S-MATRIX APPROACH TO HADRON GAS

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XII POLISH WORKSHOP ON RHIC
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IN COLLABORATION WITH

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S-MATRIX APPROACH

HADRON RESONANCE GAS MODEL

- Confinement

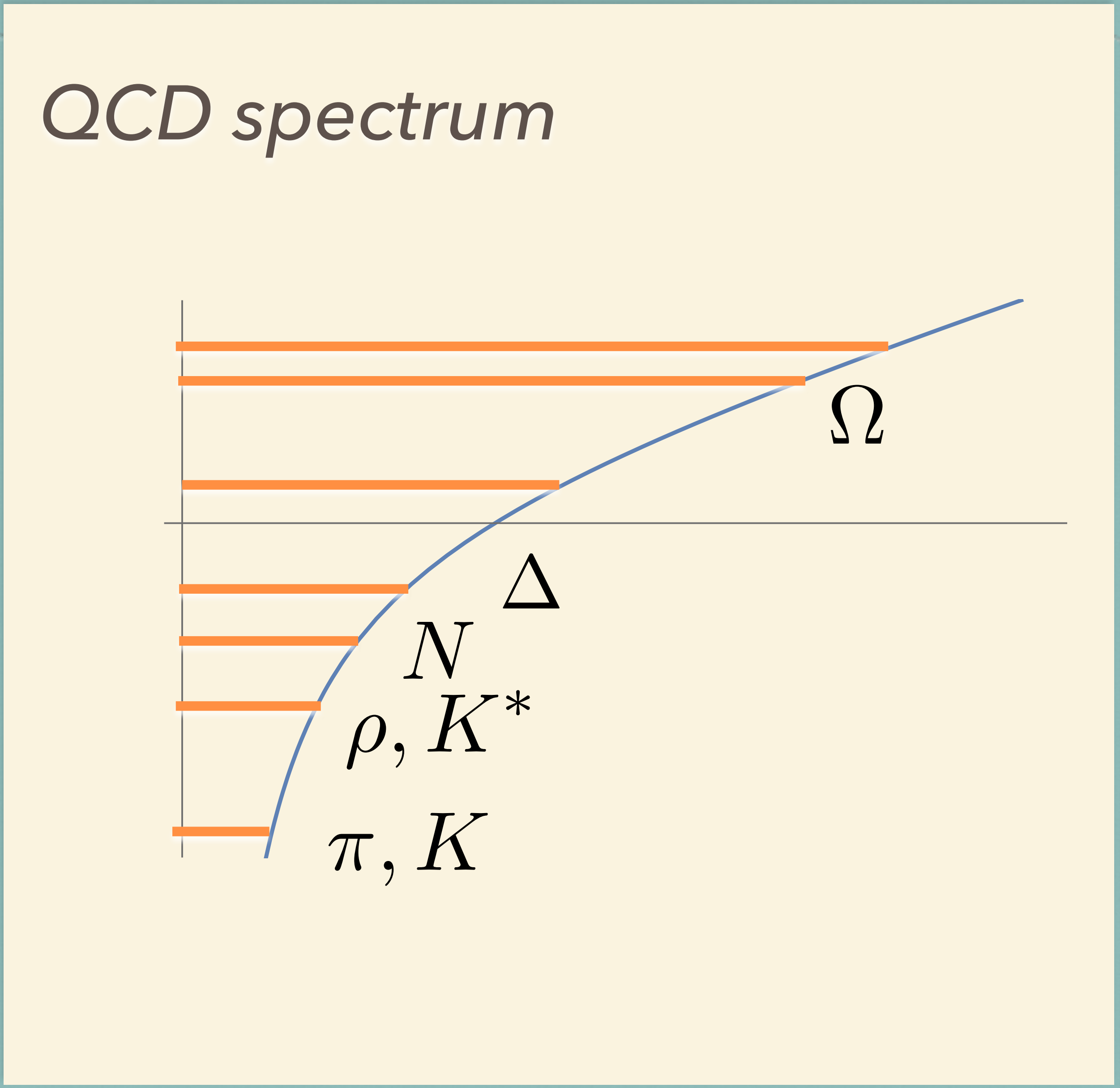
physical quantities → hadronic states representation

$$Z = \sum_{\alpha=B,M} \langle \alpha | e^{-\beta H} | \alpha \rangle$$

HADRON RESON MODEL

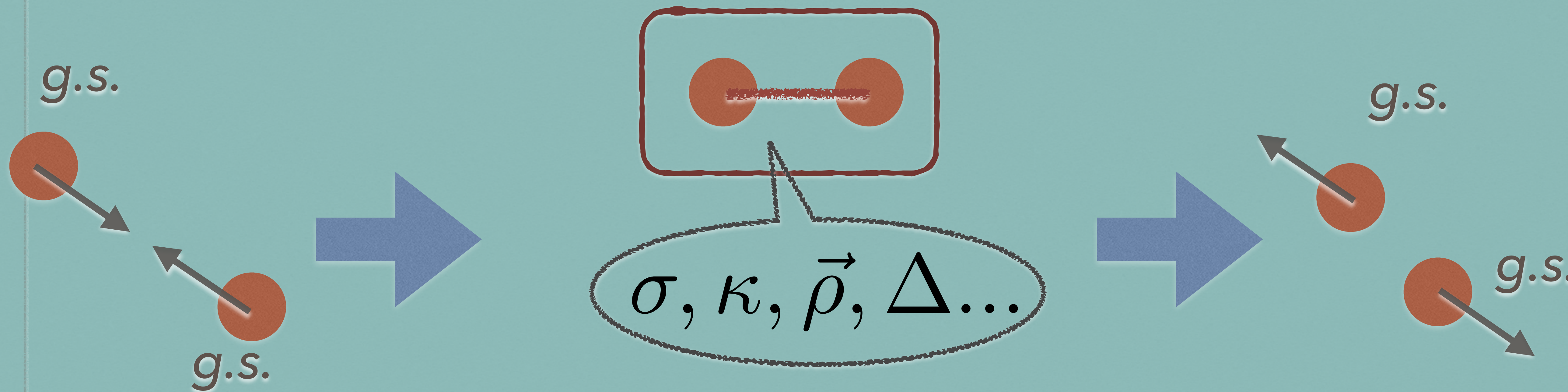
- Confinement

physical quantities



$$Z = \sum_{\alpha=B,M} \langle \alpha | e^{-\beta H} | \alpha \rangle$$

S-MATRIX APPROACH



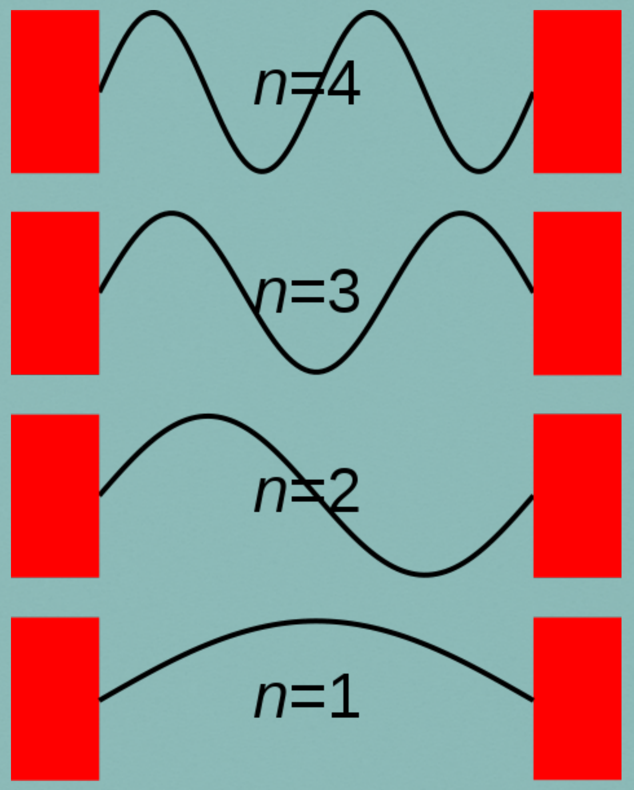
+ *repulsive channels*

interaction:
attractive + repulsive

PHASE SHIFT AND DENSITY OF STATES

particle in a box

$$\psi \sim \sin(k^{(0)}x) \qquad k^{(0)} = \frac{n\pi}{L}$$



PHASE SHIFT AND DENSITY OF STATES

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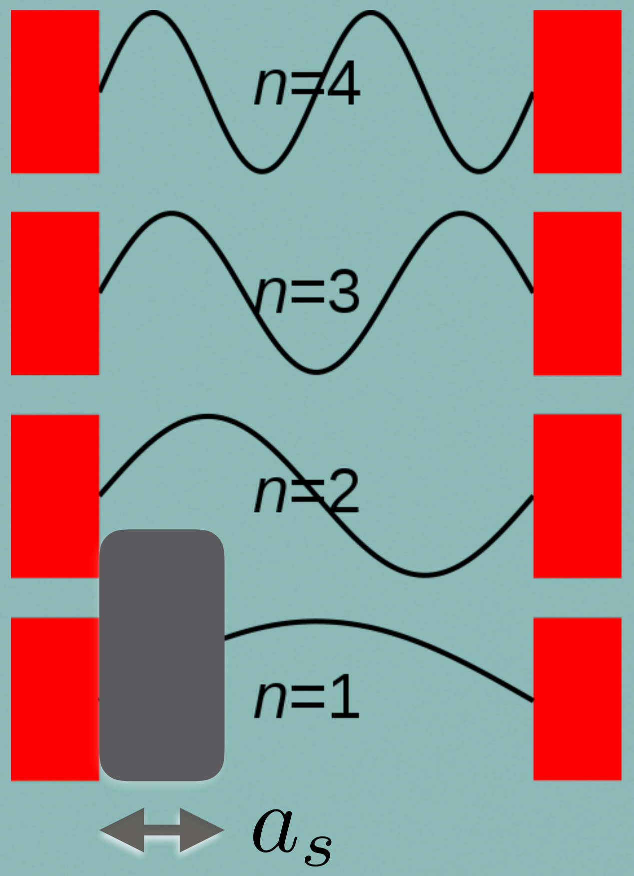


in the presence of a scattering potential

$$\psi \sim \sin(kx + \delta(k))$$

density of states

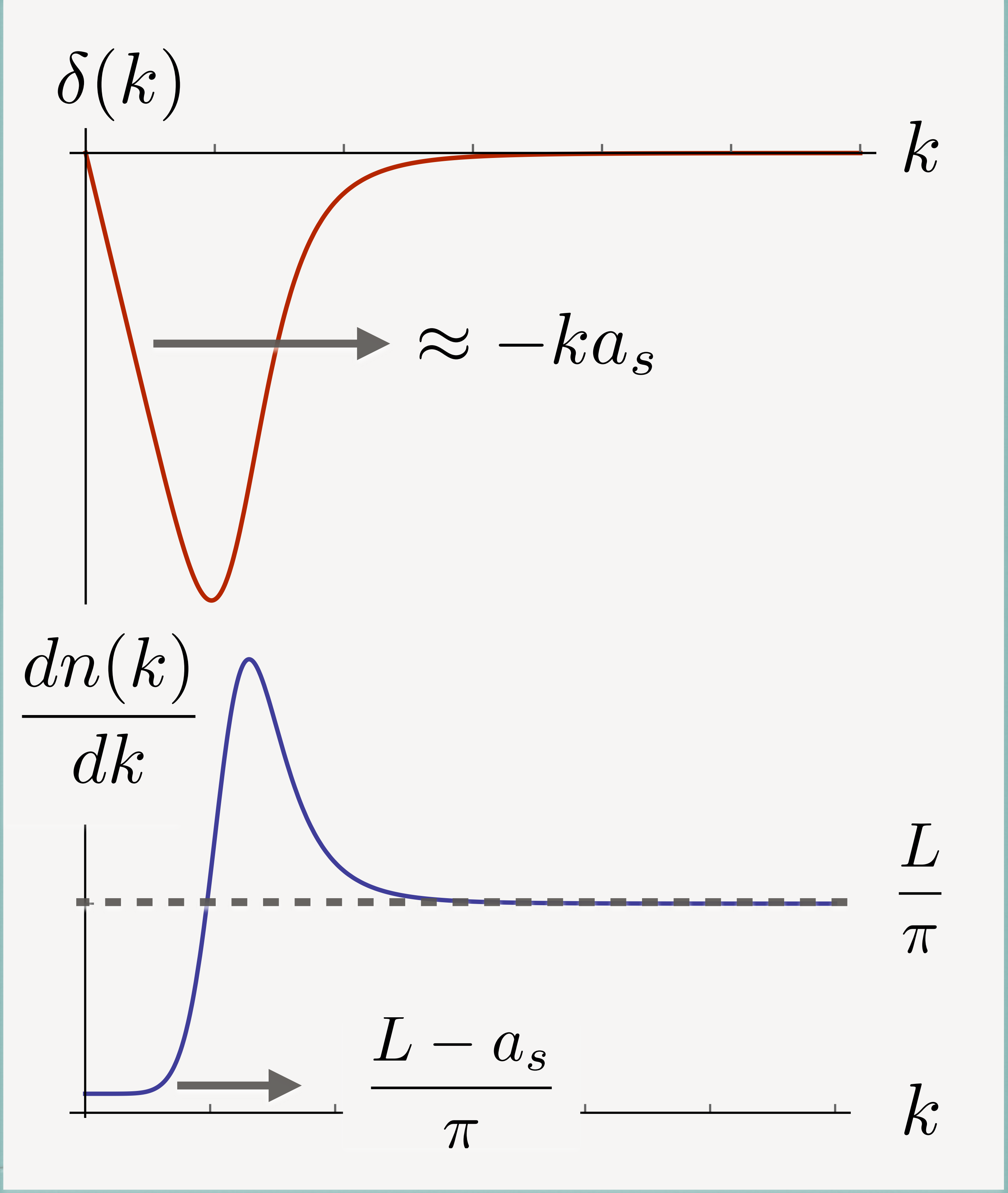
$$kL + \delta(k) = n\pi \quad \Rightarrow \quad \frac{dn(k)}{dk} = \frac{L}{\pi} + \frac{1}{\pi}\frac{d\delta}{dk}$$

PHASE SHIFT AND DENSITY OF STATES

$$\frac{dn(k)}{dk} = \frac{L}{\pi} + \frac{1}{\pi}\frac{d\delta}{dk}$$

change in d.o.s. due to int.

Effect of repulsive interaction:
pushing states from low k to high k



phase shift and d.o.s. (schematics)

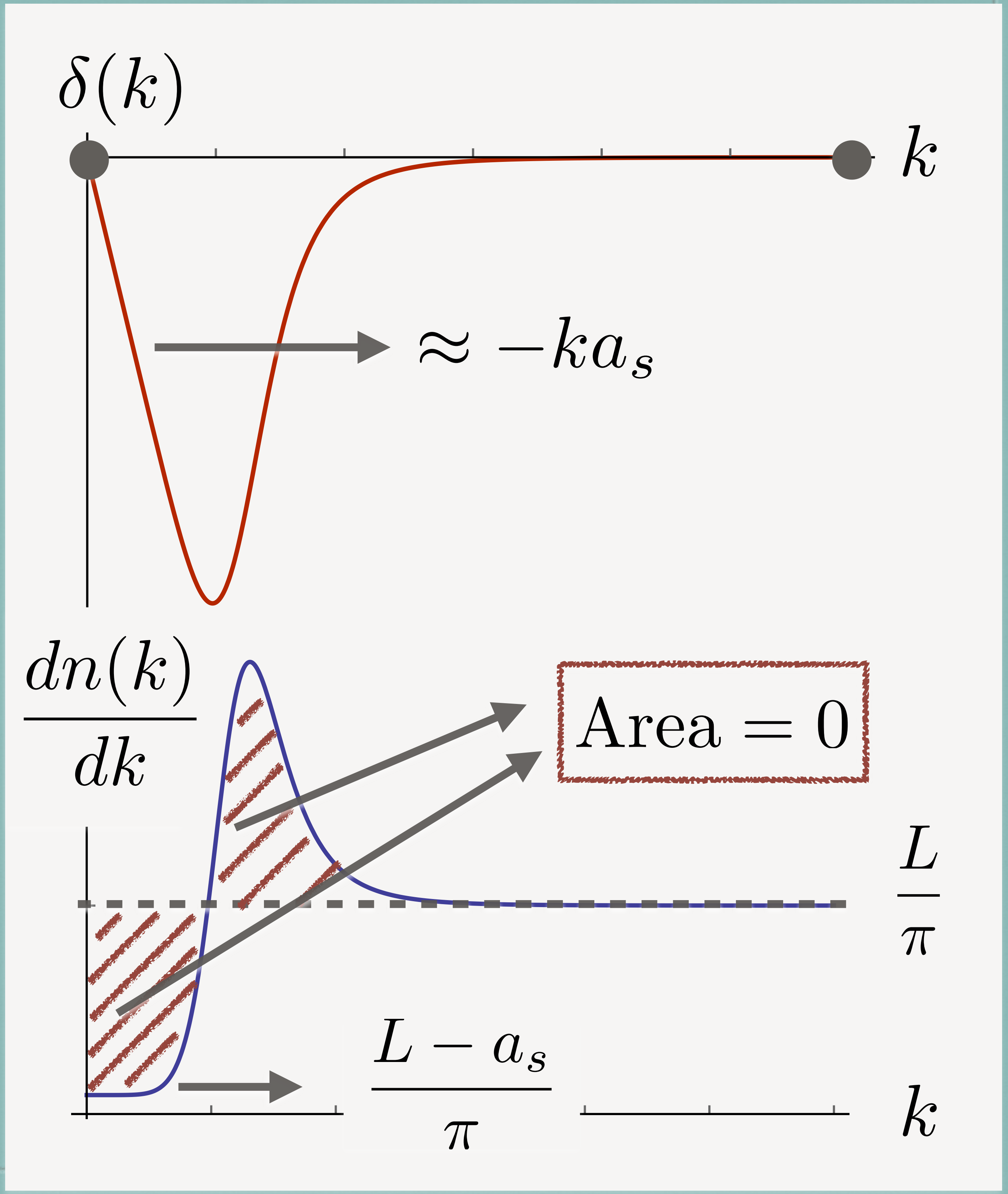
PHASE SHIFT AND DENSITY OF STATES

$$\frac{dn(k)}{dk} = \frac{L}{\pi} + \frac{1}{\pi}\frac{d\delta}{dk}$$

sum rule
(Levinson's theorem)

$$\int_0^\infty dk\,\frac{1}{\pi}\delta' = \frac{\delta(\infty)-\delta(0)}{\pi}$$

n_{int}



phase shift and d.o.s. (schematics)

FORMULATION

given the exact phase shift δ_l

from theory
or
from experiment

thermodynamics

$$B_l = 2\frac{d}{dq}\delta_l$$

eff. spectral function

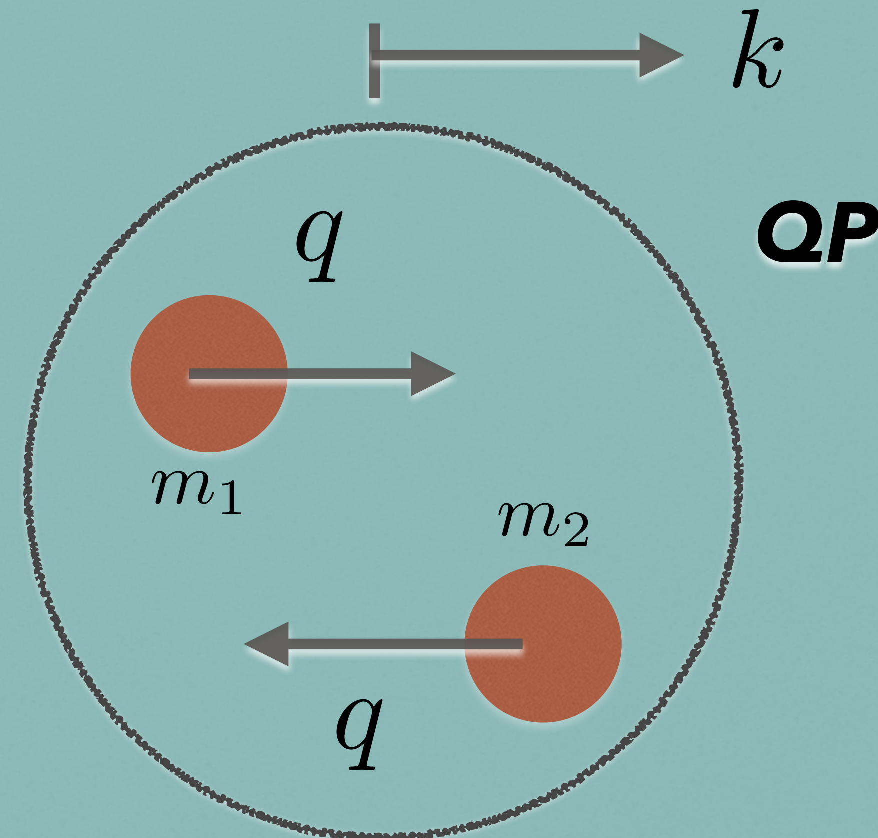
$$P = P^{(0)} + \Delta P^{B.U.}$$

free gas + interaction

FORMULATION

dynamical ***statistical (thermal weight)***

$$\Delta P^{\mathrm{B.U.}} = (2l+1)\int \frac{dq}{2\pi} B_l(q) \int \frac{d^3k}{(2\pi)^3} T \ln(1+e^{-\beta E(k,q,m_i)})$$



$$E = \sqrt{k^2 + M(q)^2}$$

$$M(q) = \sqrt{q^2+m_1^2} + \sqrt{q^2+m_2^2}$$

$$B_l = 2\frac{d}{dq}\delta_l$$

$$M(q) = \sqrt{q^2+m_1^2} + \sqrt{q^2+m_2^2}$$

APPLICATION
PION + NUCLEON + DELTA SYSTEM

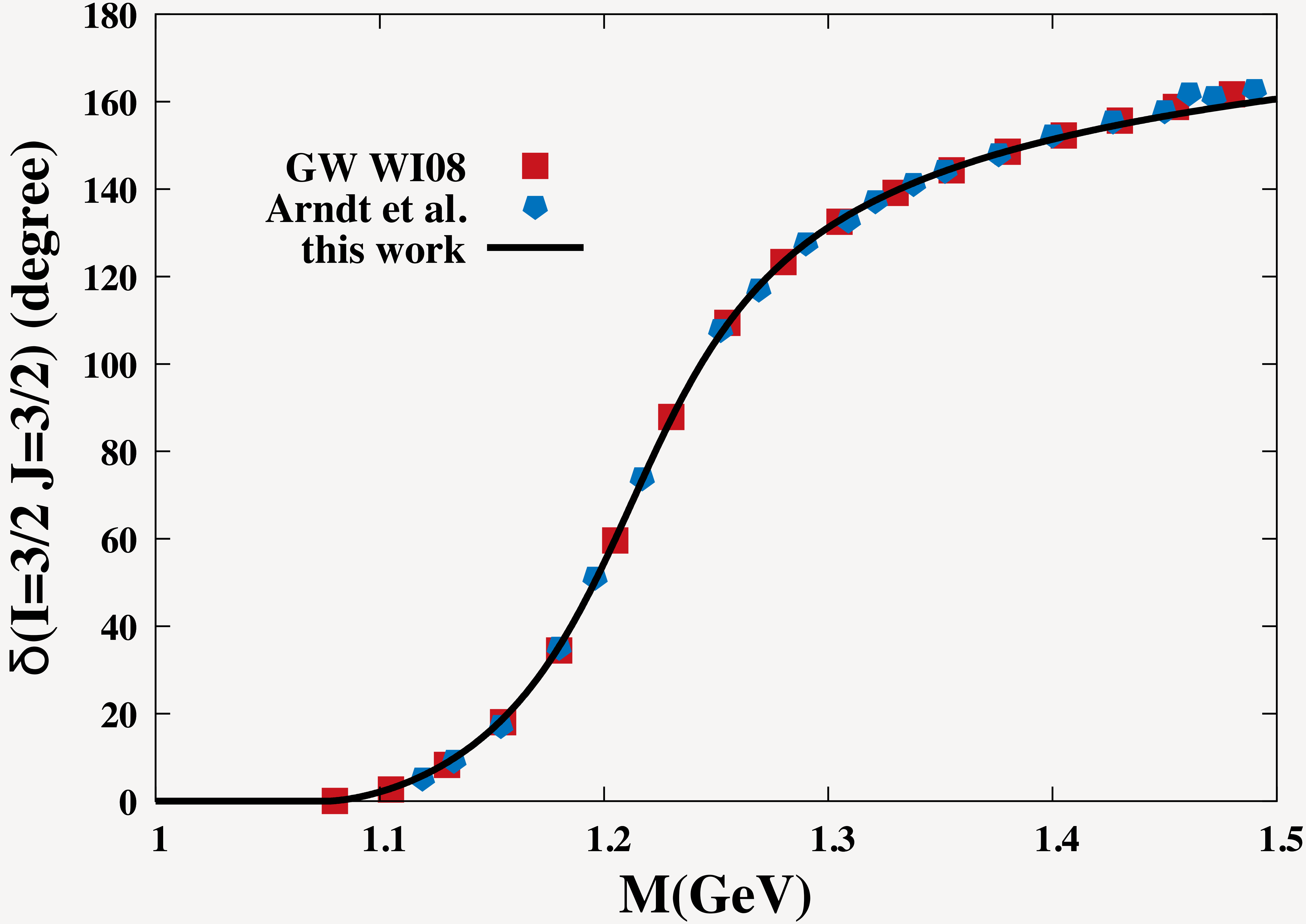
WHAT'S IN A NAME? THAT WHICH WE CALL A RESONANCES?

- A resonance is MORE than a MASS and a **WIDTH**

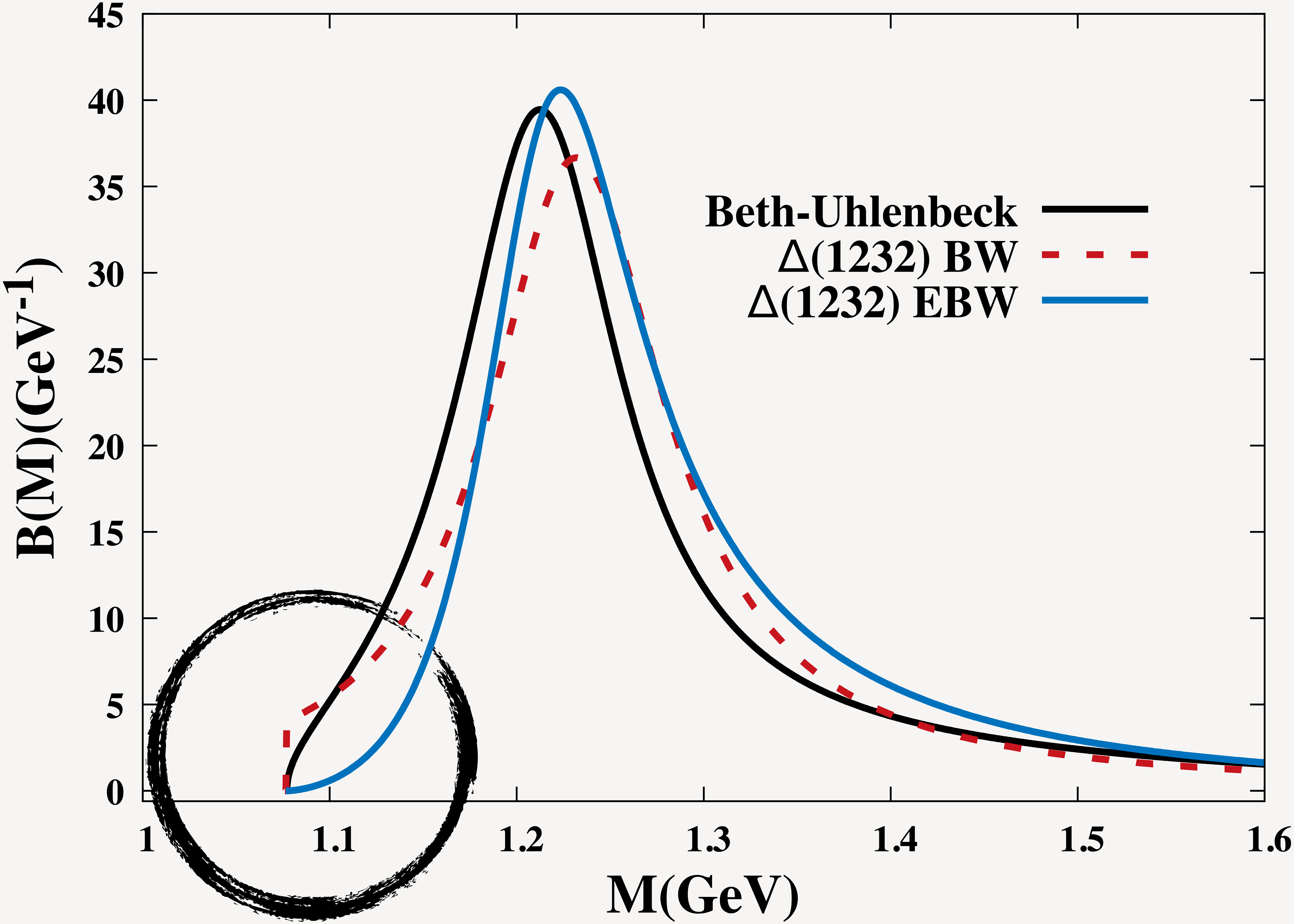
$\Delta(1232)\ 3/2^+$

$I(J^P) = \frac{3}{2}(\frac{3}{2}^+)$

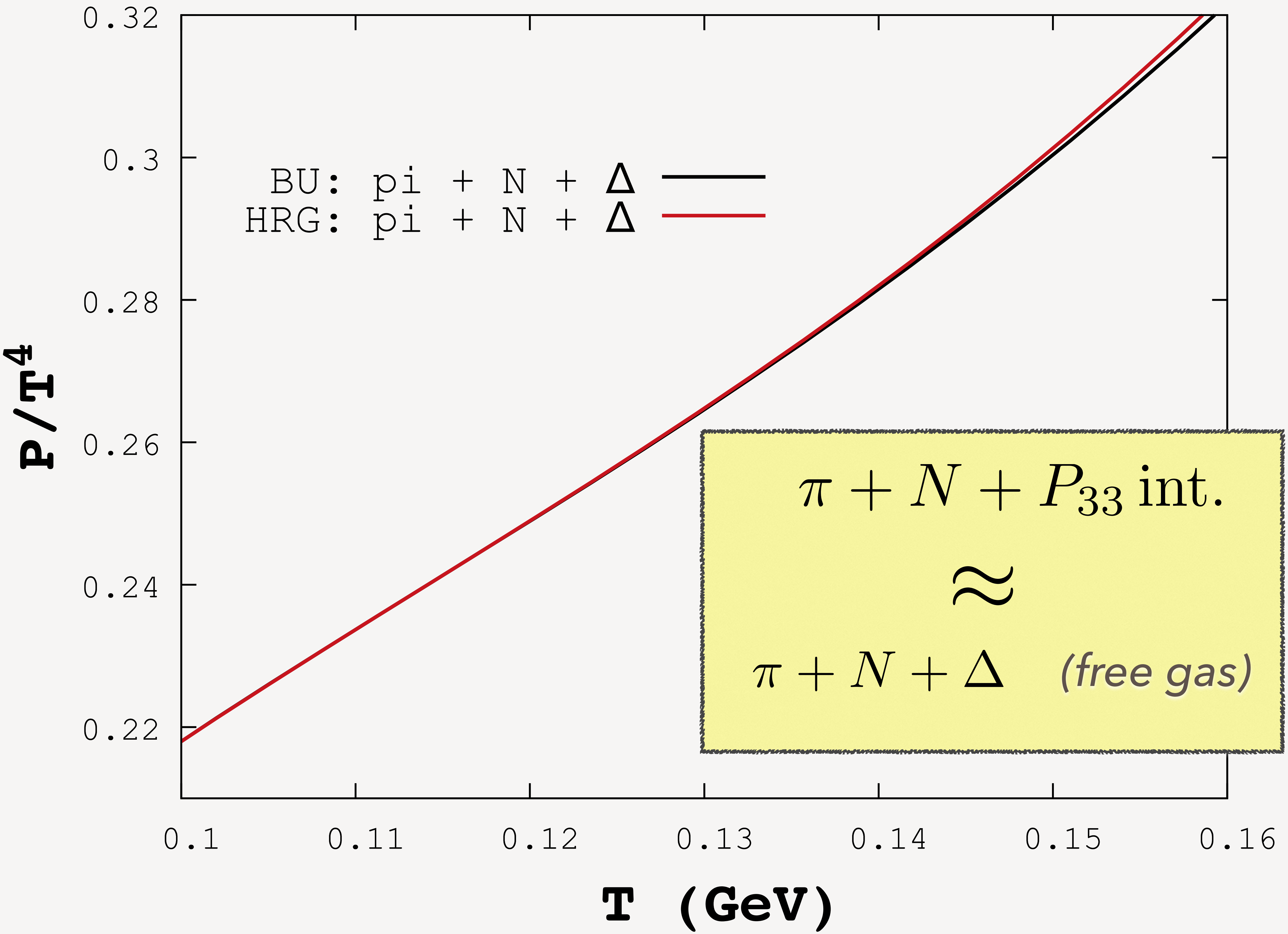
Breit-Wigner mass (mixed charges) = 1230 to 1234 ($\approx$ 1232) MeV
Breit-Wigner full width (mixed charges) = 114 to 120 ($\approx$ 117) MeV
Re(pole position) = 1209 to 1211 ($\approx$ 1210) MeV
-2Im(pole position) = 98 to 102 ($\approx$ 100) MeV



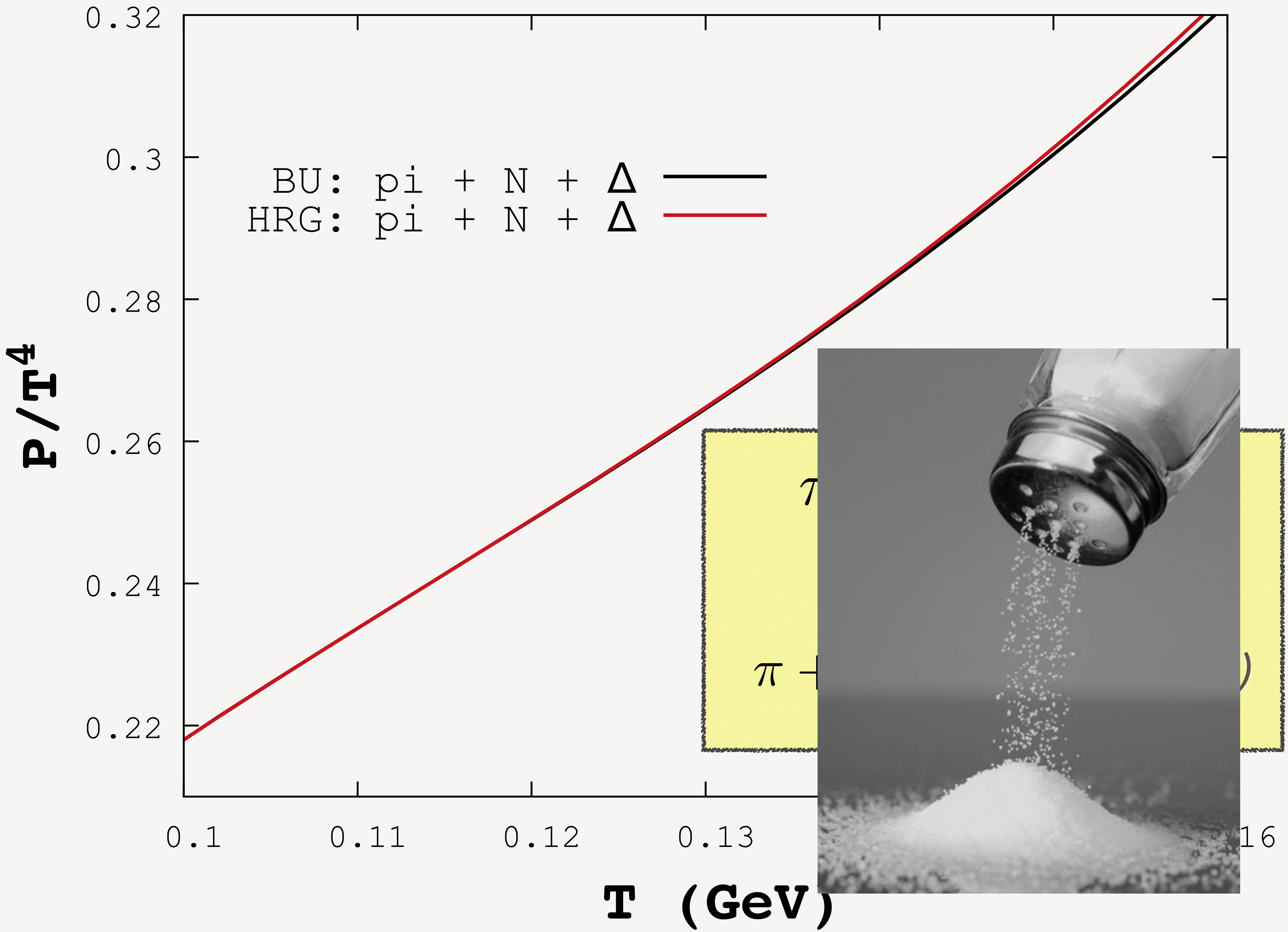
GW WI08
Arndt et al.
this work
δ(I=3/2 J=3/2) (degree)
M(GeV)
0
20
40
60
80
100
120
140
160
180
1
1.1
1.2
1.3
1.4
1.5



45
40
35
30
25
20
15
10
5
0
1
1.1
1.2
1.3
1.4
1.5
1.6
B(M)(GeV-1)
M(GeV)
Beth-Uhlenbeck
Δ(1232) BW
Δ(1232) EBW



P/T⁴
0.32
0.3
0.28
0.26
0.24
0.22
0.1
0.11
0.12
0.13
0.14
0.15
0.16
T (GeV)
BU: pi + N + Δ
HRG: pi + N + Δ
$\pi + N + P_{33}$ int.
$\approx$
$\pi + N + \Delta$ (free gas)



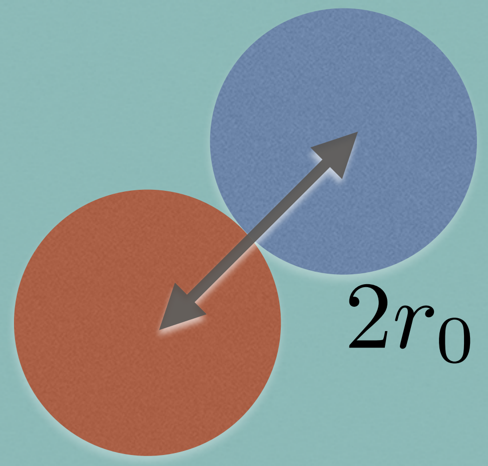
BU: pi + N + Δ
HRG: pi + N + Δ
P/T^4
T (GeV)
0.32
0.3
0.28
0.26
0.24
0.22
0.1
0.11
0.12
0.13

REPULSIONS AND EXCLUDED VOLUMES

MODELING SHORT-RANGE REPULSION BETWEEN HADRONS

- Excluded volume approach

  eigenvolume v_0



- Beth-Uhlenbeck approach

  1) QM problem of a hard-core potential -> $\delta_l(q\,a_S)$
  2) Thermodynamics by S-matrix

FORMULATION #1

$$P(T,\mu) = P^{id}(T, \tilde{\mu} = \mu - v_0 P(T,\mu))$$

To be solved self-consistently.

non-linear effects

FORMULATION #1

$$P(T,\mu) = P^{id}(T, \tilde{\mu} = \mu - v_0 P(T,\mu))$$

$$\mu = \mu_B B + \mu_S S + \mu_Q Q$$

conserved charges

FORMULATION #1

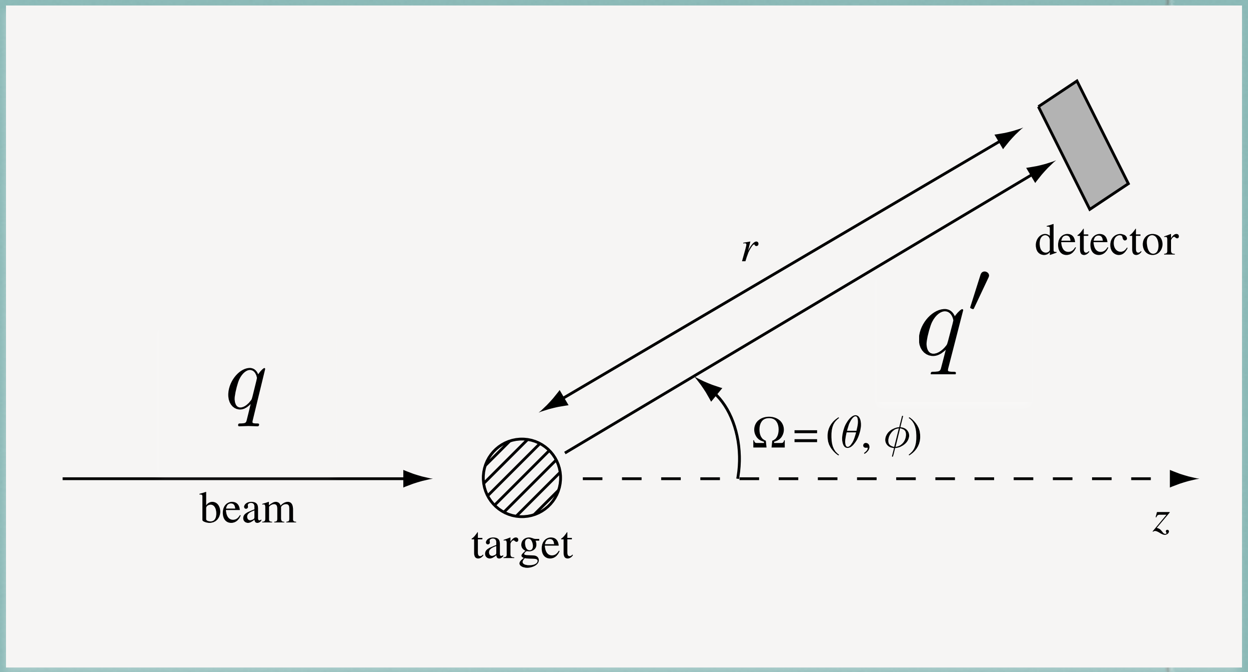
$$P(T,\mu) = P^{id}(T, \tilde{\mu} = \mu - v_0 P(T,\mu))$$

$$\mu = \mu_B B + \mu_S S + \mu_Q Q$$ ⟷ conserved charges

μ_{sym} ⟷ multiplicity

FORMULATION #2

- Starting point:
  hard-core potential in QM



$$V = \infty \qquad r < a$$
$$= 0 \qquad r > a$$

Momentum q enters through the scattering Schroedinger equation with a centrifugal term (l -dependence)

$$\tan(\delta_l) = \frac{j_l(qa)}{n_l(qa)}$$

$$\psi^q(r \to \infty) \longrightarrow e^{iqr\cos(\theta)} + \frac{e^{iqr}}{r} \sum_l (2l+1)\, P_l \,\frac{e^{i\delta_l}}{q}\, \sin(\delta_l)$$

FORMULATION #2

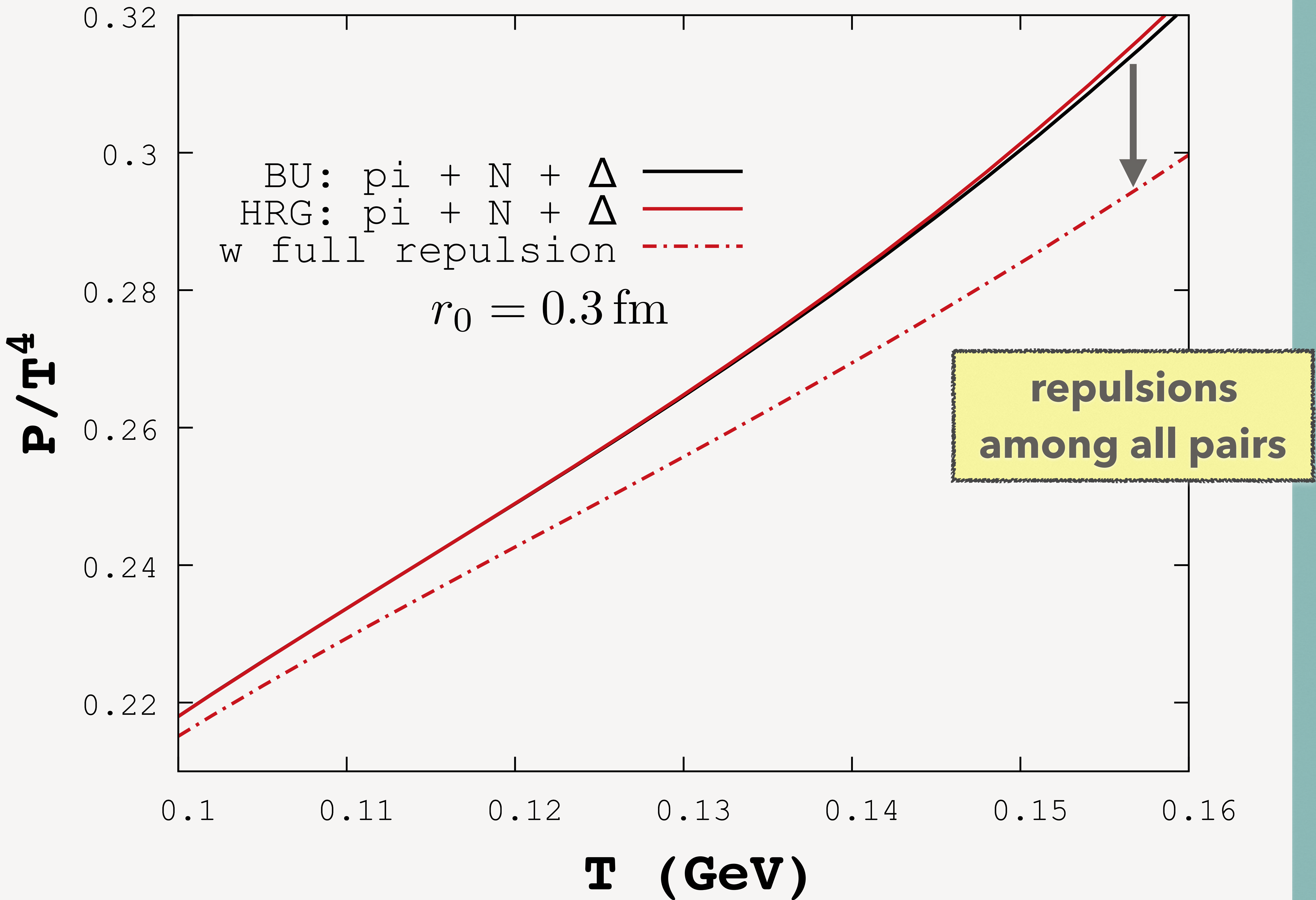
$$\tan(\delta_l) = \frac{j_l(qa)}{n_l(qa)}$$

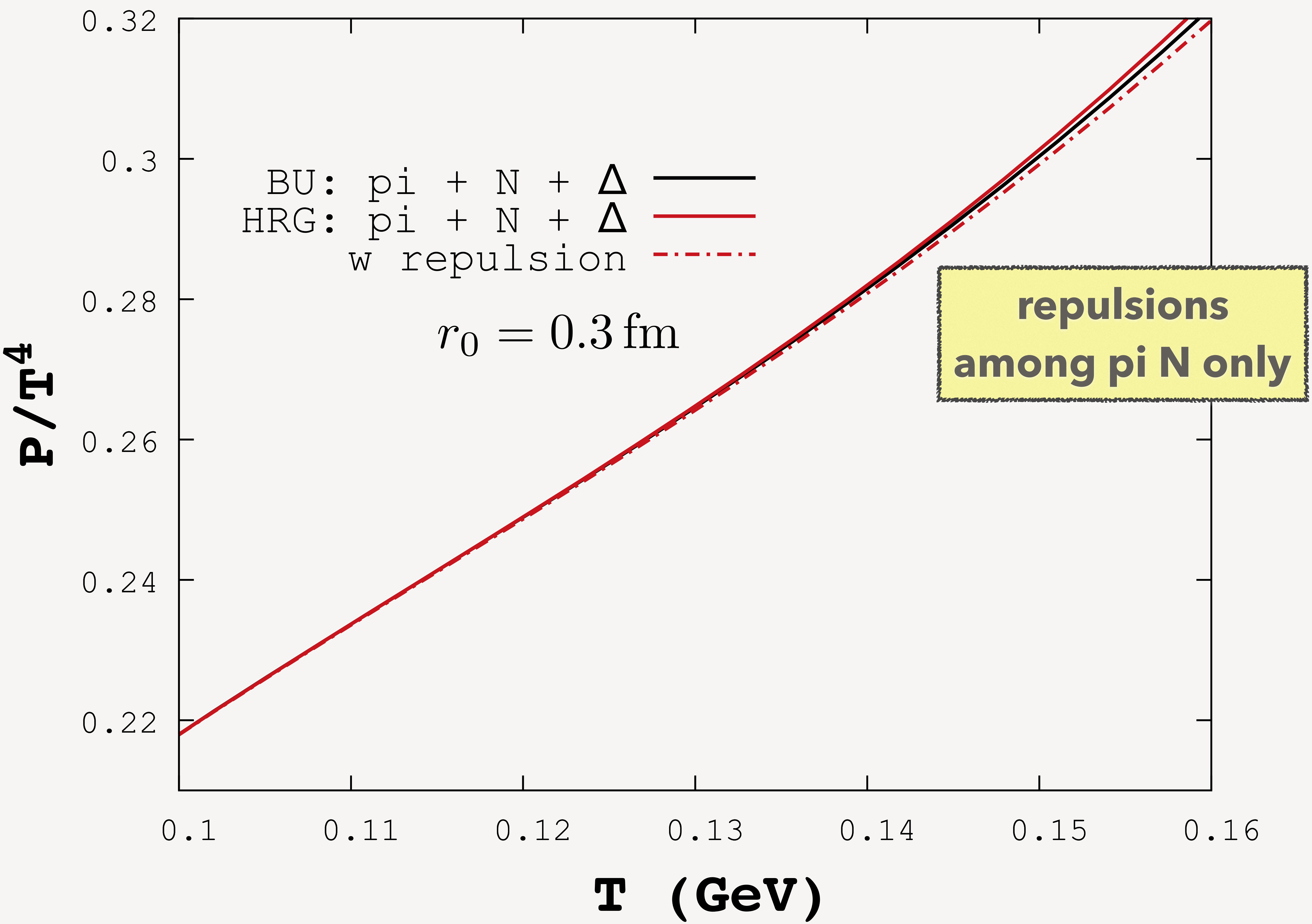
for small $x = qa$ *(near threshold)*

$$\tan(\delta_l) \to \frac{-x^{2l+1}}{(2l+1)((2l-1)!!)^2}$$

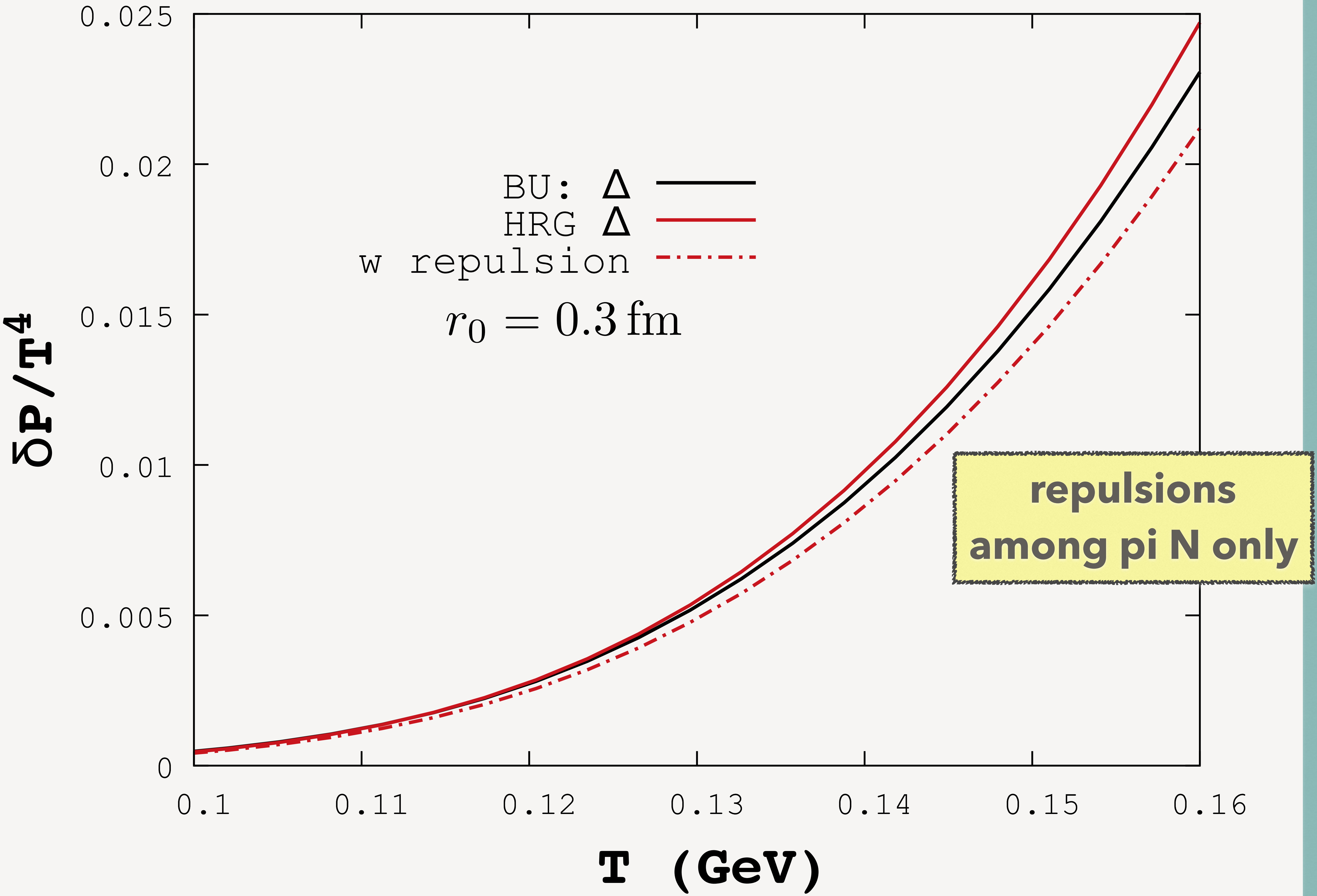
$$\delta_l \propto (q\,a)^{2l+1}$$

(near threshold)

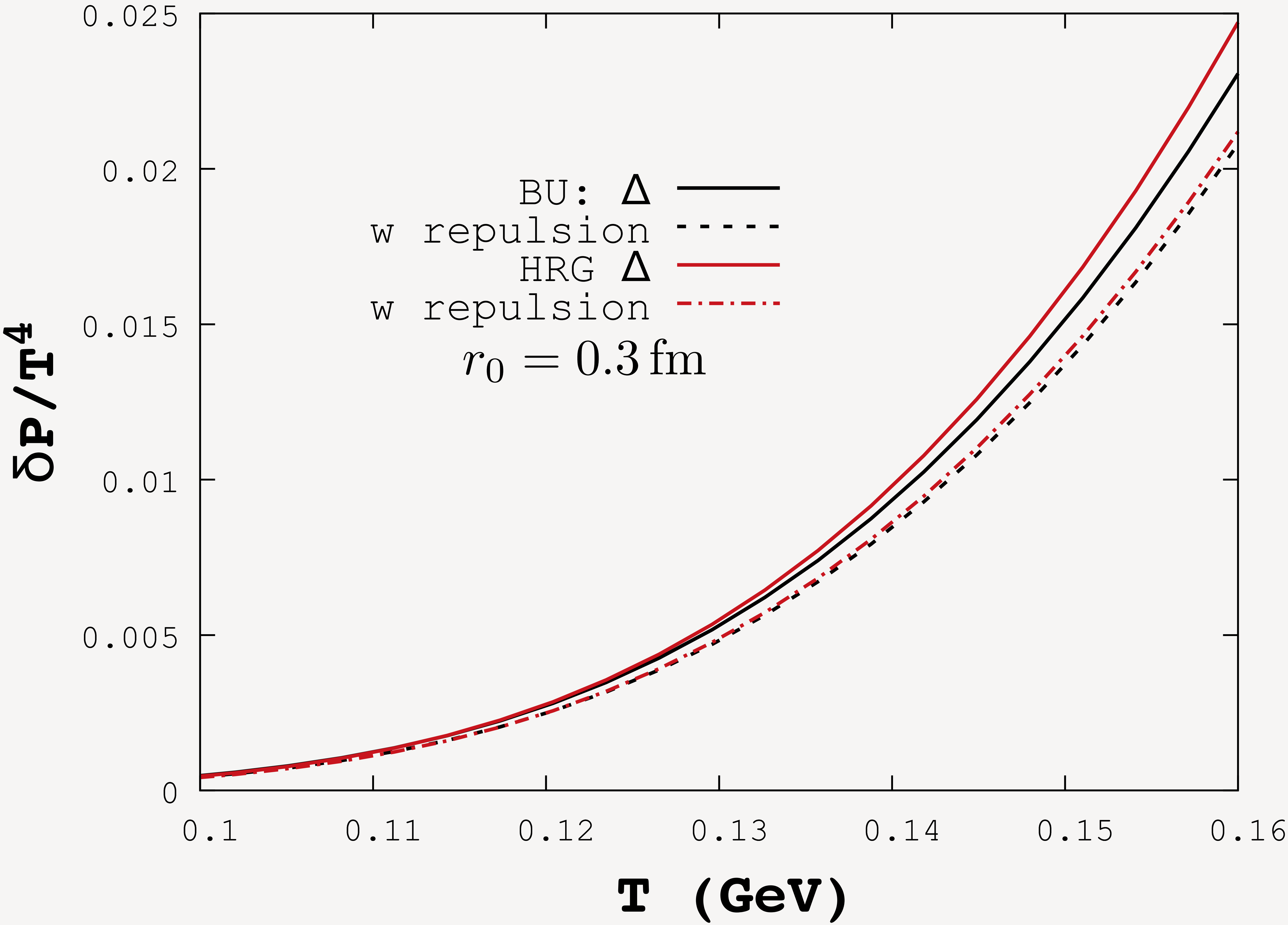

0.32
0.3
0.28
0.26
0.24
0.22
P/T^4
0.1
0.11
0.12
0.13
0.14
0.15
0.16
T (GeV)
BU: pi + N + Δ
HRG: pi + N + Δ
w full repulsion
$r_0 = 0.3\,\mathrm{fm}$
repulsions among all pairs

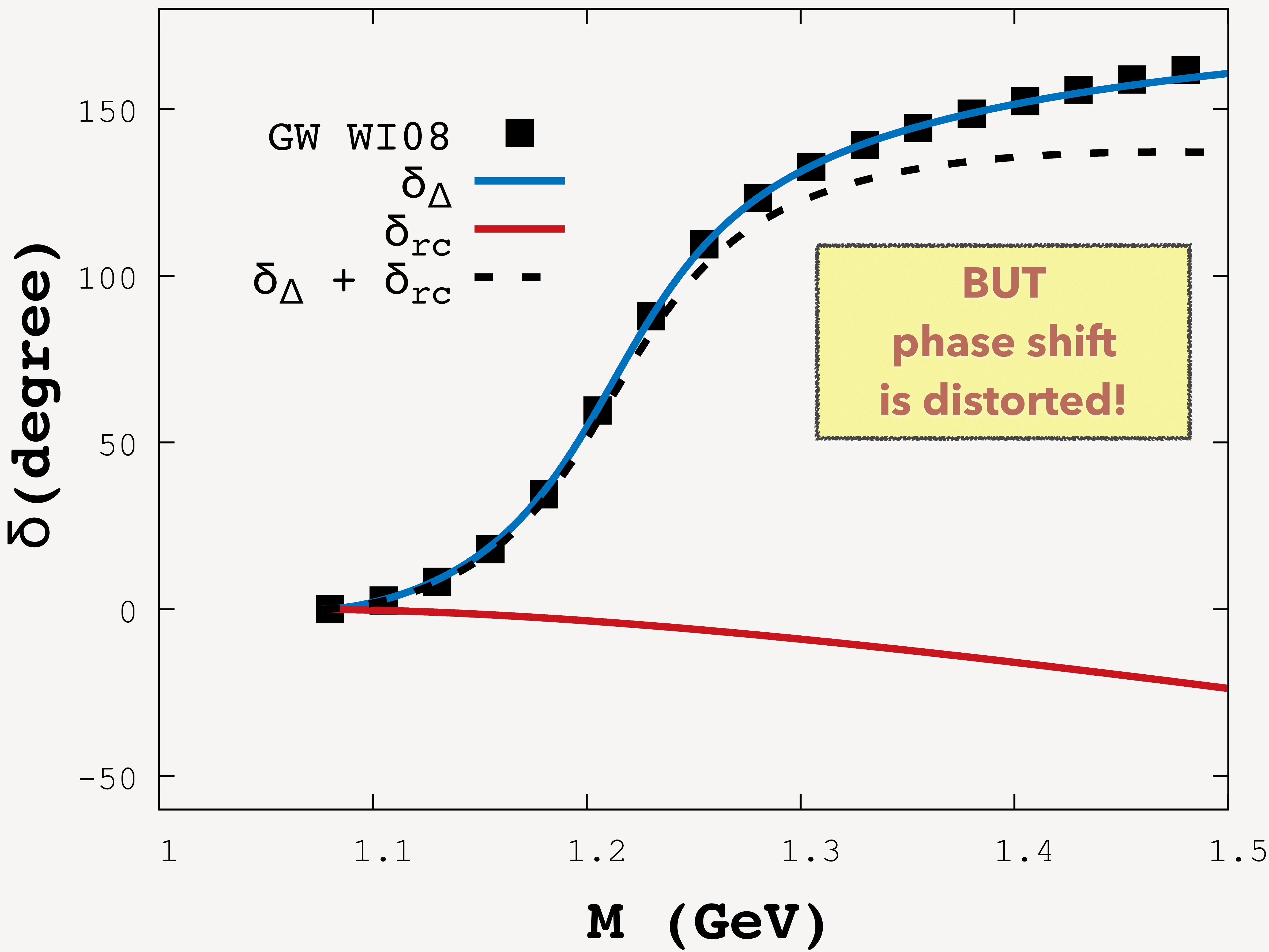


BU: pi + N + Δ
HRG: pi + N + Δ
w repulsion
$r_0 = 0.3\,\mathrm{fm}$
repulsions
among pi N only
P/T^4
T (GeV)
0.32
0.3
0.28
0.26
0.24
0.22
0.1
0.11
0.12
0.13
0.14
0.15
0.16

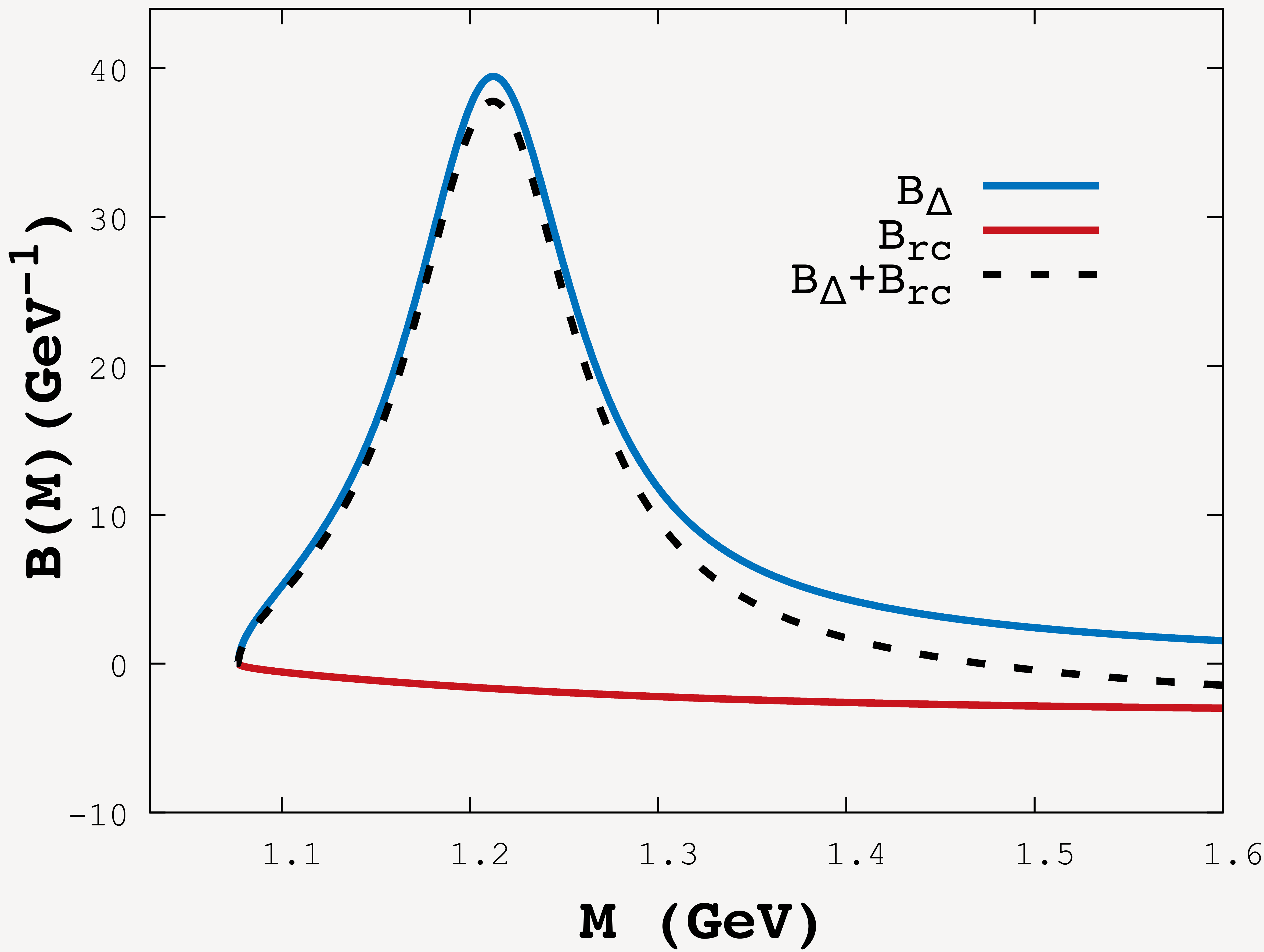


0.025
0.02
0.015
0.01
0.005
0
0.1
0.11
0.12
0.13
0.14
0.15
0.16
δP/T⁴
T (GeV)
BU: Δ
HRG Δ
w repulsion
$r_0 = 0.3\,\mathrm{fm}$
repulsions among pi N only

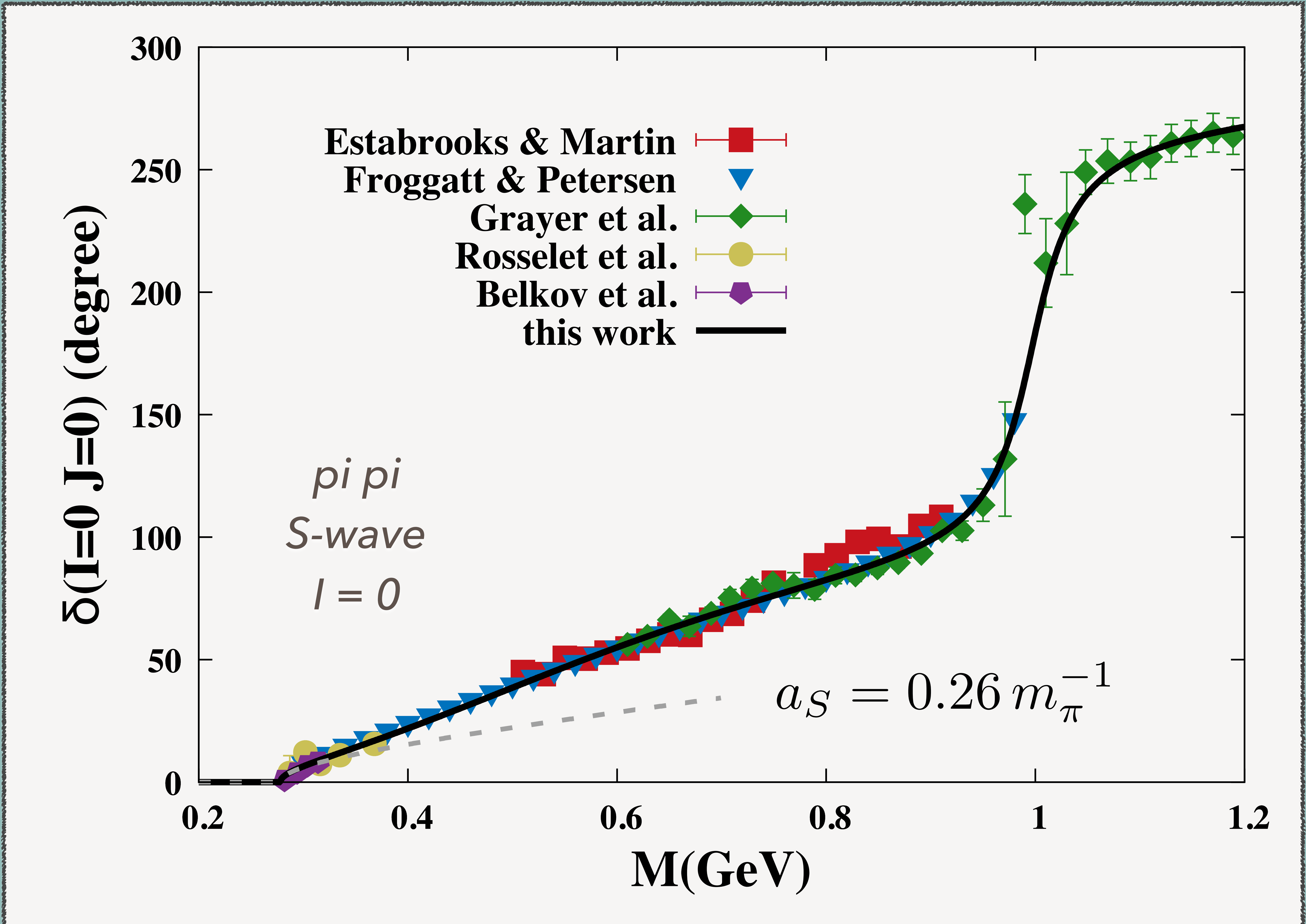

BU: Δ
w repulsion
HRG Δ
w repulsion
$r_0 = 0.3\,\mathrm{fm}$
δP/T⁴
T (GeV)
0
0.005
0.01
0.015
0.02
0.025
0.1
0.11
0.12
0.13
0.14
0.15
0.16



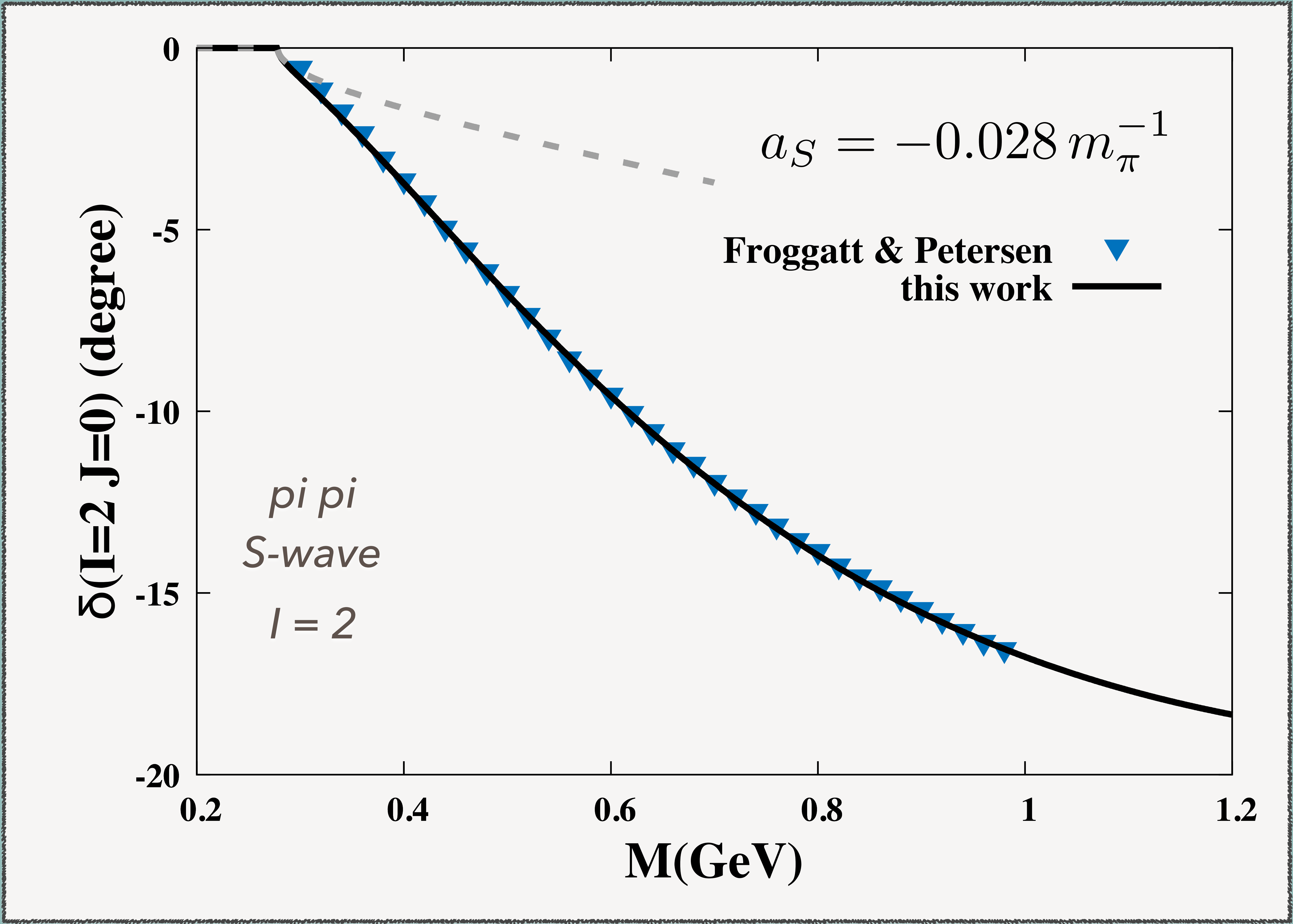
GW WI08
δ_Δ
δ_{rc}
$\delta_\Delta + \delta_{rc}$
BUT
phase shift
is distorted!
150
100
50
0
-50
1
1.1
1.2
1.3
1.4
1.5
δ(degree)
M (GeV)

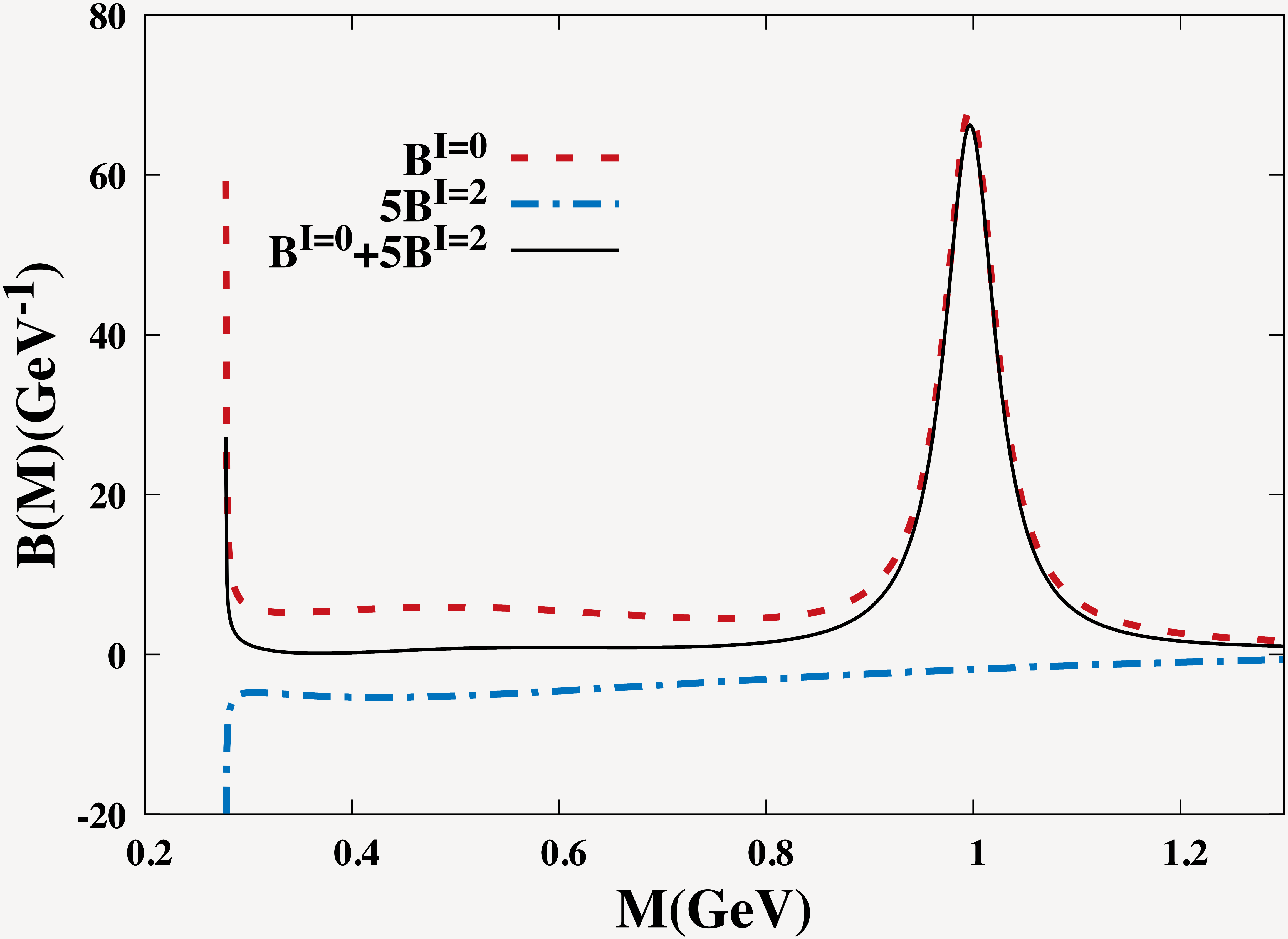

B(M) (GeV^{-1})
40
30
20
10
0
-10
1.1
1.2
1.3
1.4
1.5
1.6
M (GeV)
B_Δ
B_{rc}
$B_\Delta+B_{rc}$

PI PI SCATTERING

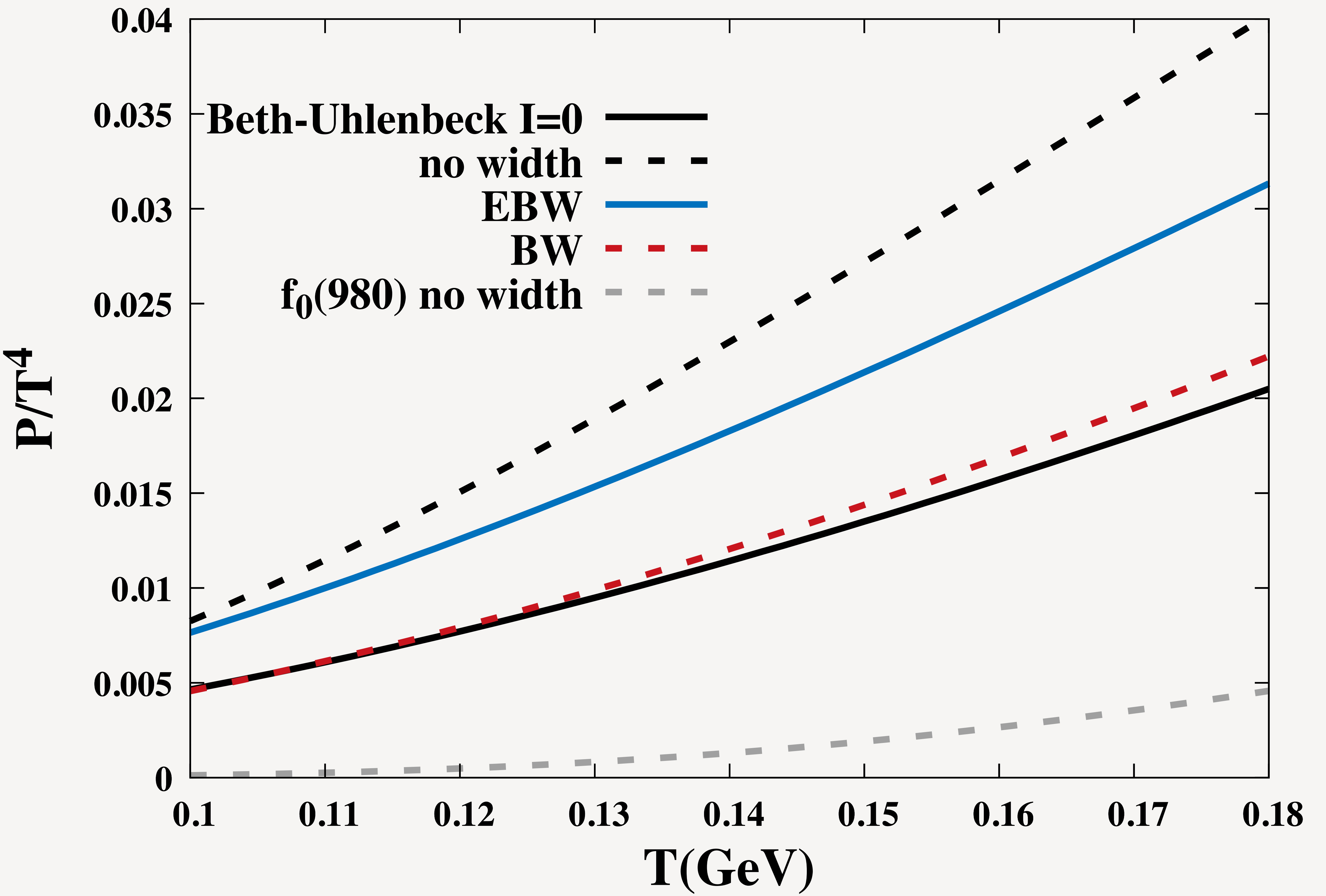


Estabrooks & Martin
Froggatt & Petersen
Grayer et al.
Rosselet et al.
Belkov et al.
this work
pi pi
S-wave
I = 0
$a_S = 0.26\, m_\pi^{-1}$
δ(I=0 J=0) (degree)
M(GeV)
0
50
100
150
200
250
300
0.2
0.4
0.6
0.8
1
1.2

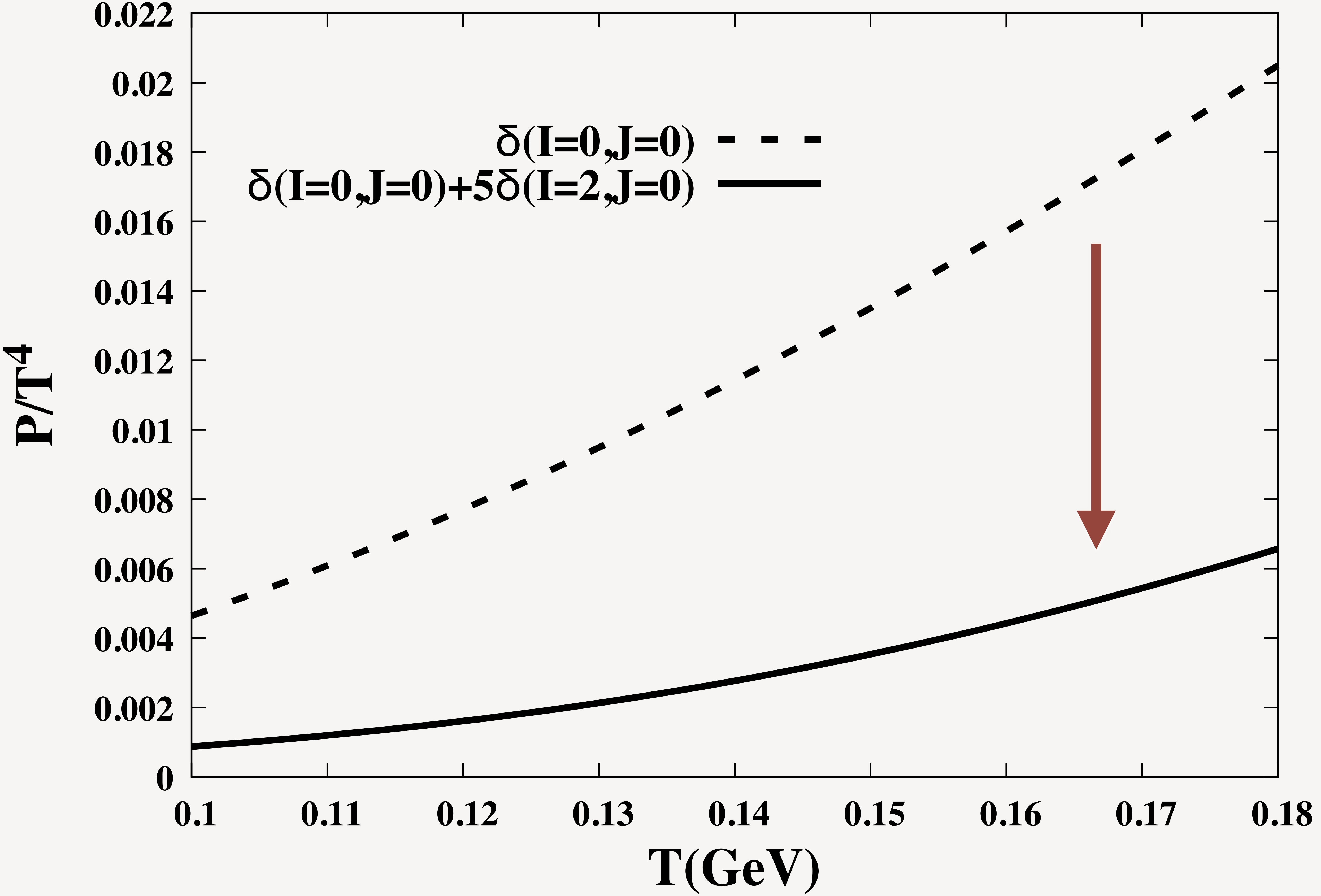

0
-5
-10
-15
-20
0.2
0.4
0.6
0.8
1
1.2
M(GeV)
δ(I=2 J=0) (degree)
$a_S = -0.028\, m_\pi^{-1}$
Froggatt & Petersen
this work
pi pi
S-wave
I = 2



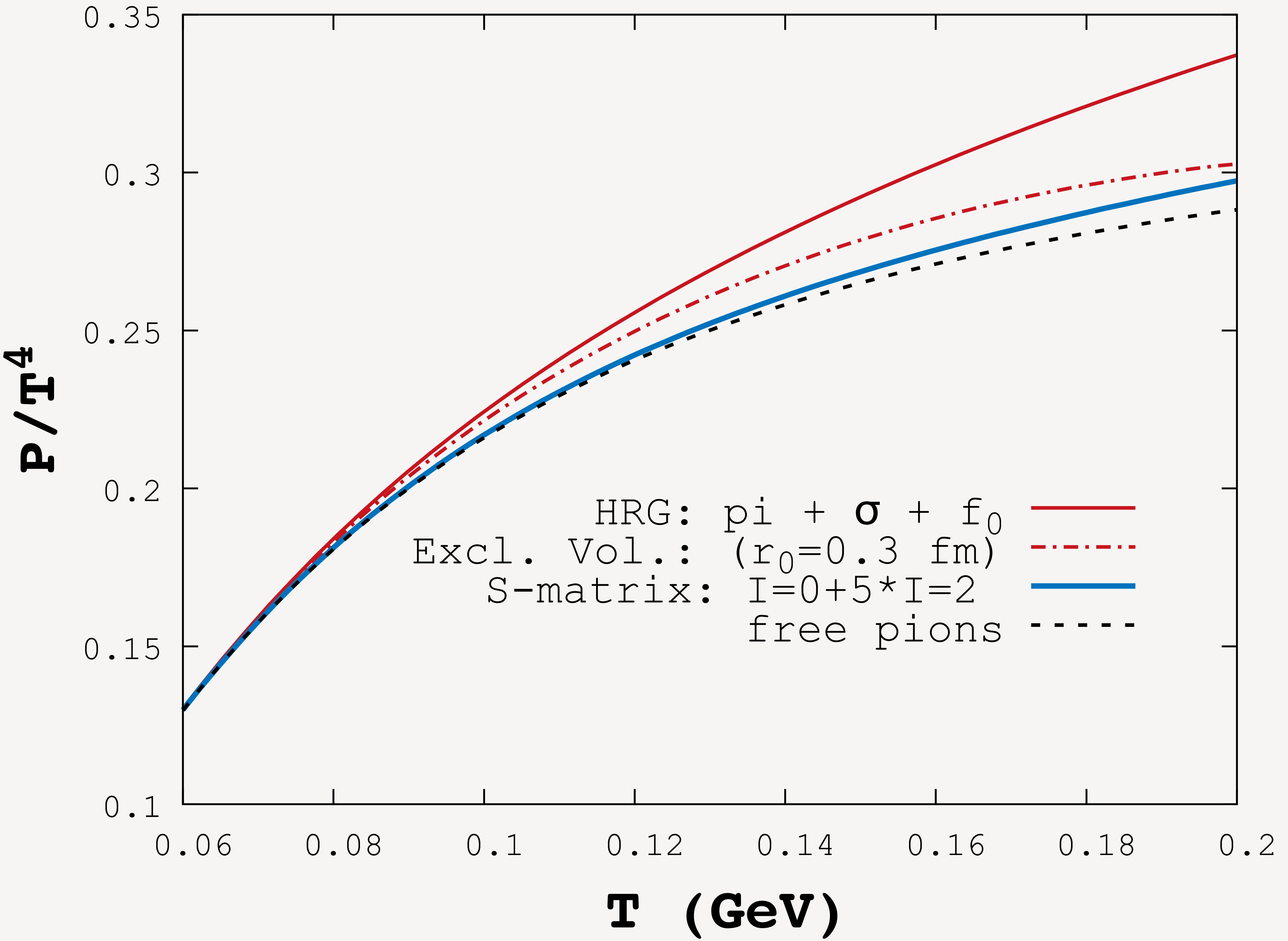
80
60
40
20
0
-20
0.2
0.4
0.6
0.8
1
1.2
M(GeV)
B(M)(GeV-1)
BI=0
5BI=2
BI=0+5BI=2



Beth-Uhlenbeck I=0
no width
EBW
BW
f0(980) no width
P/T4
T(GeV)
0
0.005
0.01
0.015
0.02
0.025
0.03
0.035
0.04
0.1
0.11
0.12
0.13
0.14
0.15
0.16
0.17
0.18



δ(I=0,J=0)
δ(I=0,J=0)+5δ(I=2,J=0)
P/T^4
T(GeV)
0
0.002
0.004
0.006
0.008
0.01
0.012
0.014
0.016
0.018
0.02
0.022
0.1
0.11
0.12
0.13
0.14
0.15
0.16
0.17
0.18



0.35
0.3
0.25
0.2
0.15
0.1
0.06
0.08
0.1
0.12
0.14
0.16
0.18
0.2
P/T4
T (GeV)
HRG: pi + σ + f0
Excl. Vol.: (r0=0.3 fm)
S-matrix: I=0+5*I=2
free pions

SUMMARY

- Repulsion strengths are channel-dependent.
- They are automatically incorporated in phase shifts, no need for additional excluded volume
- Flexibility in modeling
e.g. scattering length is isospin dependent => channel dependent!

$$a^{\pi\pi}_{I=0} > a^{\pi\pi}_{I=2}$$