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vBag – vector interaction in an extended bag model.

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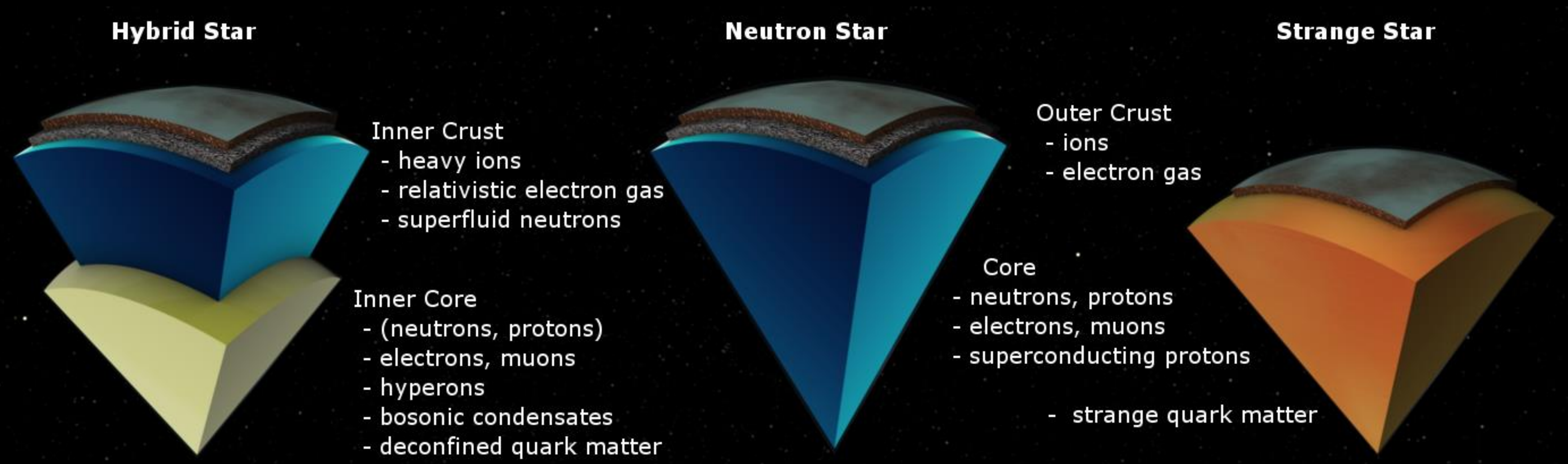
Neutron Stars = Quark Cores?

- Variety of scenarios regarding inner structure: with or without QM
- Question whether/how QCD phase transition occurs is not settled
- Most honest approach: take both (and more) scenarios into account and compare to available data



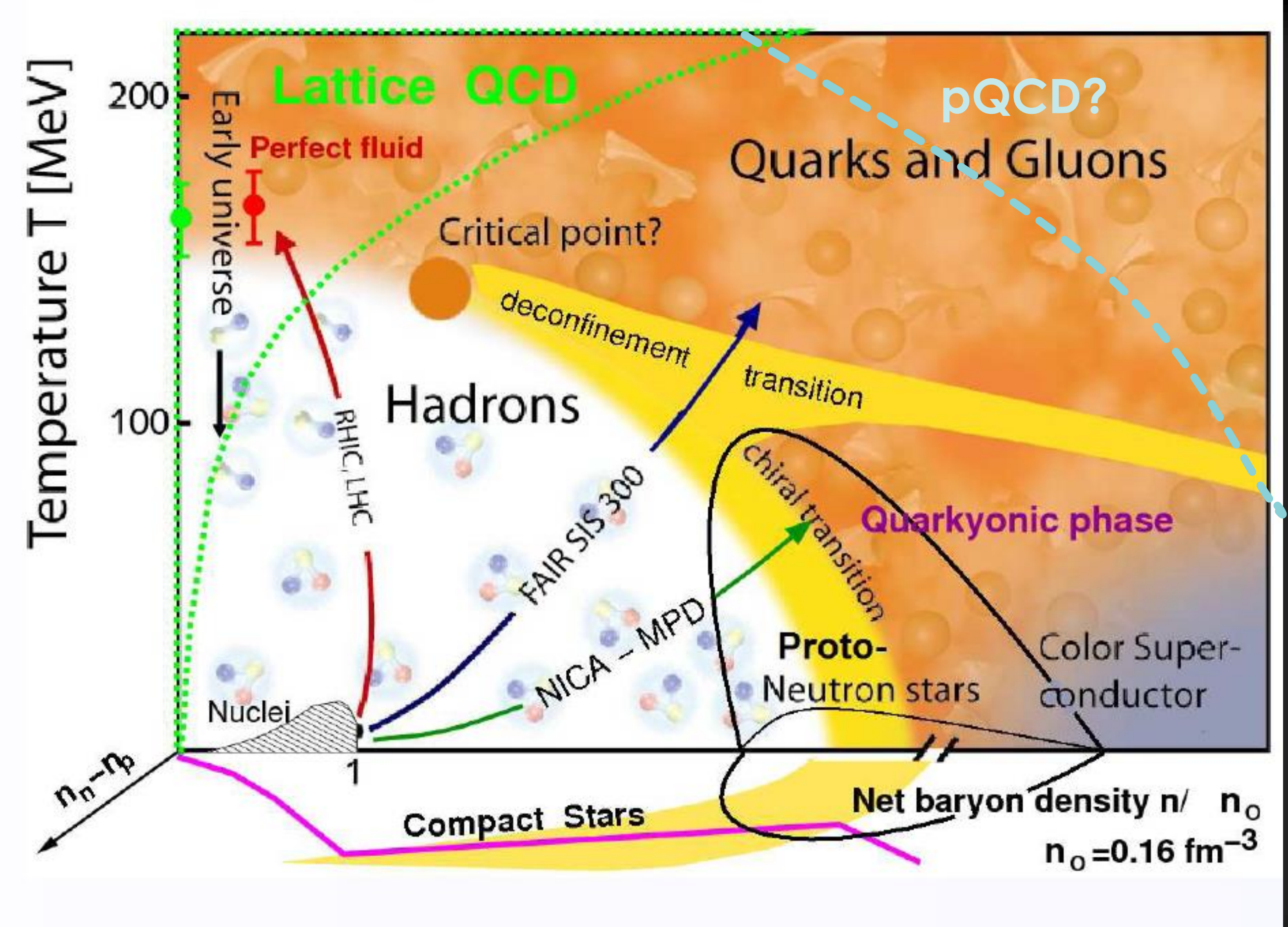
QCD Phase Diagram

- **dense hadronic matter**

<u>HIC in collider experiments</u>
Won't cover the whole diagram
Hot and 'rather' symmetric

<u>NS as a 2nd accessible option</u>
Cold and 'rather' asymmetric

Problem is more complex than
It looks at first gaze



Quark Matter

What is so special about quarks?

Confinement: No isolated quark has ever been observed
Quarks are confined in baryons and mesons

Dynamical Mass Generation:

Proton 940 MeV, 3 constituent quarks with each 5 MeV
→ 98.4% from .... somewhere?

and then this:
eff. quark mass in proton: 940 MeV/3 ≈ 313 MeV
eff. quark mass in pion : 140 MeV/2 = 70 MeV

quark masses generated by interactions only
‚out of nothing'
interaction in QCD through (self interacting) gluons
dynamical chiral symmetry breaking (DCSB)
is a distinct nonperturbative feature!

Confinement and DCSB are connected. Not trivially seen from QCD Lagrangian.
Investigating quark-hadron phase transition requires nonperturbative approach.

Quark Matter

Confinement and DCSB are features of QCD.
It would be too nice to account for these phenomena when describing QM in Compact Stars...

Current reality is:

Bag-Model :

While Bag-models certainly account for confinement (constructed to do exactly this) they do not exhibit DCSB (quark masses are fixed).

Chodos, Jaffe et al: Baryon Structure (1974)
Farhi, Jaffe: Strange Matter (1984)

NJL-Model :

While NJL-type models certainly account for DCSB (applied, because they do) they do not (trivialy) exhibit confinement.

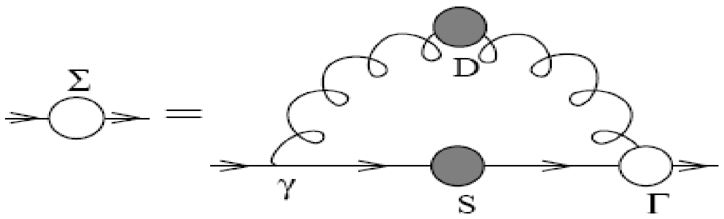
Nambu, Jona-Lasinio (1961)

Modifications to address these shortcomings exist (e.g. PNJL)
Still holds: Inspired by, but not based on QCD.

Lattice QCD still fails at T=0 and finite μ

Dyson-Schwinger Approach

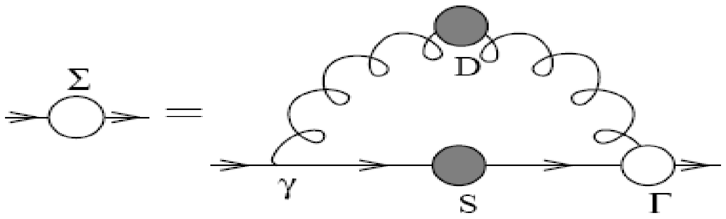
Derive gap equations from QCD-Action. Self consistent self energies.
Successfuly applied to describe meson and hadron properties
Extension from vacuum to finite densities desirable
→ EoS within QCD framework
→ **THIS TALK: Bag and NJL model as simple limits within DS approach**



Dyson Schwinger Perspective

One particle gap equation(s)

$$S^{-1}(p;\mu) = i\vec{\gamma}\vec{p} + i\gamma_4(p_4 + i\mu) + m + \Sigma(p;\mu)$$



Self energy -> entry point for simplifications

$$\Sigma(p;\mu) = \int_\Lambda \frac{d^4q}{(2\pi)^4} g^2 D_{\rho\sigma}(p-q)\gamma_\rho \frac{\lambda^a}{2} S(q)\Gamma^a_\sigma(p;q)$$

General (in-medium) gap solutions

$$S^{-1}(p;\mu) = i\vec{\gamma}\vec{p}A(p;\mu) + i\gamma_4(p_4 + i\mu)C(p;\mu) + B(p;\mu)$$

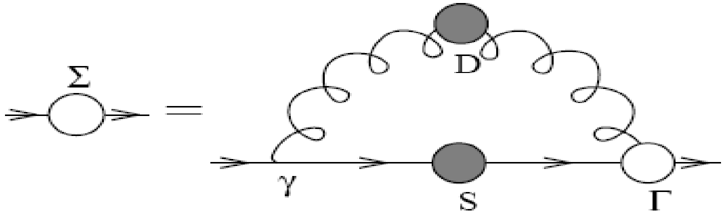
DSE -> NJL model

$$g^2 D_{\rho\sigma}(p-q) = \frac{1}{m_G^2}\delta_{\rho\sigma},$$

Gluon contact interaction in configuration space (other models exist)

$$\Gamma_\rho^a(p;q) = \frac{\lambda^a}{2}\gamma_\rho.$$

Rainbow approximation



$$\vec{p}^2 A_p = \vec{p}^2 + \frac{8N_c}{9m_G^2}\int_\Lambda \frac{d^4q}{(2\pi)^4}\frac{\vec{p}\vec{q}A_q}{\vec{q}^2 A_q^2 + \widetilde{q}_4^2 C_q^2 + B_q^2},$$

$$A = 1$$

$$B_p = m + \frac{16N_c}{9m_G^2}\int_\Lambda \frac{d^4q}{(2\pi)^4}\frac{B_q}{\vec{q}^2 A_q^2 + \widetilde{q}_4^2 C_q^2 + B_q^2},$$

$$B_\mu = m + \frac{4N_c}{9m_G^2} n_s(T,\mu^*,B),$$

$$\widetilde{p}_4^2 C_p = \widetilde{p}_4^2 + \frac{8N_c}{9m_G^2}\int_\Lambda \frac{d^4q}{(2\pi)^4}\frac{\widetilde{p}_4\widetilde{q}_4 C_q}{\vec{q}^2 A_q^2 + \widetilde{q}_4^2 C_q^2 + B_q^2},$$

$$\mu = \mu^* - \frac{2N_c}{9m_G^2} n_v(T,\mu^*,B),$$

$$\widetilde{p}_4 C = p_4 + i(\mu + \omega_\mu) \equiv \hat{p}_4$$

Thermodynamical Potential

DS: steepest descent

$$P[S] = \mathrm{Tr}\ln[S^{-1}] - \frac{1}{2}\mathrm{Tr}[\Sigma S].$$

NJL model is easily understood as a particular approximation of QCD's DS gap equations

$$P_{FG} = \mathrm{Tr}\ln S^{-1} = 2N_c \int_\Lambda \frac{d^4p}{(2\pi)^4} \ln(\vec{p}^2 + \hat{p}_4^2 + B_\mu^2)$$

$$P_I = -\frac{1}{2}\mathrm{Tr}\Sigma S = \frac{3}{4} m_G^2 \omega_\mu^2 - \frac{3}{8} m_G^2 \phi_\mu^2$$

Compare to NJL type model with following Lagrangian (interaction part only):

$$\mathcal{L}_\mathrm{I} = \mathcal{L}_\mathrm{S} + \mathcal{L}_\mathrm{V} = G_s \sum_{a=0}^{8} (\bar{q}\tau_a q)^2 + G_v(\bar{q}i\gamma_0 q)^2.$$

$$\Omega_q = \Omega_q^0 + \frac{\phi^2}{4G_s} - \frac{\omega^2}{2G_v} - \Omega_q(T = \mu = 0)$$

$$\phi_\mu = 2G_s N_c n_s(T, m_f^*, \mu_f^*)$$

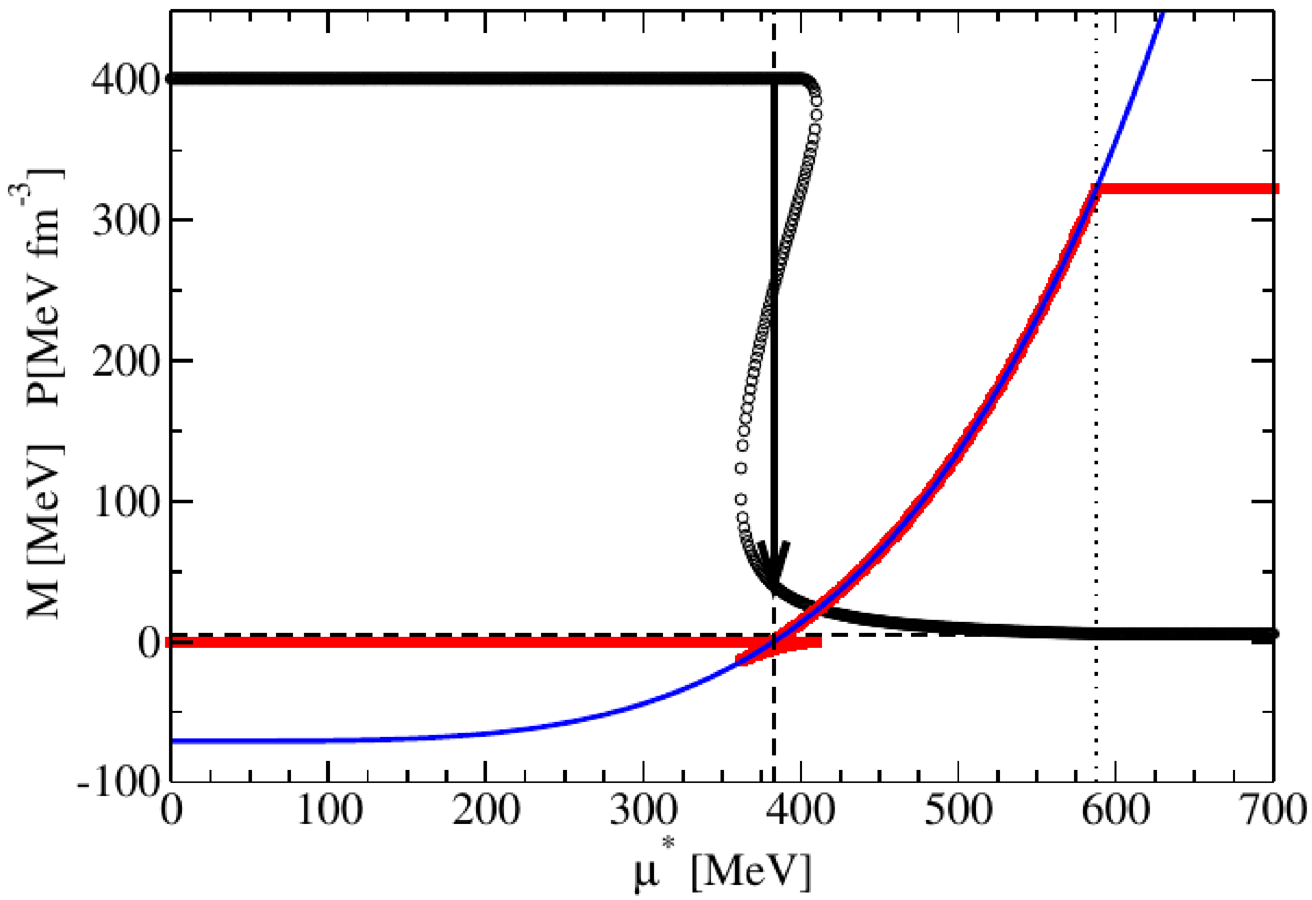
$$\omega_\mu = -2G_s N_c n_v(T, m_f^*, \mu_f^*)$$

$$\frac{\partial \Omega_q}{\partial \phi_\mu} = \frac{\partial \Omega_q}{\partial \omega_\mu} = 0.$$

Bag Model from NJL perspective (T.Klahn, T.Fischer, ApJ, accepted)

obvious differences between NJL and Bag:

- **DχSB**
- confinement
- vector interaction



u,d-quark

Mass

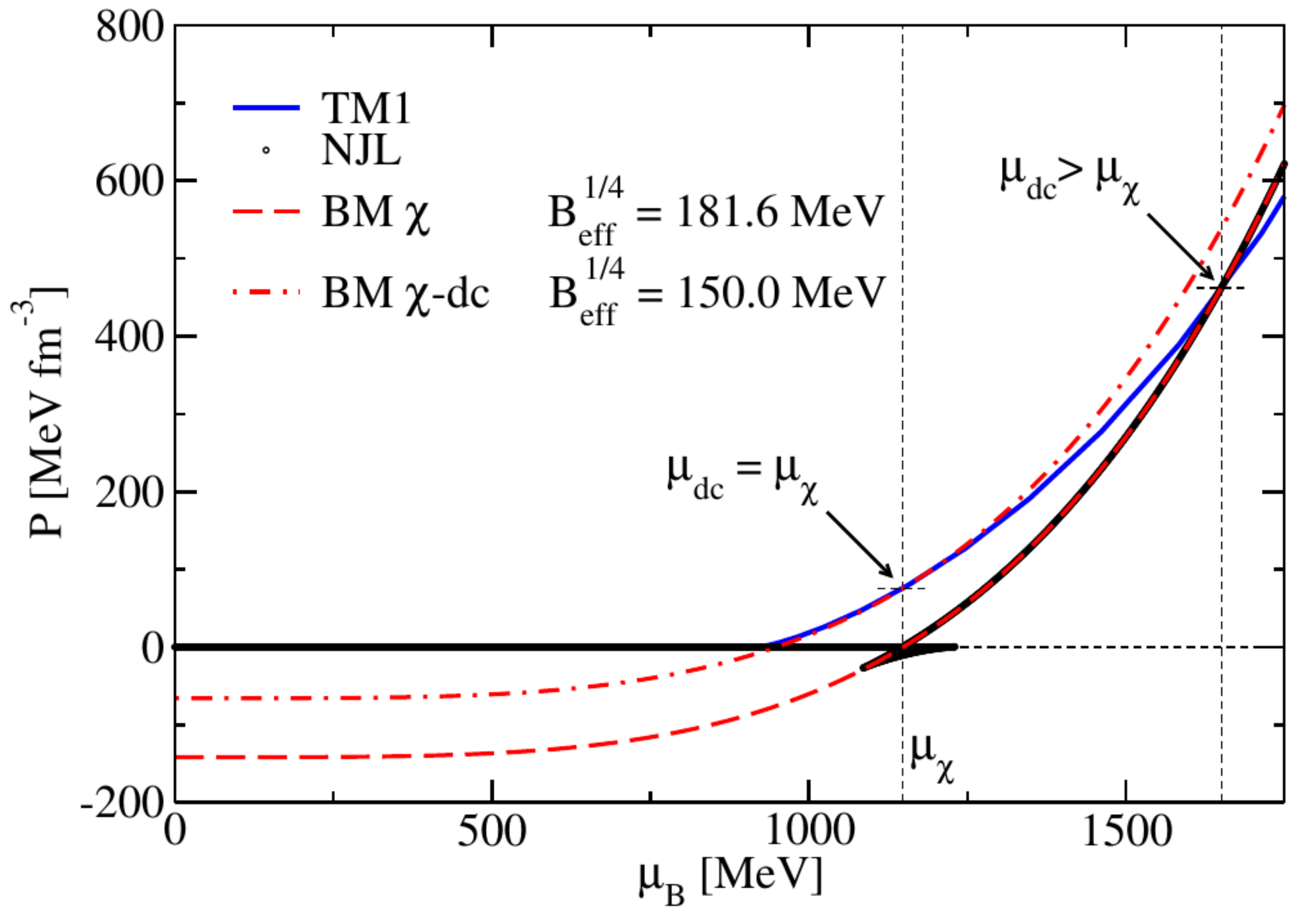
Pressure NJL

Pressure Ideal Gas - Bag

Bag Model from NJL perspective

obvious differences between NJL and Bag:

- DχSB
- **confinement**
- vector interaction



confinement

Pressure Quark NJL/Bag
Pressure Nuclear Matter

Obviously not zero at χ transition
Reduce χ bag pressure – by hand

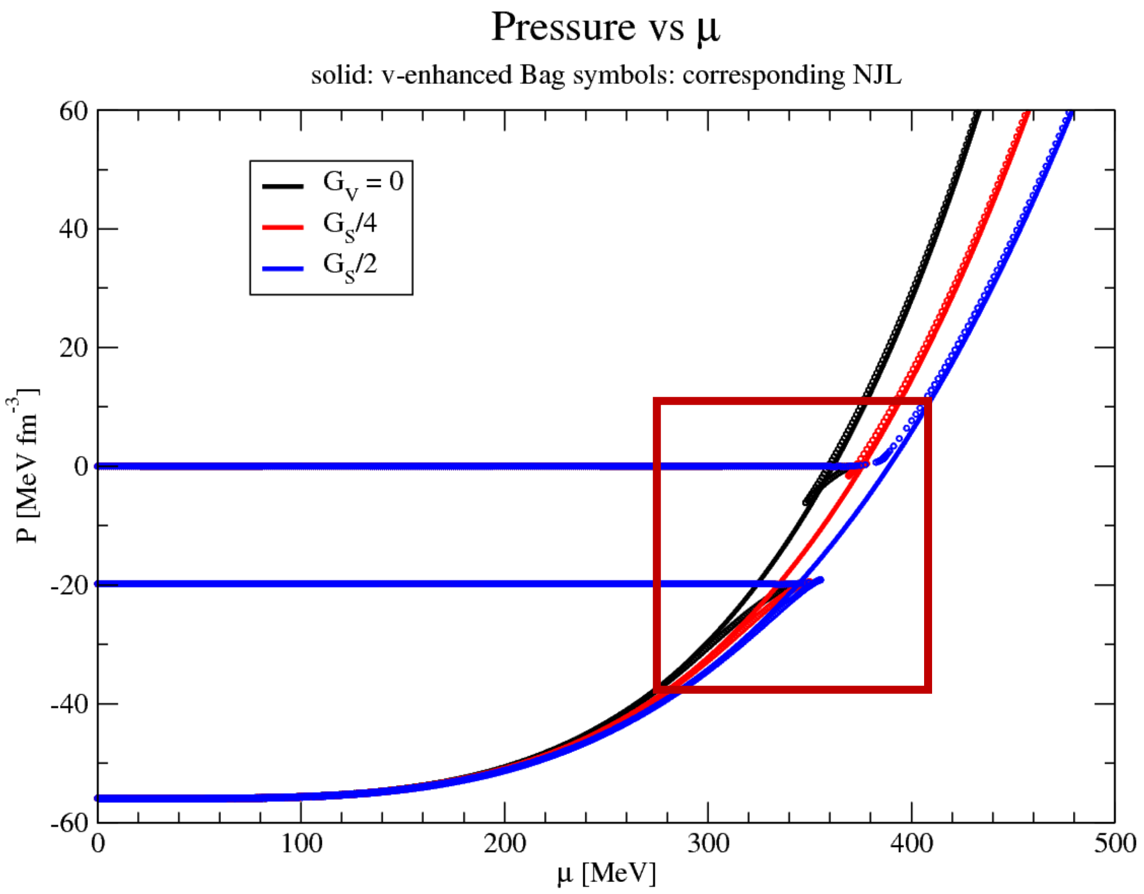
Bag Model from NJL perspective

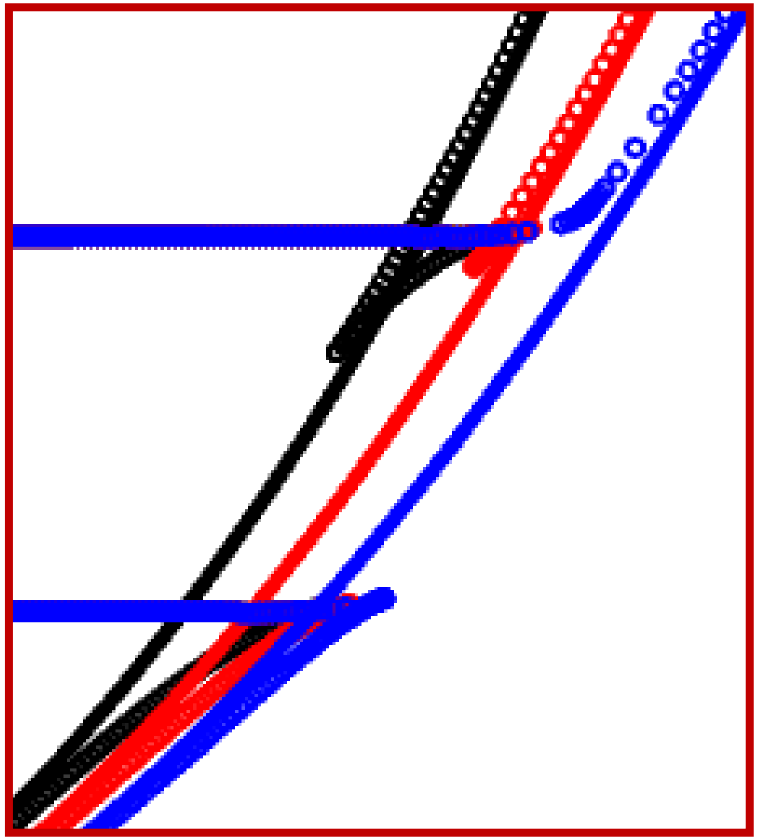
obvious differences between NJL and Bag:

- DχSB
- confinement
- **vector interaction**

$$B_\mu = m + \frac{4N_c}{9m_G^2} n_s(T, \mu^*, B),$$

$$\mu = \mu^* - \frac{2N_c}{9m_G^2} n_v(T, \mu^*, B),$$





vBag: vector interaction enhanced bag model

Chiral + Vector:

$$P^i_{BM}(\mu_i) = P_{kin}(\mu^*_i) + \frac{K_v}{2} n^2_v(\mu^*_i) - P^i_{BAG}$$

$$\varepsilon^i_{BM}(\mu_i) = \varepsilon_{kin}(\mu^*_i) + \frac{K_v}{2} n^2_v(\mu^*_i) + P^i_{BAG}$$

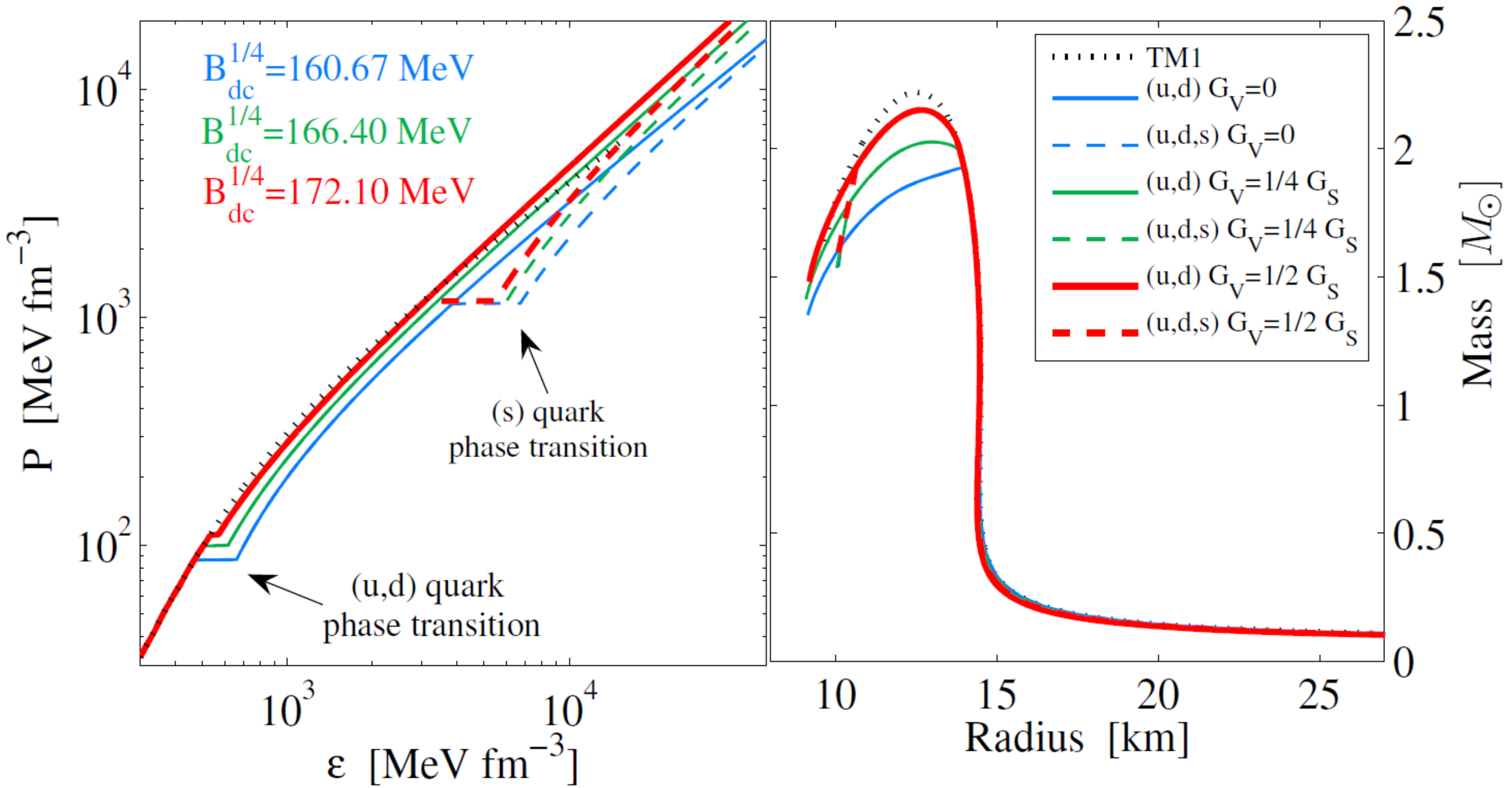
$$\mu_i = \mu^*_i + K_v n_v(T, \mu^*_i)$$

'Confinement':

$$P = \sum_f P^{kin}_f - B_{eff} \text{ with } B_{eff} = \sum_f B^f_\chi - B_{\rm dc}$$

And, of course, chiral+vector+'confinement' (Klahn & Fischer arXiv:1503.07442 ApJ accepted)

Neutron Stars with QM core – vBAG vs BAG



Conclusions Part I

Vector enhanced bag like model can be derived from NJL - which can be obtained from DS gap equations

Bag model character: bare quark masses
effective bag pressure

Difference:
chiral bag pressure as consequence of DχSB, flavor dependence
confining bag pressure with opposite sign (binding energy)
accounts for vector interaction -> stiff EoS, promising for astrophysical applications

What NJL couldn't: bag pressure due to deconfinement -> subtracted by hand without harm to td consistence

Advantage of the model: extremely simple to use, no regularization required

$$P^i_{BM}(\mu_i) = P_{kin}(\mu_i^*) + \frac{K_v}{2} n_v^2(\mu_i^*) - P^i_{BAG}$$

$$\varepsilon^i_{BM}(\mu_i) = \varepsilon_{kin}(\mu_i^*) + \frac{K_v}{2} n_v^2(\mu_i^*) + P^i_{BAG}$$

$$\mu_i = \mu_i^* + K_v n_v(T, \mu_i^*)$$

$$P = \sum_f P_f^{kin} - B_{eff} \text{ with } B_{eff} = \sum_f B_\chi^f - B_{\rm dc}$$

Conclusions Part II

vBAG:

- vector interaction resolves the problem of too soft bag model EoS w/o perturbative corrections
- No problem at all to obtain stable hybrid neutron star configurations
- Standard BAG models bag constant is understood to mimic confinement, DχSB is absent
- vBAG introduces effective bag constant with similar values to original BAG

$$B_{eff} = \sum_f B_\chi^f - B_{\rm dc}$$

- However, positive value due to chiral transition, deconfinement actually reduces B
- Absolutely stable strange matter likely ruled out due to DχSB

- NJL and Bag model result from particular approximations within Dyson-Schwinger approach
  rainbow approximation (quark-gluon vertex) + contact interaction (gluon propagator)
- Consequence: both models lack momentum dependent gap solutions

A little teaser of our current work…

Munczek/Nemirowsky -> NJL's complement

Wigner Phase

$$\frac{\mathcal{G}(k^2)}{k^2} = 8\pi^4 D\delta^4(k) + \frac{4\pi^2}{\omega^6} D k^2 \mathrm{e}^{-k^2/\omega^2} + 4\pi \frac{\gamma_m \pi}{\frac{1}{2}\ln\left[\tau + \left(1 + k^2/\Lambda_{\mathrm{QCD}}^2\right)^2\right]} \mathcal{F}(k^2)$$

$B_W = 0, A_W = C_W$:

$$C_W(p,\mu) = \frac{1}{2}\left(1 + \sqrt{1 + \frac{2\eta^2}{p_3^2 + (p_4 + i\mu)^2)}}\right)$$

Nambu Phase

$A_N = C_N$.

$\Re(\tilde{p}^2) < \frac{\eta^2}{4}$:

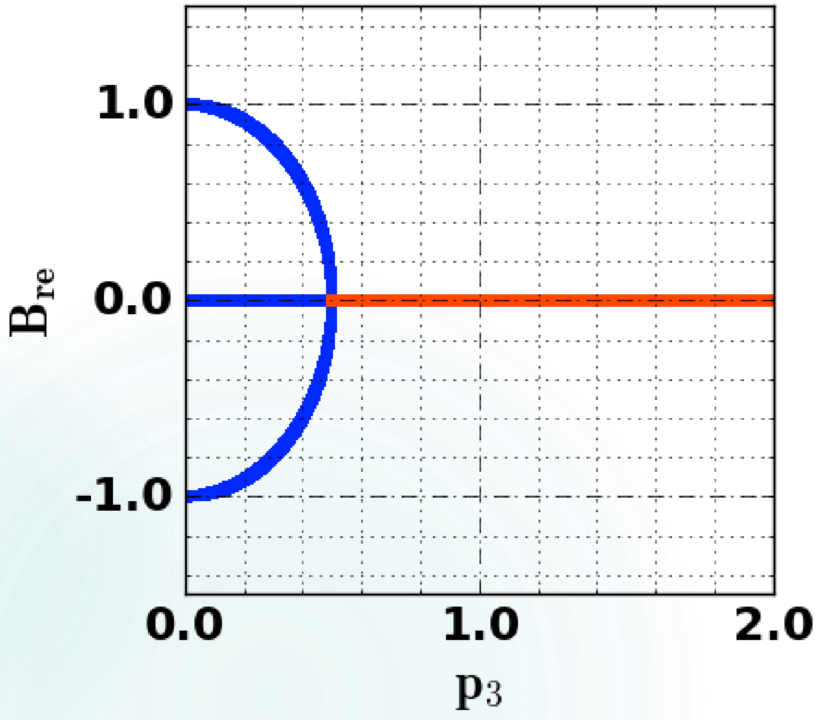
$$B_N(p,\mu) = \sqrt{\eta^2 - 4(p_3^2 + (p_4 + i\mu)^2))}$$
$$C_N(p,\mu) = 2$$

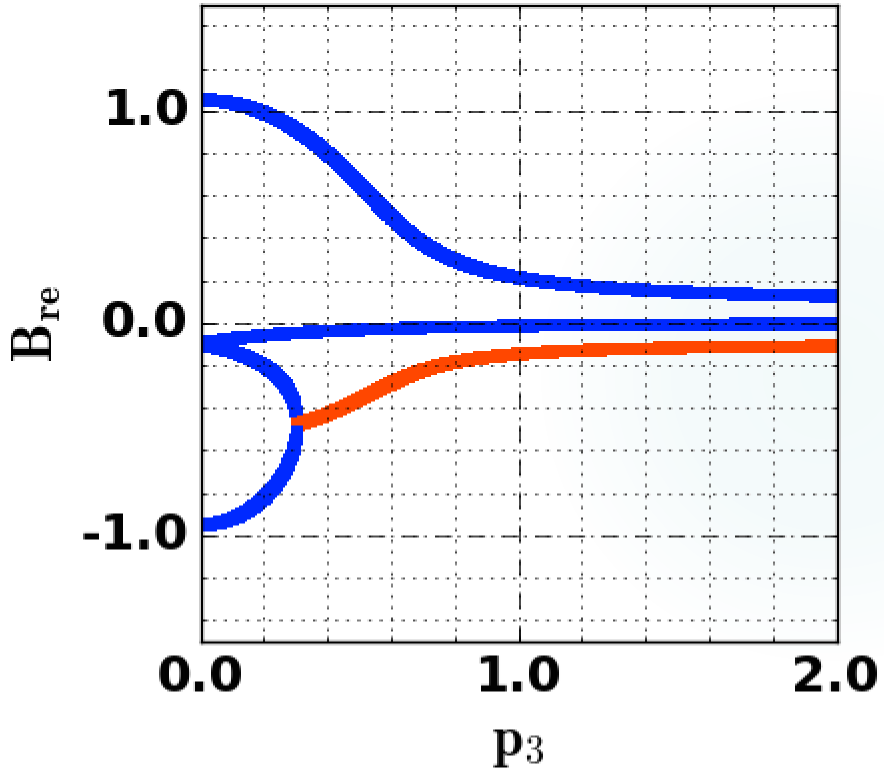
$\Re(\tilde{p}^2) > \frac{\eta^2}{4}$:

$$A_N = A_W, B_N = B_W, C_N = C_W.$$

MN antithetic to NJL
NJL:contact interaction in x
MN:contact interaction in p

In-medium quark mass momentum dependence





$$A_N = 2 \qquad B_N = \sqrt{\eta^2 - 4[\vec{p}^2 + (p_4 + i\mu)^2]}$$

$$B_W = 0 \qquad A_W = \frac{1}{2}\left(1 \pm \sqrt{1 + \frac{2\eta^2}{\vec{p}^2 + (p_4 + i\mu)^2}}\right)$$

$$B = m + \eta^2 \frac{B}{[\vec{p}^2 + (p_4 + i\mu)^2]A^2 + B^2} \quad A = \frac{2B}{B + m}$$

Thank you for your attention ☺

Space, time and matter are related via Einsteins Field Equations

$$G_{\mu\nu} = -8\pi G T_{\mu\nu}$$

Einstein Tensor $G_{\mu\nu}$

defined by metric

Energy Momentum Tensor $T_{\mu\nu}$

defined by equation of state

Approximations

non rotating, spheric symmetry

$$ds^2 = g_{\mu\nu} dx^\mu dx^\mu$$

$$\rightarrow g_{00}(r)dt^2 + g_{11}(r)dr^2 + g_{22}(r)d\theta^2 + g_{33}(r,\theta)d\phi^2$$

hydrostatic equilibrium

$$-pg^{\mu\nu} + (p+\varepsilon)u^\mu u^\nu$$

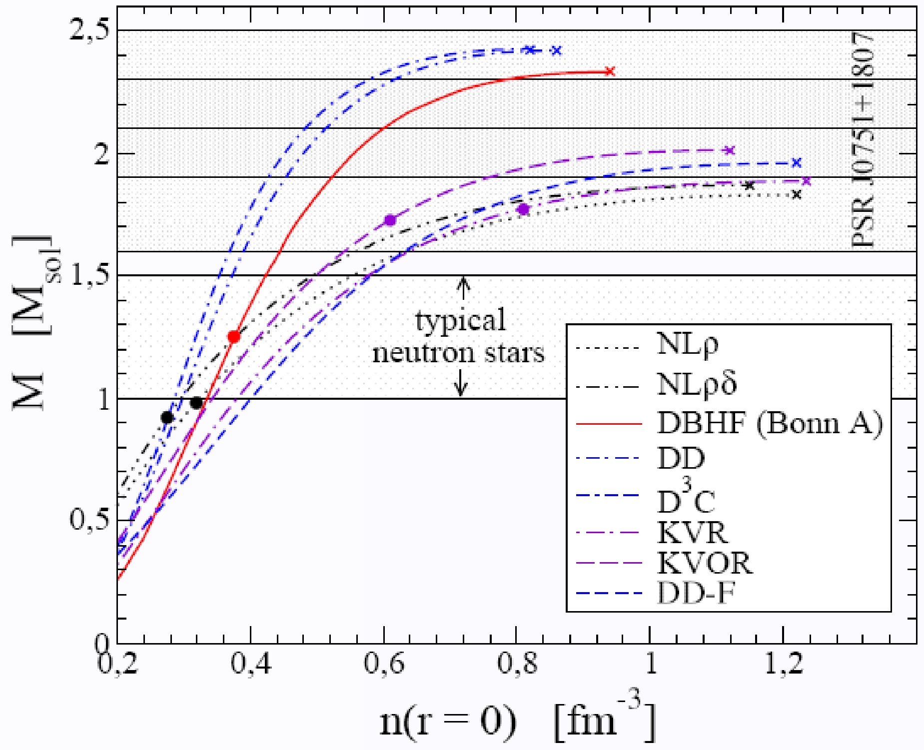
Tolman-Oppenheimer-Volkov (TOV) Equations (1939)

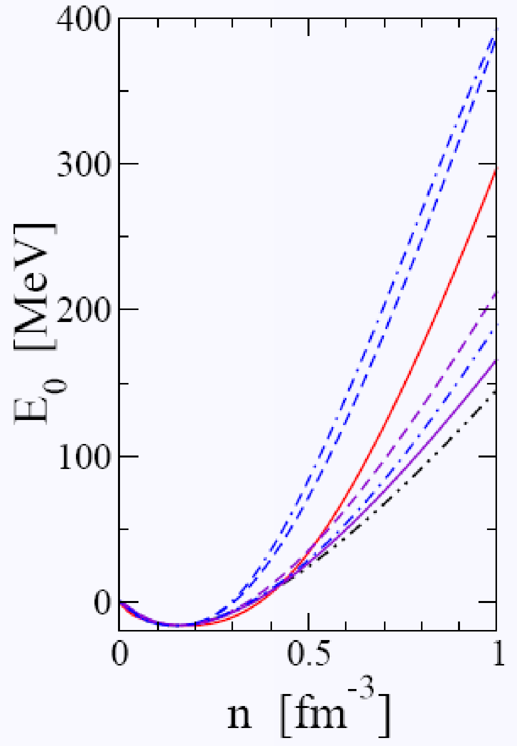
$$\frac{dp(r)}{dr} = -\frac{Gm(r)\varepsilon(r)}{r^2}\left(1+\frac{p(r)}{\varepsilon(r)}\right)\left(1+\frac{4\pi r^3 p(r)}{m(r)}\right)\left(1-\frac{2Gm(r)}{r}\right)^{-1}$$

$$m(r) = 4\pi\int_0^r dr'\, r'^2 \varepsilon(r')$$

NS masses and the (QM) Equation of State

- NS mass is sensitive mainly to the sym. EoS (In particular true for heavy NS)
- Folcloric: QM is soft, hence no NS with QM core
- Fact: QM is softer, but able to support QM core in NS
- Problem: (transition from NM to) QM is barely understood



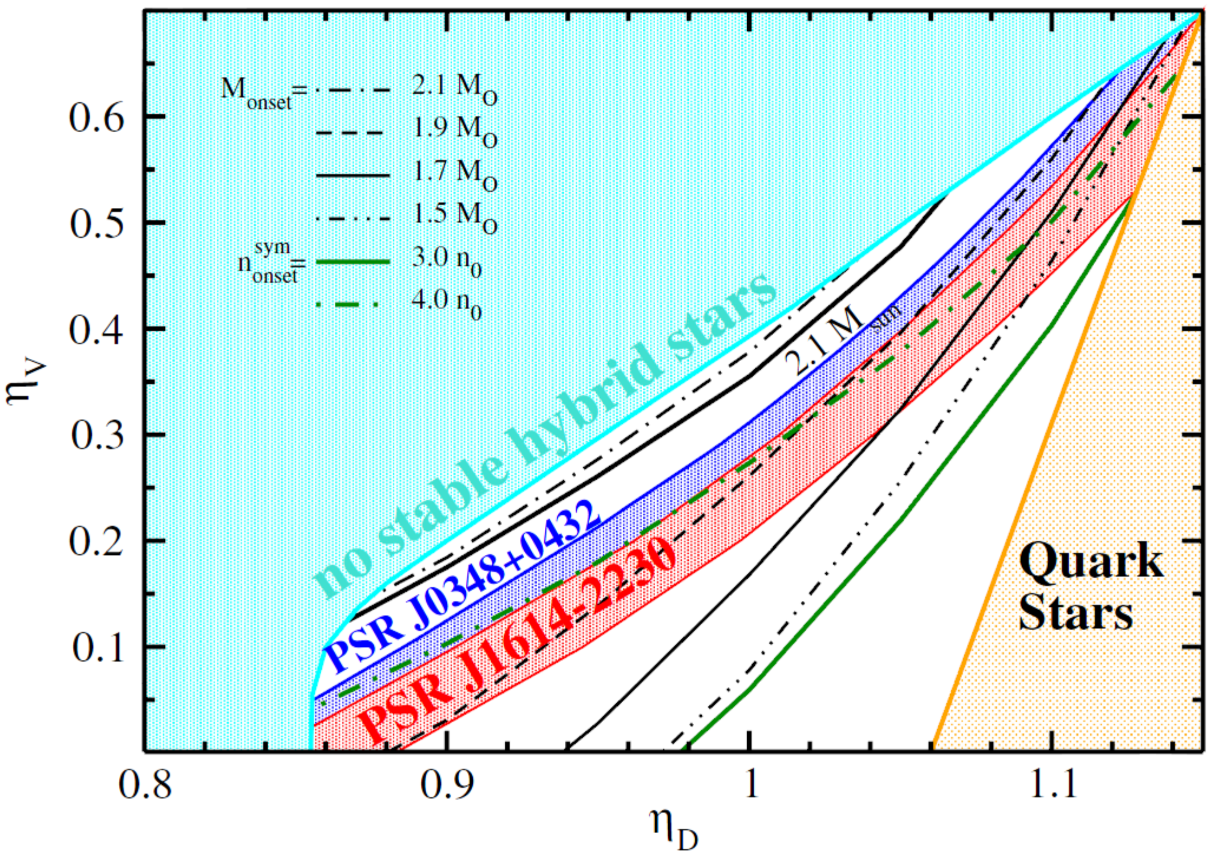


$M(n)$ correlated to $E_0(n)$

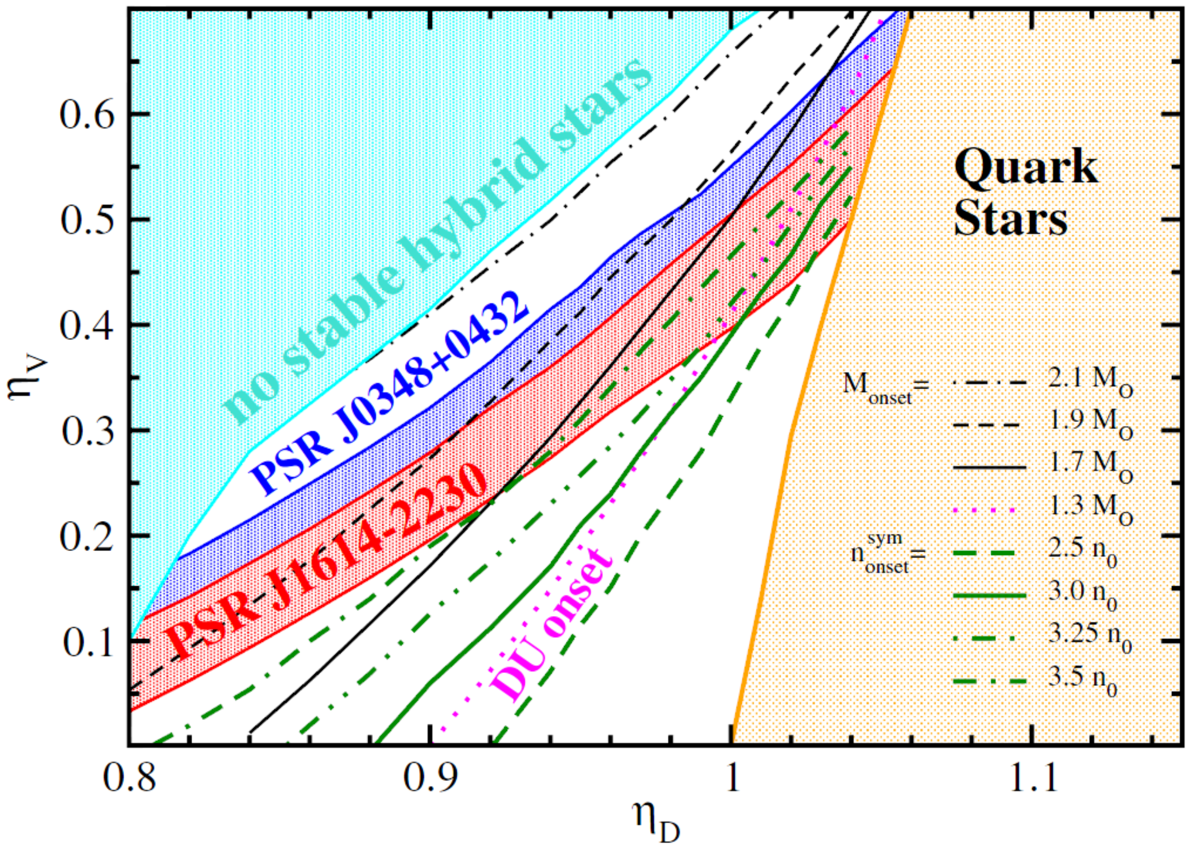
stiff: higher M_{max} at smaller densities

soft: smaller M_{max} at higher densities

NJL model study for NS (T.Klahn, R.Łastowiecki, D.Blaschke, PRD **88**, 085001 (2013))



Set A



Set B

Conclusion: NS may or may not support a significant QM core.
additional interaction channels won't change this if coupling strengths are not precisely known.

Effective gluon propagator

$$S(p;\mu)^{-1} = Z_2(i\,\vec{\gamma}\,\vec{p} + i\,\gamma_4(p_4 + i\mu) + m_{\rm bm}) + \Sigma(p;\mu)$$

$$\Sigma(p;\mu) = Z_1 \int_q^\Lambda g^2(\mu) D_{\rho\sigma}(p-q;\mu) \frac{\lambda^a}{2} \gamma_\rho S(q;\mu) \Gamma_\sigma^a(q,p;\mu)$$

Ansatz for self energy (rainbow approximation, effective gluon propagator(s))

$$Z_1 \int_q^\Lambda g^2 D_{\mu\nu}(p-q) \frac{\lambda^a}{2} \gamma_\mu S(q) \Gamma_\nu^a(q,p) \to \int_q^\Lambda \mathcal{G}((p-q)^2)\, D_{\mu\nu}^{\rm free}(p-q) \frac{\lambda^a}{2} \gamma_\mu S(q) \frac{\lambda^a}{2} \gamma_\nu$$

Specify behaviour o $\mathcal{G}(k^2)$

$$\frac{\mathcal{G}(k^2)}{k^2} = 8\pi^4 D \delta^4(k) + \frac{4\pi^2}{\omega^6} D k^2 \mathrm{e}^{-k^2/\omega^2} + 4\pi \frac{\gamma_m \pi}{\frac{1}{2}\ln\left[\tau + \left(1 + k^2/\Lambda_{\rm QCD}^2\right)^2\right]} \mathcal{F}(k^2)$$

Infrared strength (zero width + finite width contribution)

running coupling for large k

EoS (finite densities):

1st term (Munczek/Nemirowsky (1983)) delta function in momentum space → Klähn et al. (2010)

2nd term → Chen et al.(2008,2011)

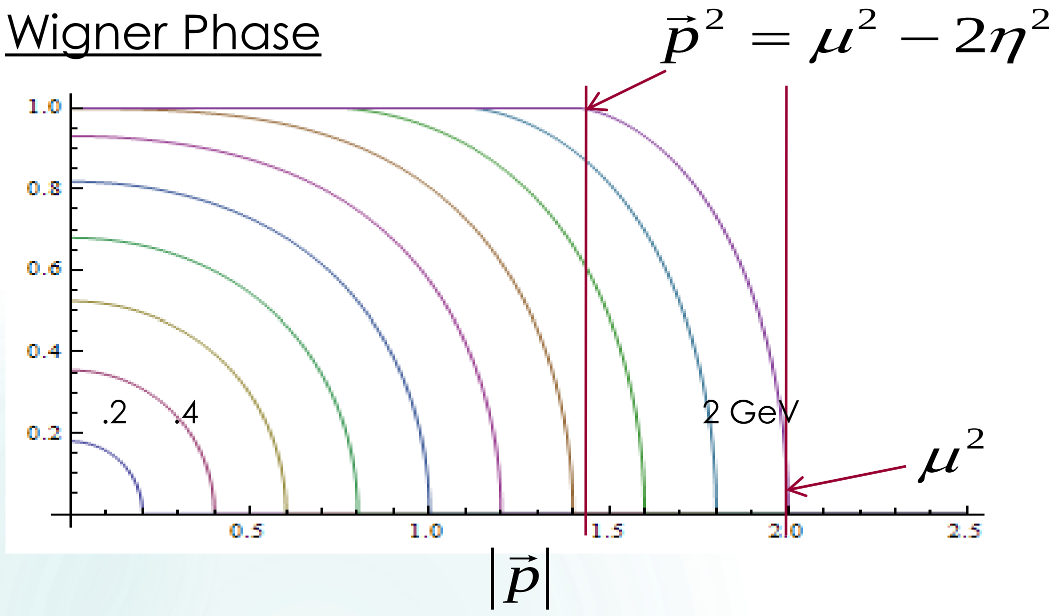
NJL model: $g^2 D_{\rho\sigma}(p-q) = \frac{1}{m_G^2} \delta_{\rho\sigma}$ delta function in configuration space = const. In mom. space

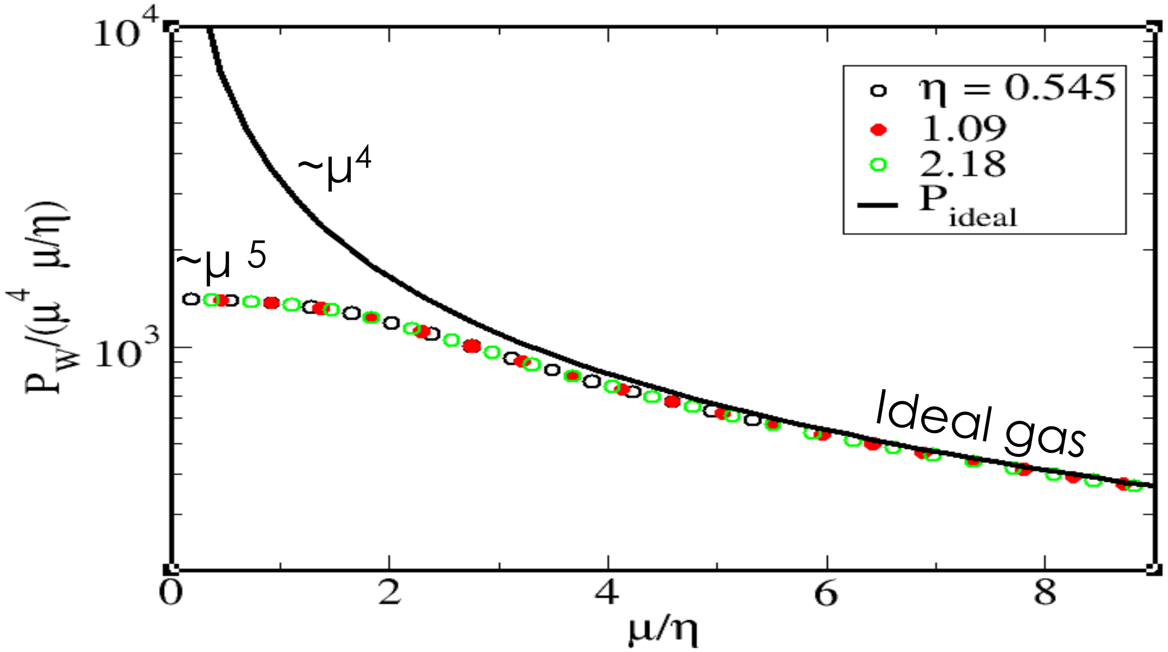
Munczek/Nemirowsky

$$f_1(|\vec{p}|;\mu) = \frac{1}{4\pi}\int_{-\infty}^{\infty} dp_4 \, \mathrm{tr_D}(-\gamma_4) S(p;\mu)$$

$$P(\mu<\eta) = P_0 + \int_0^\mu d\mu' \, n(\mu') \propto P_0 + const \times \mu^5$$

<u>Wigner Phase</u>





$\mu^2 \geq 2\eta^2$ to obtain $f_1(\vec{p}^2=0)=1$ model is scale invariant regarding μ/η

$P(\mu) \propto \mu^5$ well satisfied up to $\mu/\eta \approx 1$

$(\eta = 1.09\,\mathrm{GeV})$

‚small' chem. Potential: $f_1(\vec{p}^2=0, \mu<\eta) \propto \mu \leftarrow$ $n(\mu<\eta) = \frac{2N_c N_f}{2\pi^2}\int d^3\vec{p} \; f_1(|\vec{p}|) \propto \mu^4$

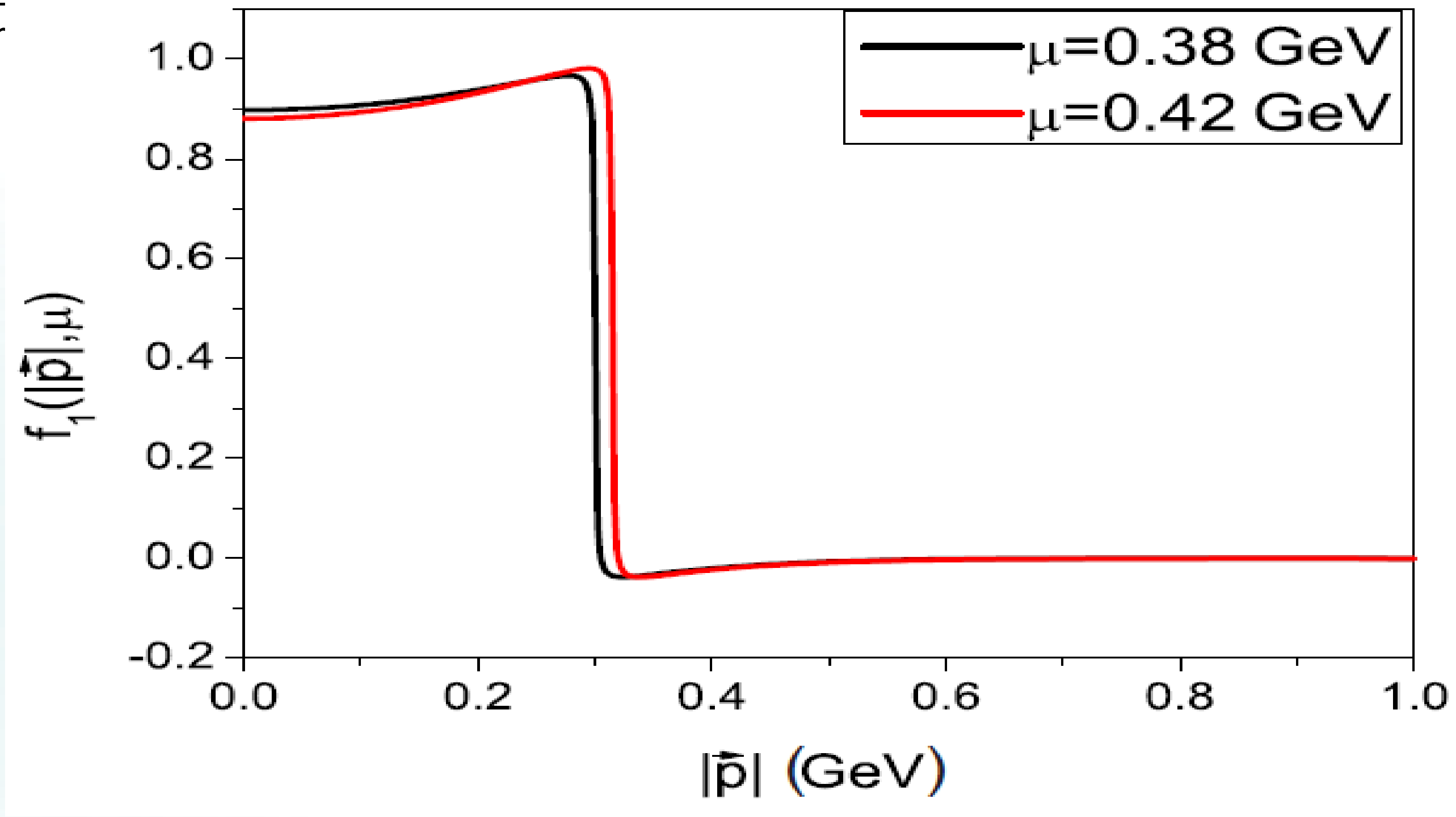
T. Klahn, C.D. Roberts, L. Chang, H. Chen, Y.-X. Liu PRC 82, 035801 (2010)

DSE – simple effective gluon coupling

$$\frac{\mathcal{G}(k^2)}{k^2} = 8\pi^4 D\delta^4(k) + \frac{4\pi^2}{\omega^6} D k^2 \mathrm{e}^{-k^2/\omega^2} + 4\pi \frac{\gamma_m \pi}{\frac{1}{2}\ln\left[\tau + \left(1 + k^2/\Lambda_{\mathrm{QCD}}^2\right)^2\right]} \mathcal{F}(k^2)$$

<u>Wigner Phase</u>

differ



Chen et al. (TK) PRD 78 (2008)