The critical behavior of hadronic matter: Comparison of lattice and bootstrap model calculations

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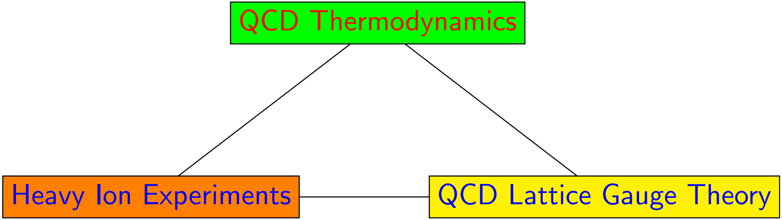
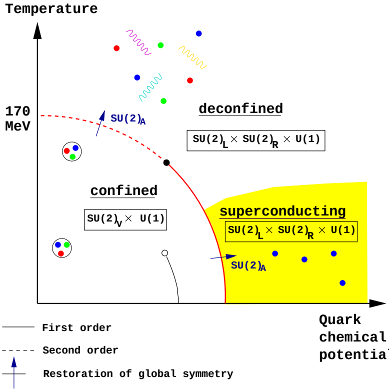
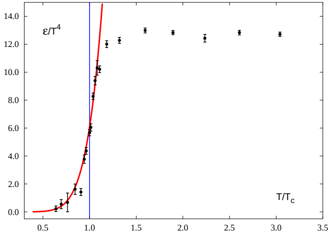
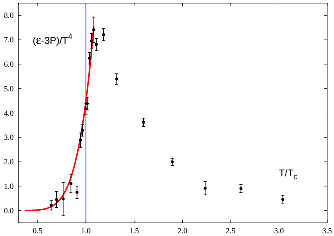
1. Introduction
2. Critical curve from the lattice calculations
   - Relation to the QCD
3. Critical curve from the statistical bootstrap model
   - Exact formula
   - Taylor expansion
4. Conclusions



- Equation of state at finite baryon density
- Critical conditions


Temperature
170 MeV
deconfined
SU(2)L × SU(2)R × U(1)
SU(2)A
confined
SU(2)V × U(1)
superconducting
SU(2)L × SU(2)R × U(1)
SU(2)A
Quark chemical potentia
First order
Second order
Restoration of global symmetry

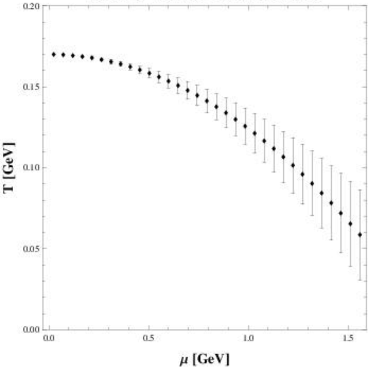

ε/T4
T/Tc


(ε-3P)/T4
T/Tc

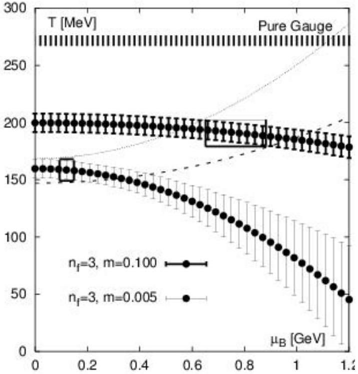
The quark mass dependence is parameterized through the relation

$$(m_H a)^2 = (m_H a)^2_{phys} + b(m_\pi a)^2$$

LGT 2−flavor critical curve



LGT 3-flavor critical curves



Rolf Hagedorn

"A fireball consists of fireballs, which in turn consist of fireballs, and so on. . . ."

Bootstrap equation

$$\rho(m, V_0) = \delta(m - m_0) + \sum_N \frac{1}{N!} \left[\frac{V_0}{(2\pi)^3}\right]^{N-1} \int \prod_{i=1}^{N} [dm_i \, \rho(m_i) \, d^3 p_i] \, \delta^4(\Sigma_i p_i - p)$$

The bootstrap input function

$$\tau_0(p^2, b, s) = \delta(p^2 - m_1^2)\theta(p_0) + \delta(p^2 - m_2^2)\theta(p_0)$$

The bootstrap equation is given (in a simplified form) as

$$\begin{aligned}\tau(p^2, b, s) =& \delta(p^2 - m_1^2)\theta(p_0) + \delta(p^2 - m_2^2)\theta(p_0) \\ & + \sum_{n=2}^{\infty} \frac{1}{n!} \int \delta^4(p - \sum^{n} p_l) \sum_{\{b_j\}\{s_i\}}^{n} \delta(b - \sum b_j)\delta(s - \sum s_i) \\ & \times \prod_{i=1}^{n} \tau(p_i^2, b_i, s_i)\, d^4 p_i\end{aligned}$$

With

$$\varphi_{n_\pi,n_N}(\mu,T) = \int d^4p\,\Big(n_\pi\delta(p^2-m_\pi^2)+n_N\delta(p^2-m_N^2)\Big)\exp(-\beta_\mu p^\mu)$$
$$= 2\pi T\left[n_\pi m_\pi K_1\left(\frac{m_\pi}{T}\right)+n_N m_N K_1\left(\frac{m_N}{T}\right)\cosh\left(\frac{\mu}{T}\right)\right],$$

and

$$\Phi(T,\mu) = \sum_b \mathrm{e}^{-\beta\, b}\int d^4p\,\mathrm{e}^{-\beta_\mu p^\mu}\,\tau(p^2,b)$$

Bootstrap equation has a form

$$2\Phi = \varphi + e^{\Phi} - 1\,,$$

meaningful only for

$$\varphi \leqslant \ln 4 - 1\,.$$

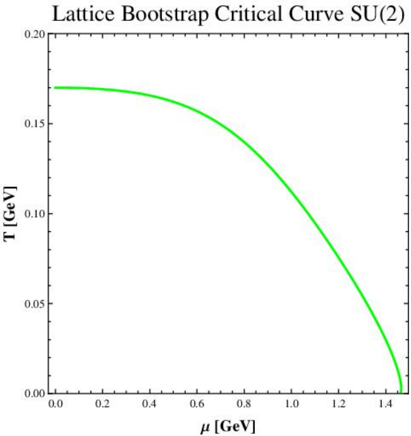
The critical curve

The critical curve $T_c(\mu)$ is given on the $\mu_B - T$ plane by the condition

$$\varphi_{n_\pi,n_N}(\mu, T) = \ln 4 - 1 \,.$$

The bootstrap input function

$$\begin{aligned}\varphi_{n_\pi,n_N}(\mu, T) &= H \int d^4p \, \left(n_\pi \delta(p^2 - m_\pi^2) + n_N \delta(p^2 - m_N^2)\right) \exp(-\beta_\mu p^\mu) \\ &= 2H\pi T \left[n_\pi m_\pi K_1\left(\frac{m_\pi}{T}\right) + n_N m_N K_1\left(\frac{m_N}{T}\right) \cosh\left(\frac{\mu}{T}\right)\right] ,\end{aligned}$$

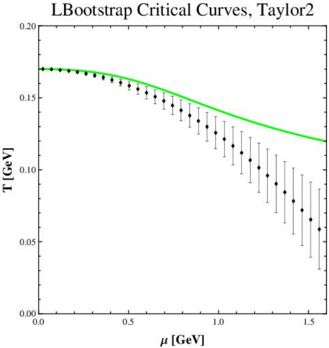


The statistical bootstrap model used on the QCD lattice system has its basis components such as they appear in lattice QCD simulation. It means, nucleon mass expressed by pion mass (all in GeV) as

$$m_N(m_\pi) = 0.94 + \frac{m_\pi^2}{0.94}$$

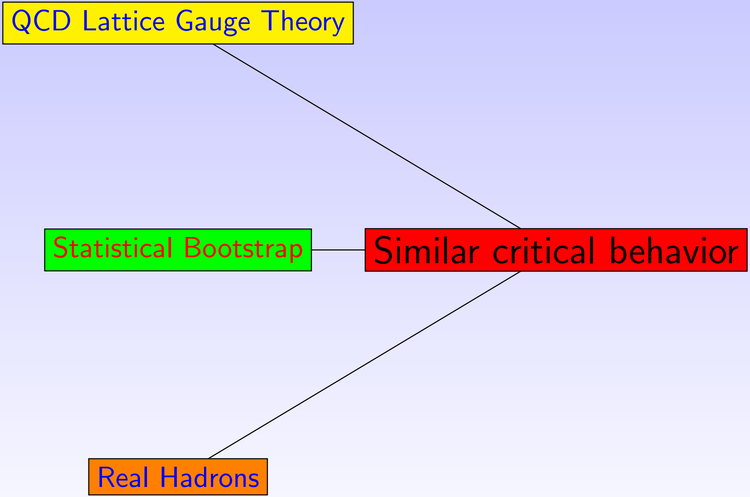
The transition temperature T_c , as a function of baryonic chemical potential for 2-flavor lattice, bootstrap system

Critical curves $T_c(\mu_q)$ from the QCD lattice calculations were obtained up to $\mathcal{O}((\mu_q/T_c)^2)$ term.



$\cosh[(\frac{\mu_B}{T})]$ replaced by the corresponding Taylor expansion truncated to the first two terms

The critical temperature 2-flavor lattice bootstrap system, compared with result of corresponding lattice simulation, both considered in power law approximation


QCD Lattice Gauge Theory
Statistical Bootstrap
Similar critical behavior
Real Hadrons