

Pushing fluid dynamics beyond its limits

Radoslaw Ryblewski



THE HENRYK NIEWODNICZAŃSKI
INSTITUTE OF NUCLEAR PHYSICS
POLISH ACADEMY OF SCIENCES

in collaboration with:

W. Florkowski (IFJ PAN & UJK, PL)

M. Strickland (Kent State U., US)

L. Tinti (Ohio State U., US)

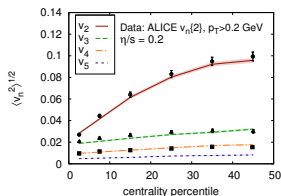
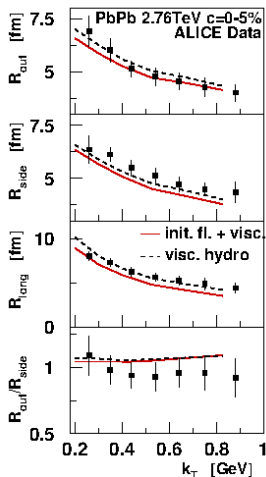
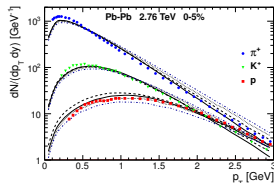
details in:

W. Florkowski, R. R., M. Strickland, L. Tinti, arXiv: 1609.06293

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- from instabilities to fluctuations*

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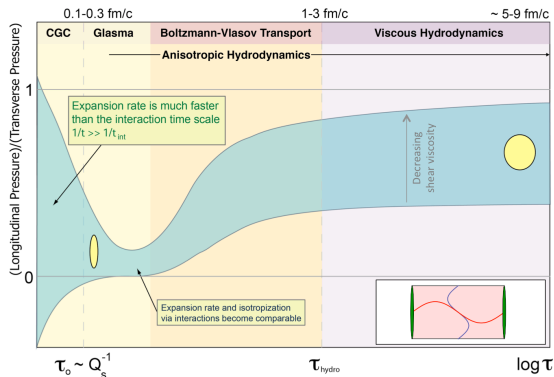
P. Bozek, Phys.Rev. C85 (2012) 064915

P. Bozek, Phys. Rev. C83 (2011) 044910

C. Gale et al. Phys. Rev. Lett. 110, 012302 (2013)

- the ultimate goal of the URHIC experiments is to understand/extract the properties of the produced matter ($\eta/s, \zeta/s, \dots$)
- predictions of dissipative fluid dynamics models agree with data surprisingly well

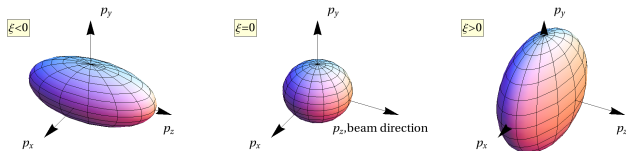
Motivation



M. Strickland, Acta Phys.Polon. B45 (2014) no. 12, 2355-2394

- for their derivation **small deviations from LE** are usually assumed
 - URHIC dynamics leads inevitably to **rapid longitudinal expansion** and **large pressure anisotropies**
- see **M. Strickland talk**
- their presence triggered intense development of the dissipative fluid dynamical models

Motivation



- in addition to the improvements of the standard perturbative frameworks the **anisotropic hydrodynamics** was invented . . .
W. Florkowski and R. R., Phys. Rev. C83, 034907 (2011), 1007.0130
M. Martinez and M. Strickland, Nucl. Phys. A848, 183 (2010), 1007.0889
- . . . and further improved
D. Bazow, U. W. Heinz, and M. Strickland, Phys. Rev. C90, 054910 (2014)
D. Bazow, U. W. Heinz, and M. Martinez, Phys. Rev. C91, 064903 (2015)
E. Molnar, H. Niemi, and D. H. Rischke, Phys. Rev. D93, 114025 (2016)
L. Tinti, Phys. Rev. C 94, 044902 (2016)
- within the underlying RTA kinetic theory **good performance of *aHydro** was shown for certain highly-symmetric flows (Bjorken, Gubser)
W. Florkowski, R. R., and M. Strickland, Nucl. Phys. A916, 249 (2013)
G. S. Denicol, U. W. Heinz, M. Martinez, J. Noronha, and M. Strickland, Phys. Rev. Lett. 113, 202301 (2014)
M. Nopoush, R. R., and M. Strickland, Phys. Rev. D91, 045007 (2015)
- the ability to **capture both close-to-equilibrium and free-streaming limits** makes it a unique tool

Motivation

- due to limited knowledge of longitudinal dynamics in URHIC boost-invariance is employed usually in simulations . . .
... although some improvements are on the way → see B. Schenke talk
- if relaxed, the gradients in the rapidity direction provide a new source of pressure corrections that affect the hydrodynamical evolution
- since aHydro is devoted to handle them more precisely we expect some quantitative differences as compared to standard dissipative approaches

Motivation

- in order to make the problem numerically and analytically well controlled we **assume conformal symmetry and (1+1)D transversely homogenous geometry**
- often fluid dynamical approaches are derived from kinetic theory
- we **use RTA collisional kernel** as the common denominator for the study (classical statistics)

General principles

- neglect conserved charges
- conservation of energy and momentum yields $\partial_\mu T^{\mu\nu} = 0$
- use Landau definition of four-velocity $U_\mu T^{\mu\nu} = \mathcal{E}U^\nu$
- the most general decomposition of $T^{\mu\nu}$ in this frame is

$$T^{\mu\nu} = \mathcal{E}U^\mu U^\nu - (\mathcal{P} + \Pi)\Delta^{\mu\nu} + \pi^{\mu\nu}$$

- temperature is defined via Landau matching $\mathcal{E}(x) = \mathcal{E}_{\text{eq}}(T(x))$
- consider conformal fluid $\mathcal{P} = \mathcal{E}/3, \Pi = 0$

$$U_\nu \partial_\mu T^{\mu\nu} = 0 \quad \Longrightarrow \quad D\mathcal{E} = -(\mathcal{E} + \mathcal{P})\theta + \pi_{\mu\nu}\sigma^{\mu\nu}$$

$$\Delta_\nu^\alpha \partial_\mu T^{\mu\nu} = 0 \quad \Longrightarrow \quad (\mathcal{E} + \mathcal{P})DU^\alpha = \nabla^\alpha \mathcal{P} - \Delta_\mu^\alpha \partial_\nu \pi^{\mu\nu}$$

$$D = U^\mu \partial_\mu, \quad \theta = \partial_\mu U^\mu, \quad \nabla^\mu = \Delta^{\mu\nu} \partial_\nu, \quad \sigma^{\mu\nu} = \partial^{(\mu} U^{\nu)}$$

$$A^{(\mu\nu)} = \Delta_{\alpha\beta}^{\mu\nu} \quad A^{\alpha\beta} \Delta_{\alpha\beta}^{\mu\nu} = \left(\Delta_\alpha^\mu \Delta_\beta^\nu + \Delta_\alpha^\nu \Delta_\beta^\mu \right) / 2 - \Delta^{\mu\nu} \Delta_{\alpha\beta} / 3$$

$$\Delta^{\mu\nu} = g^{\mu\nu} - U^\mu U^\nu$$

Dissipative hydrodynamics formulations (I)

- the conservation equations are supplemented with dynamic equations for $\pi^{\mu\nu}$
- later on we restrict ourselves to (1+1)D evolution thus we put $\omega^{\mu\nu} = 0$
- as the underlying microscopic theory we consider RTA Boltzmann equation
- **Müller-Israel-Stewart (MIS)** approach

I. Müller, Z. Phys. 198, 329 (1967)

W. Israel, Annals Phys. 100, 310 (1976)

J. M. Stewart, Proc. R. Soc. London A 357, 59 (1977)

a common version of it has the following form

A. Muronga, Phys. Rev. Lett. 88, 062302 (2002)

$$D\pi^{\langle\mu\nu\rangle} + \frac{1}{\tau_\pi} \left(\pi^{\mu\nu} - 2\eta\sigma^{\mu\nu} \right) = -\pi^{\mu\nu} \frac{T\beta_\pi}{2} \partial_\rho \left(\frac{1}{T\beta_\pi} U^\rho \right) \approx -\frac{4}{3} \pi^{\mu\nu} \theta$$

- **Baier-Romatschke-Son-Starinets-Stephanov (BRSSS)** approach

R. Baier, P. Romatschke, D. T. Son, A. O. Starinets, and M. A. Stephanov, JHEP 04, 100 (2008)

$$D\pi^{\langle\mu\nu\rangle} + \frac{1}{\tau_\pi} \left(\pi^{\mu\nu} - 2\eta\sigma^{\mu\nu} \right) = -\frac{d}{d-1} \pi^{\mu\nu} \theta + \frac{\lambda_1}{\tau_\pi \eta^2} \pi_\alpha^{\langle\mu} \pi^{\nu\rangle\alpha}$$

matching its gradient expansion to the one of RTA BE yields: $\lambda_1 = \frac{5}{7} \tau_\pi \eta$

W. Florkowski, R. R. and M. Spaliński, arXiv:1608.07558

M. P. Heller, A. Kurkela and M. Spaliński, arXiv:1609.04803

→ see M. Spalinski talk

for RTA BE one also has $\beta_\pi = \eta/\tau_\pi = 4\mathcal{P}/5$

Dissipative hydrodynamics formulations (II)

- **Denicol-Niemi-Molnar-Rischke (DNMR) approach**

G. S. Denicol, H. Niemi, E. Molnar, and D. H. Rischke, Phys. Rev. D85, 114047 (2012)

$$D\pi^{\langle\mu\nu\rangle} + \frac{1}{\tau_\pi} \left(\pi^{\mu\nu} - 2\eta\sigma^{\mu\nu} \right) = -\frac{\delta_{\pi\pi}}{\tau_\pi} \pi^{\mu\nu} \theta - \frac{\tau_{\pi\pi}}{\tau_\pi} \pi_\alpha^{\langle\mu} \sigma^{\nu\rangle\alpha} + \frac{\varphi_7}{\tau_\pi} \pi_\alpha^{\langle\mu} \pi^{\nu\rangle\alpha}$$

for RTA BE: $\frac{\delta_{\pi\pi}}{\tau_\pi} = \frac{4}{3}$, $\frac{\tau_{\pi\pi}}{\tau_\pi} = \frac{10}{7}$, $\phi_7 = 0$

same result obtained by Jaiswal with modified Chapman-Enskog method

A. Jaiswal, Phys. Rev. C87, 051901 (2013)

- **anisotropic hydrodynamics (aHydro)** employing Tinti *anisotropic matching principle*

L. Tinti, Phys.Rev. C94 (2016) 044902

for the special case of RTA BE EOM read

$$D\pi^{\langle\mu\nu\rangle} + \frac{1}{\tau_{\text{eq}}} \pi^{\mu\nu} = -\left(\sigma_{\rho\sigma} + \frac{1}{3} \theta \Delta_{\rho\sigma} \right) \int dP \frac{p^{\langle\mu} p^{\nu\rangle} p^\rho p^\sigma f_\alpha}{(p \cdot U)^2} - 2\pi_\alpha^{\langle\mu} \sigma^{\nu\rangle\alpha} + 2\mathcal{P} \sigma^{\mu\nu} - \frac{5}{3} \theta \pi^{\mu\nu}$$

$$f_\alpha(x, p) = k \exp \left[-\frac{1}{\Lambda(x)} \sqrt{p_\mu \Xi^{\mu\nu}(x) p_\nu} \right]$$

$$\Xi^{\mu\nu} = \phi_U U^\mu U^\nu + \xi^{\langle\mu\nu\rangle}$$

Implementation of (1+1)D non-boost-invariant expansion

- consider the transversally homogenous system
- in hyperbolic coordinates choose the complete orthonormal basis

$$U^\mu = (\cosh(\eta + \theta_{\parallel}(\tau, \eta)), 0, 0, \sinh(\eta + \theta_{\parallel}(\tau, \eta)))$$

$$X^\mu = (0, 1, 0, 0)$$

$$Y^\mu = (0, 0, 1, 0)$$

$$Z^\mu = (\sinh(\eta + \theta_{\parallel}(\tau, \eta)), 0, 0, \cosh(\eta + \theta_{\parallel}(\tau, \eta)))$$

$$\Delta^{\mu\nu} = -\sum_I I^\mu I^\nu \quad I \in \{X, Y, Z\}$$

- the symmetry implies significant simplifications

$$\pi^{\mu\nu} = \sum_I \pi_I I^\mu I^\nu \quad \pi_X = \pi_Y = \pi_s(\tau, \eta)/2 \quad \pi_Z = -\pi_s(\tau, \eta)$$

$$\mathcal{P}_I = I_{\mu\nu} T^{\mu\nu} \quad \mathcal{P}_T = \mathcal{P} + \frac{\pi_s}{2} \quad \mathcal{P}_L = \mathcal{P} - \pi_s$$

- energy-momentum conservation equations read

$$D\mathcal{E} = -(\mathcal{E} + \mathcal{P}_L)\theta, \quad D_L \mathcal{P}_L = -(\mathcal{E} + \mathcal{P}_L)\theta_L$$

$$D_L = Z^\mu \partial_\mu, \quad \theta_L = \partial_\mu Z^\mu$$

Dissipative hydrodynamics equations in (1+1)D setup (I)

- Müller-Israel-Stewart (MIS) approach

$$D\pi_s + \frac{1}{\tau_\pi} \left(\pi_s - \frac{4\eta}{3} \theta \right) = -\frac{4}{3} \theta \pi_s$$

- Baier-Romatschke-Son-Starinets-Stephanov (BRSSS) approach

$$D\pi_s + \frac{1}{\tau_\pi} \left(\pi_s - \frac{4\eta}{3} \theta \right) = -\frac{4}{3} \theta \pi_s - \frac{5}{14\eta} \pi_s^2$$

- Denicol-Niemi-Molnar-Rischke (DNMR) approach

$$D\pi_s + \frac{1}{\tau_\pi} \left(\pi_s - \frac{4\eta}{3} \theta \right) = -\frac{38}{21} \theta \pi_s$$

Dissipative hydrodynamics equations in (1+1)D setup (II)

- anisotropic hydrodynamics (aHydro) approach

$$f_{\alpha}(x, p) = k f_{iso} \left[-\frac{1}{\Lambda(x)} \sqrt{(p_{\mu} U^{\mu})^2 + \xi (p_{\mu} Z^{\mu})^2} \right]$$

- thermodynamic variables, $f_{iso}(x) = e^{-x}$

$$\begin{aligned} \mathcal{E} &= \mathcal{E}_{eq}(\Lambda) \mathcal{R}(\xi) = 3\mathcal{P}_{eq}(\Lambda) \mathcal{R}(\xi) \\ \mathcal{P}_L &= \mathcal{P}_{eq}(\Lambda) \mathcal{R}_L(\xi) \\ \mathcal{P}_T &= \mathcal{P}_{eq}(\Lambda) \mathcal{R}_T(\xi) \end{aligned}$$

- energy-momentum conservation

$$\mathcal{R}(\xi) D \ln \mathcal{P}_{eq}(\Lambda) + \mathcal{R}'(\xi) D \xi + \frac{2}{3} \left[\mathcal{R}_T(\xi) + \mathcal{R}_L(\xi) \right] \theta = 0$$

$$\mathcal{R}_L(\xi) D_L \ln \mathcal{P}_{eq}(\Lambda) + \mathcal{R}'_L(\xi) D_L \xi + 2 \left[\mathcal{R}_T(\xi) + \mathcal{R}_L(\xi) \right] \theta_L = 0$$

- anisotropy evolution

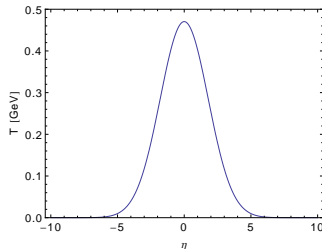
$$\left[\mathcal{R}_T(\xi) - \mathcal{R}_L(\xi) \right] \left[D \ln \mathcal{P}_{eq}(\Lambda) + \frac{1}{\tau_{eq}} \right] + \left[\mathcal{R}'_T(\xi) - \mathcal{R}'_L(\xi) \right] \left[D \xi - 2(1 + \xi) \theta \right] = 0$$

Setup

- we have to solve 3 coupled PDE for:

$T(\tau, \eta), \theta_{\parallel}(\tau, \eta), \pi_s(\tau, \eta)$ (viscous) or

$\Lambda(\tau, \eta), \theta_{\parallel}(\tau, \eta), \xi(\tau, \eta)$ (anisotropic)



- initial setup:

$\mathcal{E}(\tau_0, \eta) = \mathcal{E}_0 \exp(-\eta^2/(2a^2))$ with $a = 0.9, \mathcal{E}_0 = 100 \text{ GeV/fm}^3$ at $\tau_0 = 0.3 \text{ fm}$

$\alpha T^4 = \mathcal{E}$ $\alpha = 15.63$

$\theta_{\parallel}(\tau_0, \eta) = b \eta$ $b = 10^{-10}$

$\pi_s(\tau_0, \eta) = c \mathcal{P}(\tau_0, \eta)$ $c = 10^{-10}$

$\frac{\pi_s(\tau_0, \eta)}{\mathcal{P}(\tau_0, \eta)} = 1 - \frac{\mathcal{R}_L(\xi(\tau_0, \eta))}{\mathcal{R}(\xi(\tau_0, \eta))} \rightarrow \xi(\tau_0, \eta)$

- technicalities:

large spatial gradients \rightarrow weighted LAX (wLAX) algorithm with $\lambda_{\text{LAX}} = 0.01$

[M. Martinez, R. R., and M. Strickland, Phys. Rev. C85, 064913 \(2012\)](#)

standard fourth-order Runge-Kutta for temporal update

Results

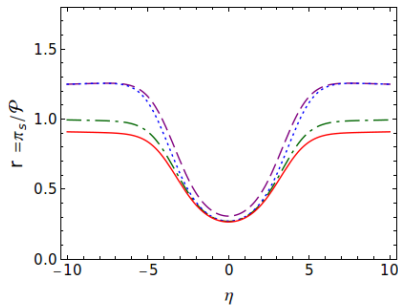
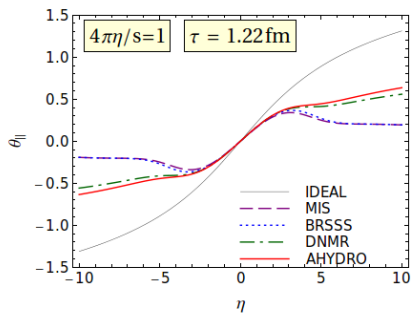
- $\eta/s = 1/(4\pi)$ ¹

$$R^{-1} = \sqrt{\pi_{\mu\nu}\pi^{\mu\nu}}/\mathcal{P} = \sqrt{3/2}\pi_s/\mathcal{P} = \sqrt{3/2}r$$
$$\tau_{\text{shock}}^{\text{BRSSS}} < \tau_{\text{shock}}^{\text{MIS}} < \tau_{\text{shock}}^{\text{DNMR}} \ll \tau_{\text{shock}}^{\text{AHYDRO}}$$

all formulations crash eventually unless shock-dealing procedures are employed

¹Animation: open in Adobe Reader.

Results



$$\text{BRSSS: } \sigma^{\langle\mu\alpha} \sigma_{\alpha}^{\nu\rangle} \rightarrow \pi^{\langle\mu\alpha} \pi_{\alpha}^{\nu\rangle} \text{ through } \pi^{\mu\alpha} = 2\eta\sigma^{\mu\alpha}$$

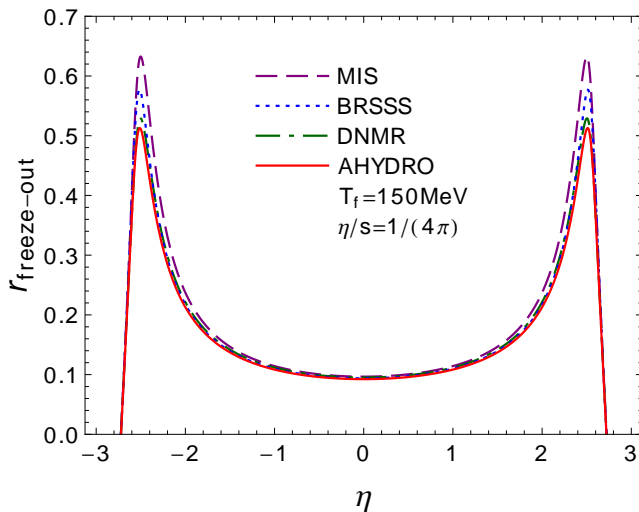
$$\text{DNMR: } \sigma^{\langle\mu\alpha} \pi_{\alpha}^{\nu\rangle}$$

Results

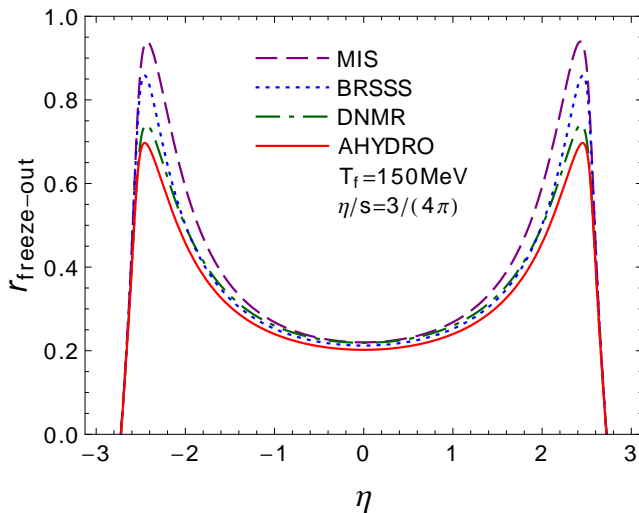
- $\eta/s = 3/(4\pi) \quad ^2$

²Animation: open in Adobe Reader.

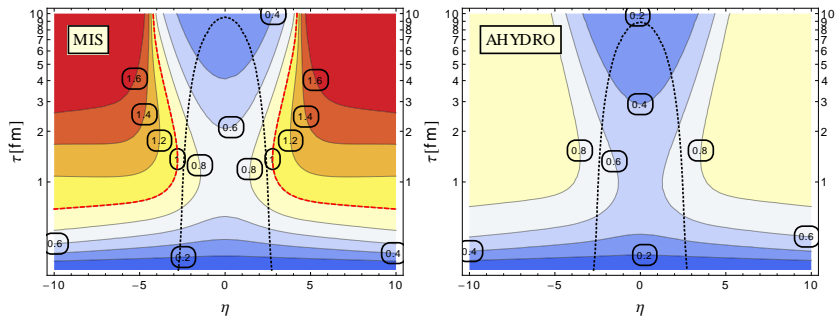
Results



Results



Results



Summary

- negative pressure is present in standard hydrodynamic approaches - no such situation allowed in underlying kinetic theory (if fields are neglected)
- pressure in aHydro is always positive thus the dynamics is self-regularized at large spacetime rapidity
- negative pressure at large spacetime rapidity results in matter pile-up and faster development of a shock front
- aHydro substantially delays the formation of a shock
- different treatment of the shear-to-shear coupling terms in the EOM of the shear-stress tensor produces, contrary to common assumptions, a significant difference in the evolution of the system away from equilibrium

Thank you for your attention!