image: www.iter.org





MPC online optimization problem

- **MPC Quadratic Program** – hard input and soft state constraints

$$\min_{\mathbf{z},\mathbf{s}} \frac{1}{2}\mathbf{z}^T\mathbf{H}\mathbf{z} + \mathbf{c}^T\mathbf{z} + \frac{1}{2}\mathbf{s}^T\mathbf{W}\mathbf{s} + \mathbf{w}^T\mathbf{s}$$
$$\text{subject to} \quad \begin{aligned} \mathbf{C}_x\mathbf{z} &\preccurlyeq \mathbf{b}_x + \mathbf{s} \\ \mathbf{C}_u\mathbf{z} &\preccurlyeq \mathbf{b}_u \\ \mathbf{s} &\preccurlyeq \mathbf{0} \end{aligned}$$

- **Dual Fast Gradient Method**

$$\begin{aligned} \boldsymbol{\nu}^k &= \mathbf{v}^k + \beta^k(\mathbf{v}^k - \mathbf{v}^{k-1}) \\ \mathbf{y}^k &= -\mathbf{H}^{-1}\left(\mathbf{C}^T\boldsymbol{\nu}^k + \mathbf{c}\right) \\ \mathbf{v}^{k+1} &= \boldsymbol{\nu}^k + \mathbf{C}\mathbf{y}^k - \widetilde{\mathrm{prox}}_{h,\mathbf{W},\mathbf{w}}\left(\boldsymbol{\nu}^k + \mathbf{C}\mathbf{y}^k\right) \end{aligned}$$

$$\widetilde{\mathrm{prox}}_{h,\mathbf{W},\mathbf{w}}(\mathbf{t})_i = \begin{cases} t_i & \text{if } t_i \le b_i \\ b_i & \text{if } t_i > b_i \text{ and } i \text{ hard} \\ b_i & \text{if } b_i + w_i \ge t_i > b_i \text{ and } i \text{ soft} \\ \dfrac{t_i + W_{ii}b_i - w_i}{W_{ii} + 1} & \text{if } b_i + w_i < t_i \text{ and } i \text{ soft} \end{cases}$$

Iterative algorithm

Microparallelization

(matrix-vector multiplication within iterations)

Several approaches to speeding up M*v multiplication

TL;DR: Speed-up via HLS achieved but not automatically