Relaxation rates and phase transitions

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JHEP **1606**, 047 (2016)
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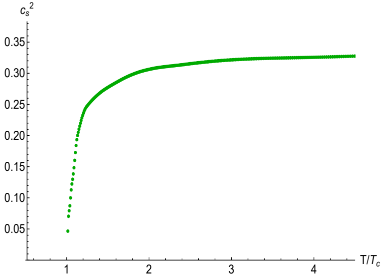
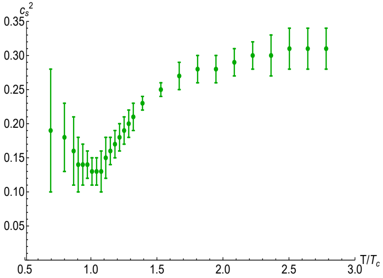
Phase structure at strong coupling

- Systems at strong coupling exhibit various phase structures
- Pure gluon system $\longrightarrow$ 1^{st} order phase transition (left)
- Gluons + quarks $\longrightarrow$ smooth crossover (right)



G. Boyd *et.al.* Nucl. Phys. B **469**, 419 (1996)



S. Borsanyi *et al.* JHEP **1009**, 073 (2010)

Phase structure at strong coupling

- Lattice methods do not reach real time dynamics easily
- Use other methods to model strongly coupled phase transitions
- Compute the spectrum of linearized perturbations
- Compute transport coefficients and non-hydrodynamic modes
- Check linear stability

Method:

Use a string theory based approach to formulate models at strong coupling!

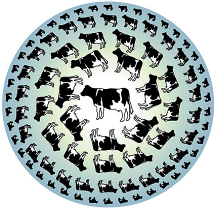
Questions

- Does spinodal instability appear for a system with a 1^{st} order phase transition?
- Does dynamical instability has to be accompanied by a thermodynamical instability?
- How do non-hydrodynamic degrees of freedom behave in the critical region?
- Do diffusive modes appear?

Method:

Use a string theory based approach to formulate models at strong coupling!

- Holographic principle
  Quantum gravity in d dimensions must have a number of DOF which scales like that of QFT in $d-1$ dimensions
  't Hooft and Susskind '93



- String Theory realization: *AdS/CFT correspondence*
  Theory is *conformal* and *supersymmetric* Maldacena '97
- Extensions to *non-supersymmetric* and *non-conformal* field theories are possible
- Applications: elementary particle physics and condensed matter physics

Bottom-up models

- Equilibrium state in QFT $\longleftrightarrow$ black hole in dual spacetime
  Field theory temperature $\longleftrightarrow$ Hawking temperature
  Field theory entropy $\longleftrightarrow$ Bekenstein-Hawking entropy

  E. Witten, Adv. Theor. Math. Phys. **2**, 505 (1998)

- Assuming AdS/CFT dictionary, try to model gravity+matter background to approach as closely as possible to your favourite physics

- The simplest case: gravity + single scalar self-interacting field

U. Gursoy, *et.al.* JHEP **0905**, 033 (2009)
S. S. Gubser, A. Nellore, Phys. Rev. D **78** (2008) 086007

Phase transitions in holography

- Finite T states correspond to various black hole solutions in the dual spacetime
- Phase structure is determined by the choice of gravity Lagrangian parameters
- It is possible to tune these parameters to mimic
  $\rightarrow$ crossover, e.g. QCD
  $\rightarrow$ 1^{st} order phase transition, e.g. pure gluon systems
  $\rightarrow$ 2^{nd} order phase transition

U. Gursoy, *et.al.* JHEP **0905**, 033 (2009)
S. S. Gubser, A. Nellore, Phys. Rev. D **78** (2008) 086007

Quasinormal modes (QNMs)

- QNMs are the solutions of linearized fluctuation equations that correspond to poles of holographic retarded Green's functions
- In general

$$\omega_n(k) = \Omega_n(k) - i\Gamma_n(k)$$

  where $n = 1, 2, 3, \ldots$ $\Omega_n(k)$—oscillation frequency, $\Gamma_n(k)$—damping rate.
- Stable modes have $\Gamma_n(k) > 0$
- A convenient normalization is: $q = \frac{k}{2\pi T}$, $\varpi = \frac{\omega}{2\pi T}$

P. K. Kovtun, A. O. Starinets, Phys. Rev. D **72**, 086009 (2005)

Hydrodynamics

- The hydrodynamic mode is defined by

$$\lim_{k \to 0} \omega_{\mathrm{H}}(k) = 0$$

- The sound mode

$$\omega(k) = \pm\, c_s\, k - \frac{i}{2\,T} \left(\frac{4}{3} \frac{\eta}{s} + \frac{\zeta}{s} \right) k^2 + O(k^3)$$

η—shear viscosity, ζ—bulk viscosity, s—entropy density, c_s—speed of sound, T—temperature

P. K. Kovtun, A. O. Starinets, Phys. Rev. D **72**, 086009 (2005)

- In holographic models also high order hydro computation is possible

M. Lublinsky, E. Shuryak, Phys. Rev. C **76**, 021901 (2007)

Spinodal instability

- When $c_s^2 < 0$ we have purely damped hydro-modes

$$\omega \approx \pm i|c_s|\, k - \frac{i}{2T}\left(\frac{4}{3}\frac{\eta}{s} + \frac{\zeta}{s}\right) k^2 = \pm i|c_s|\, k - i\Gamma_s k^2$$

  so for small enough k we have $\mathrm{Im}\ \omega > 0$

- This mode is present for a finite range of $0 < k < k_{\mathrm{max}}$
- The maximum momentum for the unstable mode is $k_{\mathrm{max}} = |c_s|/\Gamma_s$
- This appears for systems with a 1^{st} order phase transition; *spinodal* instability

P. Chomaz, M. Colonna, J. Randrup, Phys. Rept. **389**, 263 (2004)

Examples of spinodal instabilities

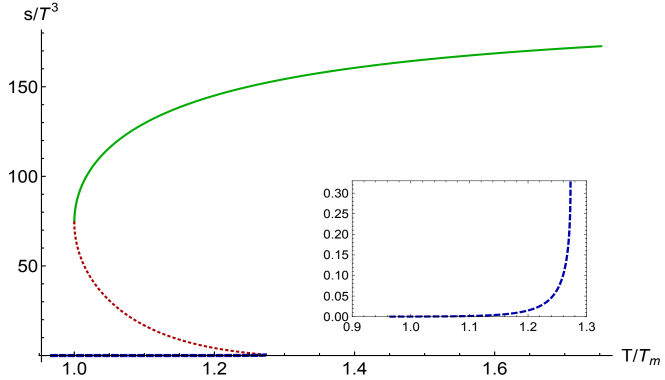
- Water: superheated liquid and supercooled vapour

- Spinodal instability in nuclear matter liquid-gas transition

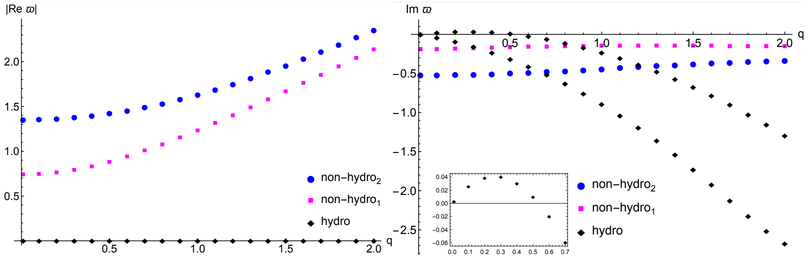
  P. Chomaz, M. Colonna, J. Randrup, Phys. Rept. **389**, 263 (2004)

Example I: First order phase transition

- Transition between two different black hole solutions
- An example of holographic 1st order phase transition
- There exists a critical temperature $T_c \simeq 1.05\,T_m$
- For the unstable region (red dashed line) we have $c_s^2 < 0$



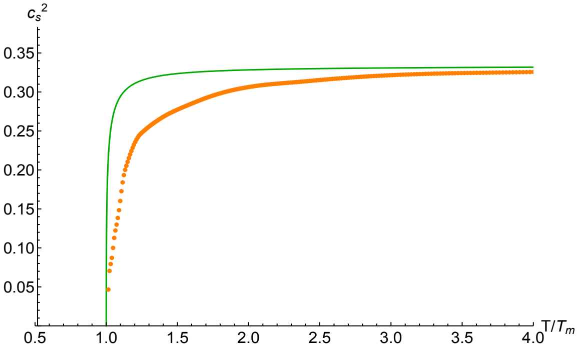
Example I: Holographic spinodal instability



- Modes for $T \simeq 1.06\,T_m$ where $c_s^2 \simeq -0.1$
- The hydrodynamic mode follows the thermodynamic instability
- Scale of the bubble $=$ k for which $\mathrm{Im}\ \omega$ is maximal
- The maximal value of $\mathrm{Im}\ \omega$ is called the growth rate
- Non-hydrodynamic modes have weak k-dependence

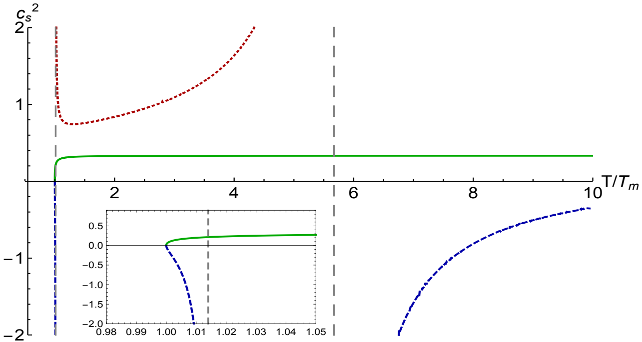
Example II: Confining model IHQCD

- Transition between black hole and horizon-less geometry
  S. W. Hawking, D. N. Page, Commun. Math. Phys. **87**, 577 (1983)
- Holographic 1^{st} order phase transition



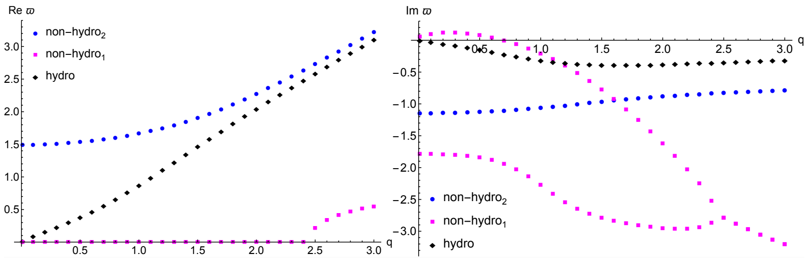
G. Boyd *et.al.* Nucl. Phys. B **469**, 419 (1996)

Example II: Full holographic scan



- Below T_m no black hole solution exists
- Green line - stability region, blue dashed line - spinodal region
- Red dashed line - „dynamically unstable" region

Example II: Dynamical instability



- Quasinormal modes at $T = 1.027\,T_m$
- System displays dynamical instability in spite of thermodynamical stability!
- The system is unstable against uniform ($k = 0$) perturbations
- Possible implications for thermalization time

  U. Gursoy, A. Jansen, W. van der Schee, Phys. Rev. D **94**, no. 6, 061901 (2016)

Summary

- Thermodynamic instability $\rightarrow$ dynamical instability
- Converse doesn't seem to be true!

  U. Gursoy, A. Jansen, W. van der Schee, Phys. Rev. D **94**, no. 6, 061901 (2016)
- Non-trivial phase structure limits the applicability of hydrodynamics
- In most cases non-hydro degrees of freedom have very weak dependence on k $\rightarrow$ *„ultralocality"*
- Extensions to lower couplings and comparison to kinetic theory

  S. Grozdanov, N. Kaplis, A. O. Starinets, JHEP **1607**, 151 (2016)
- Experimental evidences in cold atom systems

  J. Brewer, P. Romatschke, Phys. Rev. Lett. **115**, no. 19, 190404 (2015)