Toward hydrodynamics as an effective theory

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Abstract

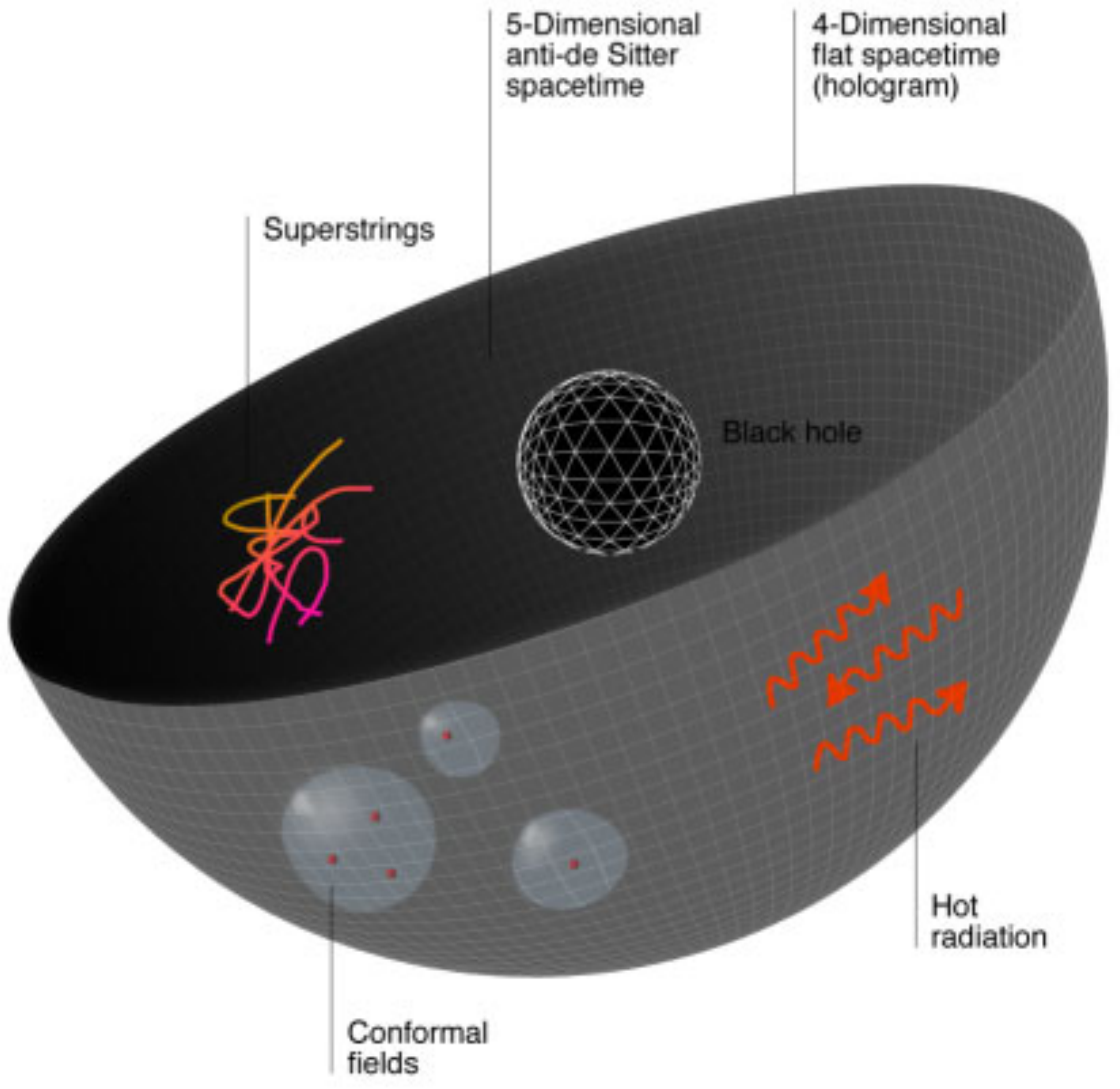
Model studies of equilibration using the AdS/CFT correspondence have lead to a number of insights concerning relativistic hydrodynamics; their validity extends well beyond the specific case of N=4 supersymmetric Yang-Mills theory. I will summarise these and assess their impact from the perspective of the physics of quark-gluon plasma.

Introduction

- Task: understand hydrodynamics as an effective theory **matching QCD**
- Useful input from **first principles calculations** using AdS/CFT:
    A. Hydro works early ("**hydrodynamization**")
    B. Complete 2nd order theory (**BRSSS** replacing/completing MIS)
    C. Gradient expansions are **asymptotic**
- Recent progress based on kinetic theory

Strongly coupled picture (AdS/CFT)

- Holographic relation between N=4 supersymmetric Yang-Mills theory and string theory
- Strongly coupled Yang-Mills maps to classical gravity
- Equilibrium states map to black hole spacetimes
- Perturbations of equilibrium: **quasinormal modes**

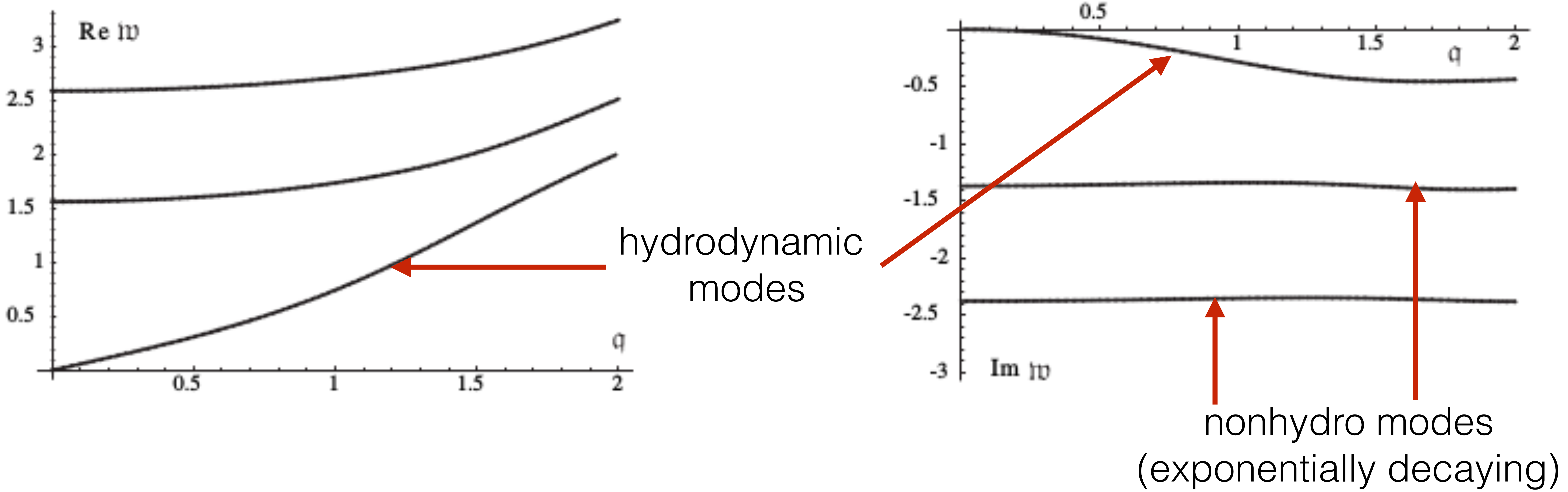


Strong coupling picture from AdS/CFT

Quasinormal modes:

$$\delta\Phi \sim \exp\left(-i(\omega t - \vec{k}\cdot\vec{x})\right)$$

Damped when $Im(\omega) < 0$



Hydrodynamics: an effective description of the hydro modes alone?

[Kovtun, Starinets hepth/0506184]

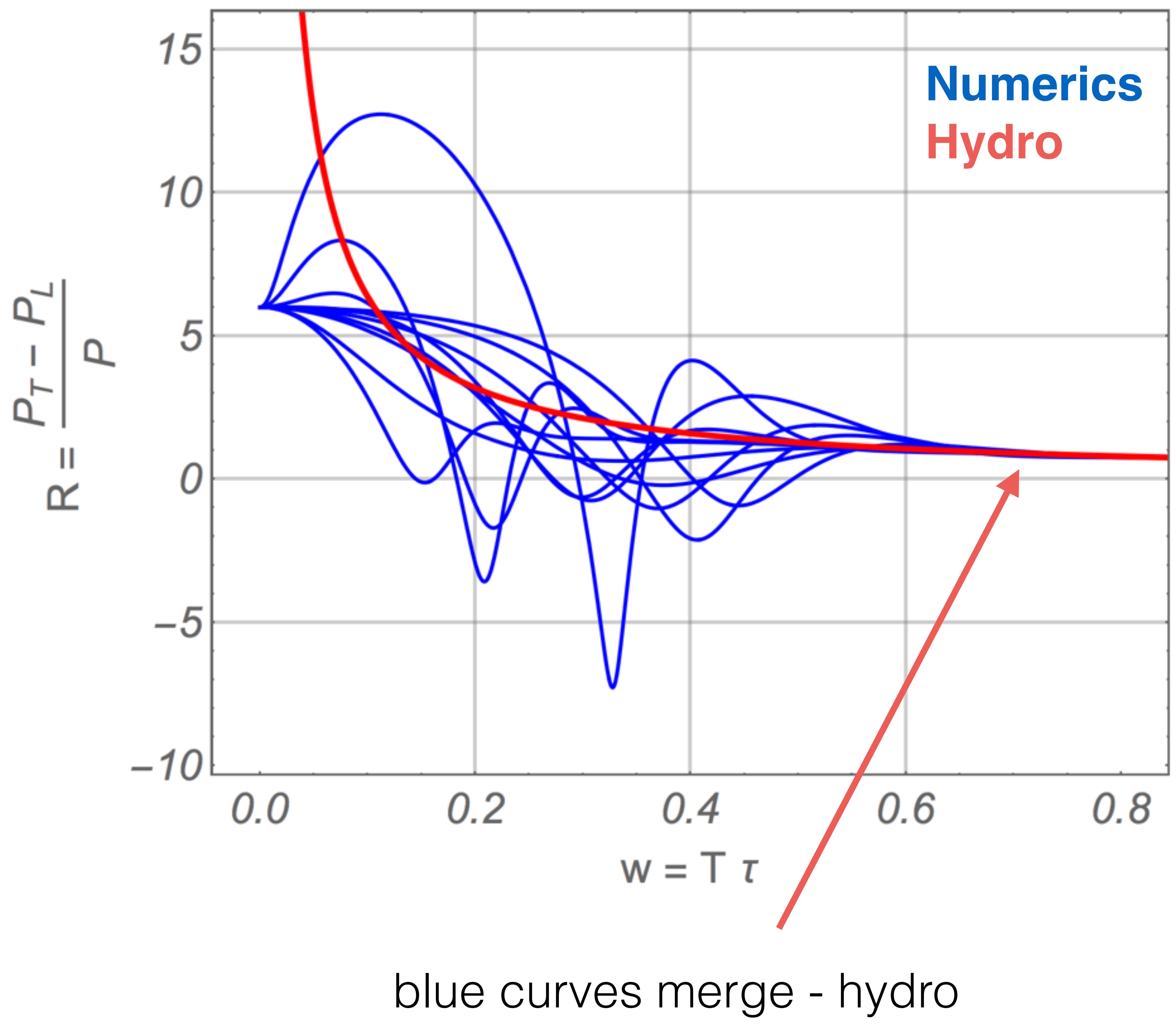
Hydrodynamization from holography

- Hydro works great even at large pressure anisotropies (~ 60%).
- Effects of nonhydro modes are plainly visible at small times
- "Hydrodynamization" for

$$\tau T \sim 0.7$$

- Applicability of hydro determined by decay nonhydro modes

[Heller et al.1302.0697]
[Jankowski et al.1411.1969]



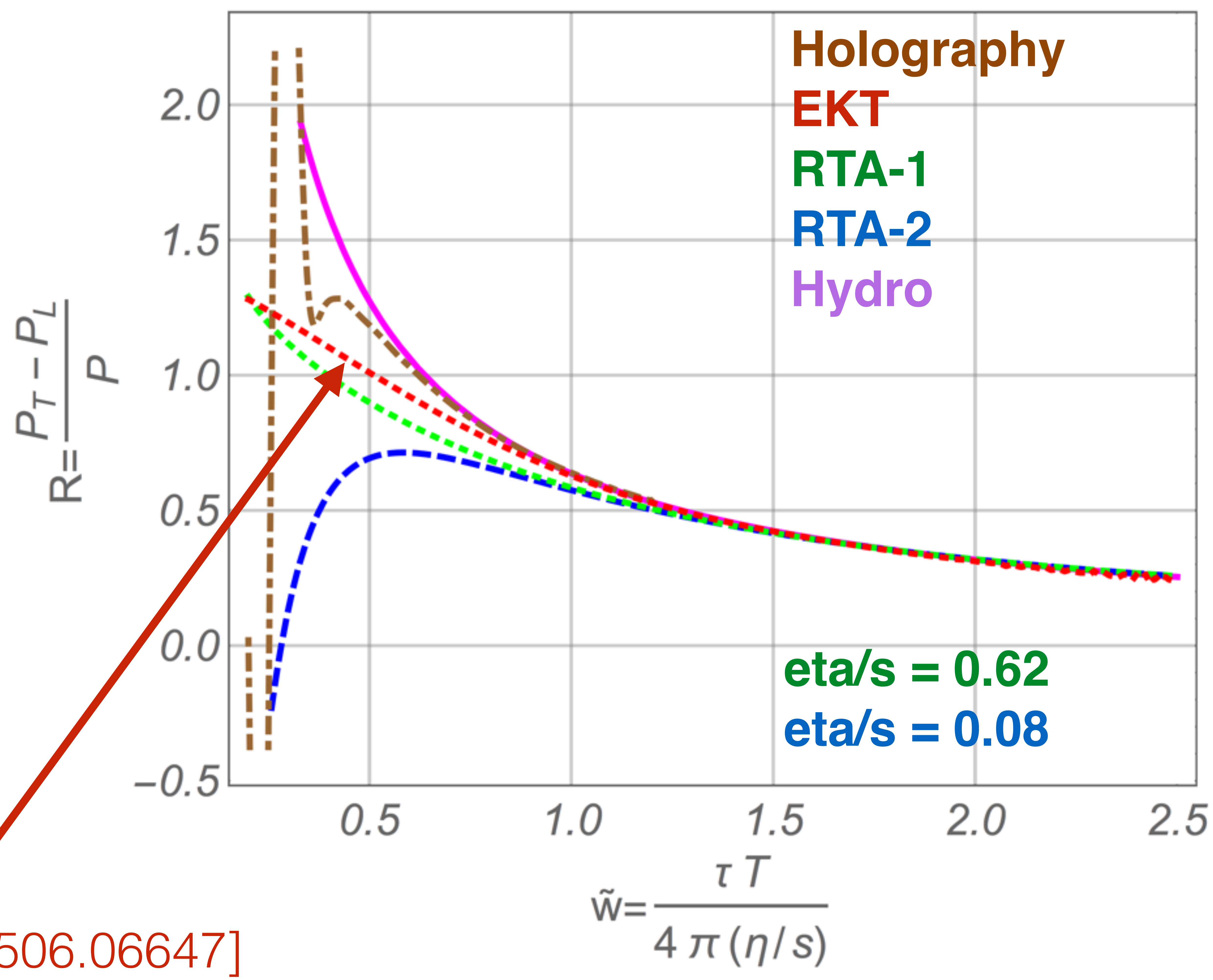
Hydrodynamization from kinetic theory

Boltzmann eqn, 2 collision kernels:

- Relaxation time approximation
- Effective kinetic theory (AMY)

Note:

- Hydrodynamization
- Boring at early time



[Kurkela et al.1506.06647]

[Heller et al.1609.04803]

The gradient expansion

Define the flow velocity by

some scalar functions of
microscopic parameters

$$\langle T^{\nu}_{\mu} \rangle u^{\mu} = -\mathcal{E} u^{\nu}$$

On general grounds

$$\langle T^{\mu\nu} \rangle = \mathcal{E} u^{\mu} u^{\nu} + \mathcal{P}(g^{\mu\nu} + u^{\mu} u^{\nu}) - \eta \sigma^{\mu\nu} + \tau_{\Pi} \mathcal{D}(\eta \sigma^{\mu\nu}) + \ldots$$

This:

- allows **matching** phenomenological and microscopic descriptions
- defines what we mean by transport coefficients

by comparing with the corresponding expansions in hydrodynamics.

Hydrodynamic theories

$$T^{\mu\nu} = \mathcal{E} u^\mu u^\nu + \mathcal{P}(\mathcal{E})(g^{\mu\nu} + u^\mu u^\nu) + \Pi^{\mu\nu}$$

$$\nabla_\alpha T^{\alpha\beta} = 0$$

- Navier Stokes: $\Pi^{\mu\nu} = -\eta(\mathcal{E})\sigma^{\mu\nu}$
- Mueller; Israel & Stewart; **BRSSS**: $(\tau_\Pi \mathcal{D} + 1)\, \Pi^{\mu\nu} = -\eta\sigma^{\mu\nu} + \ldots$
- Anisotropic hydrodynamics
- Extensions including leading QNM

complete set of 2nd order terms

nohydrodynamic modes act as a **regulator** ensuring causality when $T\tau_\Pi \geq 2\eta/s$

They all generate gradient expansions **with terms of all orders**, which can be **matched** to microscopic calculations.

Gradient expansion for Bjorken flow

- Large proper time expansion $T \sim \tau^{-1/3}$

$r_n \sim n!$

- Pressure anisotropy

$w \equiv \tau T$

$$\mathcal{R} \equiv \frac{\mathcal{P}_T - \mathcal{P}_L}{\mathcal{P}} = \sum_{n=1}^{\infty} r_n w^{-n}$$

- The singularities of the analytic continuation (using Pade approximants) of the Borel transform

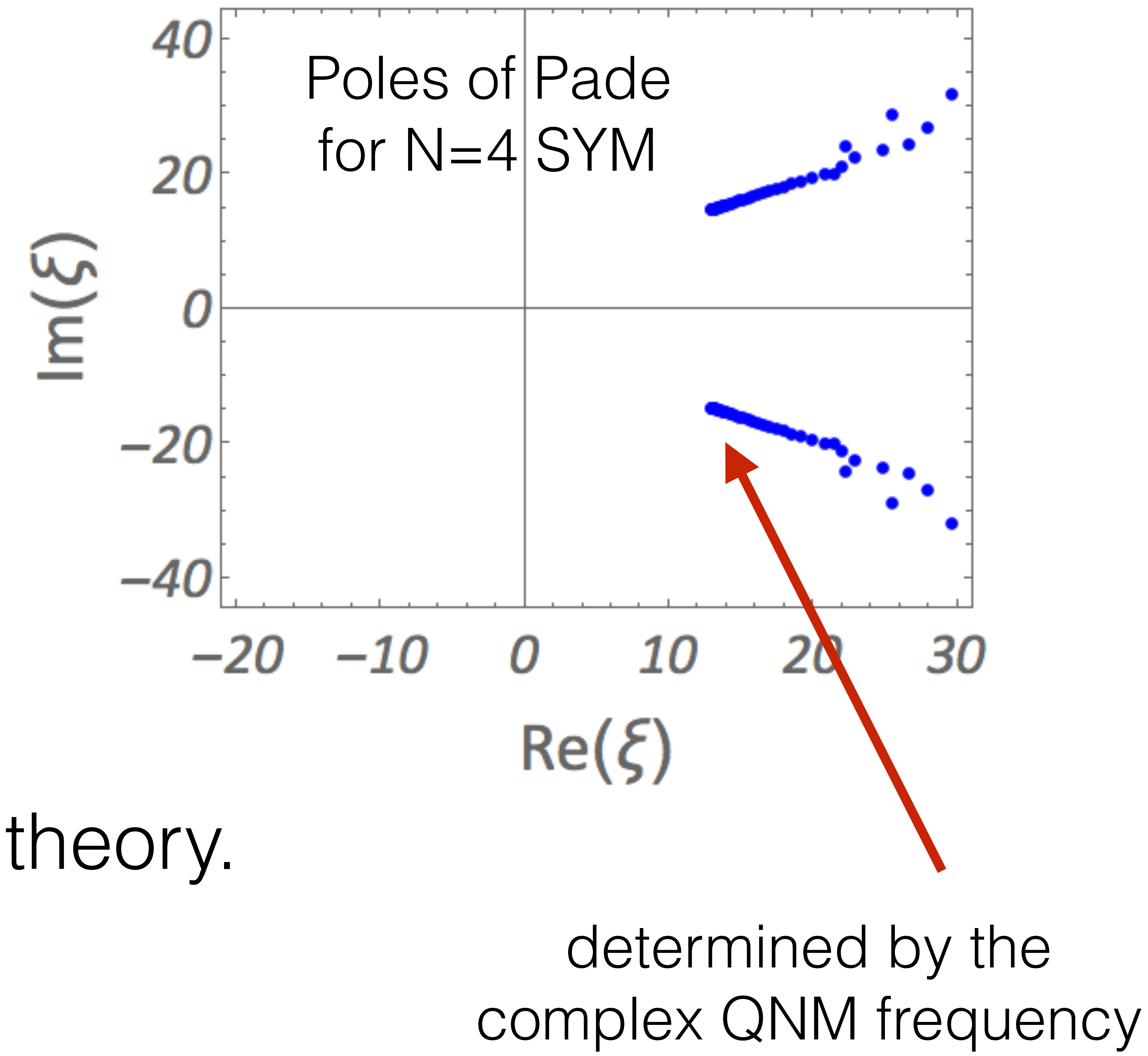
$$\mathcal{R}_B(\xi) = \sum_{n=1}^{\infty} \frac{r_n}{n!} \xi^n$$

contain **information about nonhydrodynamic modes** (both at the microscopic level and in hydrodynamics).

NHM from asymptotic behaviour

This connection has been checked in cases where it is understood what the nonhydrodynamic sector looks like:

- at the microscopic level: N=4 SYM
- at the the level of hydrodynamics:
    A. MIS/BRSSS (purely damped NHM)
    B. extensions with oscillatory NHM



Some results have also been obtained in kinetic theory.

[Heller et al.1302.0697, Heller, MS 1503.07514, Aniceto, MS 1601.06358]

Kinetic theory in the RTA

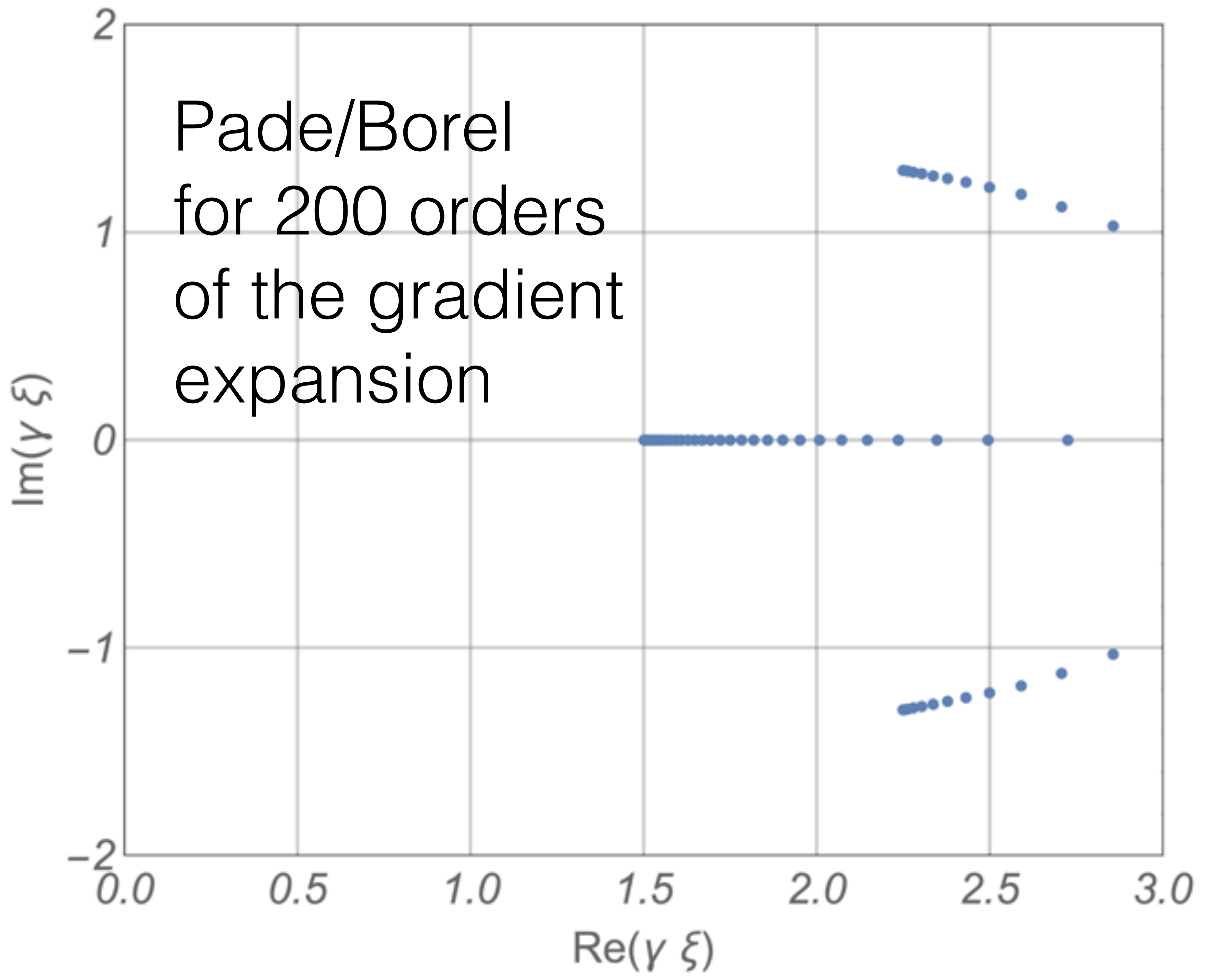
Microscopic calculation: RTA with relaxation time $\mathcal{T} = \gamma/T$ (conformal):

$$\mathcal{R} = \frac{8}{15}\gamma w^{-1} + \frac{32}{105}\gamma^2 w^{-2} - \frac{416}{525}\gamma^3 w^{-3} + \ldots$$

Hydrodynamic descriptions

- MIS ← universal
- BRSSS, DNMR, AHYDRO
- Jaiswal's 3rd order hydro

AHYDRO, DNMR: a little better than BRSSS.



[Heller et al.1609.04803, Florkowski et al.1608.07558]

Summary

- The emergence of hydrodynamic behaviour **at the microscopic level** is governed by the decay of nonhydrodynamic modes

- Nonhydrodynamic modes appear also in relativistic hydrodynamic theories, where they serve as a **regulator** for causality and set limits of applicability (how early, how small)

- Information about nonhydrodynamic modes is encoded in the **large order behaviour** of the gradient expansion

- Hydrodynamic theories can be **engineered** to match the gradient expansion of a given microscopic theory