Participant number fluctuations for higher moments of a multiplicity distribution

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based on

arXiv: **1606.05358** and ongoing analysis with **M. Mackowiak-Pawlowska**

12-th Polish Workshop on Relativistic Heavy-Ion Collisions

4-6 November 2016, Kielce

Independent participant (wounded nucleon) model

The multiplicity of some particles N created in a collision is the sum of the contributions from N_P participants

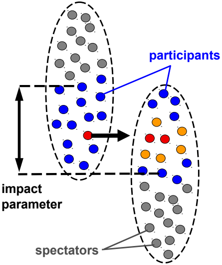
$$N = n_1 + n_2 + \ldots + n_{N_P}$$

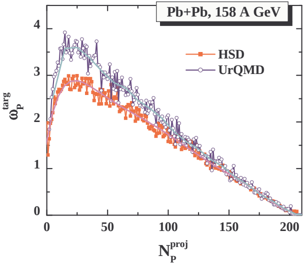
The participants are **identical**

$$\langle n_i \rangle = \langle n_j \rangle = \langle n_1 \rangle = \frac{\langle N \rangle}{\langle N_P \rangle}$$

and **independent**

$$\langle n_i n_j \rangle = \langle n_i \rangle \langle n_j \rangle$$





Konchakovski, Gorenstein, Bratkovskaya, Greiner JPG (2010)

Then the scaled variance for multiplicity fluctuations

$$\omega = \frac{\langle N^2 \rangle - \langle N \rangle^2}{\langle N \rangle} = \omega_1 + \langle n_1 \rangle \, \omega_P \,, \qquad \text{(Bialas, Bleszynski, Czyz, NPB (1976))}$$

is the sum of the **fluctuations** from **one participant** ω_1 and the **fluctuations** of **participant number** ω_P times the mean multiplicity of particles of interest from one participant $\langle n_1 \rangle$.

... was quite **ambiguous**. The scaled variance in nucleus-nucleus (A+A) collisions as the function of N_{P} was qualitatively explained by the fluctuations of participants both at **SPS** and at **RHIC**. However, more recent data of **NA49** and **NA61/SHINE** show that

$$\omega^{\mathrm{ch}}_{\mathrm{Pb+Pb}} \lesssim \omega^{\mathrm{ch}}_{\mathrm{p+Pb}} \lesssim \omega^{\mathrm{ch}}_{\mathrm{p+p}} \qquad \text{at the } \mathbf{SPS}, \qquad \text{(NA61/SHINE 1510.00163)} \tag{1}$$

while one would expect the opposite dependence from the participant model.

$$\omega^{\mathrm{ch}}_{\mathrm{Pb+Pb}} = \omega_1 + \langle n^{\mathrm{ch}}_{\mathrm{Pb+Pb}}\rangle\, \omega_{\mathrm{P}}\,, \qquad \text{where } \langle n^{\mathrm{ch}}_{\mathrm{Pb+Pb}}\rangle = \langle N^{\mathrm{ch}}_{\mathrm{Pb+Pb}}\rangle \big/ \langle N_{\mathrm{P}}\rangle\,. \tag{2}$$

- If Eq. (1) holds, then **the sources are not protons !** $\omega_1 \neq \omega_{\mathrm{p+p}}$
- It must be clear from the **LHC** data, because $\omega_{\mathrm{p+p}}$ grows with collision energy much faster then $\omega_{\mathrm{A+A}}$ due to **KNO scaling** in **p+p**.
- The analysis of the **ALICE** (Pb+Pb) and the **CMS** (p+p) data on fluctuations in the same acceptance window $|\Delta\eta| < 0.8$ gives

$$\omega^{\mathrm{ch}}_{\mathrm{Pb+Pb}} \simeq 3 < \omega^{\mathrm{ch}}_{\mathrm{p+p}} \simeq 7 \qquad \text{at the } \mathbf{LHC}, \qquad \text{(V.B. 1606.05358)}$$

- Then we have two unknowns, ω_1 and ω_{P}, in (2), and ω_1 **can not be uniquely defined**.

The raw moments $\langle N^k \rangle$ are directly related to central **moments** of a distribution $P(N)$

$$m_k = \sum (N - \langle N \rangle)^k P(N) .$$

Their combination gives the **scaled variance**, the **normalized skewness**, and the **normalized kurtosis**:

$$\omega = \frac{m_2}{\langle N \rangle} , \qquad S\sigma = \frac{m_3}{m_2} , \qquad \kappa\sigma^2 = \frac{m_4}{m_2} - 3\, m_2 , \qquad \text{where } \sigma^2 = m_2 .$$

They describe the **width**, the **asymmetry**, and the **sharpness** of a distribution with a single maximum, correspondingly.

Higher moments depend even stronger on the fluctuations of participants (V.B. 1606.05358):

$$S\sigma = \frac{\omega_1\, S_1 \sigma_1 + \langle n_1 \rangle\, \omega_{\mathbf{P}} [\, 3\,\omega_1 + \langle n_1 \rangle\, S_{\mathbf{P}}\, \sigma_{\mathbf{P}} \,]}{\omega_1 + \langle n_1 \rangle\, \omega_{\mathbf{P}}} ,$$

$$\kappa\sigma^2 = \frac{\omega_1\, \kappa_1 \sigma_1^2 + \omega_{\mathbf{P}} \,[\, \langle n_1 \rangle^3 \kappa_{\mathbf{P}}\, \sigma_{\mathbf{P}}^2 + \langle n_1 \rangle\, \omega_1 \,(\, 3\,\omega_1 + 4\, S_1 \sigma_1 + 6 \langle n_1 \rangle\, S_{\mathbf{P}}\, \sigma_{\mathbf{P}} \,)\,]}{\omega_1 + \langle n_1 \rangle\, \omega_{\mathbf{P}}} .$$

- At the **LHC** $0 \lesssim \omega_1 \lesssim 12$ and
- all the situations are possible

$\omega_1 \gg \langle n \rangle\, \omega_P$ 'Maximal',

$\omega_1 \ll \langle n \rangle\, \omega_P$ 'Poisson' and

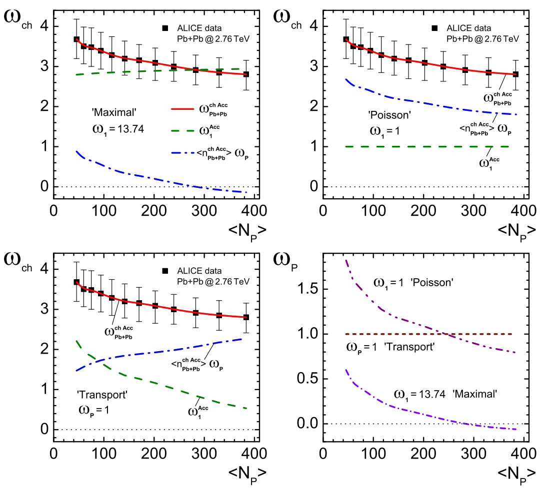
$\omega_1 \simeq \langle n \rangle\, \omega_P$ 'Transport'

For **small participant fluctuations**:

$$\omega = \omega_1 + \langle n_1 \rangle\, \omega_P$$
$$S\,\sigma \simeq S_1\,\sigma_1 + 3\langle n_1 \rangle\, \omega_P$$
$$\kappa\,\sigma^2 \simeq \kappa_1\,\sigma_1^2 + \langle n_1 \rangle\, \omega_P\,(3\omega_1 + 4S_1\,\sigma_1)$$

For **large participant fluctuations**

$$\omega \simeq \langle n_1 \rangle\, \omega_P$$
$$S\,\sigma \simeq \langle n_1 \rangle\, S_P\,\sigma_P + 3\omega_1$$
$$\kappa\,\sigma^2 \simeq \langle n_1 \rangle^2 \kappa_P\,\sigma_P^2 + 6\langle n_1 \rangle\, \omega_1\, S_P\,\sigma_P$$



V.B. 1606.05358

... is used by **STAR** and **ALICE**, and means that a value X is measured in r sub-samples, and then summed up with the relative weight of the sub-sample mean multiplicity (X. Luo for the STAR Collaboration 1106.2926):

$$X = \sum_r w_r X_r \,, \qquad w_r = \langle n_r \rangle \big/ \langle N \rangle \,,\ \ \langle N \rangle = \sum_r \langle n_r \rangle \,.$$

The width of the sub-sample $\langle n_r \rangle$ is chosen as small as possible.

- The **CBWE procedure is not working** if $\omega_P > 0$ for $\langle n_r \rangle \to 0$
- **We** (V.B. and M.M.-P.) **checked** that **this is the case** for the net-charge fluctuations in Ar+Sc at 150A GeV/c in EPOS model
- For the particular case of small fluctuations of participants ω_P, $S_P \sigma_P \ll \omega_1 / \langle n_1 \rangle$, and $\kappa_P \sigma_P^2 \ll \omega_1^2 / \langle n_1 \rangle^2$, and also small skewness of the source $S_1 \sigma_1 \ll 3 \langle n_1 \rangle \omega_P$ **one can find the scaled variance and normalized kurtosis from one source:**

$$\omega_1 \simeq \omega - \frac{S\,\sigma}{3} \,, \qquad \kappa_1 \sigma_1^2 \simeq \kappa\, \sigma^2 - \omega_1\, S\, \sigma \,.$$

Conclusions

- **The sources** in the independent participant model **are not protons at the SPS and LHC**
- **Higher moments depend even stronger on the fluctuations of participants**
- The **CBWE procedure** used by the **STAR** and the **ALICE** may **not work**
- The case when **one can determine the fluctuations from one source using high order fluctuations** is found