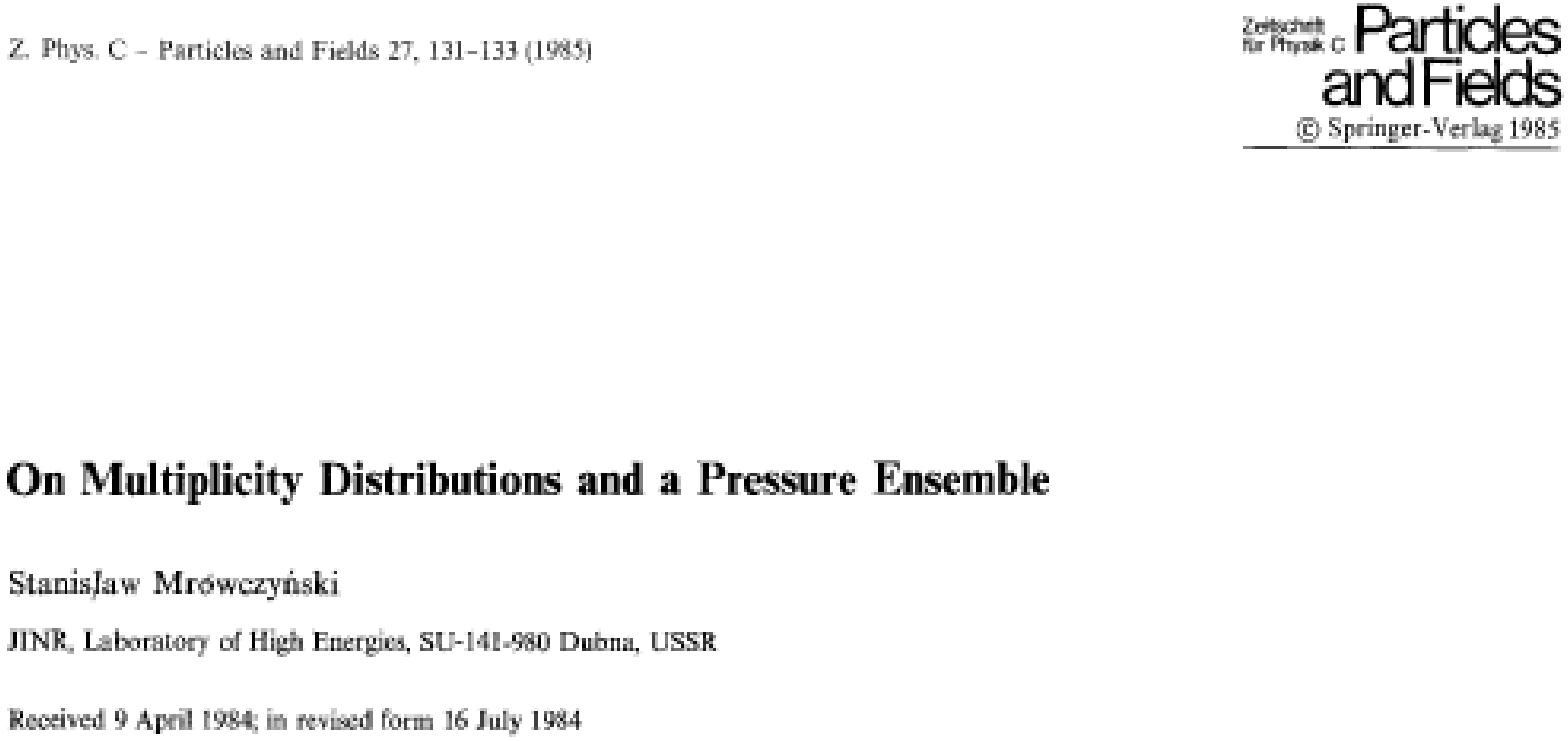
# Stanislaw Mrowczynski
# 60th Birthday

Clear formulation

Elegant solution

1985 – Doctor of Philosophy, Institute for Theoretical Physics, Kiev, Ukraine



Z. Phys. C – Particles and Fields 27, 131–133 (1985)

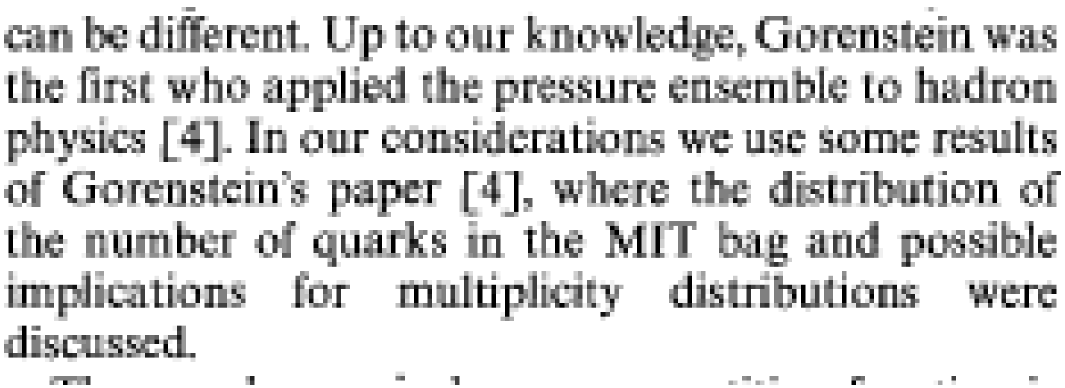
Zeitschrift für Physik C Particles and Fields

# On Multiplicity Distributions and a Pressure Ensemble

Stanisław Mrówczyński

JINR, Laboratory of High Energies, SU-141-980 Dubna, USSR

Received 9 April 1984; in revised form 16 July 1984



can be different. Up to our knowledge, Gorenstein was the first who applied the pressure ensemble to hadron physics [4]. In our considerations we use some results of Gorenstein's paper [4], where the distribution of the number of quarks in the MIT bag and possible implications for multiplicity distributions were discussed.

LETTERE AL NUOVO CIMENTO VOL. 36, N. 11 12 Marzo 1983

## On the Phase Space of Tachyons.

St. Mrówczyński
*Institute of Nuclear Research - Warsaw, Poland*

(ricevuto il 23 Agosto 1982; manoscritto revisionato ricevuto l'1 Dicembre 1982)

PACS. 03.65. – Quantum theory; quantum mechanics.

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LETTERE AL NUOVO CIMENTO VOL. 38, N. 7 15 Ottobre 1983

## On the Ideal Gas of Tachyons.

St. Mrówczyński
*Institute of Nuclear Problems - Warsaw, Poland*
*JINR, High Energy Laboratory - Dubna, USSR*

(ricevuto il 12 Luglio 1983)

PACS. 14.80. – Other and hypothetical particles.

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IL NUOVO CIMENTO VOL. 78 A, N. 4 21 Dicembre 1983

## The Phase Space of Tachyons (*).

St. Mrówczyński
*JINR - Dubna, USSR*
*Institute of Nuclear Problems - Warsaw, Poland*

(ricevuto il 9 Giugno 1983)

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IL NUOVO CIMENTO VOL. 81 B, N. 2 11 Giugno 1984

## The Ideal Gases of Tachyons (*).

St. Mrówczyński
*Institute for Nuclear Studies - 00-681 Warsaw, Hoza 69, Poland*
*JINR Laboratory of High Energy - Dubna USSR.*

(ricevuto il 5 Gennaio 1984; manoscritto revisionato ricevuto il 10 Aprile 1984)

Z. Phys. C – Particles and Fields 18, 109–111 (1983)

# Test of the Coherent Tube Model Approach to Relativistic Nucleus-Nucleus Interactions

M.Kh. Anikina[1], M. Gaździcki[2], A.I. Golokhvastov[1], M. Jędrzejczak[3], S.A. Khorozov[1], E.S. Kuznetsova[1], I. Lukstins[1], St. Mrówczyński[4], E.O. Okonov[1], T.G. Ostanevich[1], E. Skrzypczak[2], R. Szwed[2]

1 Joint Institute for Nuclear Research, Dubna USSR
2 Institute of Experimental Physics University of Warsaw, Warsaw, Poland
3 Physics Department, Białystok Branch of the University of Warsaw, Białystok, Poland
4 Institute of Nuclear Research, Warsaw, Poland

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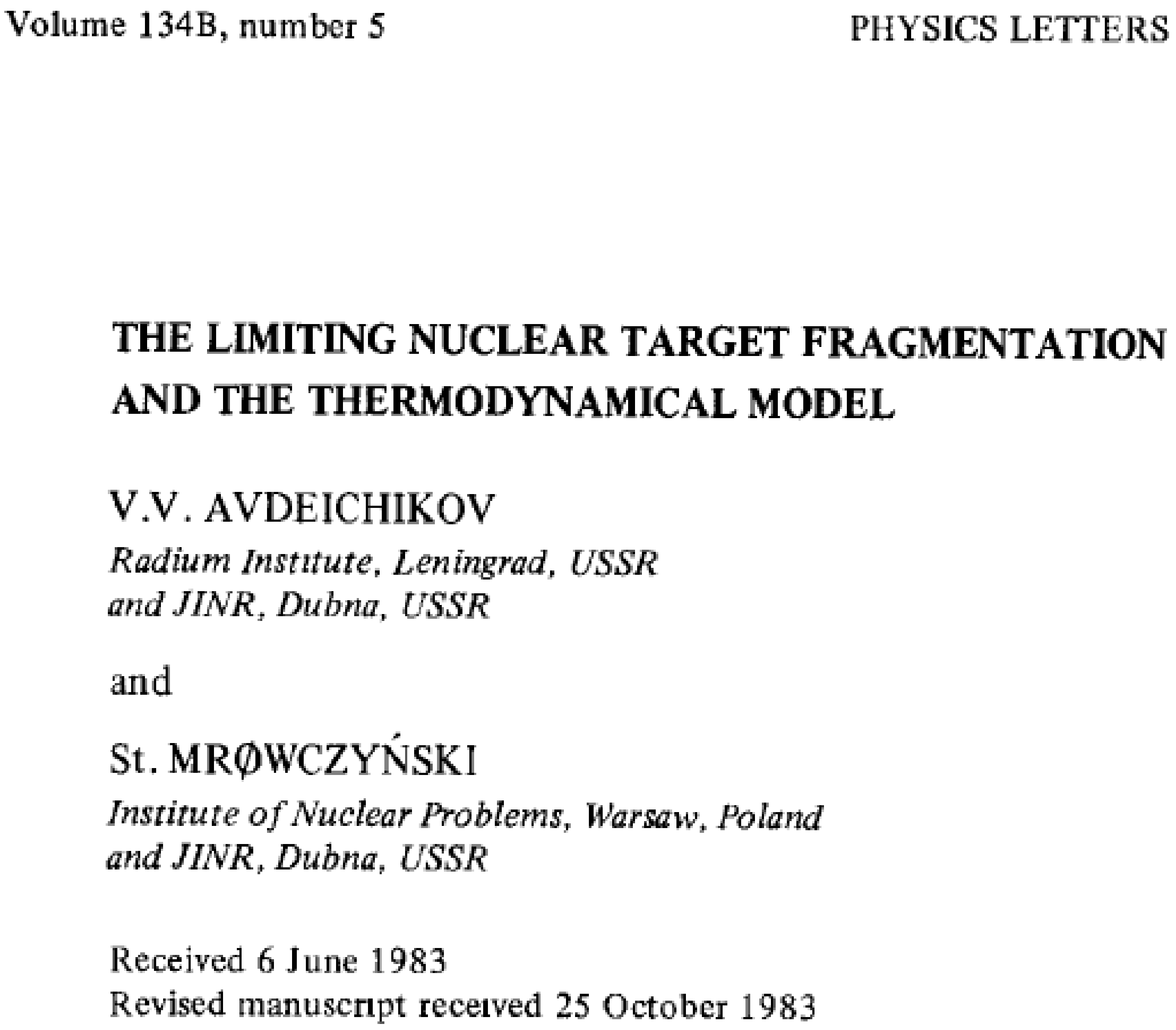
**Nuclear Physics A437** (1984) 573–589

# EMISSION OF LIGHT FRAGMENTS $^3$H, $^3$He AND $^4$He IN $^4$He–NUCLEUS COLLISIONS AT 3.33 GeV/N KINETIC ENERGY

L.I. ABASHIDZE, V.V. AVDEICHIKOV*, G.G. BEZNOGIKH, V.V. BOGATIN*,
V.A. BUDILOV, N.L. GORSHKOVA, T.F. GRABOVSKAYA, A.P. LARICHEVA,
V.D. MAISYUKOV**, St. MRØWCZYŃSKI, Yu.A. MURIN*, A. NAWROT***,
V.A. NIKITIN, P.V. NOMOKONOV, V.S. OPLAVIN*,
N.K. ZHIDKOV, I.P. ZIELIŃSKI*** and J. ZŁOMAŃCZUK****

*Laboratory of High Energies, Joint Institute for Nuclear Research, Dubna, USSR*

Received 23 August 1984



Volume 134B, number 5 PHYSICS LETTERS

# THE LIMITING NUCLEAR TARGET FRAGMENTATION AND THE THERMODYNAMICAL MODEL

V.V. AVDEICHIKOV
*Radium Institute, Leningrad, USSR
and JINR, Dubna, USSR*

and

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*Institute of Nuclear Problems, Warsaw, Poland
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Received 6 June 1983
Revised manuscript received 25 October 1983

Volume 152B, number 5,6 PHYSICS LETTERS 14 March 1985

# DIBARYONS IN NUCLEI

St. MRÓWCZYŃSKI [1]
*JINR, Laboratory of High Energies, Dubna, USSR*

Received 23 October 1984

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PHYSICAL REVIEW C VOLUME 30, NUMBER 1 JULY 1984

# Difficulties of the thermodynamical model approach to pion production in relativistic ion collisions

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*Institute of Experimental Physics, University of Warsaw, Warsaw, Poland and High Energy Laboratory,*
*Joint Institute for Nuclear Research, Dubna, USSR*

St. Mrówczyński
*Institute for Nuclear Studies, Warsaw, Poland and High Energy Laboratory,*
*Joint Institute for Nuclear Research, Dubna, USSR*

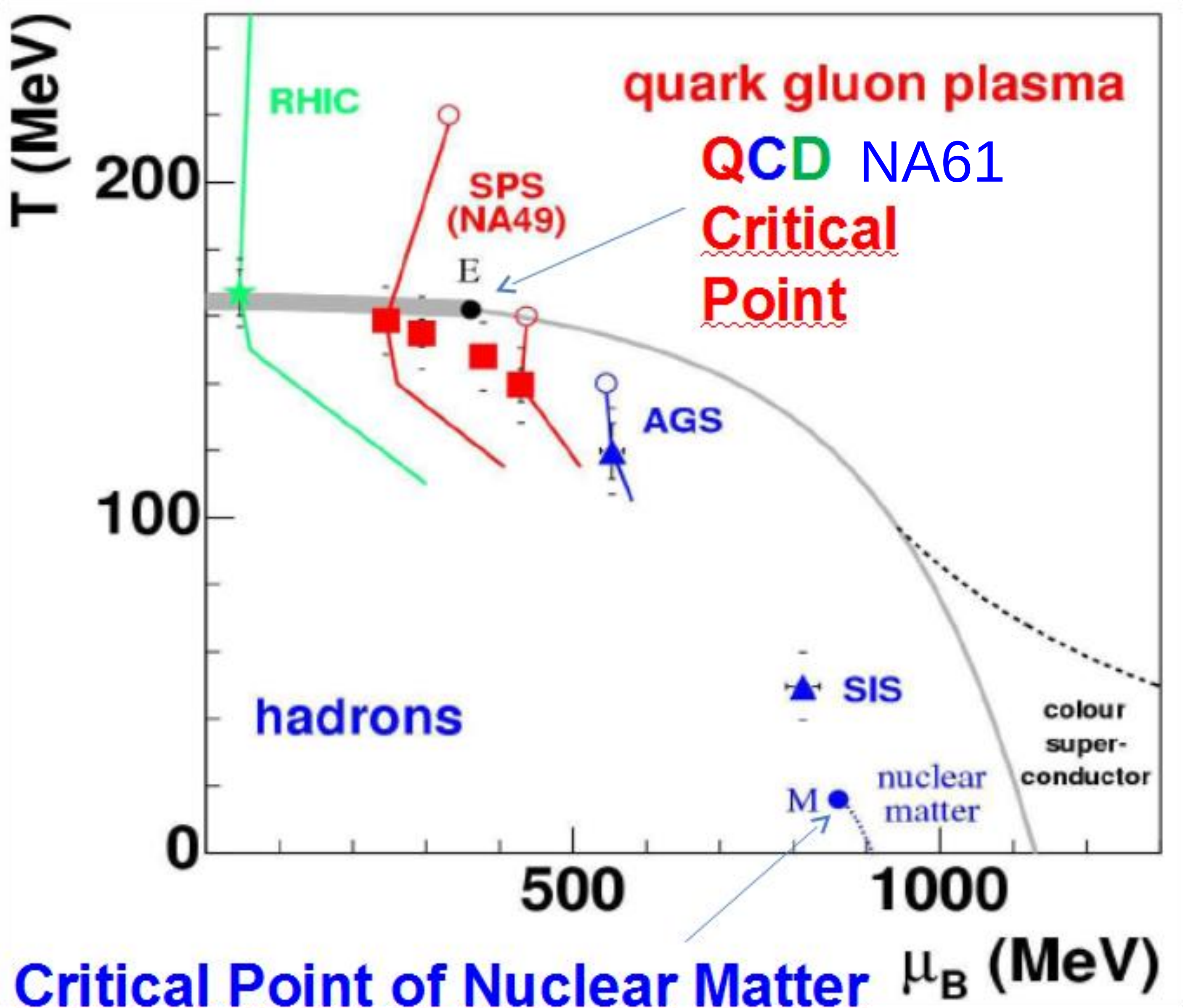
(Received 17 November 1983)

# Van der Waals equation of state: from nuclear matter to lattice QCD

Mark Gorenstein

Bogolyubov Institute for Theoretical Physics

I. Introduction
II. Van der Waals equation for nuclear matter
III. Critical Point and fluctuations
IV. VDW equation for HRG and lattice QCD
V. Summary



QCD NA61

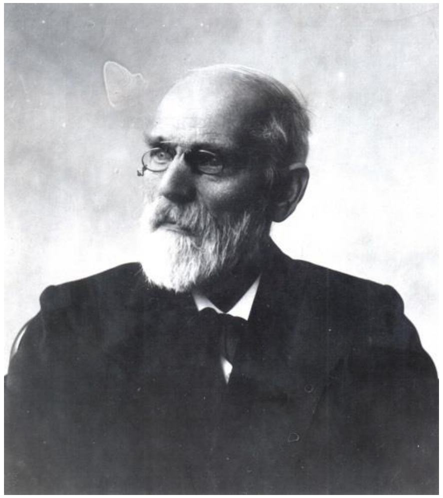
Critical

Point

Critical Point of Nuclear Matter

# II. Van der Waals Equation of State

1873, Ph. D. Thesis 1910, Nobel Prize in Physics



$$p(V,T,N) = \frac{NT}{V-bN} - a\frac{N^2}{V^2} = \frac{nT}{1-bn} - an^2,$$

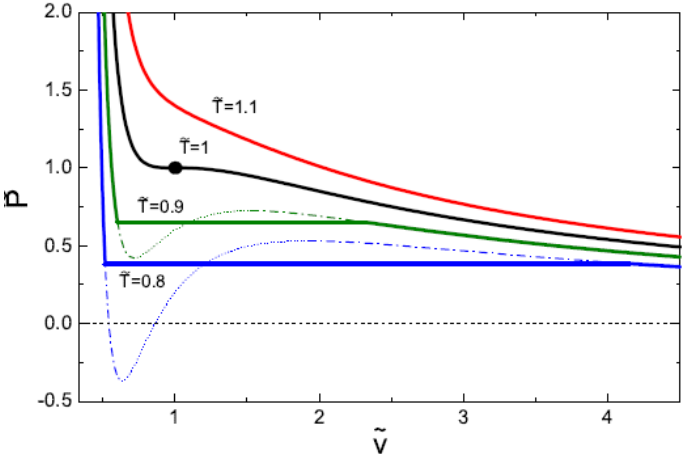
$$\frac{\partial p(T,n)}{\partial n} = 0\,, \qquad \frac{\partial p^2(T,n)}{\partial n^2} = 0$$

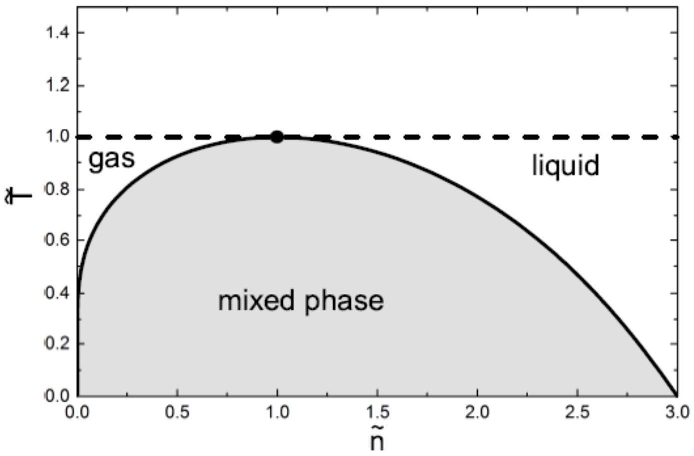
$$T_c = \frac{8a}{27b}, \quad n_c = \frac{1}{3b}, \quad p_c = \frac{a}{27b^2}$$

$$\tilde{p} = \frac{8\tilde{T}\tilde{n}}{3-\tilde{n}} - 3\tilde{n}^2$$

$$\tilde{n} = n/n_c, \quad \tilde{p} = p/p_c, \quad \tilde{T} = T/T_c,$$

$$\tilde{v} \equiv 1/n$$





$p(T,n)$

$p \cong nT$ at small $n$

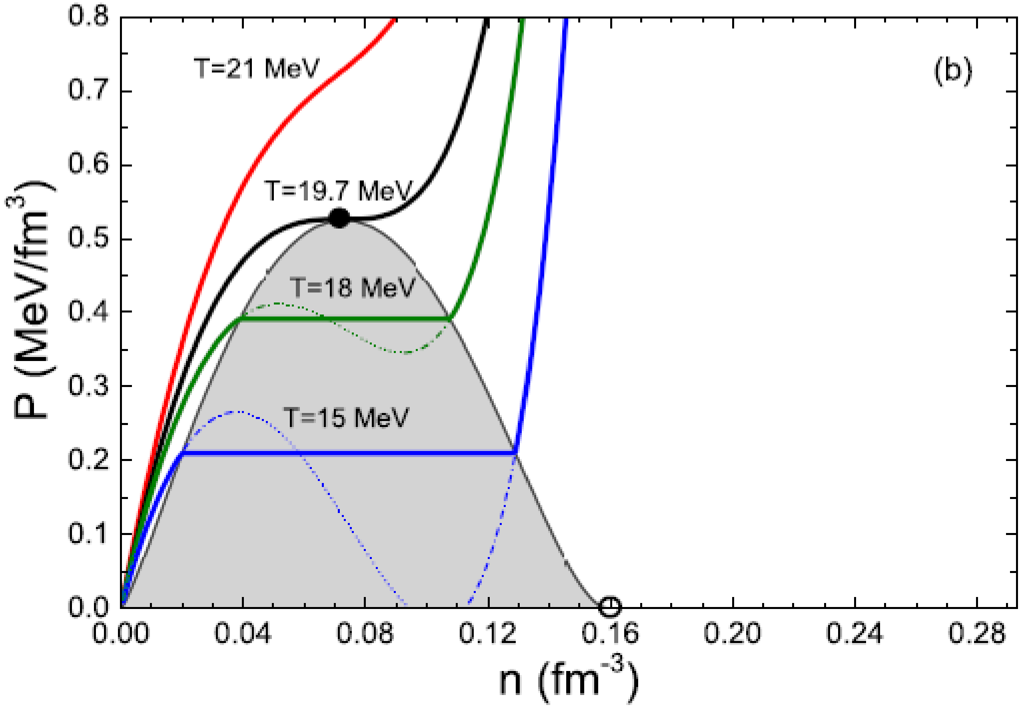
(ideal gas)

$p \leq nT$ at intermidiate $n$

(mixed phase)

$p > nT$ at large $n$

(liquid phase)



$$\frac{\partial p(T,n)}{\partial n} = 0 \,, \qquad \frac{\partial p^2(T,n)}{\partial n^2} = 0$$

$(T_c \;,\; n_c \;,\; p_c)$ Critical Point

CE $$F(V,T,N)=F(V_0,T,N)+\int_{V_0}^{V} dVp \rightarrow \mu=\frac{\partial F}{\partial N}$$ GCE

$$p(V,T,N)=\frac{nT}{1-bn}-an^2 \quad \rightarrow \quad p(T,\mu)=p_{\text{id}}\left(T,\mu^*\right)-an^2$$

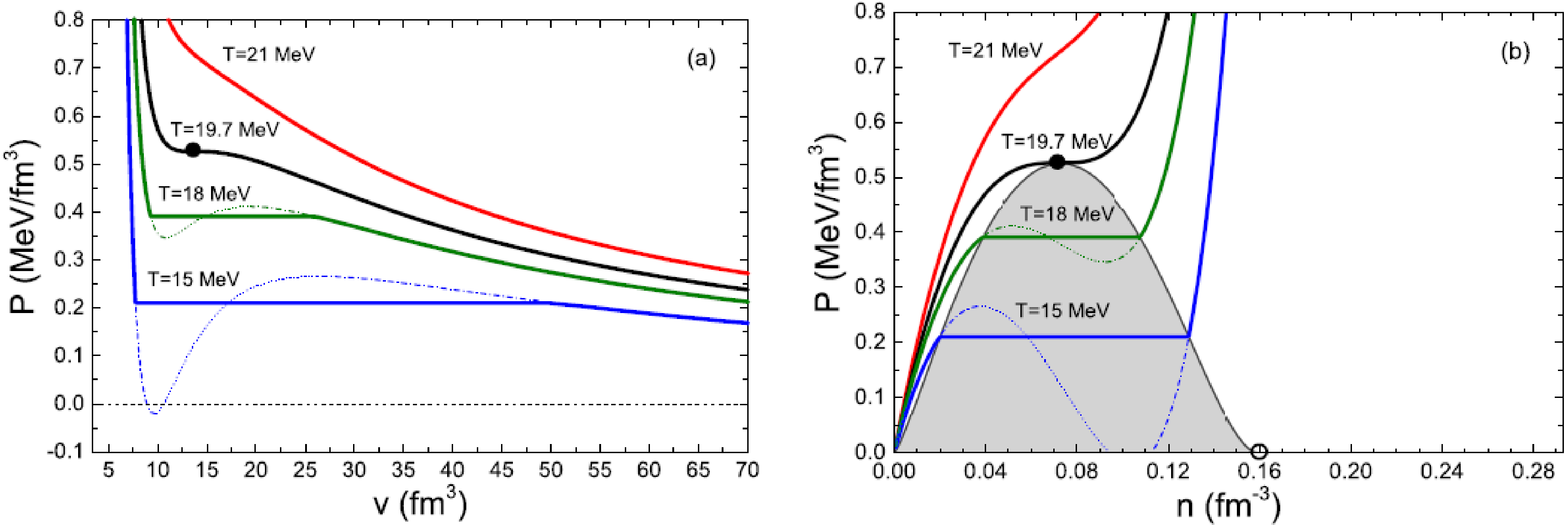
$$\mu^*=\mu-bp(T,\mu)-abn^2+2an$$

Rischke, M.I.G., Stocker, Greiner, Z. Phys. C (1991) $b>0,\ a=0$

Vovchenko, Anchishkin, M.I.G., J. Phys. A (2015) $b>0,\ a>0$

$$p_{\text{id}}\left(T,\mu\right)=\frac{d}{6\pi^2}\int_0^{\infty}k^2dk\,\frac{k^2}{\sqrt{k^2+m^2}}\left[\exp\left(\frac{\sqrt{k^2+m^2}}{T}-\mu\right)\pm 1\right]^{-1}$$

# Nuclear Matter = nucleons with van der Waals EoS



Fermi Statistics, $d = 4$, $m \cong 938$ MeV $a, b$ - ?

$$T = 0,\ \ p = 0: \quad \varepsilon / n - m \ = \ -\ 16\ \text{MeV}, \quad n = n_0 = \ 0.16\ \text{fm}^{-3}$$

$$a = 329\ \text{MeV fm}^3, \qquad b = 3.42\ \text{fm}^3 \rightarrow r = 0.59\ \text{fm}$$

$$T_c \cong 19.7\ \text{MeV},$$

$$n_c \cong 0.07\ \text{fm}^{-3}$$

Vovchenko, Anchishkin, M.I.G., Phys. Rev. C (2015)

# III. Fluctuations at the Critical point

## Scaled Variance

$$\omega[N] \equiv \frac{<N^2> - <N>^2}{<N>} = \left[\frac{1}{(1-bn)^2} - \frac{2an}{T}\right]^{-1}$$

$$a = 0, \quad b = 0 \;\rightarrow\; \omega[N] = 1$$

Vovchenko, Anchishkin, M.I.G.
J.Phys. A (2015)

$$a = 0 \rightarrow \; \omega[N] = (1-bn)^2 < 1$$

$$n \rightarrow 0 \quad \omega[N] = 1$$

M.I.G., Hauer, Nikolajenko,
Phys. Rev. C (2007)
Excluded Volume Model, i.e. a=0

# Skewness and Kurtosis

Central Moments: $<(\Delta N)^2>, \; <(\Delta N)^3>, \; <(\Delta N)^4>, \ldots$

Scaled Variance: $$\omega[N] = \frac{<(\Delta N)^2>}{<N>}, \qquad \Delta N = N - <N>$$

Skewness: $$S\sigma = \frac{<(\Delta N)^3>}{<(\Delta N)^2>},$$

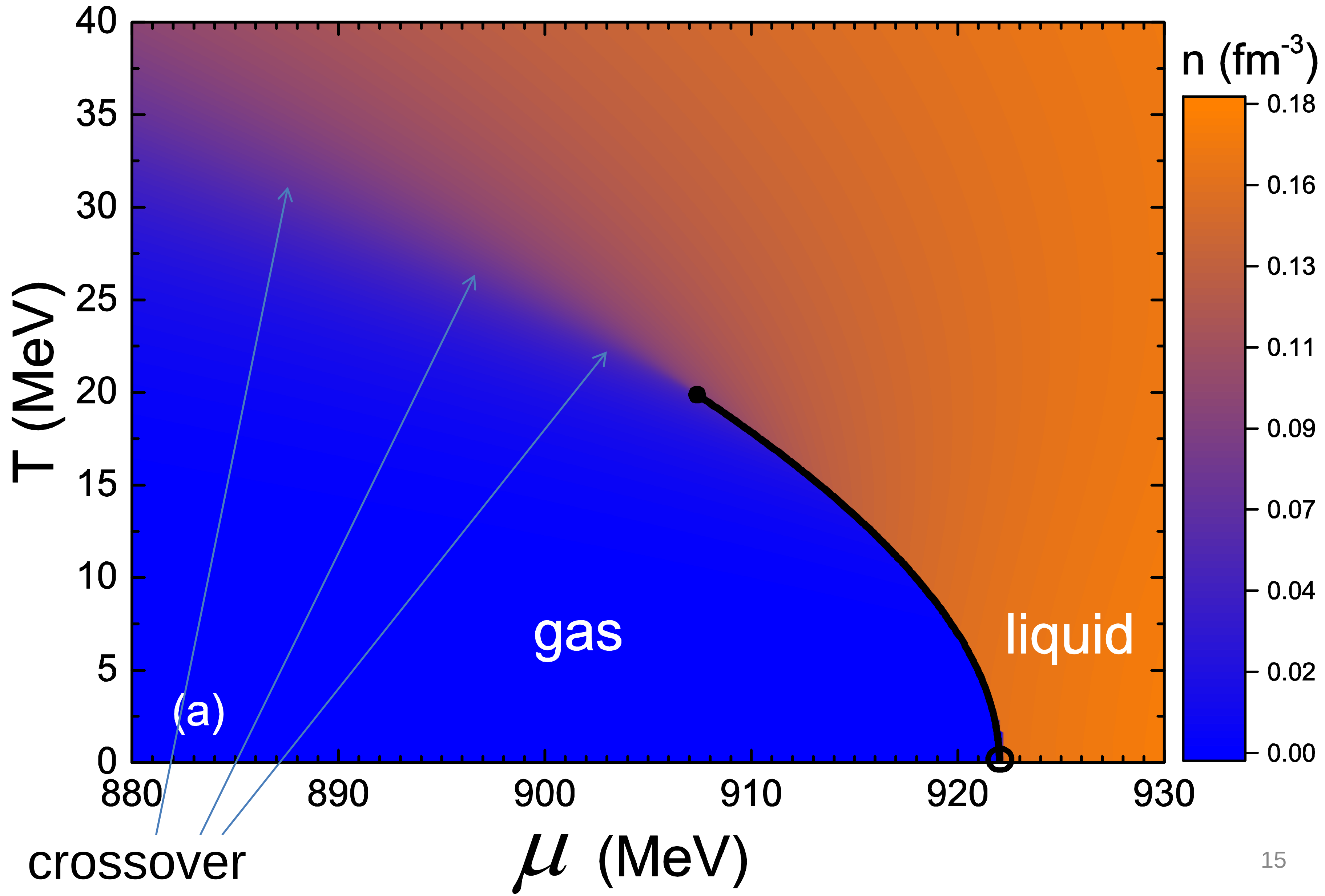
Kurtosis: $$\kappa\sigma^2 = \frac{<(\Delta N)^4 - 3<(\Delta N)^2>^2}{<(\Delta N)^2>}.$$

Cummulants: $$\mathrm{k}_n = \frac{\partial^n \left(p/T^4\right)}{\partial\left(\mu/T\right)^n}, \quad n = 1, 2, \ldots$$

$$\omega[N] = \frac{\mathrm{k}_2}{\mathrm{k}_1}, \qquad S\sigma = \frac{\mathrm{k}_3}{\mathrm{k}_2}, \qquad \kappa\sigma^2 = \frac{\mathrm{k}_4}{\mathrm{k}_2}.$$

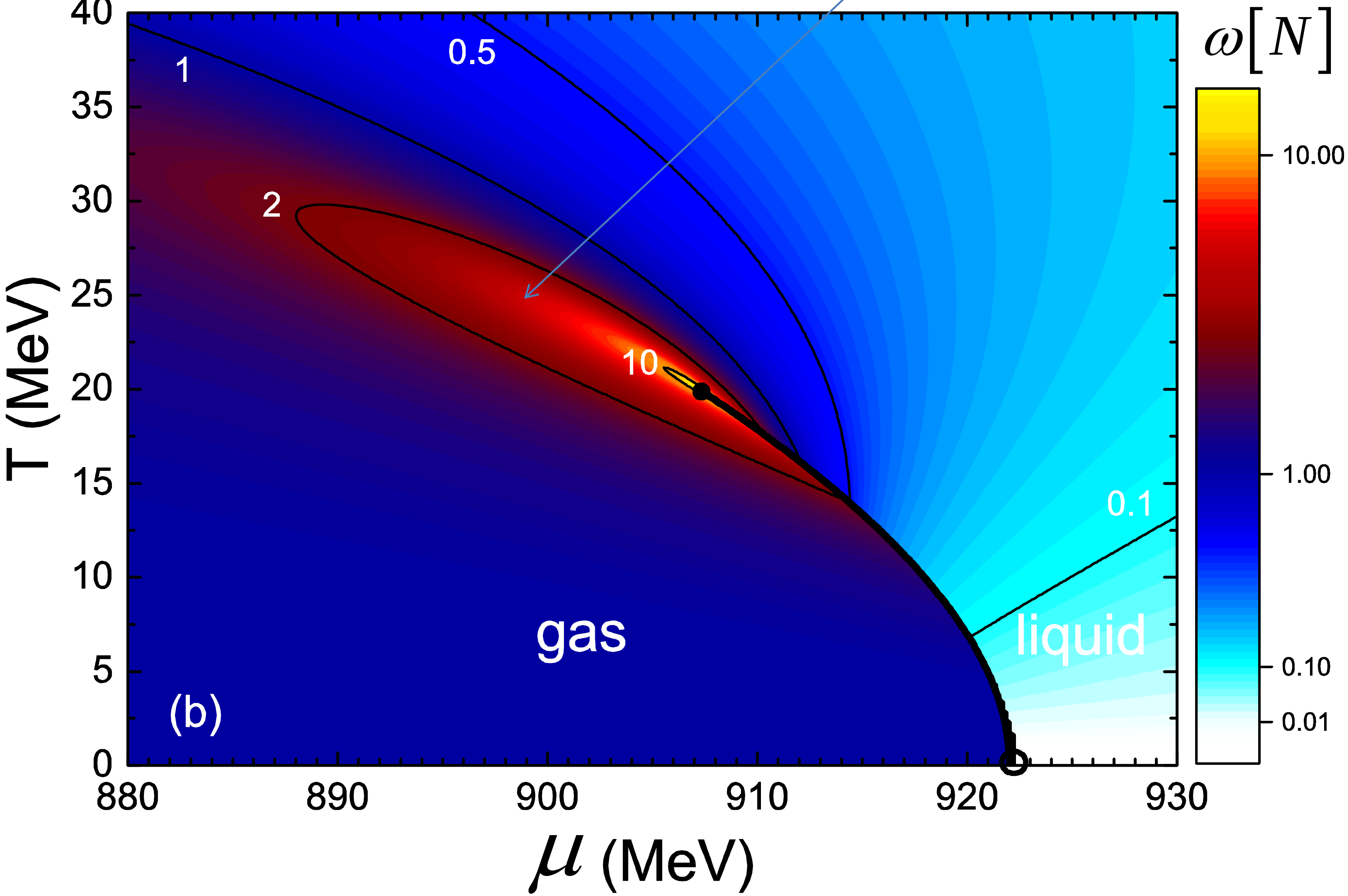
Vovchenko, Anchishkin, M.I.G., Poberezhnjuk, Phys. Rev. C (2015)

**Particle Number Density** $n(T,\mu)$ $\mu_c \cong 908\ \mathrm{MeV}$

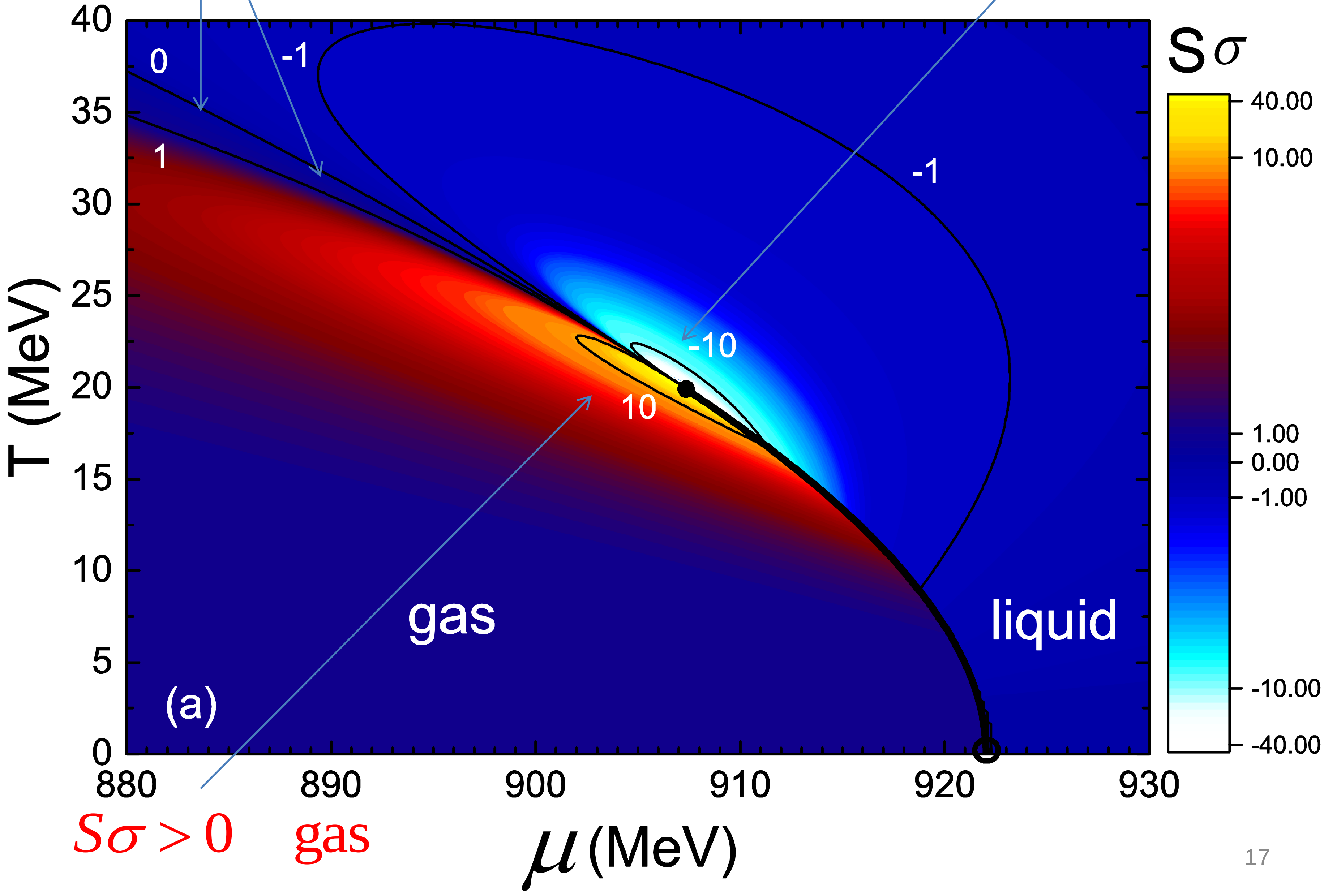


# Scaled Variance

$\omega[N]$ >>1 along crossover



# Skewness



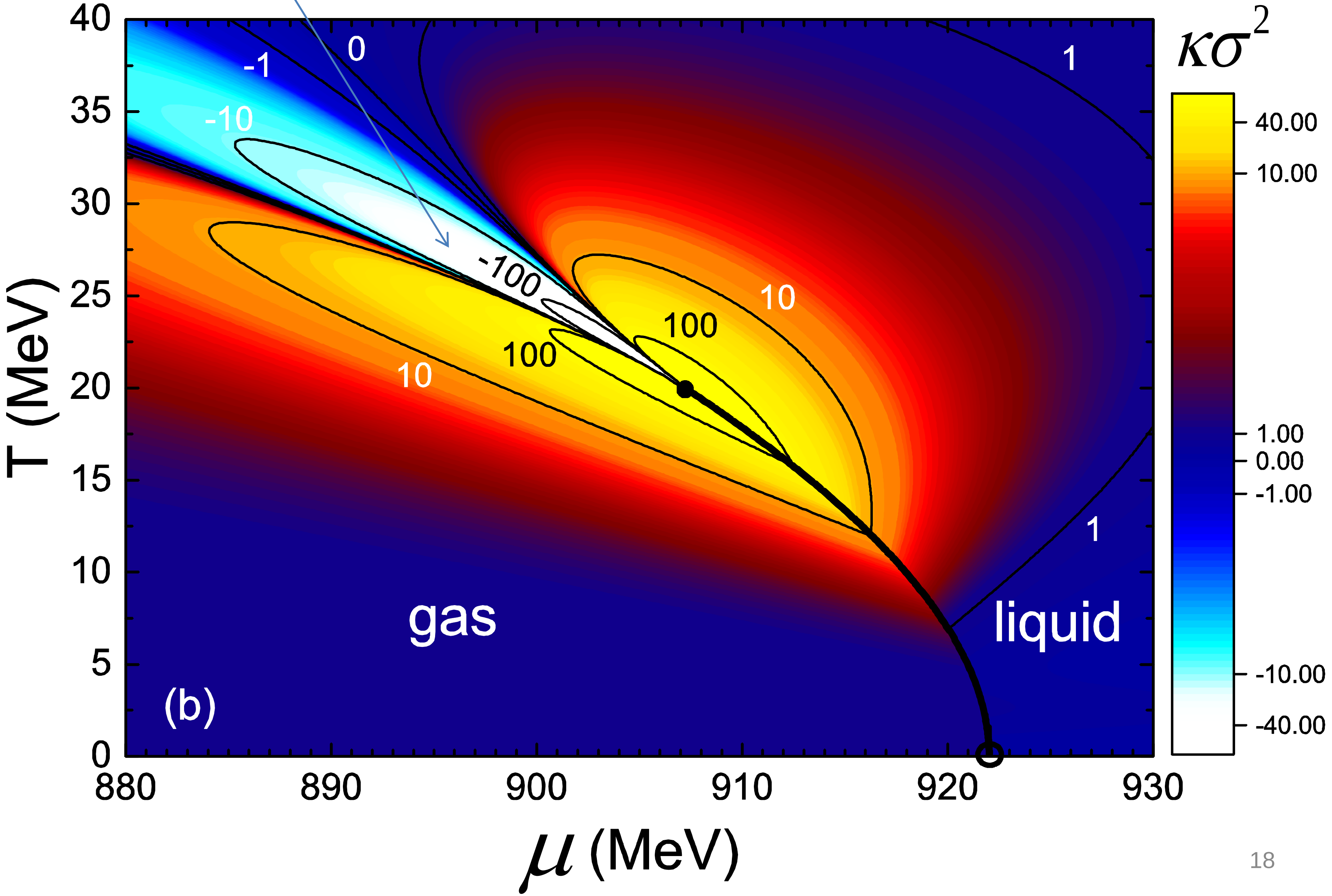
$S\sigma = 0$ crossover

$S\sigma < 0$ liquid

$S\sigma > 0$ gas

# Kurtosis

$\kappa\sigma^2 << -1$ crossover



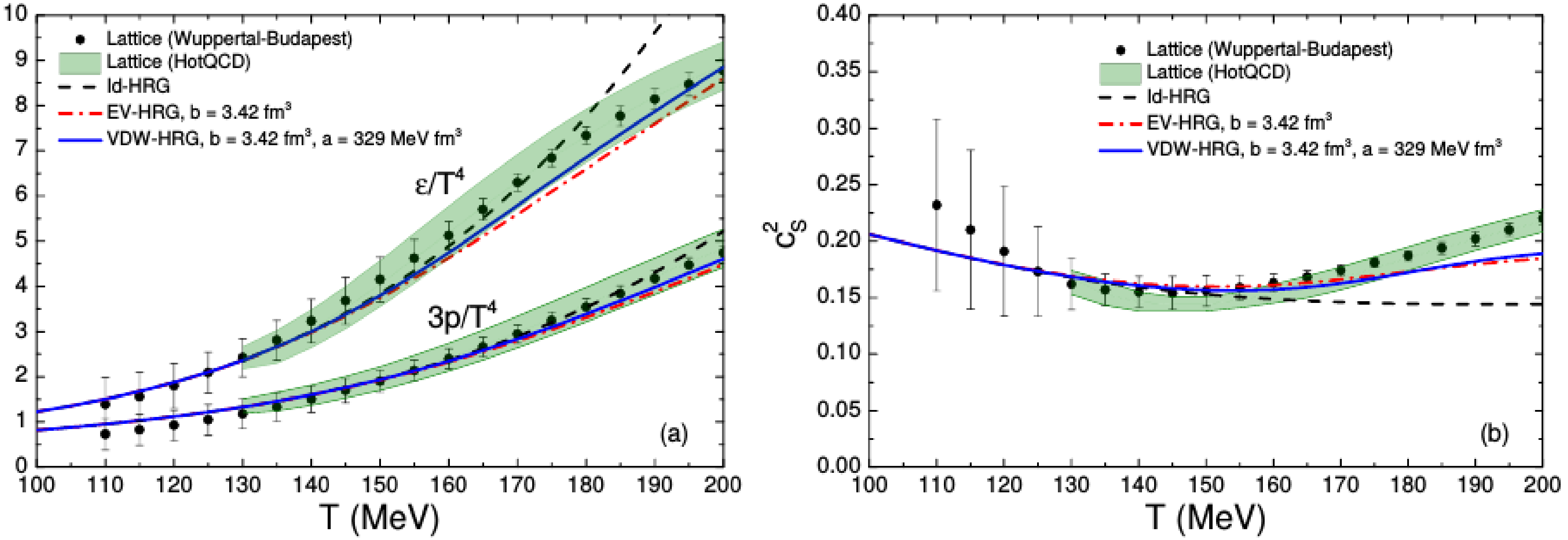
# IV. VDW equation for the HRG and lattice QCD

$$p(T,\mu) = p_M + p_B + p_{\bar{B}}, \quad p_M = \sum_{i\in M} p_i^{\rm id}\left(T,\mu_i\right),$$

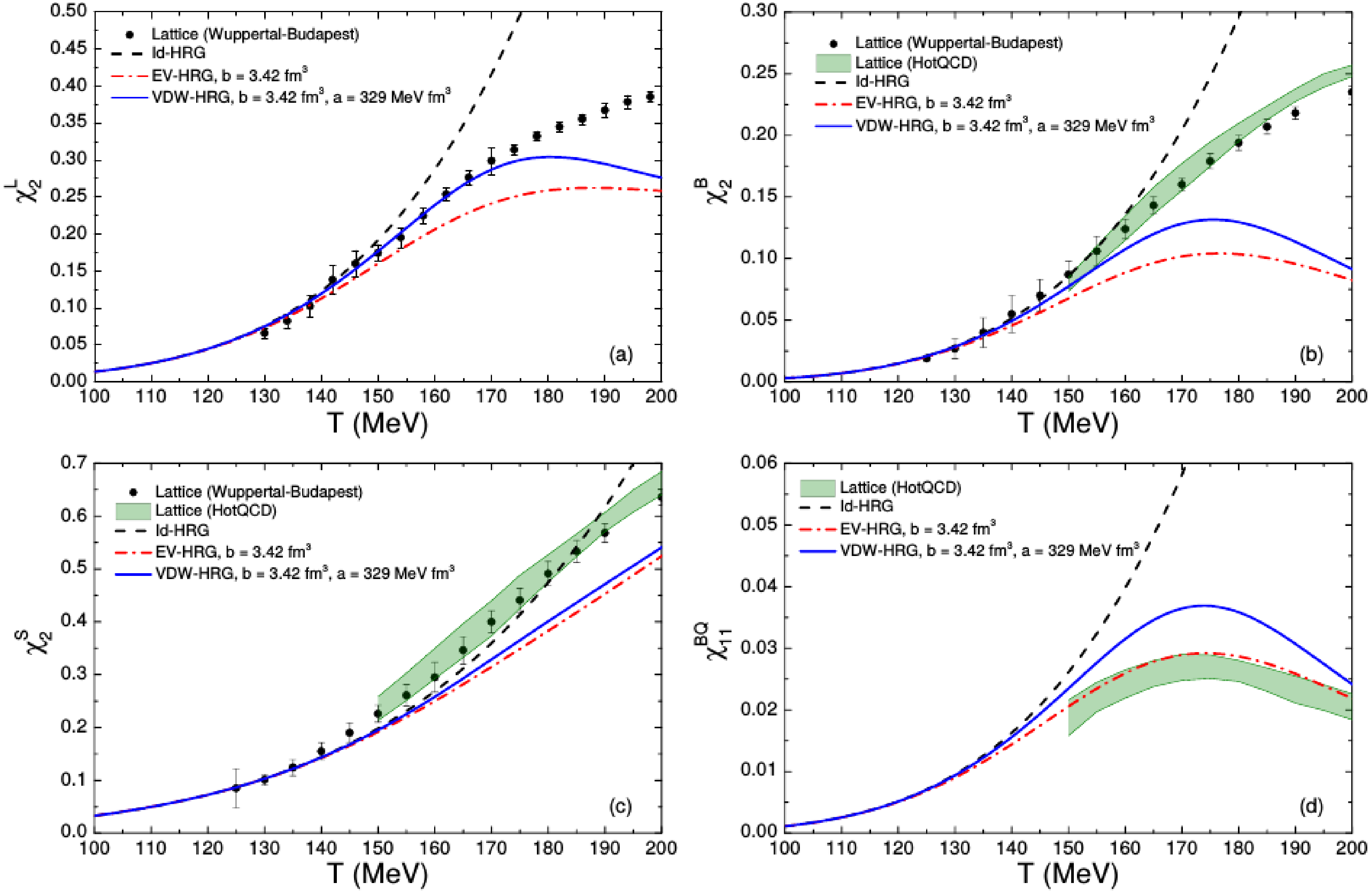
Vovchenko, M.I.G., Stoecker, arXiv:1609.03975

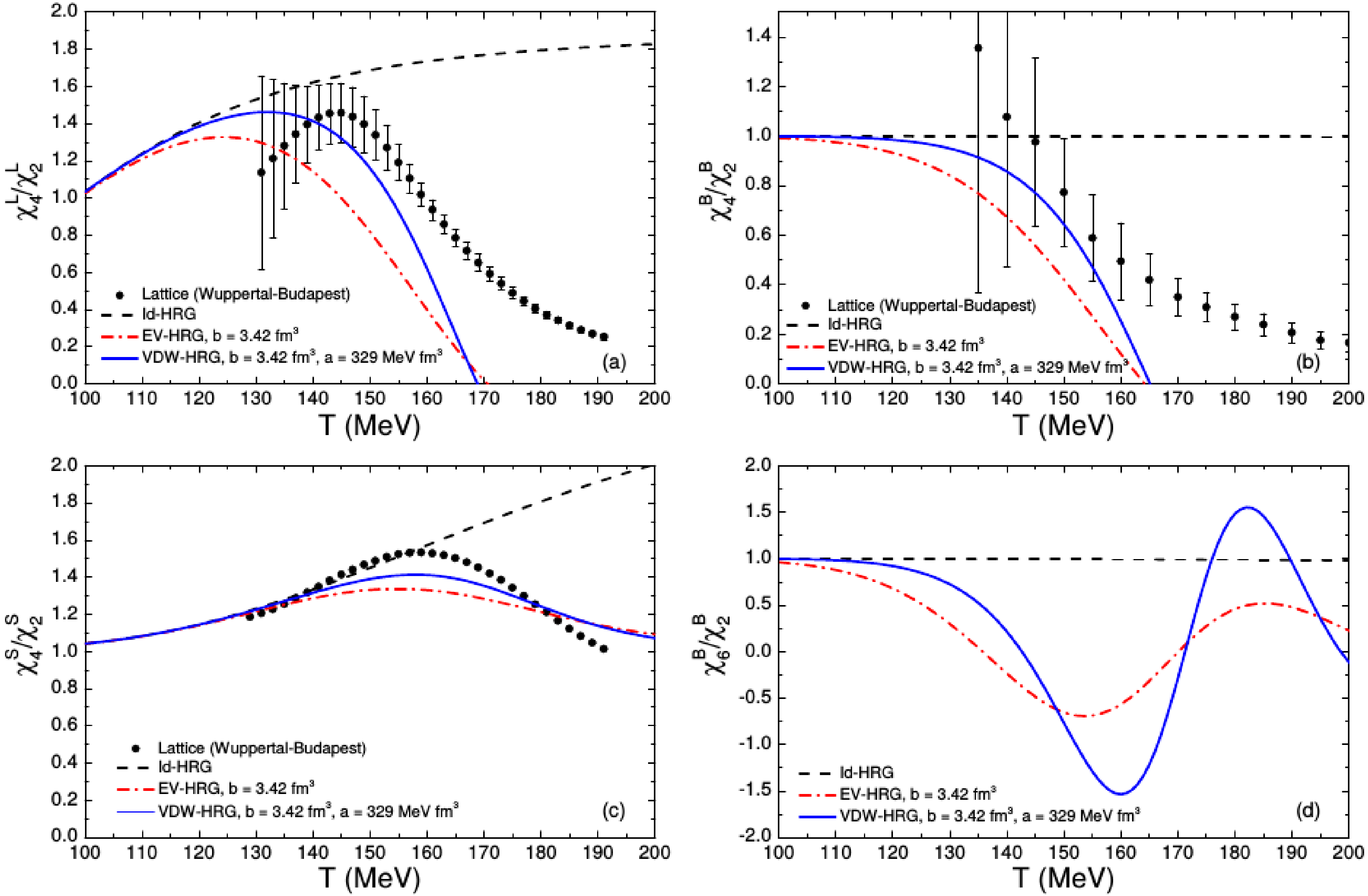
$$p_B = \sum_{i\in B} p_i^{\rm id}\left(T,\mu_i^*\right) - a n_B^2, \quad p_{\bar{B}} = \sum_{i\in B} p_i^{\rm id}\left(T,\mu_i^*\right) - a n_{\bar{B}}^2,$$

$$\mu_i^* = \mu_i - b p_B - a b n_B^2 + 2 a n_B, \quad \mu_i^* = \mu_i - b p_{\bar{B}} - a b n_{\bar{B}}^2 + 2 a n_{\bar{B}}$$



$$\chi_{lmn}^{BQS} = \frac{\partial^{lmn}(p/T^4)}{\partial\left(\mu_B/T\right)^l \partial\left(\mu_Q/T\right)^m \partial\left(\mu_S/T\right)^n}$$




$\chi_4^L/\chi_2^L$
T (MeV)
Lattice (Wuppertal-Budapest)
Id-HRG
EV-HRG, b = 3.42 fm$^3$
VDW-HRG, b = 3.42 fm$^3$, a = 329 MeV fm$^3$
(a)
$\chi_4^B/\chi_2^B$
T (MeV)
Lattice (Wuppertal-Budapest)
Id-HRG
EV-HRG, b = 3.42 fm$^3$
VDW-HRG, b = 3.42 fm$^3$, a = 329 MeV fm$^3$
(b)
$\chi_4^S/\chi_2^S$
T (MeV)
Lattice (Wuppertal-Budapest)
Id-HRG
EV-HRG, b = 3.42 fm$^3$
VDW-HRG, b = 3.42 fm$^3$, a = 329 MeV fm$^3$
(c)
$\chi_6^B/\chi_2^B$
T (MeV)
Id-HRG
EV-HRG, b = 3.42 fm$^3$
VDW-HRG, b = 3.42 fm$^3$, a = 329 MeV fm$^3$
(d)

# Summary

1. Van der Waals Equation of State for Nuclear Matter

   Provides an analytical example of the systems with 1st order liquid-gas phase transition and critical point.

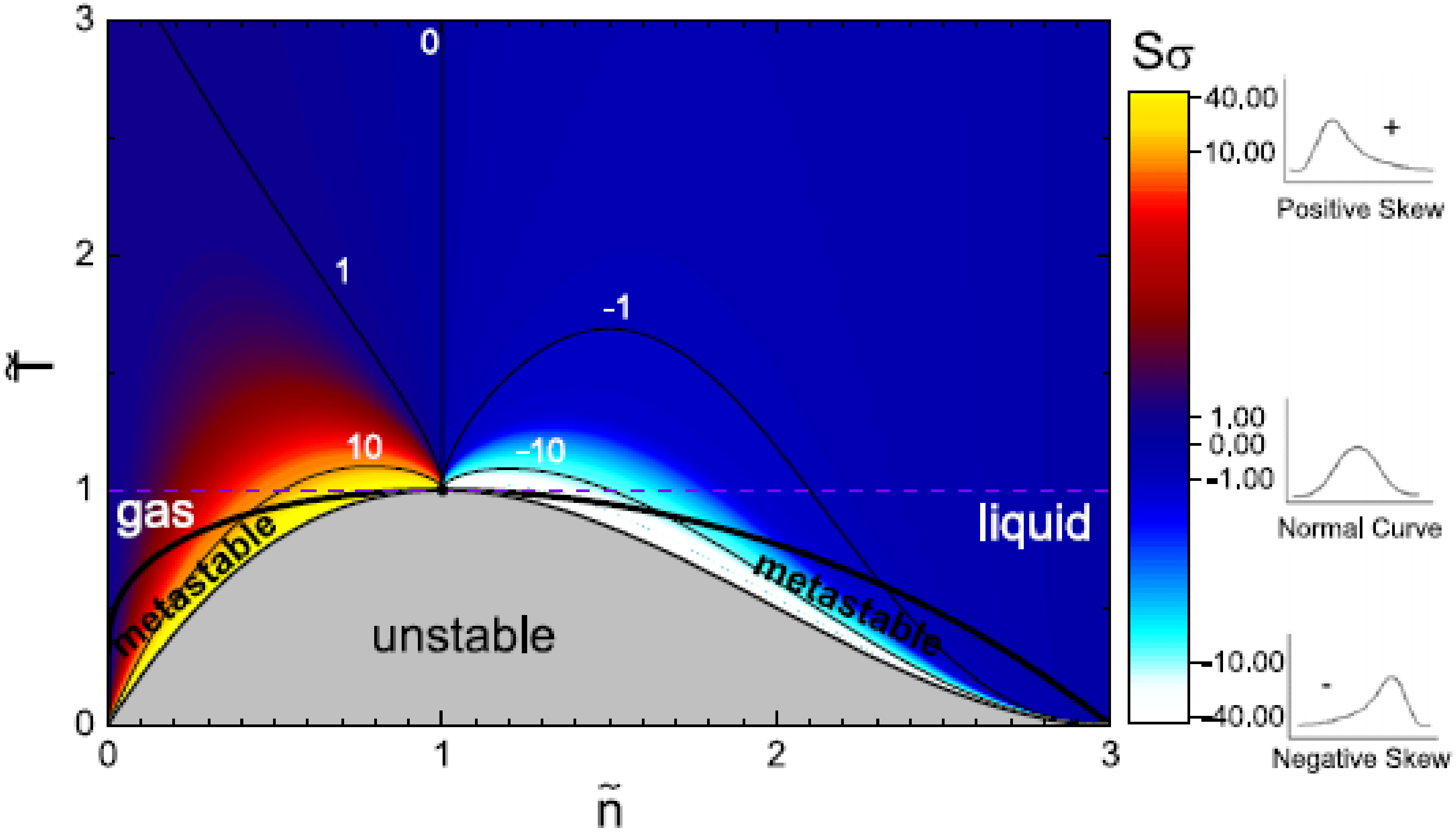
2. Particle Number Fluctuations:

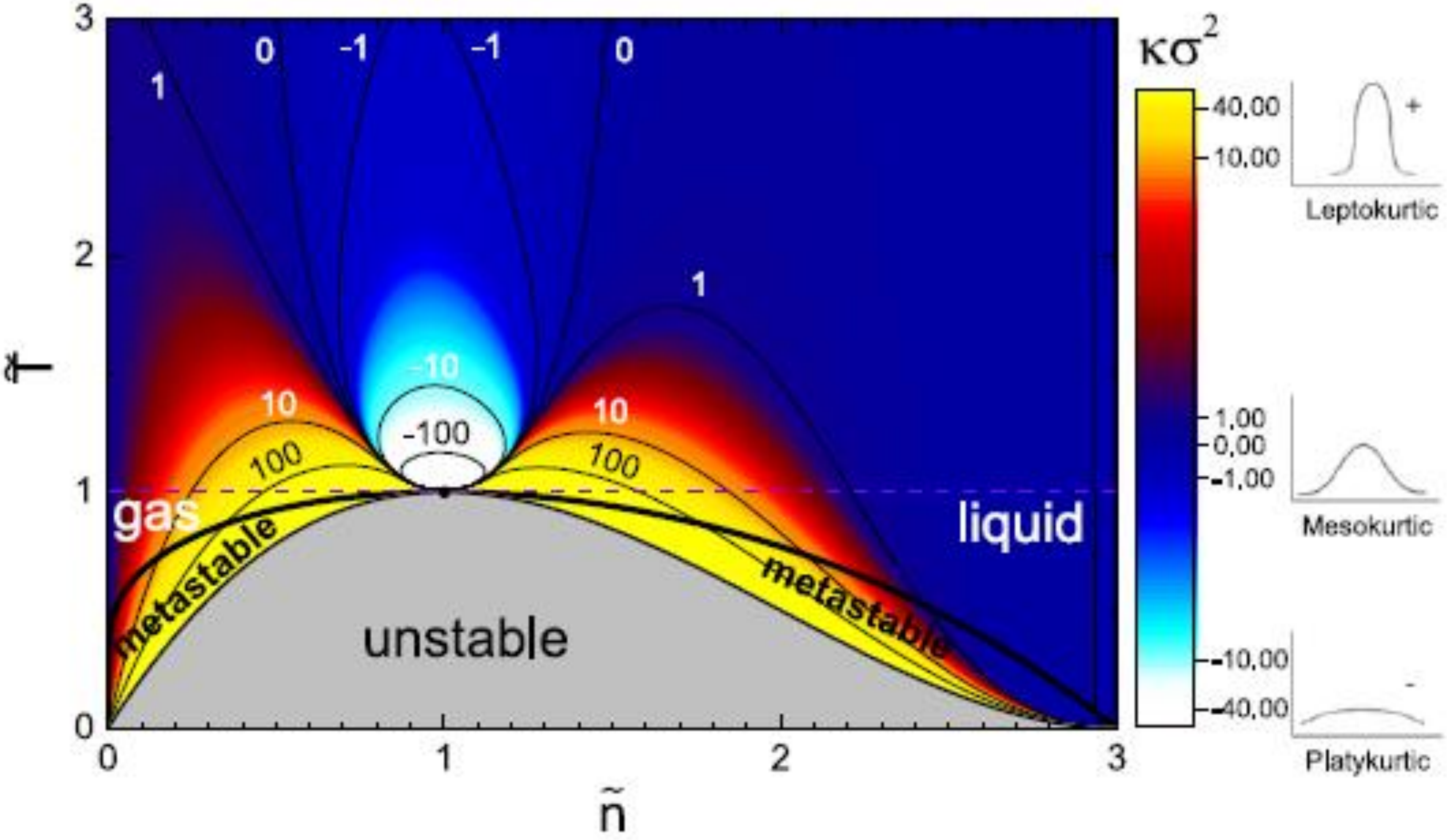
   Scaled Variance goes to infinity at the CP.

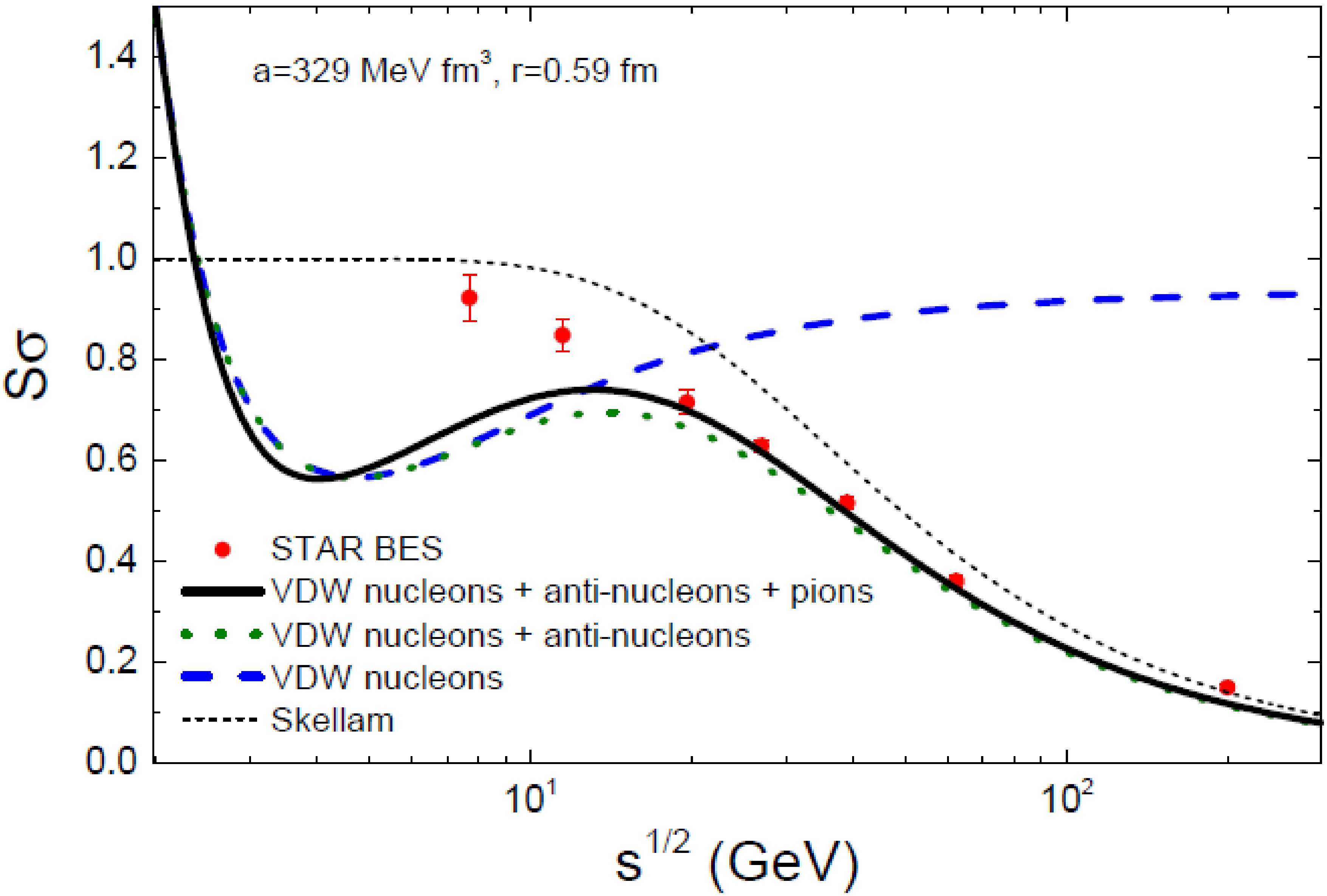
   For Skewness and Kurtosis the CP is a point of essential singularity:

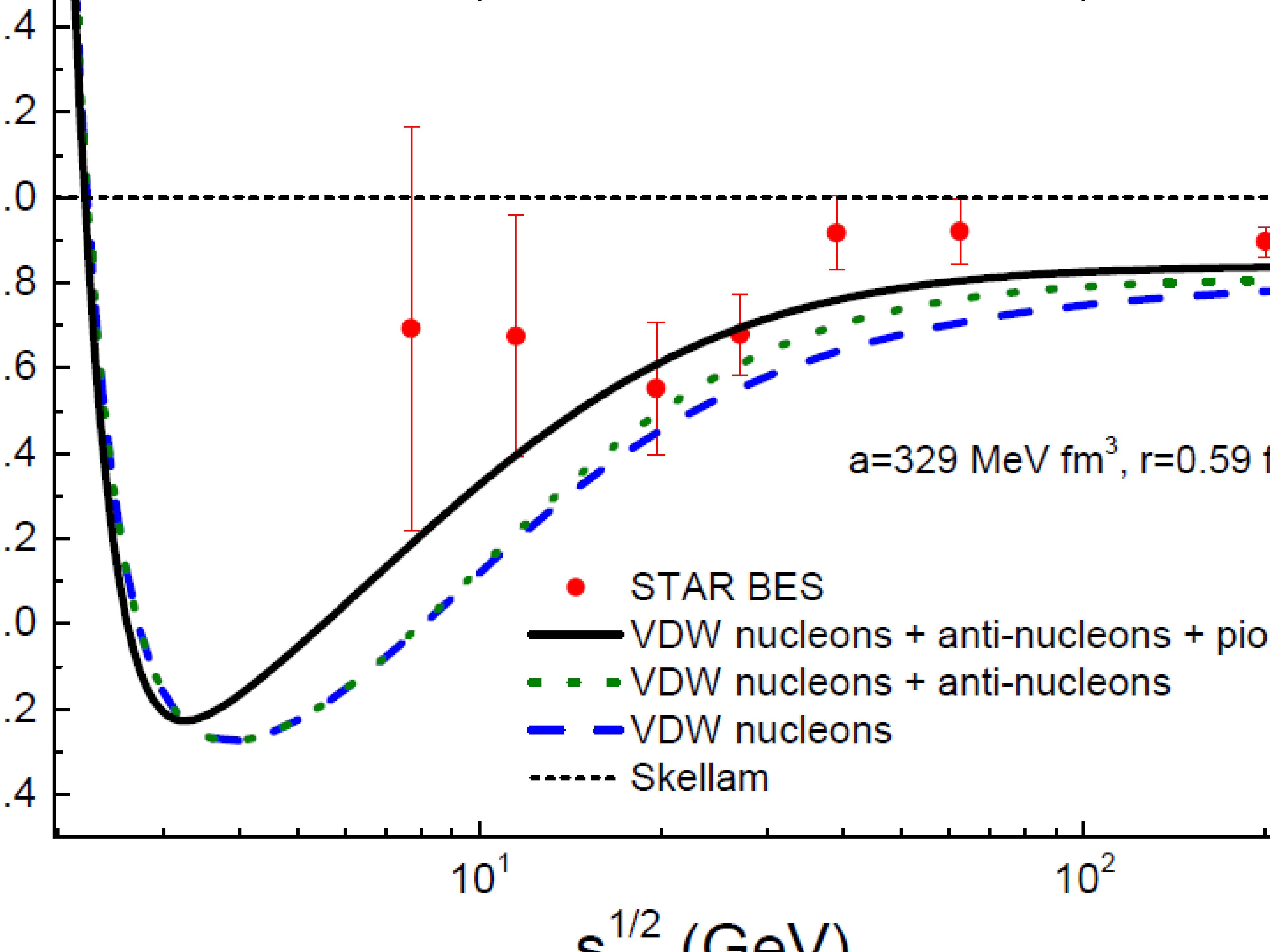
   i.e., the limiting singular values depend on the path of approach to the CP

3. Interacting Hadron Resonance Gas in the temperature region T=150-200 MeV


3
2
1
0
0
1
2
3
$\tilde{T}$
$\tilde{n}$
0
1
-1
10
-10
gas
liquid
metastable
metastable
unstable
Sσ
40,00
10,00
1,00
0,00
-1,00
-10,00
-40,00
+
Positive Skew
Normal Curve
-
Negative Skew


3
2
1
0
$\tilde{T}$
0
1
2
3
$\tilde{n}$
1
0
-1
-1
0
1
-10
-100
10
100
10
100
gas
liquid
metastable
metastable
unstable
$\kappa\sigma^2$
40,00
10,00
1,00
0,00
-1,00
-10,00
-40,00
+
Leptokurtic
Mesokurtic
-
Platykurtic


a=329 MeV fm$^3$, r=0.59 fm
Sσ
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0
$10^1$
$10^2$
$s^{1/2}$ (GeV)
STAR BES
VDW nucleons + anti-nucleons + pions
VDW nucleons + anti-nucleons
VDW nucleons
Skellam


.4
.2
.0
.8
.6
.4
.2
.0
.2
.4
$10^1$
$10^2$
$s^{1/2}$ (GeV)
a=329 MeV fm$^3$, r=0.59 f
STAR BES
VDW nucleons + anti-nucleons + pio
VDW nucleons + anti-nucleons
VDW nucleons
Skellam