Entanglement entropy and parity fluctuations

Case of neutral kaons

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Quantum entanglement

Verbally:

Quantum states of two or more objects can be described only with reference to each other, even when separated by large distances

Term coined by Schrödinger in his letter to Einstein (1935, soon after EPR paper) using term *Vershränkung*

Einstein to Born (1947): *spukhafte Fernwirkung* or *spooky action at a distance*

Quantum entanglement: how to quantify

State $|\psi\rangle$ (any orthonormal basis)

and density matrix $\hat{\rho} = |\psi\rangle\langle\psi|$

Define the **von Neumann entropy** $\mathsf{S}(\hat{\rho}) = -\mathrm{Tr}(\hat{\rho}\ln\hat{\rho})$

Bipartite division: the subsystem A and its remainder B

Reduced density matrix $\hat{\rho}_\mathsf{A} = \mathrm{Tr}_\mathsf{B}\,\hat{\rho}$

Tracing over remainder's degrees of freedom

and the **entanglement entropy** $\mathsf{S}_\mathsf{A} = -\mathrm{Tr}(\hat{\rho}_\mathsf{A}\ln\hat{\rho}_\mathsf{A})$

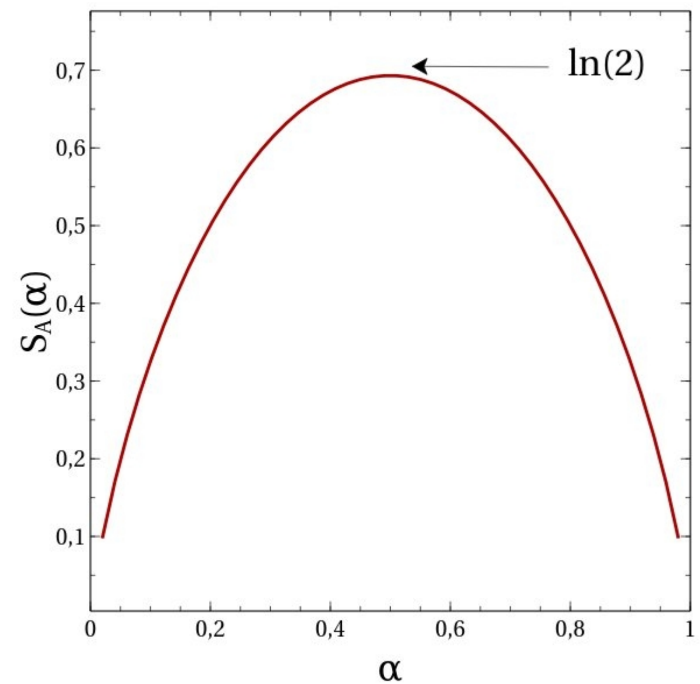
Example: two electron spins A and B in the state

$$|\psi\rangle = \sqrt{\alpha}\,|\uparrow\rangle_A|\downarrow\rangle_B - \sqrt{1-\alpha}\,|\downarrow\rangle_A|\uparrow\rangle_B \qquad 0 \leq \alpha \leq 1$$

Entanglement entropy

$$S_A = -(1-\alpha)\ln(1-\alpha) - \alpha\ln\alpha$$

is maximal for maximally entangled state $\alpha = 0.5$ and zero for factorized states $\alpha = 0, 1$



α=0,1: spontaneous factorization hypothesis, Furry 1936

Fluctuations in entangled states

Consider fluctuations of number of particles $\hat{N}_A$ in state $|\psi\rangle$

$$\mathcal{V}\mathrm{ar}(N_A) = \langle \hat{N}_A^2 \rangle - \langle \hat{N}_A \rangle^2$$

expected values taken in state $|\psi\rangle$

Variance is just the **second cumulant** C_2 where in general

$$C_n = (-i\frac{\partial}{\partial\lambda})^n \ln \chi(\lambda)|_{\lambda=0}$$

where $\chi(\lambda)$ is the cumulant generating function (may be time-dependent)

$$\chi(\lambda) = \langle \exp(i\,\lambda\,\hat{N}_A) \rangle$$

But $\chi(\lambda, t)$ also generates probability distribution

$$\chi(\lambda, t) = \sum_m p_m(t)\, e^{i\lambda m}$$

where $p_m(t)$ is probability that for given particle m „spin-ups" is transfered to „spin-downs"

$$|\uparrow\rangle_A \longrightarrow |\downarrow\rangle_A$$

Back to our example:

$$\mathcal{V}ar(N_{\uparrow_A}(\alpha)) = 1 - \alpha \qquad \mathcal{V}ar(N_{\uparrow_B}(\alpha)) = \alpha$$

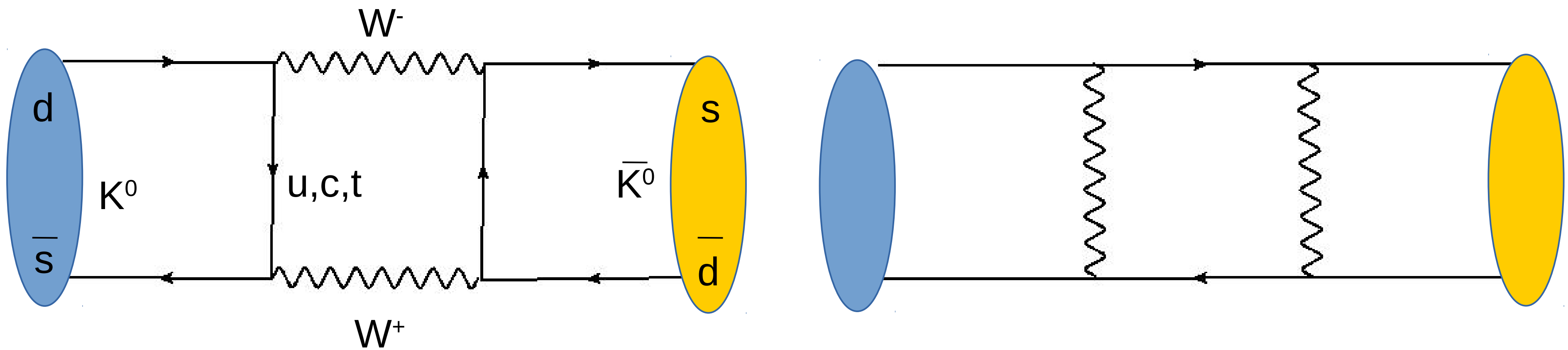
For $\alpha = 0, 1$ spins are in factorized state and no fluctuations occur

For $0 < \alpha < 1$ both spins fluctuate

Illuminating application: entangled pairs of neutral kaons

A quick reminder about K^0 mesons

$K^0, \bar{K}^0$ are eigenstates of strong Hamiltonian but weak force allows them to mix



$m_K = 0.498$ GeV

Weak force also make them to decay
Principal hadronic decay modes are into pions

$$K^0, \bar{K}^0 \to 2\pi \qquad\qquad K^0, \bar{K}^0 \to 3\pi$$

Since $\mathrm{CP}(2\pi) = +1, \ \mathrm{CP}(3\pi) = -1$

so $K^0, \bar{K}^0$ are **not CP eigenstates**

Define orthonormal CP eigenstates:

$$|K_1\rangle_{CP=+1} = \frac{1}{\sqrt{2}}(|K^0\rangle - |\bar{K}^0\rangle)$$
$$|K_2\rangle_{CP=-1} = \frac{1}{\sqrt{2}}(|K^0\rangle + |\bar{K}^0\rangle)$$

$$K_1 \to 2\pi$$
$$K_2 \to 3\pi$$

In nature, weak force also violates CP such that physical states K_S and K_L (eigenstates of $H_{\text{strong \& weak}}$) do not have definite CP

But CP violation is a small effect (2% level) and let's **assume $K_{S,L}$ are identical to $K_{1,2}$**

Their lifetimes are **much different** (phase space)

$$\tau_S = 0.9 \times 10^{-10}\ \mathrm{s}, \qquad \tau_L = 0.5 \times 10^{-7}\ \mathrm{s}$$

Their masses are **slightly different** (1 part per 10 trillion)

$$\Delta m = m_L - m_S = (3.506 \pm 0.006) \times 10^{-15}\ \mathrm{GeV}$$

but still $1/\Delta m \sim 10^{-9}$ s

Therefore neutral kaons nicely oscillate but K_S dies out much faster than K_L

$$|K_{S,L}(t)\rangle = |K_{S,L}\rangle e^{-i\lambda_{S,L}}, \qquad \lambda_{S,L} = m_{S,L} - i\,\Gamma_{S,L}/2$$

Pairs of entangled kaons in real experiment: case of KLOE (now KLOE-2)

$$K_L(K_S) \leftarrow \Phi(1020) \rightarrow K_S(K_L)$$

Produced copiously in rest at symmetric $e^+ e^-$ Φ-factory DAΦNE (LNF Frascati)

$J^{PC}(\Phi)=1^{--}$ ▸ initial state antisymmetric

$$|\psi_I\rangle \sim |K_S\rangle|K_L\rangle e^{-i\lambda_S t_1} e^{-i\lambda_L t_2} - |K_L\rangle|K_S\rangle e^{-i\lambda_L t_1} e^{-i\lambda_S t_2}$$

For simplicity, consider identical final states, e.g. $\pi^+ \pi^-$

Standard way of investigating interference phenomena

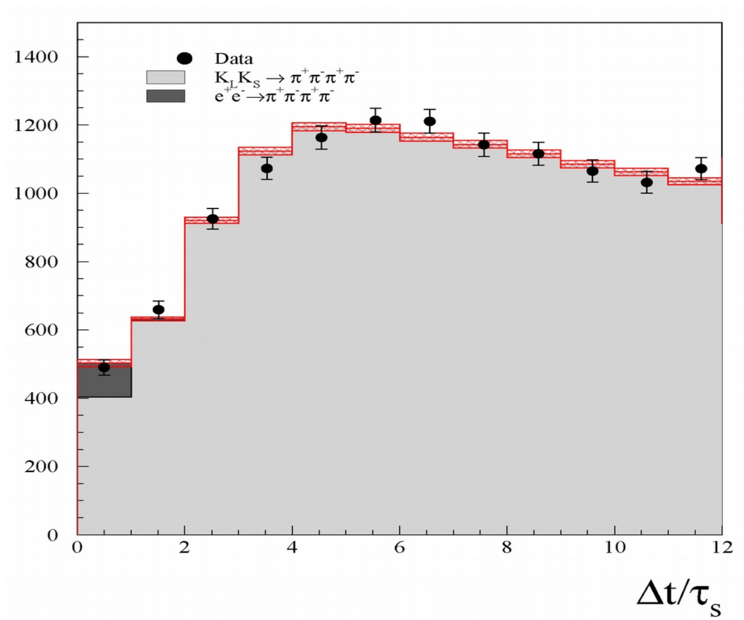
$$I(\Delta t) = e^{-\Delta t/\tau_L} + e^{-\Delta t/\tau_S} - 2e^{-\Delta t/2(1/\tau_L + 1/\tau_S)} \cos(\Delta m \, \Delta t)$$

$2(1-\zeta)$

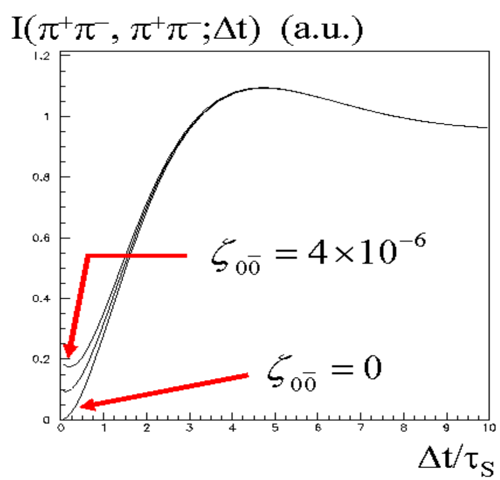
ζ=1 Furry hypothesis of spontaneous factorization:
statistical mixture of states, no inteference

ζ=0 standard QM

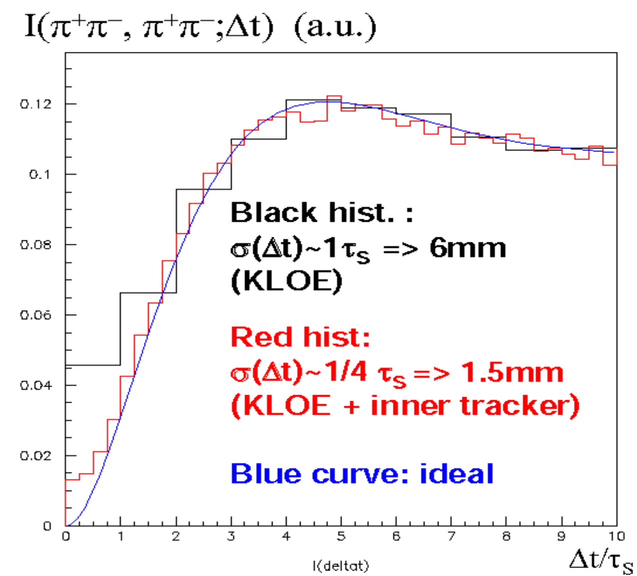
Experimental value (KLOE, 2010)
$\zeta = 0.003 \pm 0.019$



Possible signal of decoherence concentrated at very small Δt



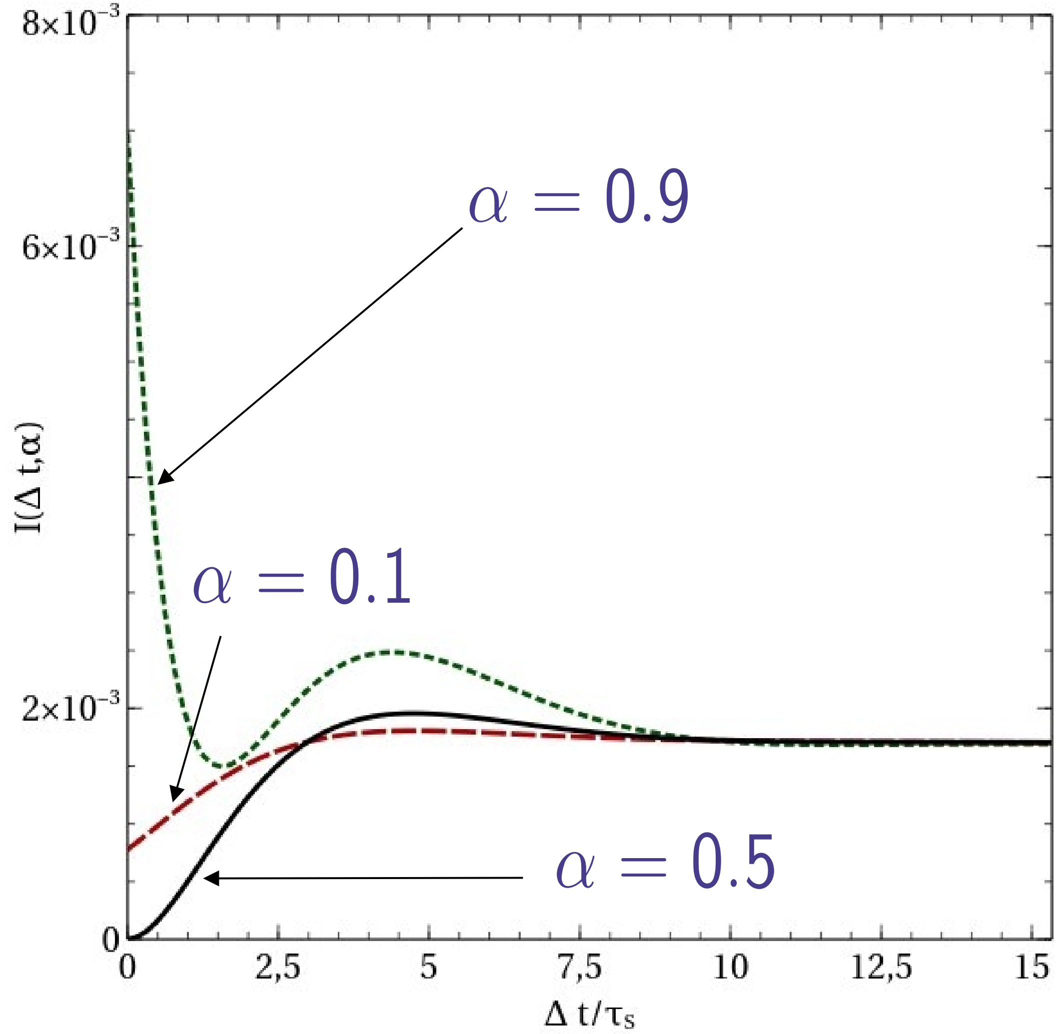
Theoretical distribution



Reconstructed distribution (MC)

Self-consistent approach to study entropy and fluctuations in entagled kaons using density matrix

$$|\psi_I\rangle = \sqrt{\alpha}|K_S\rangle|K_L\rangle e^{-i\lambda_S t_1} e^{-i\lambda_L t_2}$$
$$- \sqrt{1-\alpha}|K_L\rangle|K_S\rangle e^{-i\lambda_L t_1} e^{-i\lambda_S t_2}$$



Intensity, duly normalized 0 - ∞

Strong dependence of shape on α for small time diffs

$$I(\Delta t) = (1-\alpha)e^{-\Delta t/\tau_L} + \alpha e^{-\Delta t/\tau_S}$$
$$- 2\sqrt{\alpha(1-\alpha)}e^{-\Delta t/2(1/\tau_L + 1/\tau_S)} \cos(\Delta m \, \Delta t)$$

In case of neutral kaons, fluctuations $C_2 = Var(N_{KS})$ are fluctuations of CP parity of the left-hand side particle, (neglecting CP-violation effects)

Since K^0 are unstable and K_L, K_S lifetimes differ, fluctuations of given parity and degree of entanglement of pairs (S_{ent}) are time-dependent

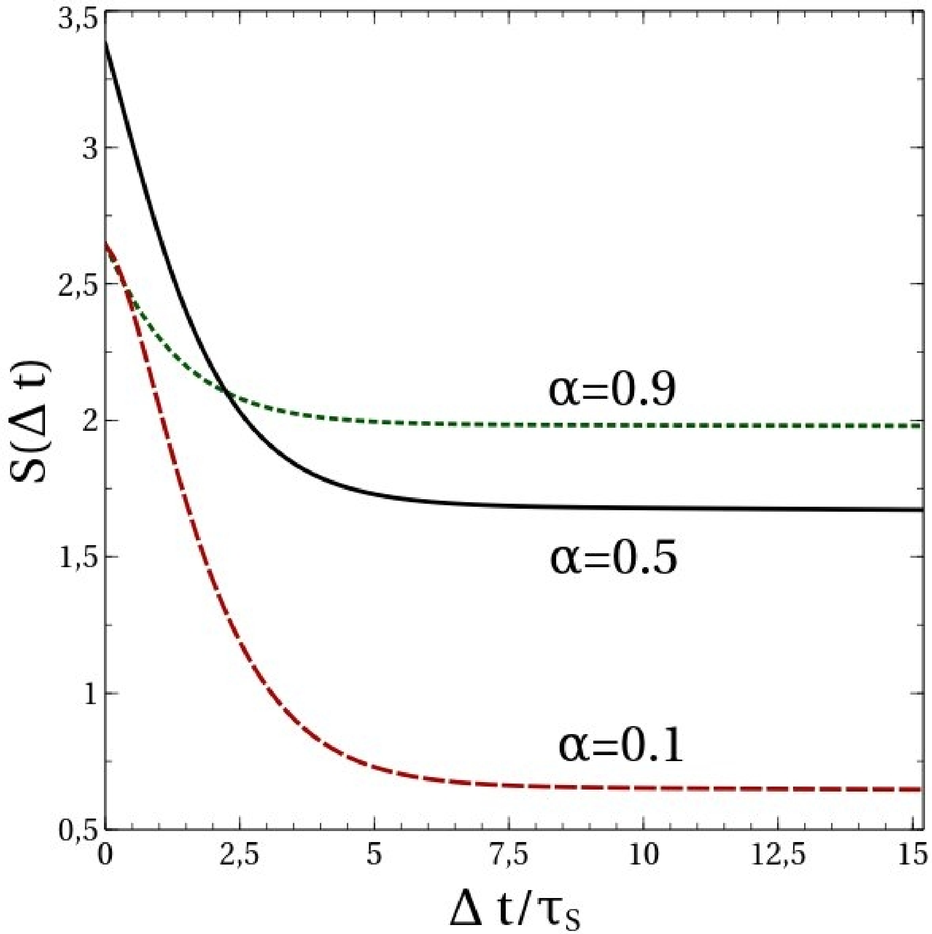
Also, maximal attainable entanglement of kaon pair is time-dependent

Dynamics of kaon decays affects entanglement of a pair and fluctuations of CP of a given particle

Entanglement entropy for K_L K_S

$$S(\Delta t, \alpha) = \frac{2}{1/\tau_S + 1/\tau_L} \times$$

$$\alpha(1 - \ln\alpha + \frac{\Delta t}{\tau_L})e^{-\frac{\Delta t}{\tau_L}} + (1-\alpha)[1 - \ln(1-\alpha) + \frac{\Delta t}{\tau_S}]e^{-\frac{\Delta t}{\tau_S}}$$



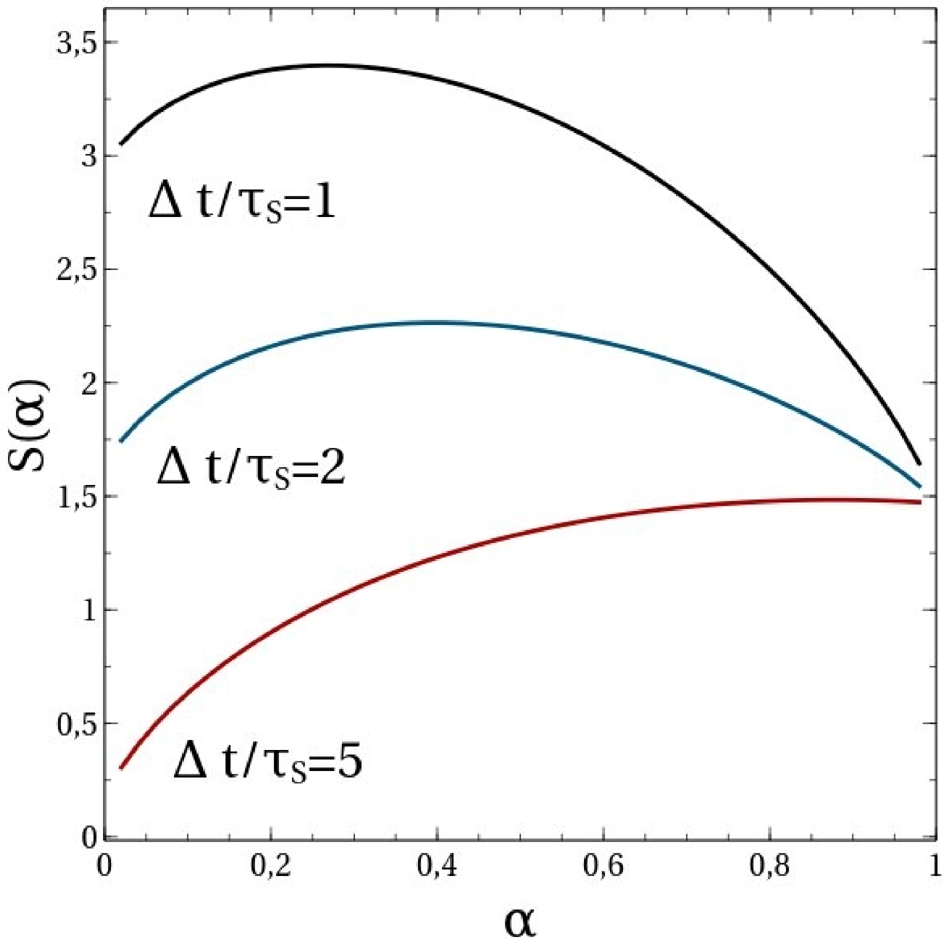
Time dependence slows down when K_S die out

Strong dependence on α

Entanglement entropy, cont.

$$S(\Delta t, \alpha) = \frac{2}{1/\tau_S + 1/\tau_L} \times$$

$$\alpha(1 - \ln\alpha + \frac{\Delta t}{\tau_L})e^{-\frac{\Delta t}{\tau_L}} + (1-\alpha)[1 - \ln(1-\alpha) + \frac{\Delta t}{\tau_S}]e^{-\frac{\Delta t}{\tau_S}}$$

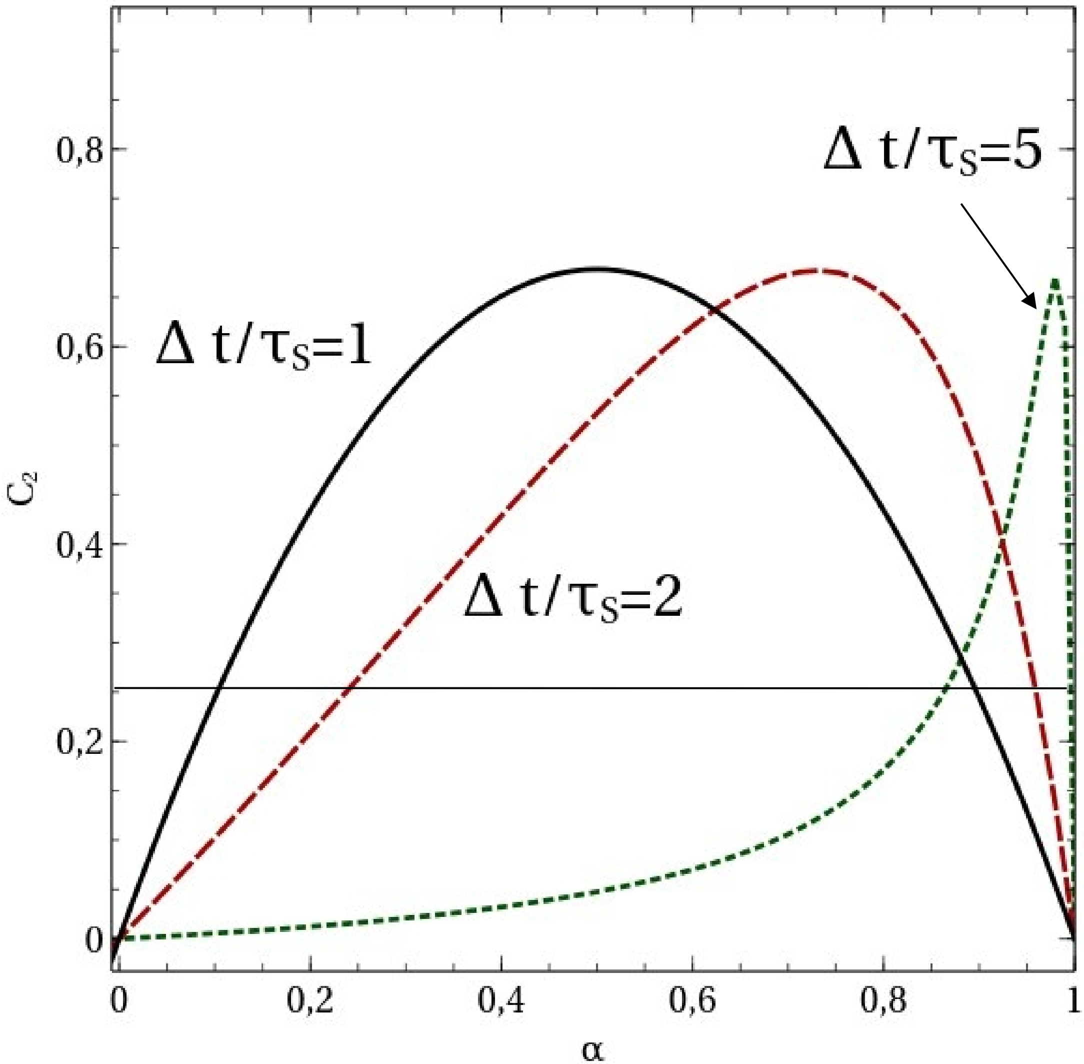


Maximum entropy is no longer at α=0.5 and its location drifts in time diff from lower to higher α, just because of time-dependence of coeffs at

$$|K_{S(L)}\rangle|K_{L(S)}\rangle$$

Fluctuations

$$C_2(\Delta t, \alpha) = \frac{\alpha(1-\alpha)e^{-\Delta t/(1/\tau_L + 1/\tau_S)}}{[\alpha e^{-\Delta t/\tau_L} + (1-\alpha)e^{-\Delta t/\tau_S}]^2}$$



Maximal fluctuations for higher α as time diff increases;
same effect as for S

$$S = a_2 C_2 + a_4 C_4 + \ldots$$

Variance for equiprobable
0 or 1 particles

This represents a new, not yet implemented, approach to entanglement in experiments investigating it

In KLOE, following the way discussed above:

- Fit decay intensity spectrum using entanglement parameters and thus determine them
- Use them to calculate entanglement entropy; Is it maximal? If not – interesting finding
- Use them to calculate variance of CP+1 (or CP-1); Does it differ from equiprobable? If not – very interesting

But also in other experiments investigating entangled pairs of mesons

Further developments and remarks

For some multidimensional systems, entanglement entropy may not be easy to detemine (nontrivial function of $\hat{\rho}$)
Easier to determine are reduced Renyi entropies

and then
$$S_q(\hat{\rho}_A) = \frac{1}{1-q} \ln[\mathrm{Tr} \hat{\rho}_A^q]$$

$$S(\hat{\rho}_A) = \lim_{q \to 1} S_q(\hat{\rho}_A)$$

Both can be expressed in terms of series of cummulants, thus relating entropies to fluctuations and higher-order cummulants: PR B85(2012)035409

Further developments and remarks, cont.

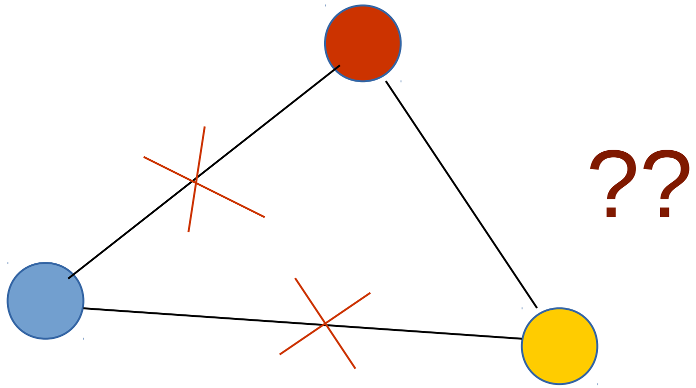
Interesting generalizations to **tripartite** entanglement where could quantify nature of quantum correlations in triplets.

Example: orthopositronium decays o-Ps $\rightarrow \gamma_A \gamma_B \gamma_C$

Determine S_{AB}, S_{BC}, S_{AC}

Destroy coherence in (AB) and (AC) pair, by detecting A

Does (BC) entanglement survives (Hopfian entanglement) or not (Borromean entanglement)



This presentation is dedicated to my friend Staszek, in appreciation of his works and with best wishes that he further develops his excellent research, teaching and erudition in physics (and elsewhere)

$$\dot{f} + \mathbf{p} \cdot \nabla f + \mathbf{F} \cdot \partial_{\mathbf{p}} f = \dot{f}_{\mathrm{coll}}$$

Fluctuations

$$C_2(\Delta t, \alpha) = \frac{\alpha(1-\alpha)e^{-\Delta t/(1/\tau_L+1/\tau_S)}}{[\alpha e^{-\Delta t/\tau_L}+(1-\alpha)e^{-\Delta t/\tau_S}]^2}$$