# Finite volume corrections and low momentum cuts in the thermodynamics of quantum gases

K. Zalewski

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**K.Redlich and K.Zalewski, (in preparation)**

J. Engels, F. Karsch and H. Satz, Nucl. Phys. B205(1982)239

A. Bhattacharyya et al., Phys.Rev. C91(2015)041901(R)
F. Karsch, K. Morita and K. Redlich, arXiv 1508.02614

# Identical bosons in one dimension

$$\psi(0) = \psi(L) = 0; \qquad E_n = \sqrt{\frac{\pi^2 n^2}{L^2} + m^2}.$$

$$\Omega(T, L, \mu) = T \sum_{n=1}^{\infty} \log[1 - e^{-\beta(E_n - \mu)}].$$

Define the continuous function of n:

$$\Omega_n(T, L, \mu) = T \log[1 - e^{-\beta(E_n - \mu)}].$$

# Thermodynamic limit and cut-off

$$\Omega_{int}(T, L, \mu) \quad = \quad T \int_0^{\infty} \Omega_n(T, L, \mu)\, dn$$

$$\Omega_{cut}(t, L, \mu, n_c) \quad = \quad T \int_{n_c}^{\infty} \Omega_n(T, L, \mu)\, dn$$

$$T \quad = \quad 120\text{MeV}, \qquad m = 140\text{MeV}, \qquad \mu = 0.$$

$$k_n \quad = \quad \frac{\pi}{L} n.$$

# Comparison of approximations

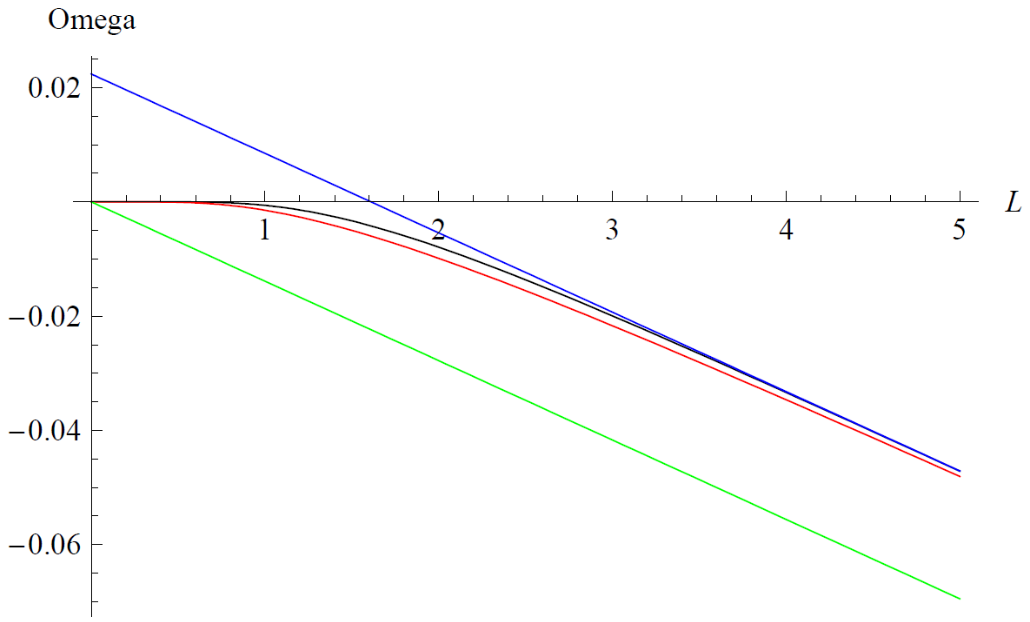



Figure 1: Comparison of the function $\Omega(120, L, 0)$ (black line) with the approximations: $\Omega_{int}(120, L, 0)$ (green line), $\Omega_{cor}(120, L, 0)$ (blue line) and $\Omega_{cut}(120, L, 0, 0.5)$ (red line)

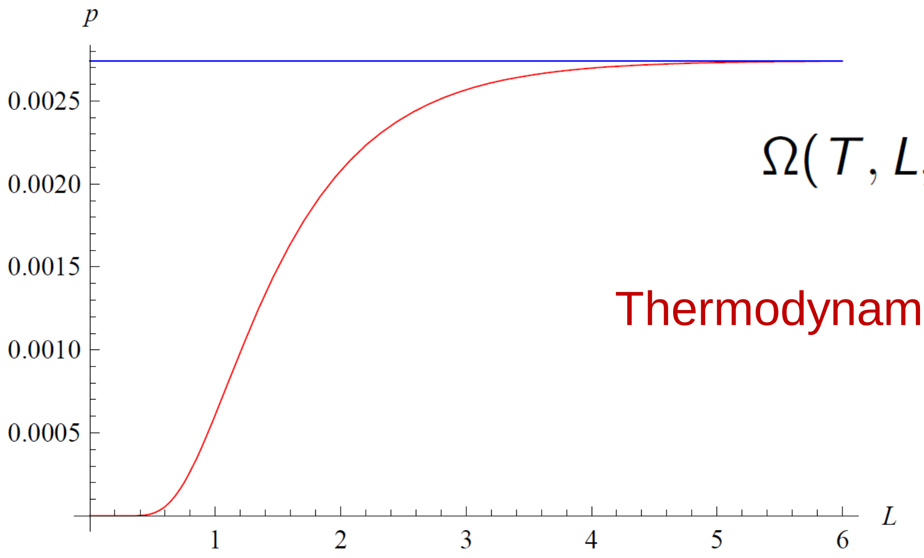
The thermodynamic limit is good within 1% for L > 166fm

# Euler-Maclaurin correction

$$\Omega_{cor}(T,L,\mu) = \Omega_{int}(T,L,\mu) - \frac{1}{2}\Omega_0(T,L,\mu),$$

$$\Omega_0(T,L,\mu) = T\log(1-e^{-\beta m}).$$

Good for L>3.7fm




$$\Omega(T,L,\mu) = -p_{int}L - \frac{1}{2}\Omega_0(T,L,\mu).$$

Thermodynamic limit for p good at L > 4.4fm.

Figure 3: Comparison of the function $p(120, L, 0)$ (red line) with the approximation $p_{int}(120, L, 0)$ (blue line).

# Cut-off correction

$$\Omega_{cut}(T, L, \mu, 0) < \Omega(T, L, \mu) < \Omega_{cut}(T, L, \mu, 1).$$

$0 < n_c(T,L,\mu) < 1$

Assuming the approximation $\Omega_{cor}$ :

$$\frac{1}{2}\Omega_0(T, L, \mu) = \int_0^{n_c} \Omega_n(T, L, \mu)\, dn = n_c\Omega_0(T, L, \mu).$$

$$n_c = \frac{1}{2}, \qquad L > 5.6\text{fm},$$

$$k_c = \frac{309}{L}\text{MeV}, \qquad k_c < 55\text{MeV}.$$

# Cut-off parameter at small volumes

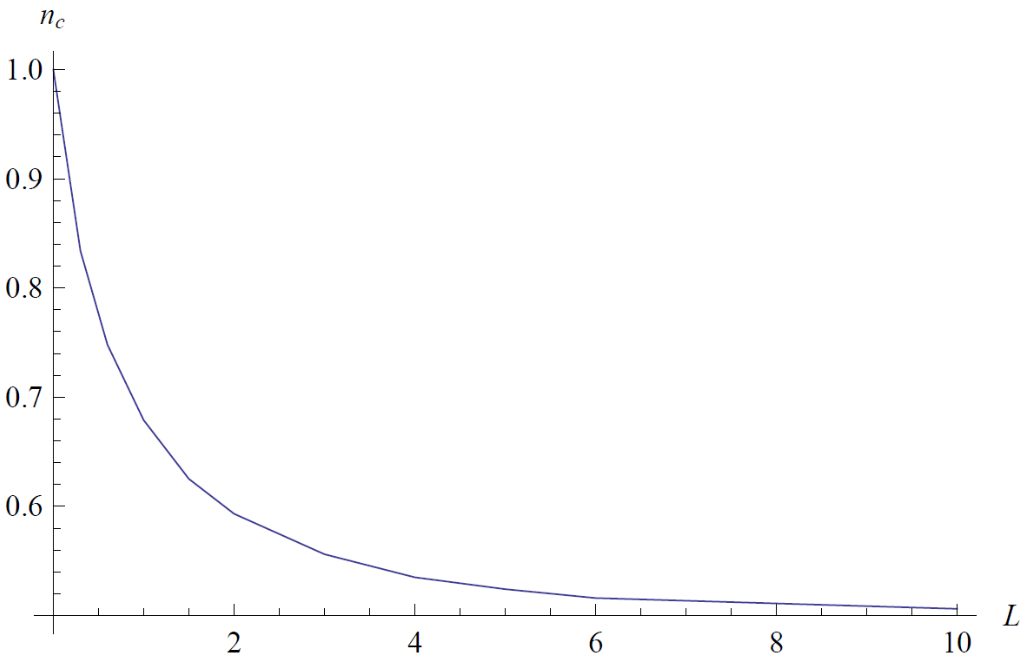



Figure 2: $L$-dependence of the cutoff parameter $n_c$

For L < 0.07 fm: $$n_c = 1 + \frac{LT}{\pi} \log \frac{LT}{\pi}.$$

# Conclusions

1. In the present model suitable momentum cut-offs exactly reproduce all the finite volume effecs.

2. The method is simple for sufficiently large volumes. In our model for L > 5.6 fm (which corresponds to $k_c$ < 55MeV).

# Einstein condensation

$$n = n_c + n_a$$

Stability condition: $$\left(\frac{\partial p}{\partial n}\right)_{T,V} \geq 0$$

Two phases