Plasma Instabilities from Hard Expanding Loops

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work in collaboration with:

Paul Romatschke (Colorado) and Mike Strickland (Kent U.)

Max Attems (now Barcelona), Andreas Ipp, and Dominik Steineder (TU Wien)

all following the Gospel According to St. Mrówczyński

History

Abelian:

E. S. Weibel: “Spontaneously growing transverse waves in a plasma due to an anisotropic velocity distribution”, PRL 2 (1959) 83-84

QGP:

- U. W. Heinz: Quark Matter 1984 (NPA418 (1984) 603c)
- St. Mrówczyński; Pokrovsky & Selikhov 1988ff
  Mrówczyński & Thoma 2000: Hard loop approach. . .
  Randrup & Mrówczyński 2003
- P. Romatschke & M. Strickland: PRD68 (2003) 036004
  P. Arnold, J. Lenaghan, G. D. Moore: JHEP 0308 (2003) 002
  St. Mrówczyński, AR, M. Strickland: PRD70 (2004) 025004

Fate of non-Abelian plasma instabilities

Numerical simulations of anisotropic hard loop effective theory:

- 1D+3V: AR, Romatschke, Strickland 2004: PRL 94 (2005)
- 3D+3V: Arnold, Moore, Yaffe: PRD72 (2005) – The Fate...
  AR, Romatschke, Strickland: JHEP 09 (2005)
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Here:

Fate in 3D+3V hard loop theory with longitudinal expansion

(boldly extrapolated to $\alpha_s \sim 0.3$ and $n(\tau_0) \propto Q_s^3 \alpha_s^{-1}$)

Hard anisotropic loop gauge boson self energy

Spectrum of unstable modes from hard loop self energy

$$\Pi^{\mu\nu}(k) = g^2 \int \frac{d^3p}{(2\pi)^3} v^\mu \partial_\beta^{(p)} f(\mathbf{p}) \left(g^{\nu\beta} - \frac{v^\nu k^\beta}{k \cdot v + i\epsilon} \right), \quad v^\mu \equiv \frac{p^\mu}{p^0}, \quad p^0 = |\mathbf{p}|$$

Special case: $f(\mathbf{p}) = f_{\rm iso}\left(\mathbf{p}^2 + \xi(\mathbf{p}\cdot\mathbf{n})^2\right)$

$\xi = 0$: isotropic; $-1 < \xi < 0$: prolate (cigar-shaped); $0 < \xi < \infty$: oblate (squashed)

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can be evaluated in closed form: [Romatschke & Strickland 2003]

Change variables $\mathbf{p}^2 + \xi(\mathbf{p} \cdot \mathbf{n})^2 = \bar{\mathbf{p}}^2$

$$\Pi^{ij}(k) = m^2 \int \frac{d\Omega}{4\pi} v^i \frac{v^l + \xi(\mathbf{v}.\mathbf{n}) n^l}{(1 + \xi(\mathbf{v}.\mathbf{n})^2)^2} \left(\delta^{jl} + \frac{v^j k^l}{k \cdot v + i\epsilon} \right)$$

$$m^2 \equiv -\frac{g^2}{2\pi^2} \int_0^\infty d\bar{p}\, \bar{p}^2 \frac{d f_{\mathrm{iso}}(\bar{p}^2)}{d\bar{p}}$$

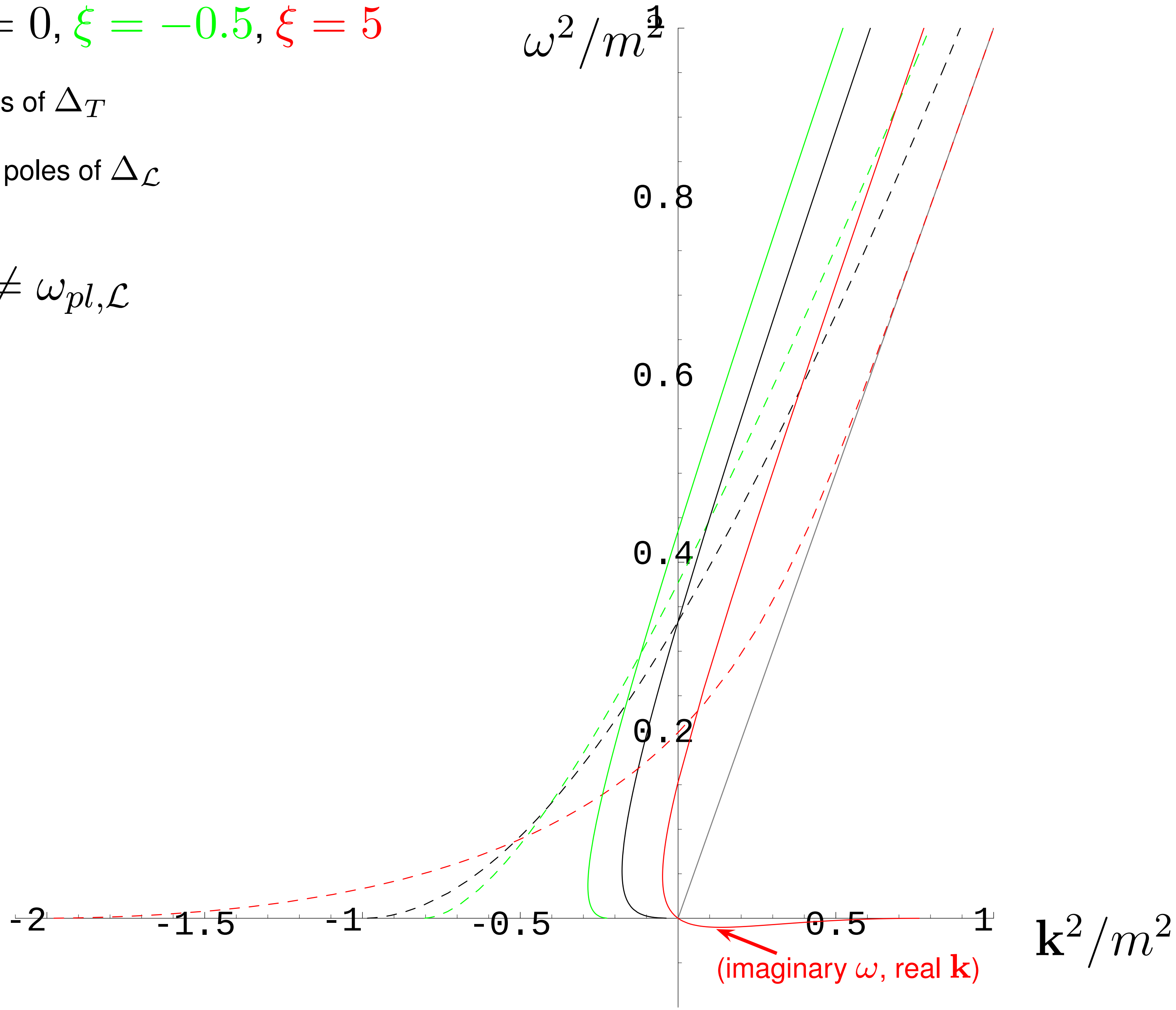
Dispersion laws in anisotropic plasma for k||n

Comparing $\xi = 0$, $\xi = -0.5$, $\xi = 5$

full lines: poles of Δ_T

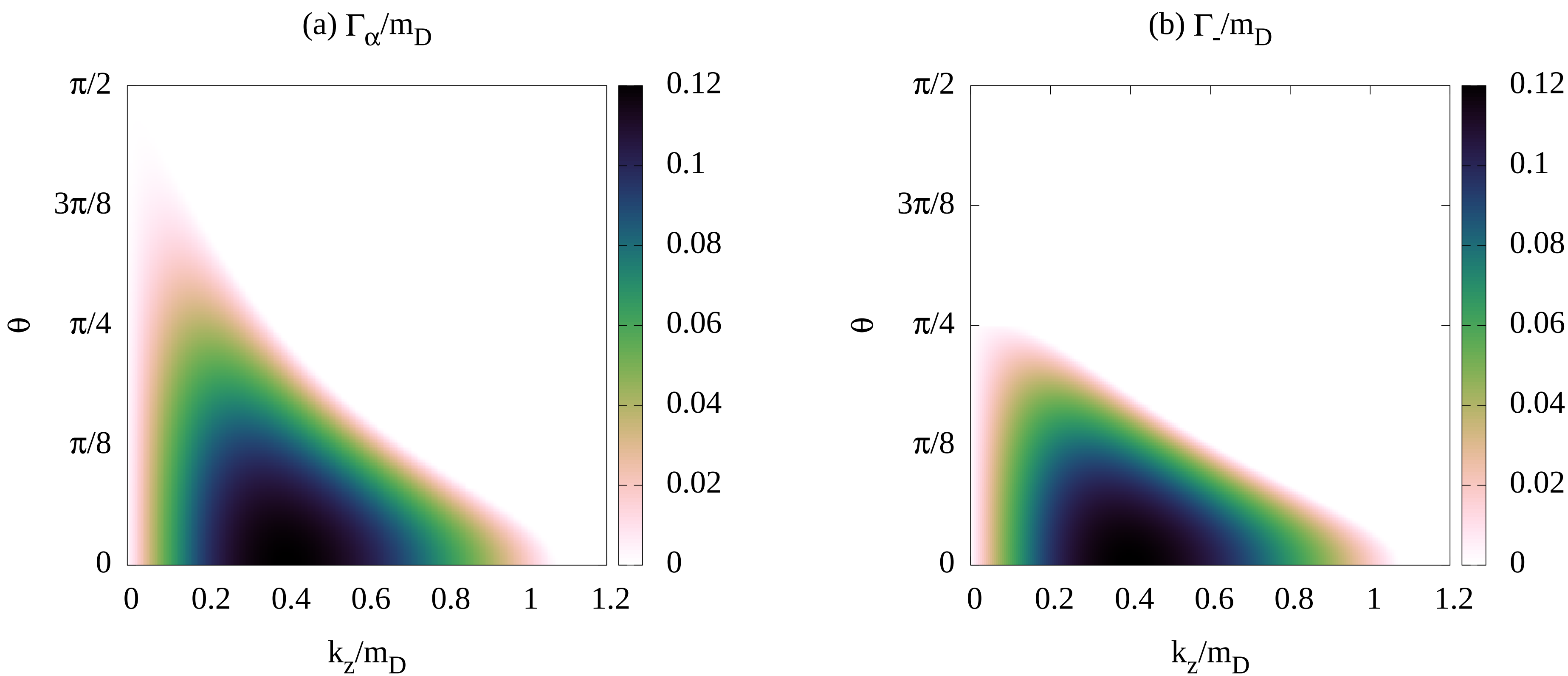
dashed lines: poles of $\Delta_{\mathcal{L}}$

$\xi \neq 0$: $\omega_{pl,T} \neq \omega_{pl,\mathcal{L}}$



Growth rates of unstable modes – general k

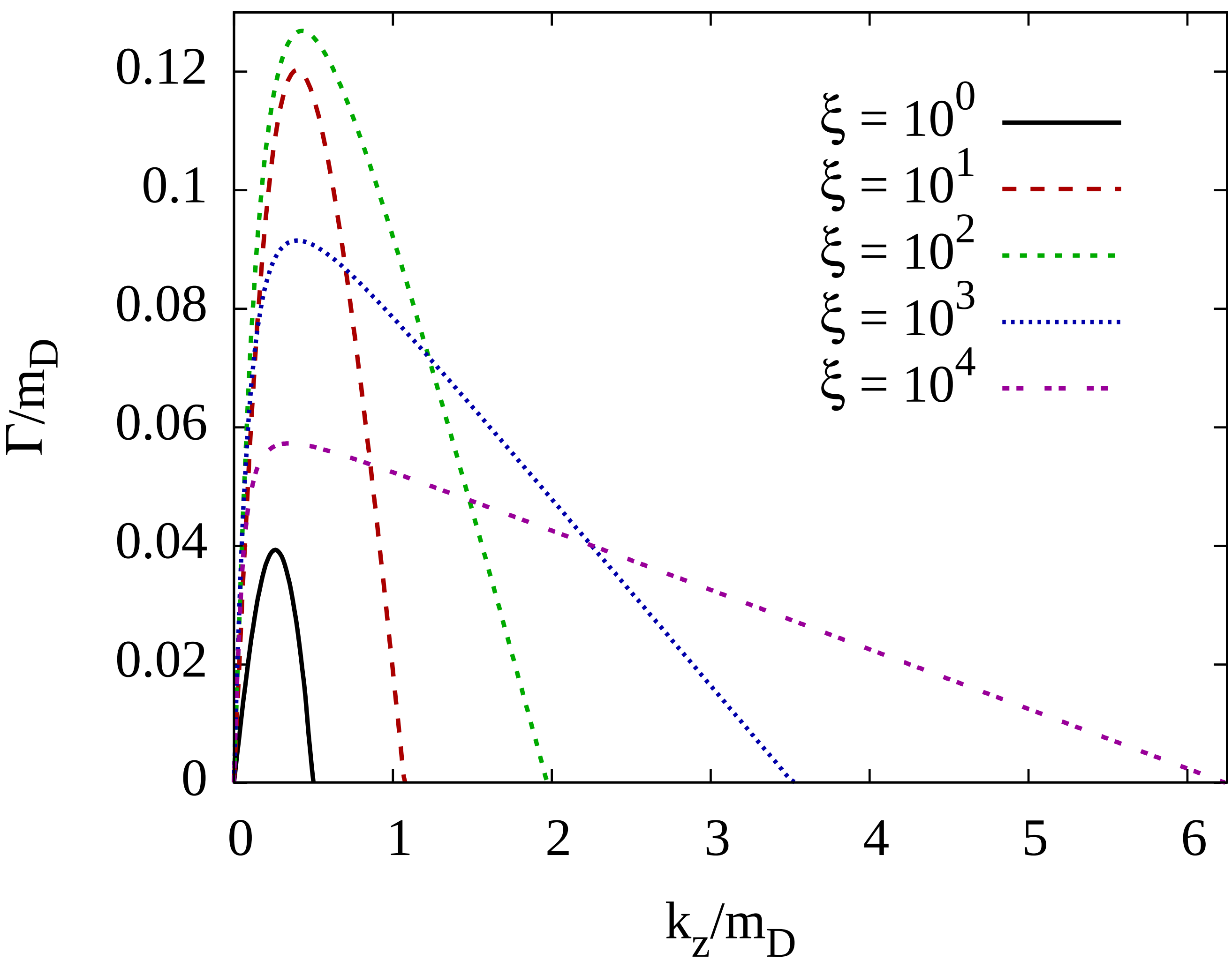
Unstable growth rates for $\xi = 10$



$\theta = \arctan(k_T/k_z)$

$m_D \ldots$ Debye mass in isotropic plasma obtained by compressing back to $\xi = 0$

Growth rates of most unstable modes (k||n)



(m_D: isotropic Debye mass before longitudinal expansion)

Hard Anisotropic Loop Effective Action

Exponentially growing non-Abelian gluon fields

⇒ linear response regime quickly left

⇒ need the (infinitely many) vertex functions required by gauge invariance

Hard (Thermal) Loop Effective Action formally still given by

[Pisarski 1993, Mrówczyński, Strickland, AR 2004]

$$S_{\rm aniso} = -\frac{g^2}{2}\int_x \int_{\mathbf{p}} \left\{ f(\mathbf{p}) F^a_{\mu\nu}(x) \left(\frac{p^\nu p^\rho}{(p\cdot D[A])^2} \right)_{ab} F_\rho{}^{b\,\mu}(x) \right\}$$

nonlinear and nonlocal!

— only useful as formal generating functional of hard anisotropic loops

Discretized Hard Loop Effective Theory

Useful:

auxiliary field formulation: [Nair; Blaizot & Iancu 1994; Mrówczyński, AR & Strickland 2004]

$$\delta f^a(x;p) = -gW^a_\mu(t,\mathbf{x};\mathbf{v})\partial^\mu_{(p)} f_0(\mathbf{p})$$

$$\boxed{[v\cdot D(A)]W_\mu(x;\mathbf{v}) = F_{\mu\gamma}(A)v^\gamma}$$

$$v^\mu \equiv p^\mu/|\mathbf{p}| = (1,\mathbf{v})$$

$$\boxed{D_\rho(A)F^{\rho\mu} = j^\mu(x) = -g^2\int \frac{d^3p}{(2\pi)^3}\frac{1}{2|\mathbf{p}|}\,p^\mu\,\frac{\partial f(\mathbf{p})}{\partial p^\nu}W^\nu(x;\mathbf{v})}$$

Hard Loop effective theory: (hard) scale $|\mathbf{p}|$ can be integrated out

Auxiliary field version: local in terms of field living also on velocity space S_2

Nonlinear response $\longrightarrow$ real-time lattice simulation

$\longrightarrow$ discretize also velocity space

$$D_\rho(A)F^{\rho\mu} = j^\mu(x) = \frac{1}{\mathcal{N}}\sum_{\mathbf{v}} v^\mu \mathcal{W}_{\mathbf{v}}(x)$$

Transversely constant modes: 1D+3V

Most unstable modes in linear response: $\mathbf{k} \parallel \mathbf{n}$

$\Longrightarrow$ no dependence on transverse coordinates;
dimensional reduction to 1 spatial dimension

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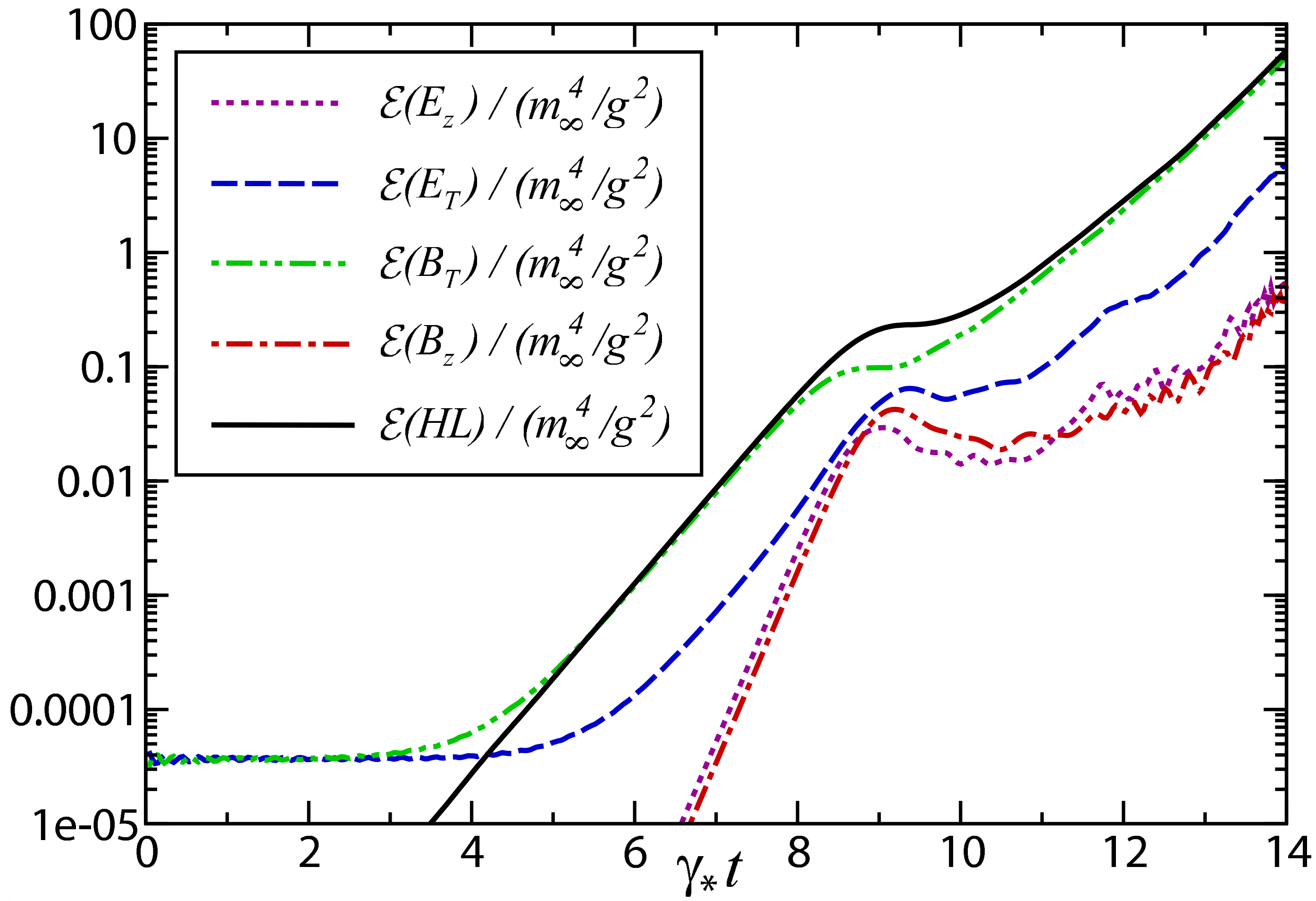
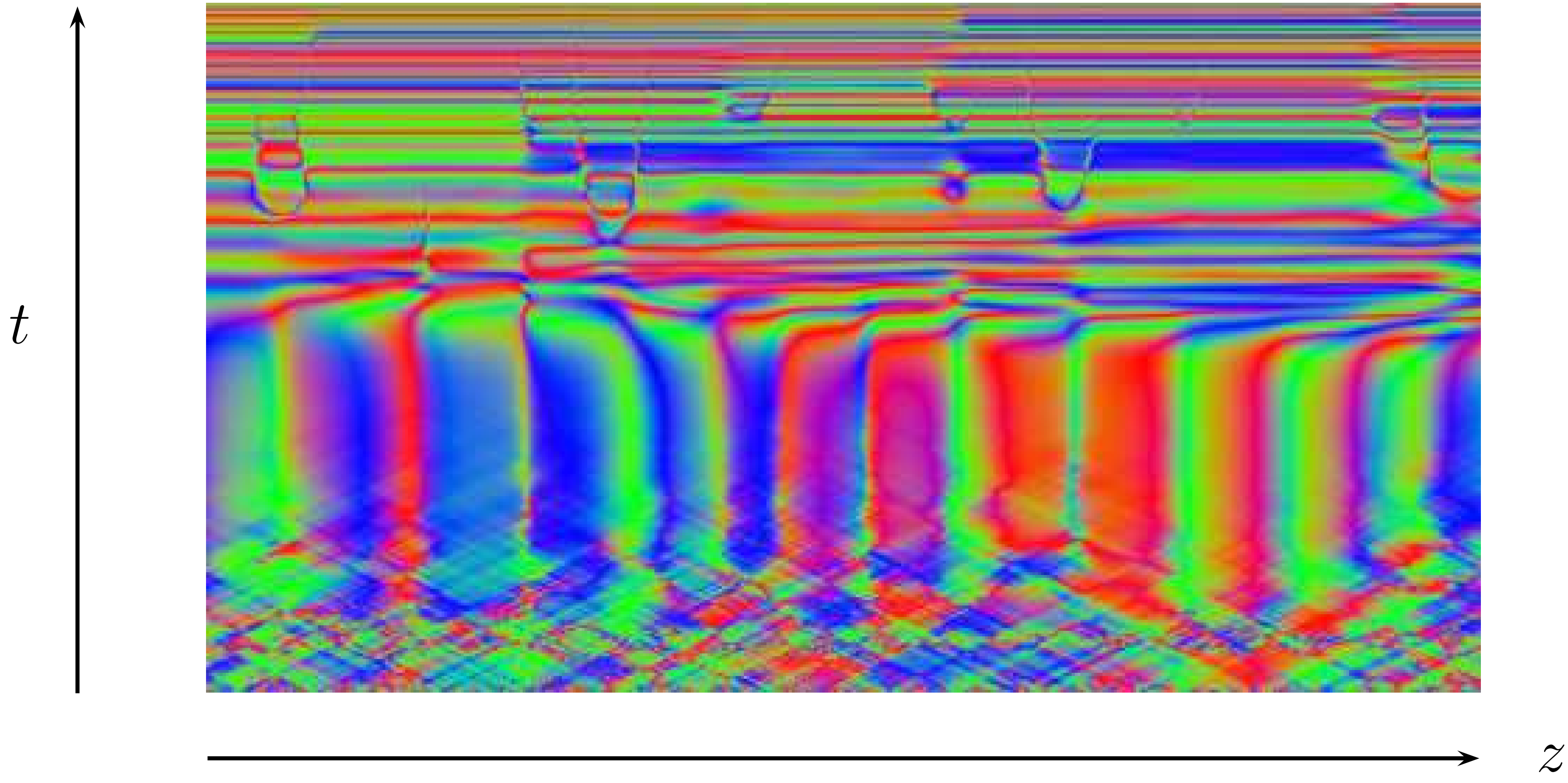
First simulation of nonabelian instabilities: [AR, Romatschke & Strickland, PRL 94 ('05) 102303]



Transversely constant modes: 1D+3V

Evolution of color degrees of freedom:
(parallel-transported color from fixed spatial point)



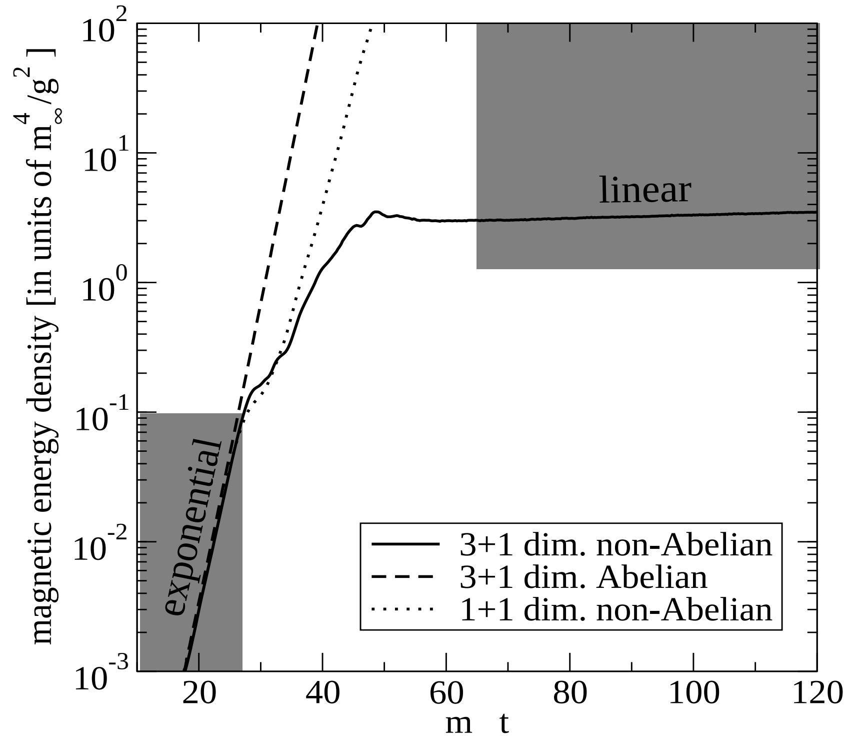
Late-time (non-linear) regime: Abelianization over extended spatial domains
– responsible for continued Abelian-like growth in non-linear regime

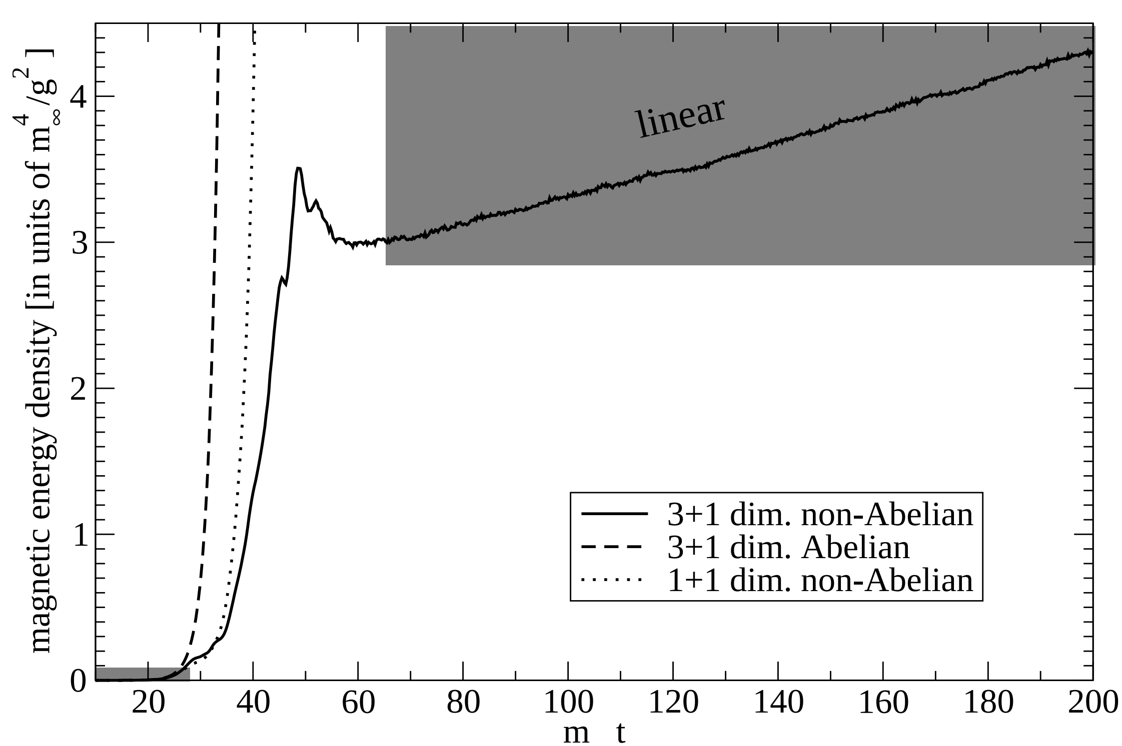
3D+3V

However: local Abelianization can be destroyed by interactions with not perfectly transversely constant modes

3D+3V

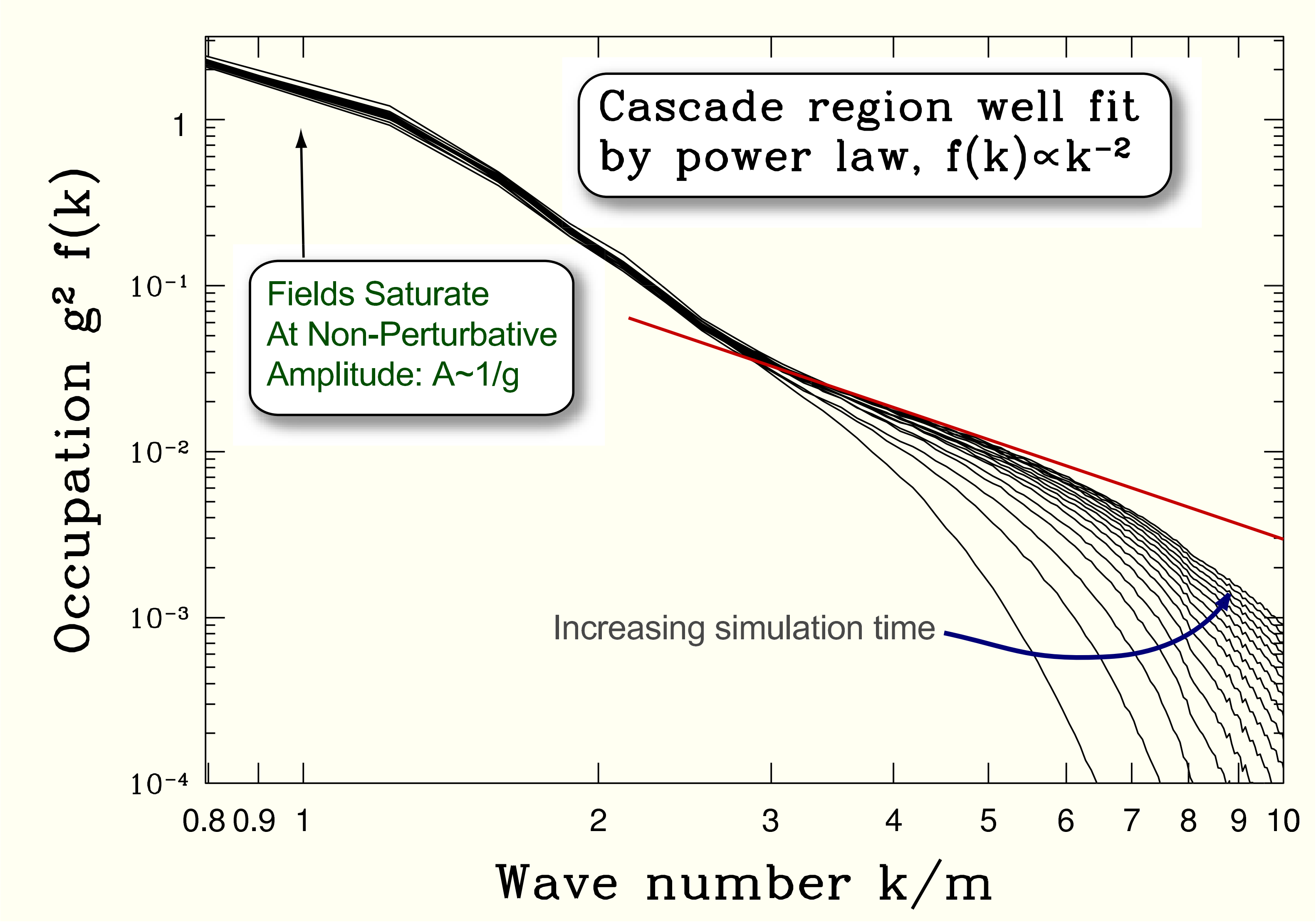
However: local Abelianization can be destroyed by interactions with not perfectly transversely constant modes
$\rightarrow$ attenuation of exponential growth to only linear in regime where backreaction still parametrically small! [Arnold, Moore & Yaffe, PRD72 (2005) 054003]
[AR, Romatschke & Strickland, JHEP 09 (2005) 041]





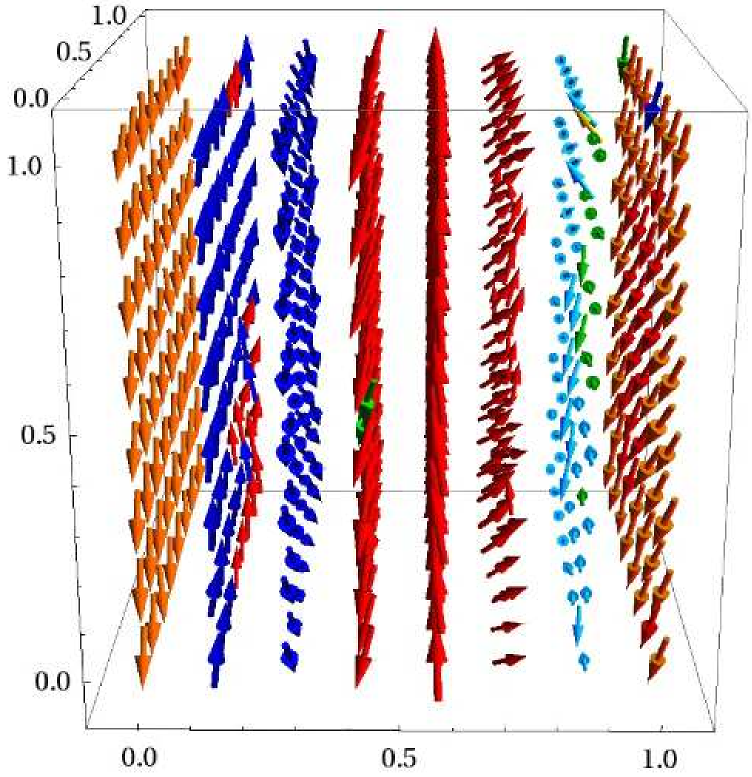
stronger anisotropies: no saturation? (when initialized with tiny seed fields)
Bödeker & Rummukainen, JHEP 07 (2007) 02

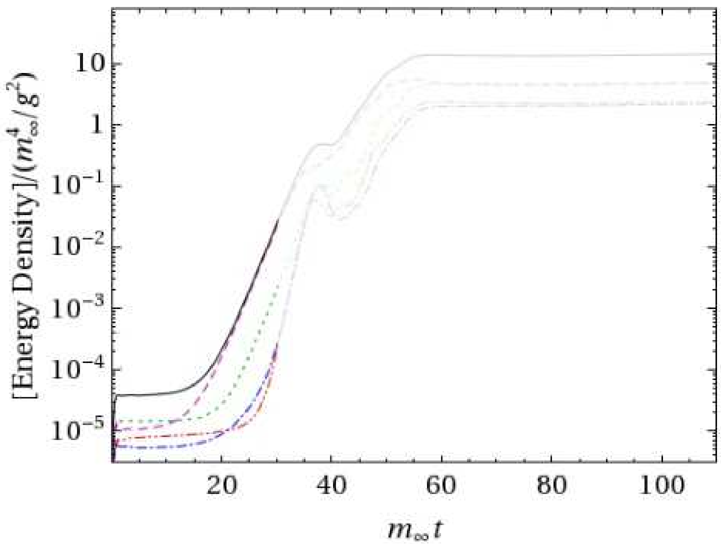
Turbulent energy cascade

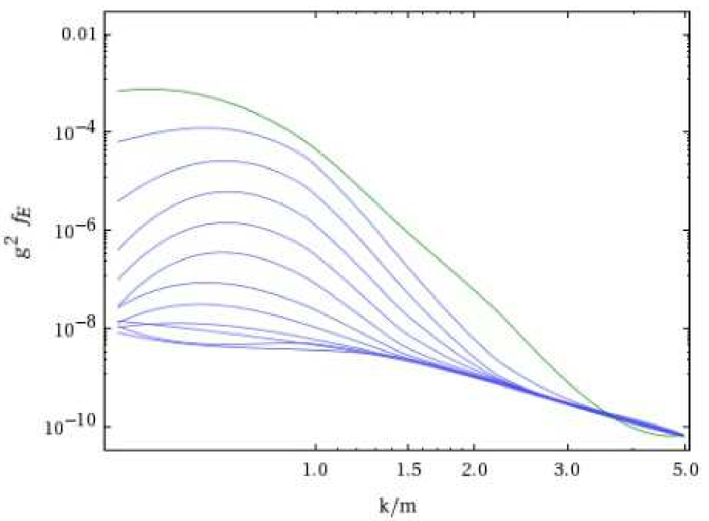


3D+3V and SU(3)

A. Ipp, AR, M. Strickland, Phys.Rev. D84 (2011) 056003







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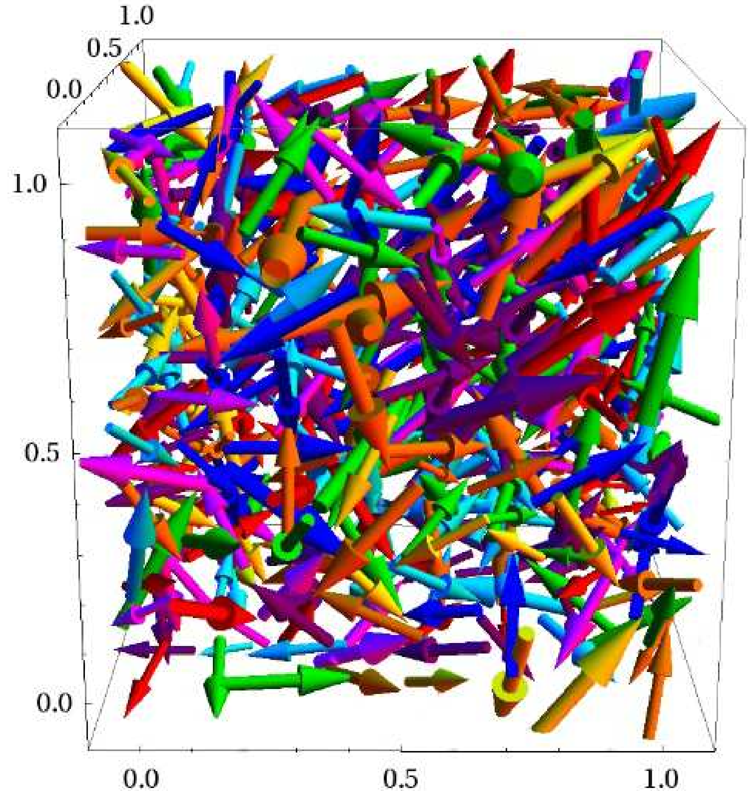

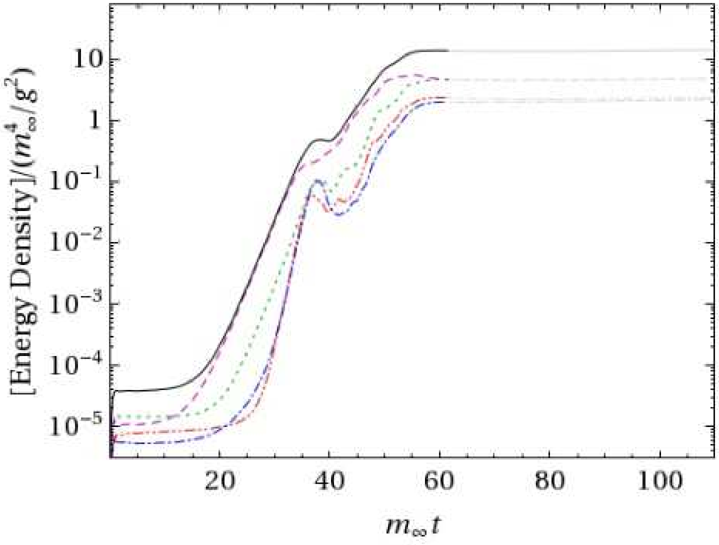

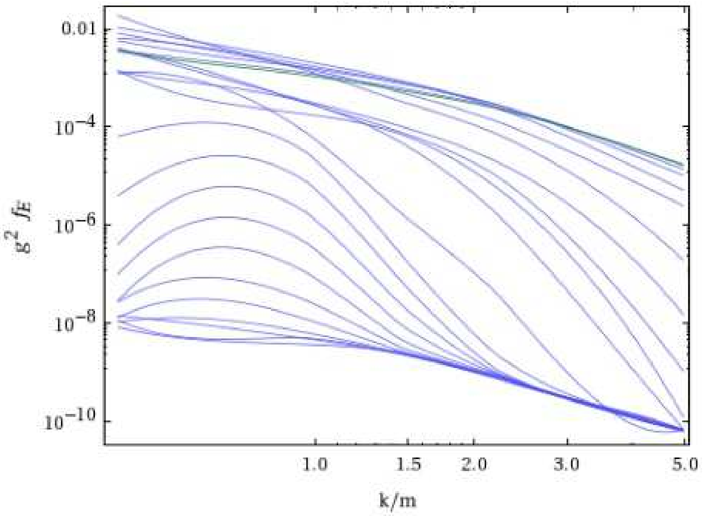

formation of turbulent cascade, again with $f \sim k^{-2}$

Plasma instabilities in Bjorken expansion

Longitudinal (Bjorken) expansion: Competition between

- increasing anisotropy (more and more modes become unstable)
- and decreasing density ($\leftrightarrow$ growth rate)

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Longitudinal (Bjorken) expansion: Competition between

- increasing anisotropy (more and more modes become unstable)
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Notation: proper time $\tau = \sqrt{t^2 - z^2}$ and space-time rapidity $\eta = \mathrm{atanh}\frac{z}{t}$

$$x^\mu \to x^\alpha = (\tau, x^i, \eta) \text{ with } g_{\alpha\beta} = (1, -1, -1, -\tau^2)$$

momentum rapidity $y = \mathrm{atanh}\frac{p^0}{p^z}$:

$$p^\mu \to p^\alpha = |\mathbf{p}_\perp|(\cosh(y-\eta), \cos\phi, \sin\phi, \tau^{-1} \underbrace{\sinh(y-\eta)}_{p'^z/|\mathbf{p}_\perp|})$$

Boost-invariant free-streaming background e.g.

$$f_0(\mathbf{p}, x) = f_{\mathrm{iso}}\left(\sqrt{p_\perp^2 + p_\eta^2/\tau_{\mathrm{iso}}^2}\right) = f_{iso}\left(\sqrt{p_\perp^2 + (p'^z\tau/\tau_{iso})^2}\right)$$

with space-time dependent anisotropy parameter $\xi(\tau) = (\tau/\tau_{iso})^2 - 1$

Hard-Expanding-Loop formalism

[Romatschke & AR, PRL 97 (2006)]

$$\boxed{v \cdot D\, W_\alpha(\tau, x^i, \eta; \phi, y)|_{\phi,y} = v^\beta F_{\alpha\beta},}$$

$$\boxed{\frac{1}{\tau} D_\alpha \left[\tau g^{\alpha\gamma}(\tau) g^{\beta\delta}(\tau) F_{\gamma\delta}\right] = j^\beta,}$$

where for $f_0(\mathbf{p}, x) = f_{\rm iso}\left(\sqrt{p_\perp^2 + p_\eta^2/\tau_{\rm iso}^2}\right)$

$$j^\alpha(\tau, x^i, \eta) = -\frac{m_D^2(\tau = \tau_{iso})}{2} \int_0^{2\pi} \frac{d\phi}{2\pi} \int_{-\infty}^{\infty} dy\, v^\alpha \left(1 + \frac{\tau^2}{\tau_{iso}^2} \sinh^2(y - \eta)\right)^{-2}$$

$$\times \left\{\cos\phi\, W_1 + \sin\phi\, W_2 - \frac{\tau}{\tau_{iso}^2} \sinh(y - \eta)\, W_\eta\right\}$$

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discretized HEL: [AR, Strickland, Attems, PRD 78 (2008): 1D+3V]

$\mathcal{W}(\tau, x^i, \eta; \phi, y)$ with discretized ϕ_n, y_m

3D+3V: (massively parallelized) on new Vienna supercomputer VSC

3D+3V HEL

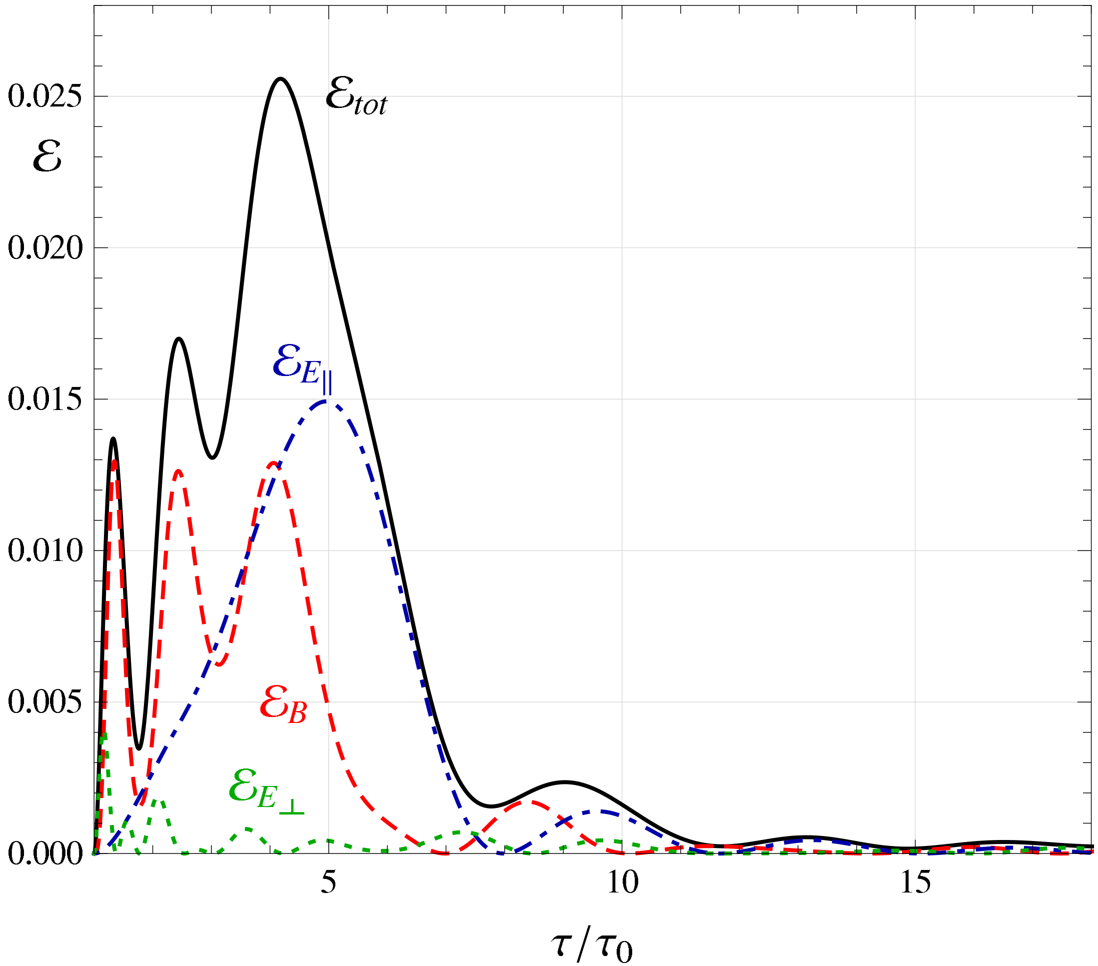
Abelian checks:

Full 3+1-dimensional semi-analytic study of linear (effectively abelian) regime

[AR & D. Steineder, Phys.Rev. D81 (2010) 085044]

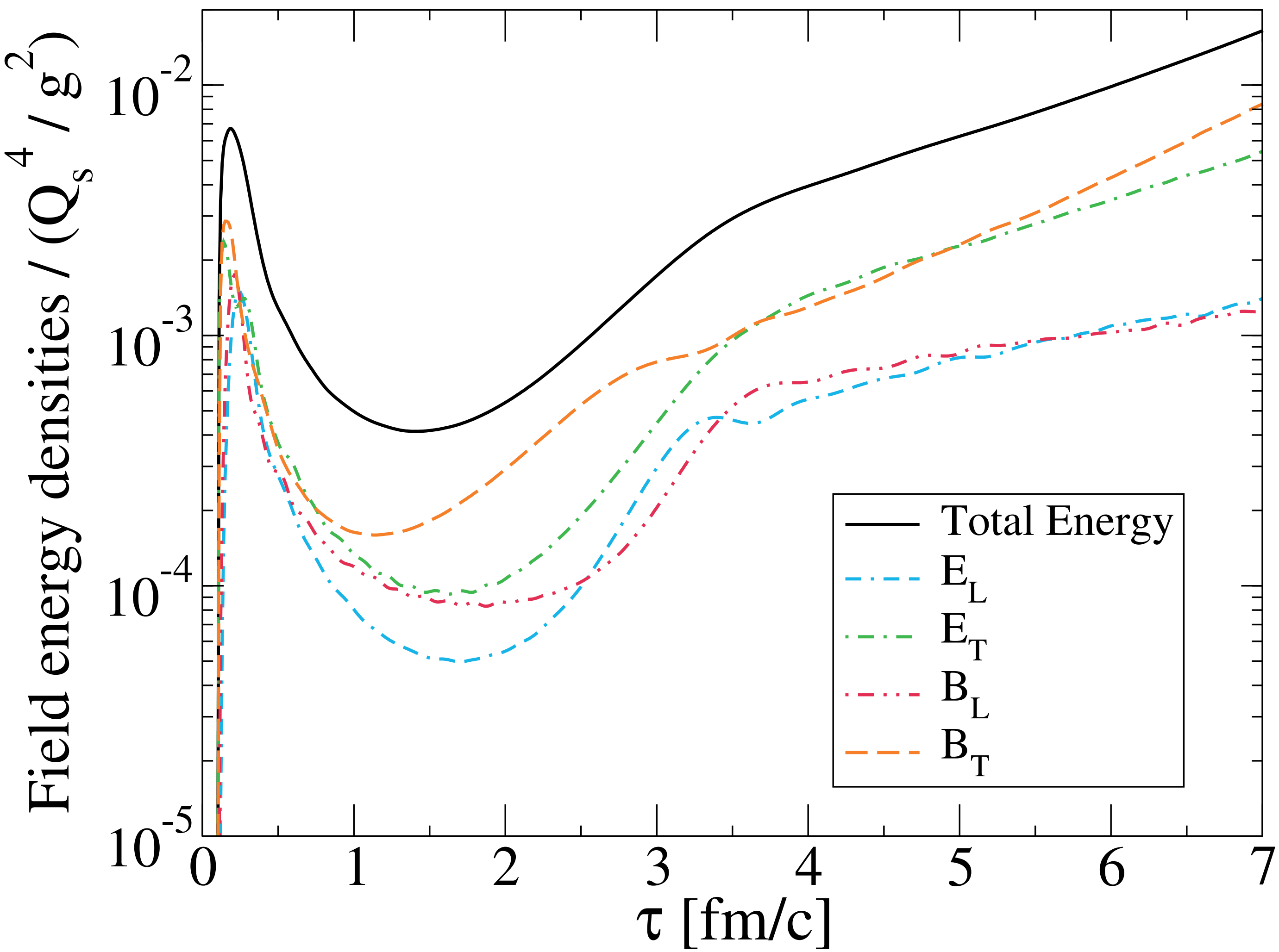
E.g.: electric (Buneman) instability for wave vector not parallel to z-axis but initially within 45^o cone to the z-axis:

(turned off by expansion because wave vector rotates away from z axis)



3D+3V HEL - Field energy densities

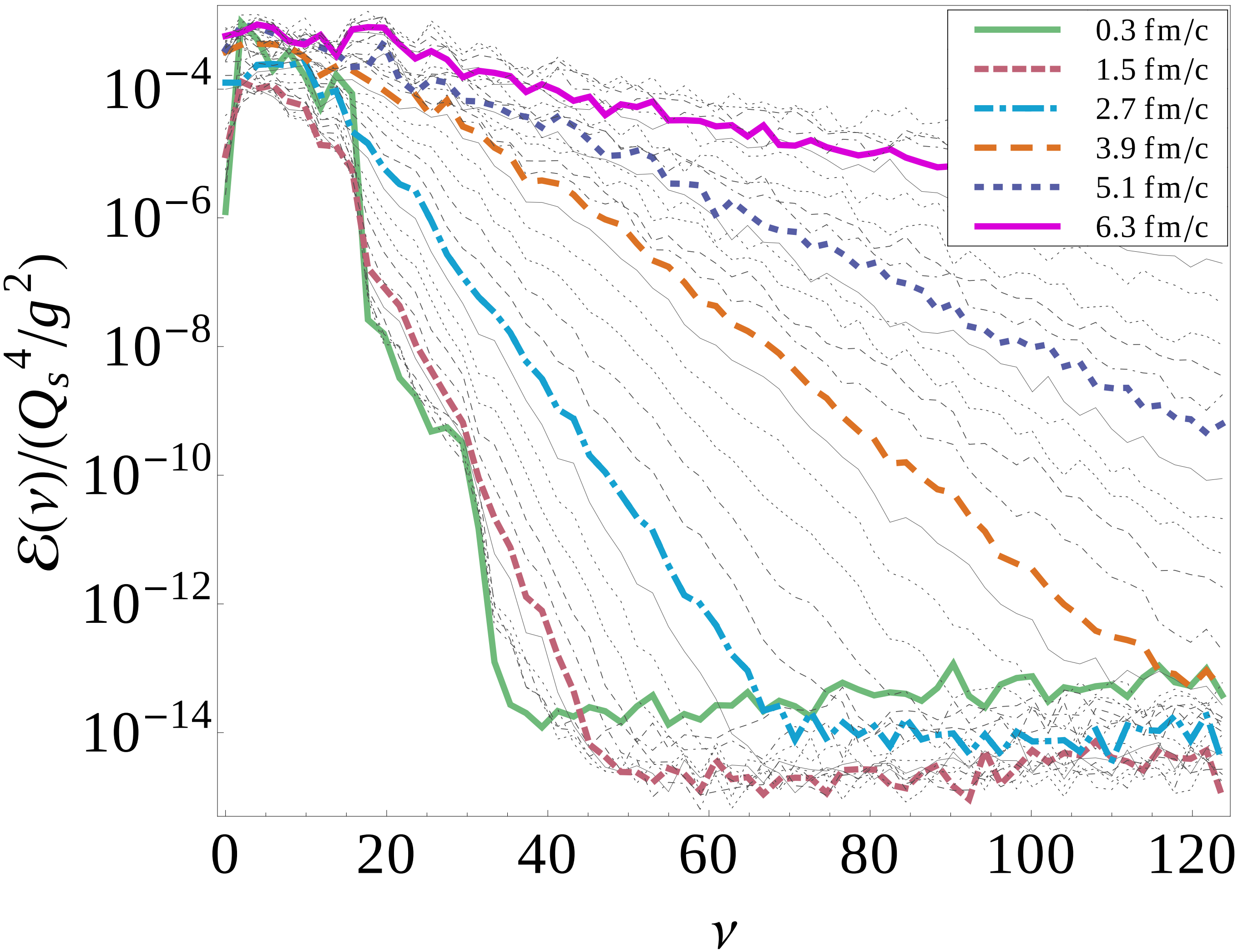
Nonabelian lattice simulations: **No saturation!** [M. Attems, AR & M. Strickland: PRD87 (2013) 025010]



scales by matching to CGC at LHC energies ($Q_s \simeq 2$ GeV)

3D+3V HEL - Longitudinal spectra

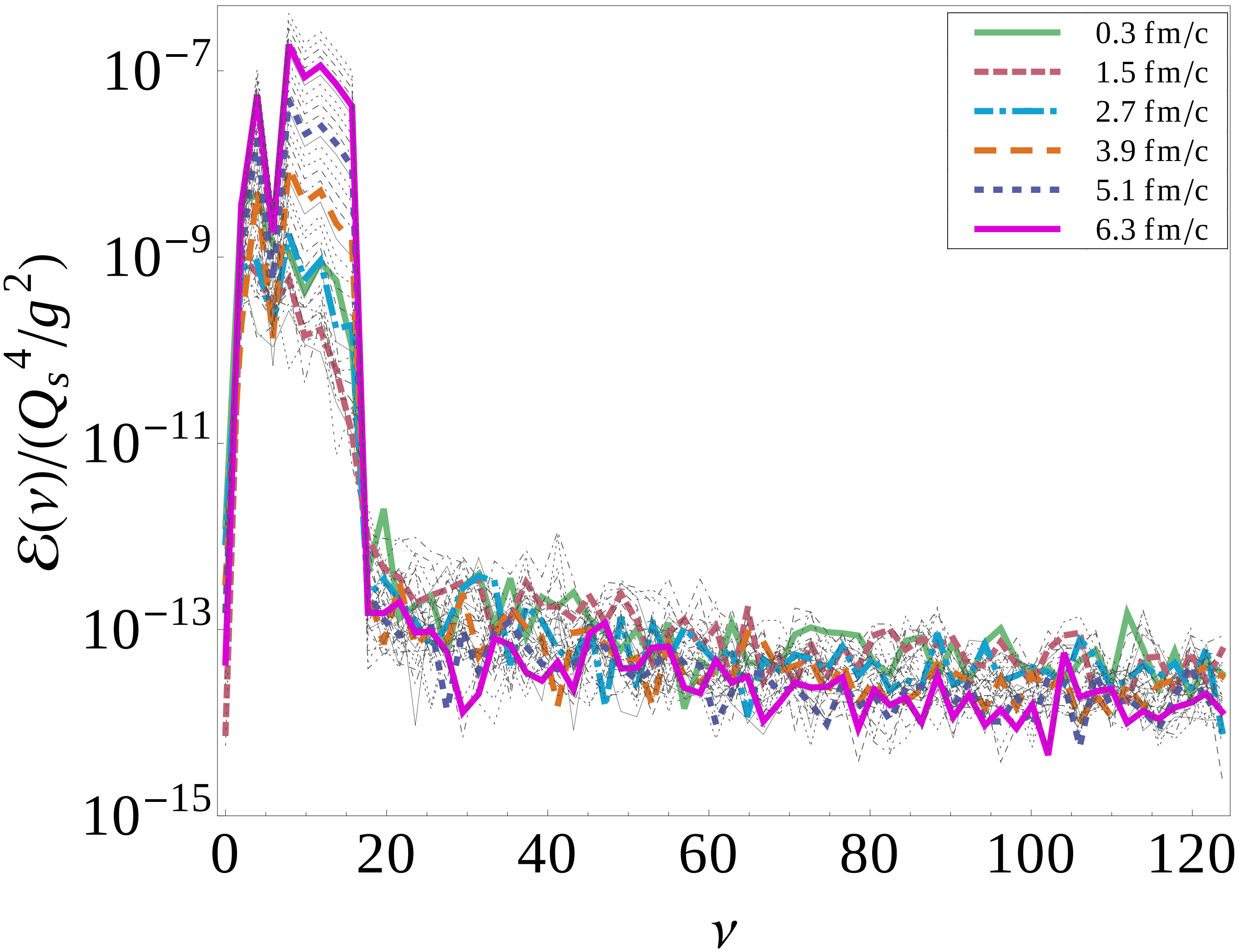
no saturation — instead of power-law: exponential!



ν: co-moving wave number in longitudinal direction

3D+3V HEL - Longitudinal spectra

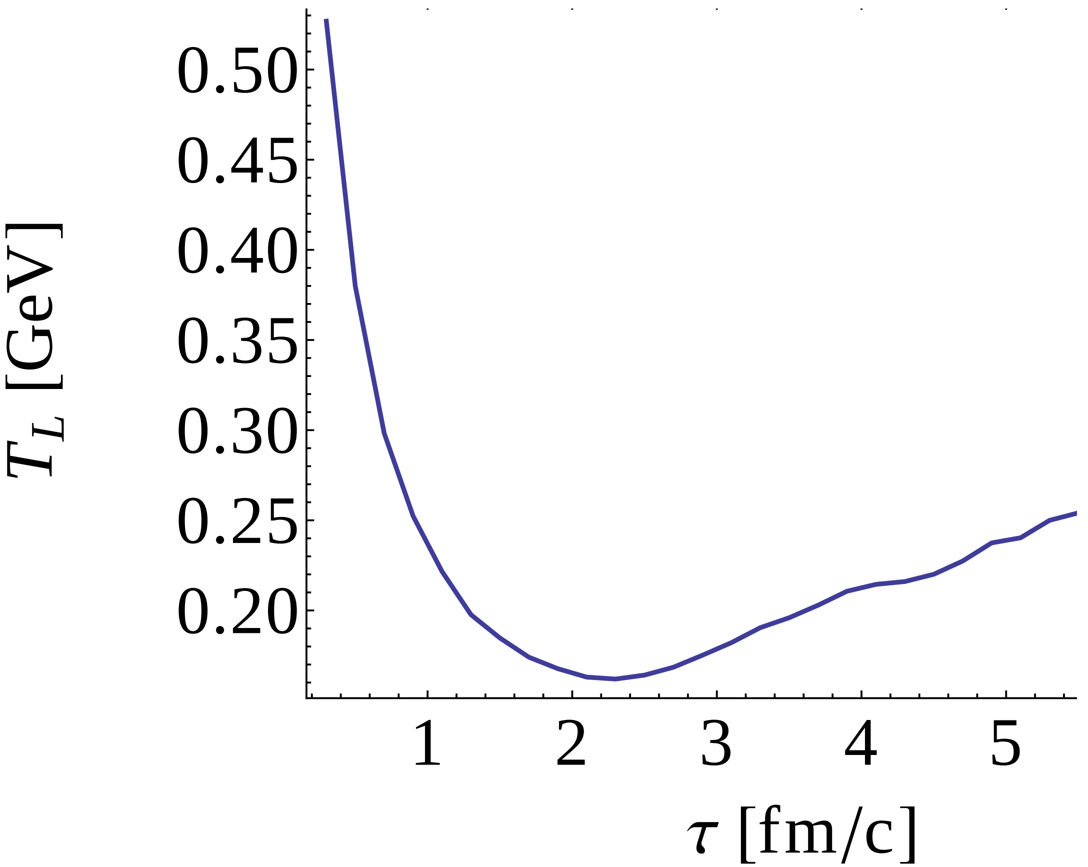
by comparison: same with Abelian gauge group



ν: co-moving wave number in longitudinal direction

3D+3V HEL - Longitudinal thermalization

Energy distribution $\mathcal{E}(\nu)$ well fitted by Boltzmann distribution
with increasing temperature for $\tau \gtrsim 2$ fm/c despite free-streaming expansion:



consistent with glasma evolution: K. Fukushima & F. Gelis, NPA874 (2012) 108
(power-law Kolmogorov behavior there only at $\tau \gtrsim 100$ fm/c !)

Conclusion

- Plasma instabilities are parametrically dominant phenomenon in anisotropic wQGP with interesting characteristic time scales

Open challenge:

- Generalization of HEL to non-free streaming of hard particles?

Conclusion

- Plasma instabilities are parametrically dominant phenomenon in anisotropic wQGP with interesting characteristic time scales
- Full 3+1-dimensional evolution of nonabelian plasma instabilities in longitudinally expanding plasma:
  - No saturation of growth in regime where backreaction on hard particles still small
  - turbulent cascade with $f \sim k^{-2}$ (only) in stationary anisotropic situation
  - HEL: instead quick *longitudinal thermalization and heating* while total pressure anisotropy remains large over several fm/c

Open challenge:

- Generalization of HEL to non-free streaming of hard particles?

Non-Abelian Discretized HEL

Hard gluon number density and initial fluctuation spectrum from **CGC** $\rightarrow$

Parameters from saturation scenario $\tau_0 \simeq Q_s^{-1}$:

$n(\tau_0) = c\frac{(N_c^2-1)Q_s^3}{4\pi^2 N_c \alpha_s (Q_s \tau_0)}$

with gluon liberation factor $c = \begin{cases} 0.5 & \text{Krasnitz '99 et al.} \rightarrow 1.1 \text{ Lappi '07 (numerical)} \\ \underline{2\ln 2} \approx 1.39 & \text{Kovchegov (analytical estimate)} \end{cases}$

$f_{\rm iso} = \mathcal{N} f_{\rm thermal}$ with (transverse) temperature $T = 0.47 Q_s$ [Krasnitz et al.]

$$\text{pure glue} \quad \rightarrow \quad \mathcal{N} = \frac{1}{\alpha_s} \frac{c}{8N_c(0.47)^3\zeta(3)} \frac{\tau_0}{\tau_{iso}} \frac{1}{Q_s \tau_0}$$

$$\rightarrow \quad \frac{\mu}{Q_s} = \frac{1}{8} m_D^2 \pi \tau_{\rm iso} Q_s^{-1} = \frac{\pi^2}{48 \cdot 0.47 \cdot \zeta(3)} c \approx \begin{cases} 0.182 & (c = 0.5) \\ \underline{0.505} & (c = 2\ln 2) \end{cases}$$

$Q_s \simeq 1$ GeV (RHIC) . . . 2 GeV (LHC) ?