Semirelativity in Semiconductors

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Band structure of narrow gap semiconductors

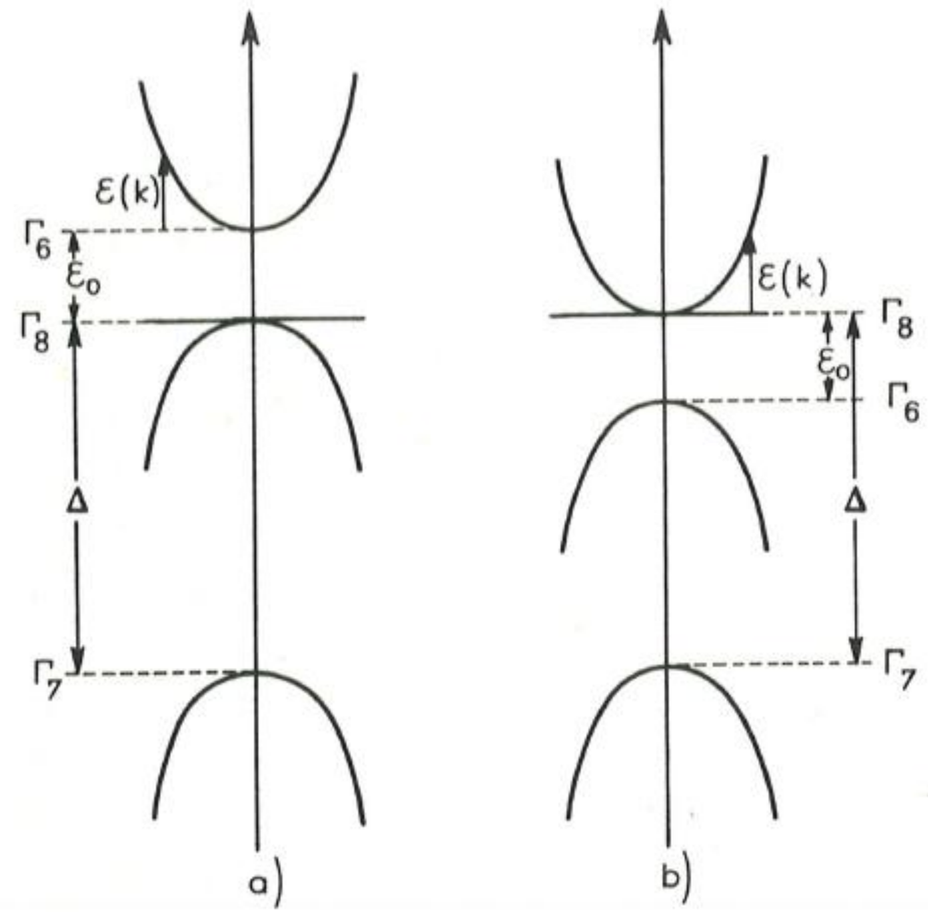


Fig. 1. Energy band structure at the Γ point of the Brillouin zone: a/ InSb-type semiconductors, b/ HgTe-type zero-gap semiconductors. In the three-level model of Γ_6, Γ_8 and Γ_7 levels the shape of the conduction band in both cases is almost the same

Band dispersion in two-band model

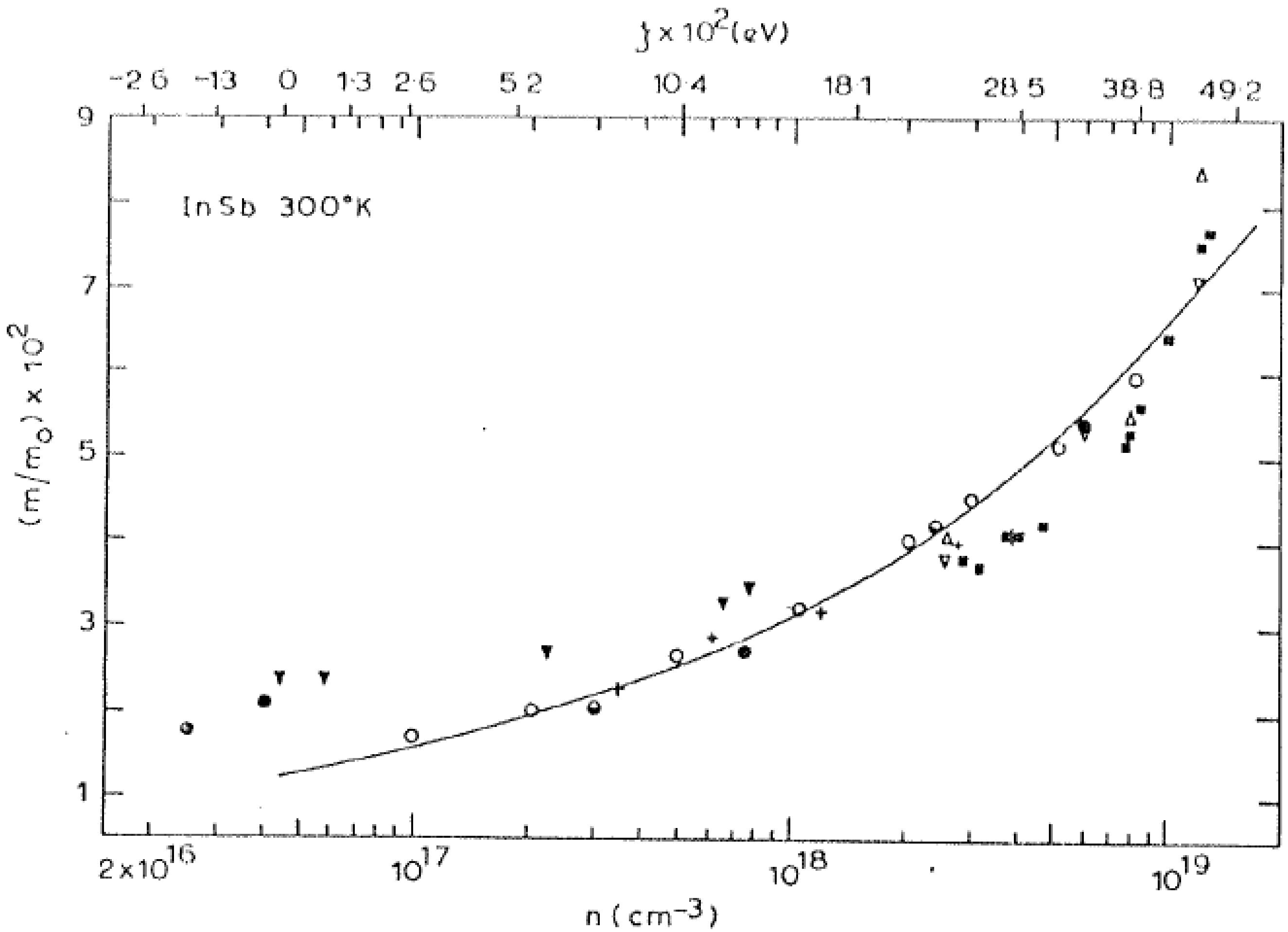
energy $$\frac{\hbar^2 k^2}{2m_0^*} = \varepsilon\left(1 + \frac{\varepsilon}{\varepsilon_g}\right)$$ $$\varepsilon = \pm\left[\left(\frac{\varepsilon_g}{2}\right)^2 + \varepsilon_g \frac{\hbar^2 k^2}{2m_0^*}\right]^{1/2}$$

$\varepsilon = 0$ at band edge $\varepsilon = 0$ at middle gap

mass $m^*(\varepsilon)$ $m^*\vec{\mathrm{v}} = \hbar\vec{\mathrm{k}}$

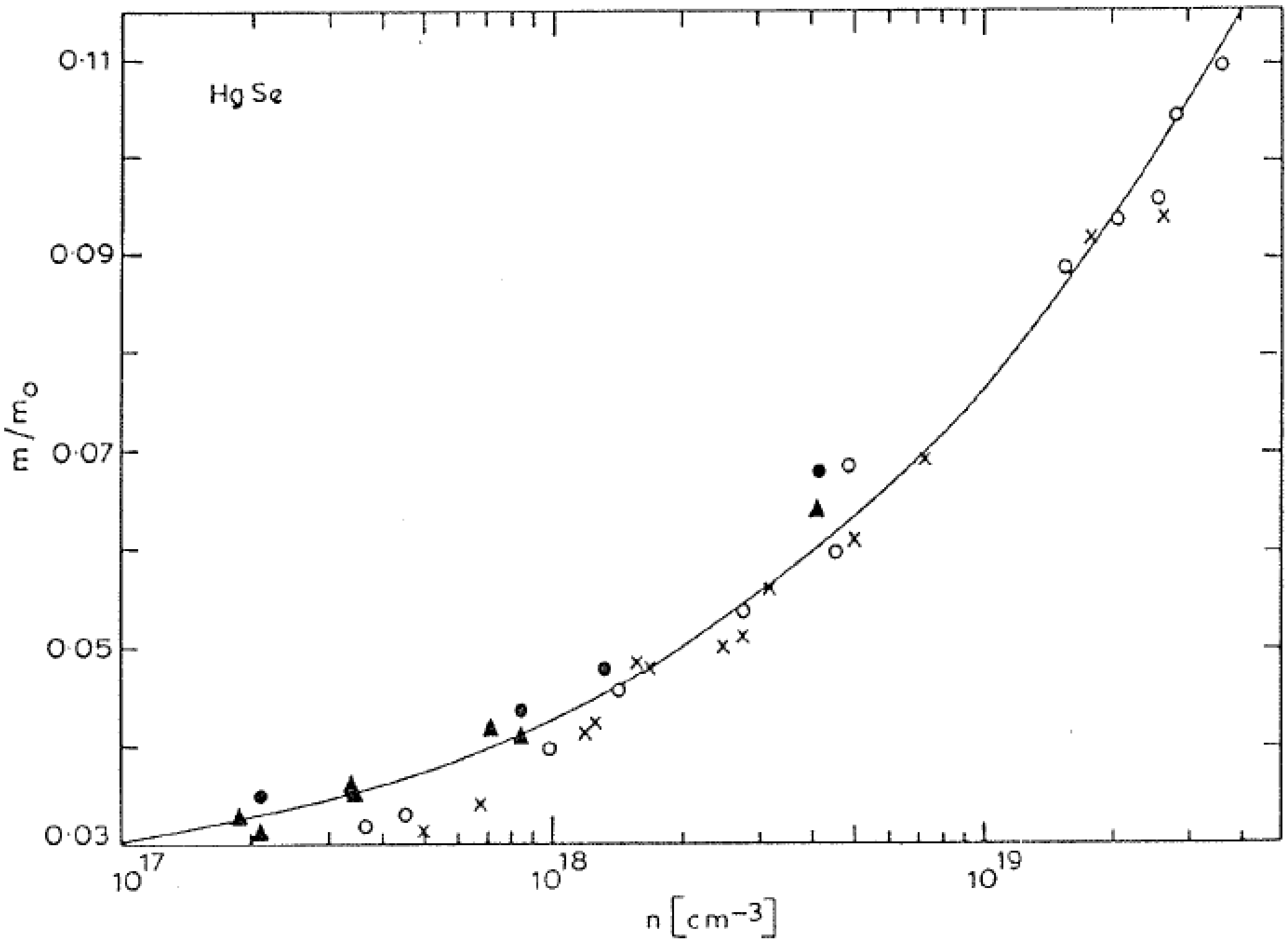
$$\frac{1}{m^*} = \frac{1}{\hbar^2 k}\frac{d\varepsilon}{dk} \qquad m^*(\varepsilon) = m_0^*\left(1 + 2\frac{\varepsilon}{\varepsilon_g}\right)$$

Electron effective mass in InSb



Electron effective mass in InSb at room temperature *versus* free electron concentration. The solid line, calculated for the Kane two-level model, represents the mass values at the Fermi energy, as indicated on the upper abscissa. At

Electron effective mass in HgSe



Electron effective mass in HgSe *versus* free electron concentration. The solid line is calculated for the Groves and Paul three-level model, including higher bands

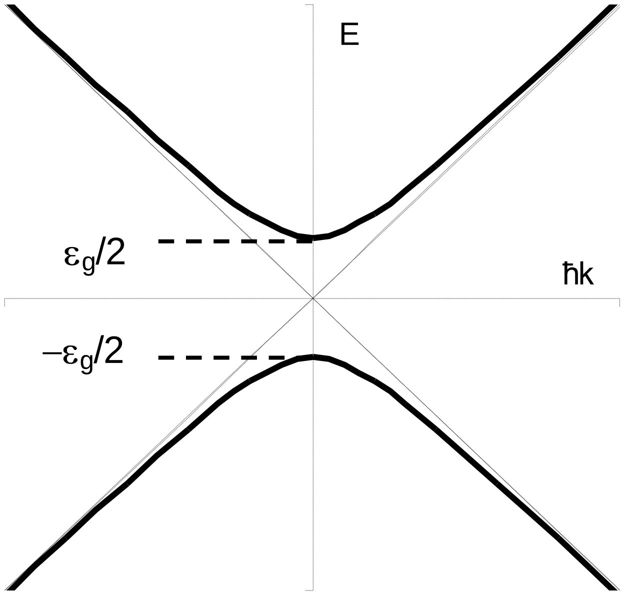
Relativistic analogy

$$\varepsilon = \pm\left[\left(\frac{\varepsilon_g}{2}\right)^2 + \varepsilon_g \frac{\hbar^2 k^2}{2m_0^*}\right]^{1/2}$$

$$\varepsilon = \pm\left[\left(\frac{2m_0c^2}{2}\right)^2 + 2m_0c^2 \frac{p^2}{2m_0}\right]^{1/2}$$

Analogy

$$\varepsilon_g \rightarrow 2m_0c^2 \qquad m_0^* \rightarrow m_0 \qquad \hbar\mathbf{k} \rightarrow \mathbf{p}$$



Relativistic analogy

maximum velocity

$$c = \left(\frac{2m_0 c^2}{2m_0} \right)^{1/2} \rightarrow \left(\frac{\varepsilon_g}{2m_0^*} \right)^{1/2} = u \approx 1 \times 10^8 \,\text{cm/s} \qquad \text{"universal"}$$

effective mass $\quad m^* \mathbf{v} = \hbar \mathbf{k}$

$$\varepsilon = \pm \left[\left(\frac{\varepsilon_g}{2} \right)^2 + \varepsilon_g \frac{\hbar^2 k^2}{2m_0^*} \right]^{1/2}$$

$$\frac{1}{m^*} = \frac{1}{\hbar^2 k} \frac{d\varepsilon}{dk} \qquad \frac{1}{m^*} = \frac{u^2}{\varepsilon} \qquad m^* = \frac{\varepsilon}{u^2}$$

$$\varepsilon = m^* u^2$$

semi-Einstein

Relativistic analogy

VOLUME 16, NUMBER 22 PHYSICAL REVIEW LETTERS 30 MAY 1966

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TWO-BAND MODEL FOR BLOCH ELECTRONS IN CROSSED ELECTRIC AND MAGNETIC FIELDS

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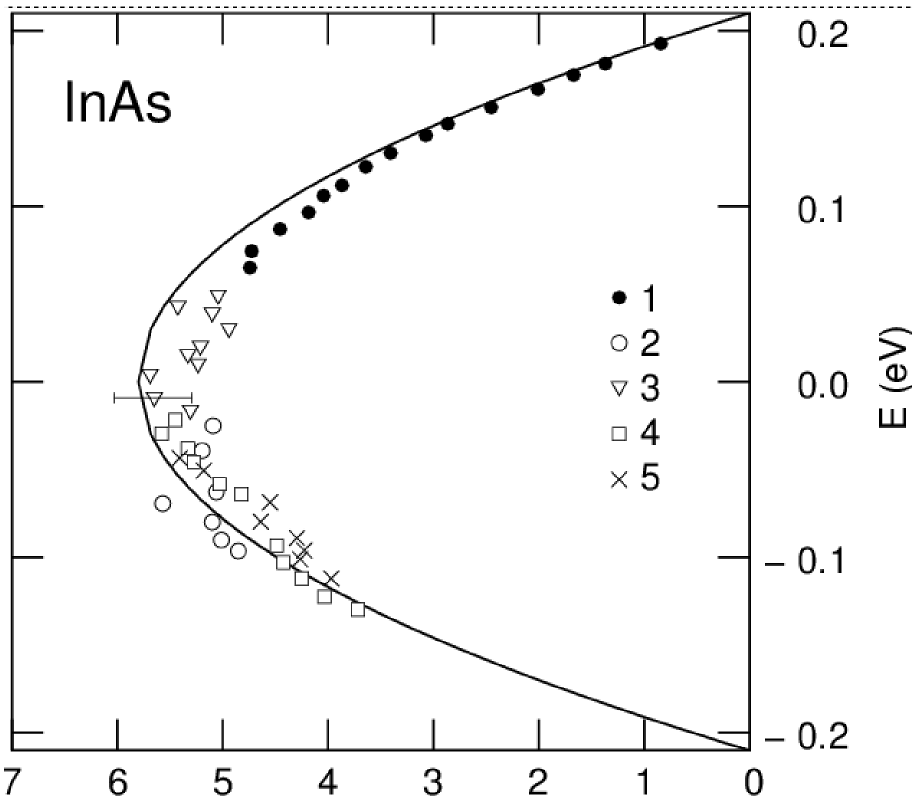
(Received 21 February 1966; revised manuscript received 10 May 1966)

The turning point between the magnetic and electric type of solutions can also be obtained from classical considerations. The dispersion relation between momentum and energy for two simple interacting bands is given by the simplified Kane formula as $\epsilon = -\epsilon_g/2 + [(\epsilon_g/2)^2 + \epsilon_g p^2/2m^*]^{1/2}$. It has the form of the relativistic relation with $2m_0c^2$ replaced by ϵ_g and m_0 by m^*. It is well known that the turning point between the magnetic and electric type of motion for a classical relativistic electron in crossed fields is given by $cE/H = c$.[13] Since c may be written as $(2m_0c^2/2m_0)^{1/2}$, we see that in our case the limiting velocity is $v = (\epsilon_g/2m^*)^{1/2}$. This gives the turning point at $(eH/m^*c)^2 = 2e^2E^2/m^*\epsilon_g$ in agreement with the quantum result.

Relativistic analogy

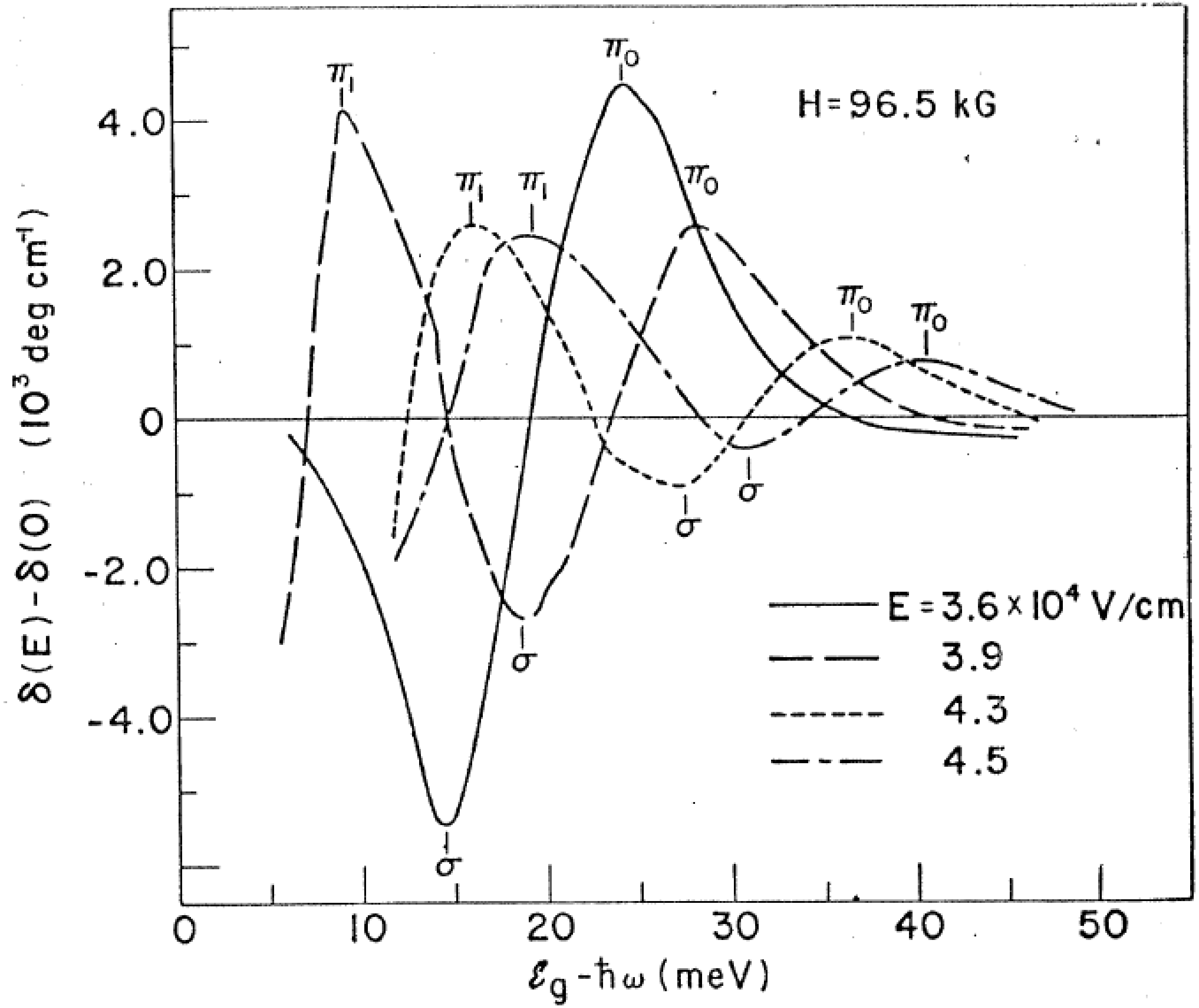
„Compton" wavelength $$\lambda_c = \frac{\hbar}{m_0 c} \rightarrow \frac{\hbar}{m_0^* u} = \lambda_Z \approx 20 - 50\,\mathrm{A}$$



$$\varepsilon = \pm \hbar u \left(\lambda_Z^{-2} + k^2 \right)^{1/2}$$

FIG. 1: Energy-wave vector dependence in the forbidden gap of InAs. Various symbols show experimental data of Parker and Mead [19], the solid line is theoretical fit using Eq. (40). The determined parameters are λ_Z = 41.5 Å and u = 1.33×10^8 cm/s. After Ref. [2].

Crossed electric and magnetic fields   E⊥H



Crossed electric and magnetic fields E⊥H

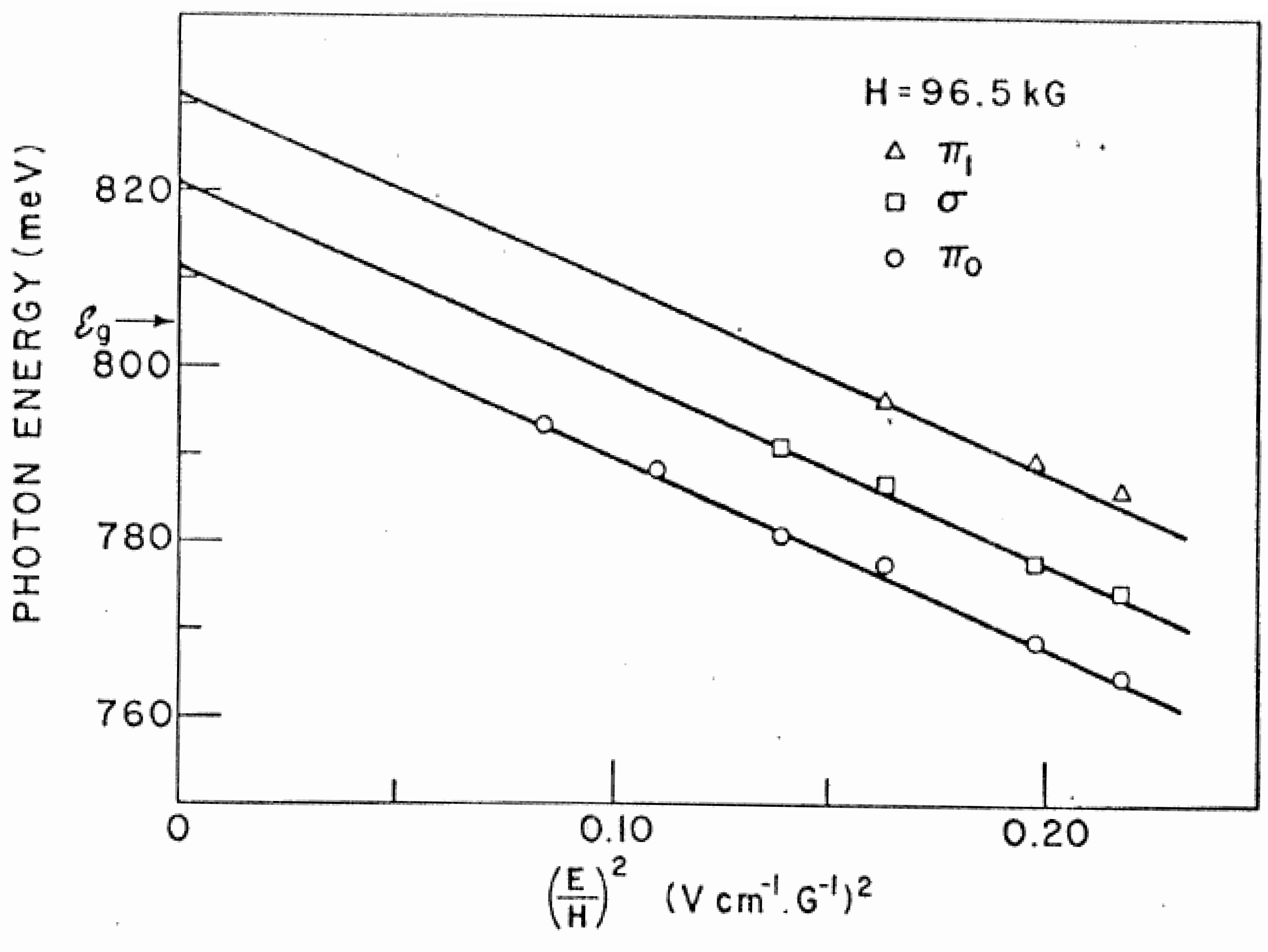


Fig. 8. Photon energies of Voigt-effect peaks in crossed fields for H = 96,5 kGs as a function of $(E / H)^2$

Crossed electric and magnetic fields E⊥H

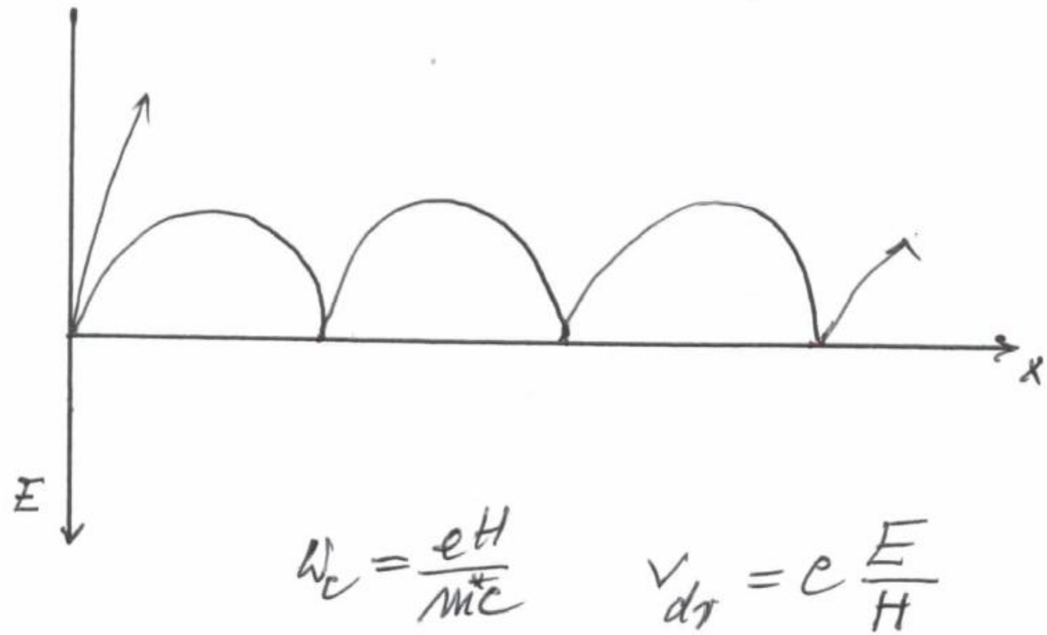
One-band **k.p**

$$\mathrm{H} = \frac{1}{2m_0^*}\left(\mathbf{p} + \frac{e}{c}\mathbf{A}\right)^2 + eEy \qquad H \parallel \mathbf{z} \qquad \mathbf{A} = \left(-Hy, 0, 0\right)$$

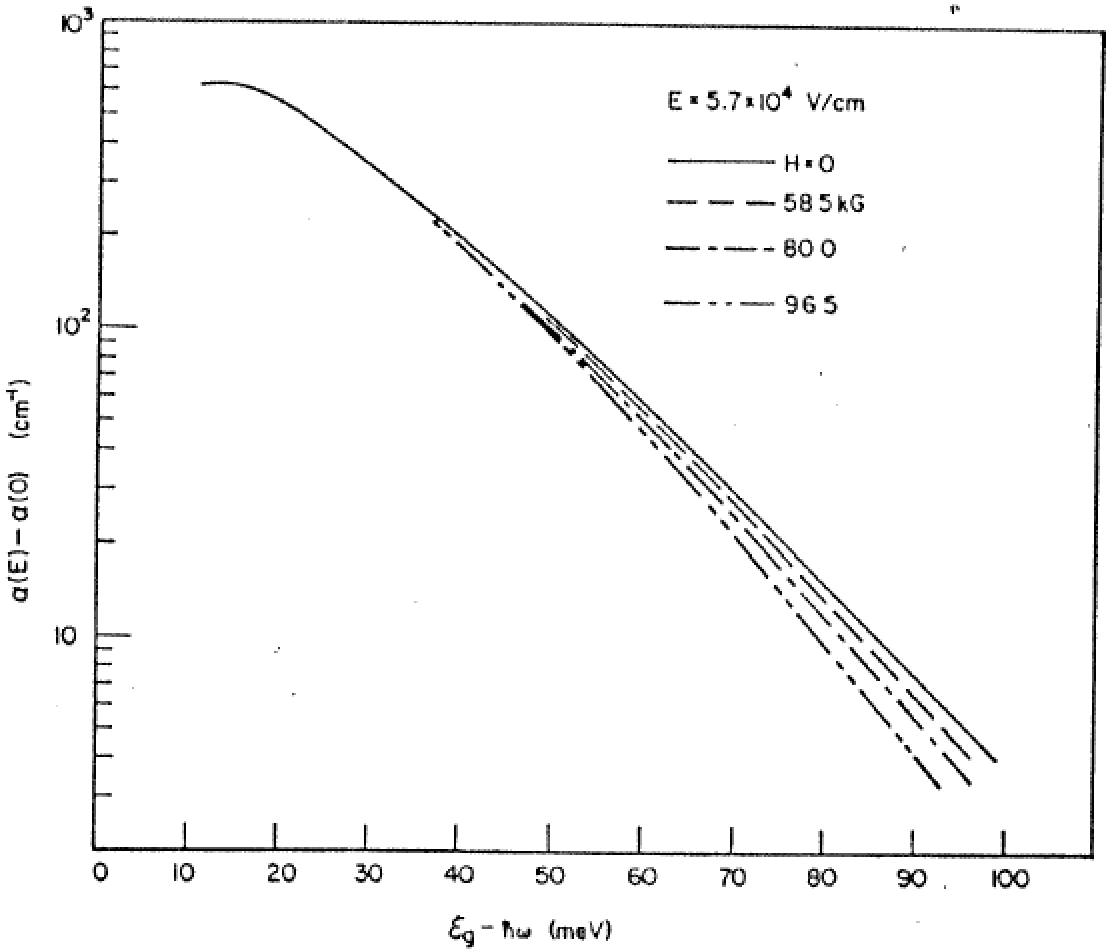
$$\varepsilon = \hbar\omega_c\left(n + \frac{1}{2}\right) + eEy_0 - \frac{1}{2}m_0^* c^2\,\frac{E^2}{H^2} + \frac{\hbar^2 k_z^2}{2m_0^*}$$

Classically

$$m_0^*\,\frac{d^2\mathbf{r}}{dt^2} = e\mathbf{E} + \frac{e}{c}\left(\mathbf{v}\times\mathbf{H}\right)$$



Crossed electric and magnetic fields E⊥H



Exponential absorption coefficient below the direct gap of germanium due to electric field $E = 5,7 \times 10^4$ V/cm for various values of transverse magnetic field.

Two-Band description

Two-band **k.p**

$$\begin{bmatrix} -\varepsilon + \frac{\varepsilon_g}{2} + eEy & a\left(\mathbf{p} + \frac{e}{c}\mathbf{A}\right) \\ a\left(\mathbf{p} + \frac{e}{c}\mathbf{A}\right) & -\varepsilon - \frac{\varepsilon_g}{2} + eEy \end{bmatrix} \begin{pmatrix} f_1 \\ f_2 \end{pmatrix} = 0 \qquad \mathbf{A} = \{-Hy, 0, 0\}$$

decisive term $\left(\frac{e^2}{c^2} H^2 - e^2 E^2\right) y^2$

E < H : magnetic motion

E > H : electric motion

Zitterbewegung

E. Schrodinger 1930

Dirac Equation $\mathrm{H} = c\vec{\alpha}\cdot\vec{p} + m_0 c^2 \beta$ α, β 4x4 matrices

velocity $\mathrm{v_i} = \dfrac{\partial H}{\partial p_i} = c\alpha_i$ α and β anticommute

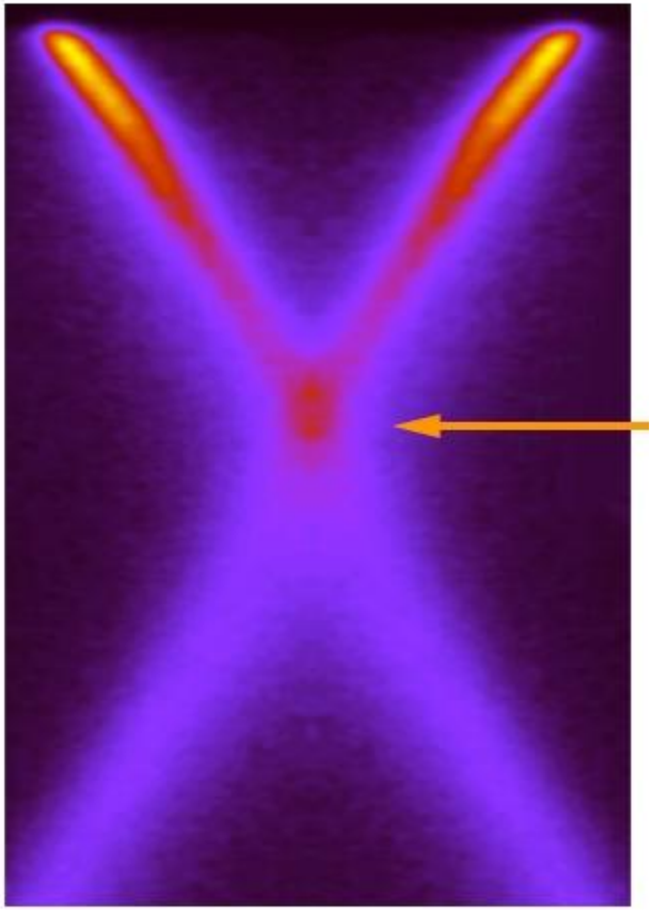
solutions $\vec{r}(t) = \vec{a} + c^2 H^{-1}\vec{p}t + \dfrac{1}{2} i c\hbar\eta_0 H^{-1} e^{-2iHt/\hbar}$ Zitterbewegung !

frequency $\hbar\omega \approx 2E \approx 2m_0 c^2$

amplitude $\dfrac{\hbar c}{E} \approx \dfrac{\hbar c}{m_0 c^2} = \dfrac{\hbar}{m_0 c} = \lambda_C \approx 4\times 10^{-3}\,\mathrm{\mathring{A}}$

two-band **k.p** theory 2005

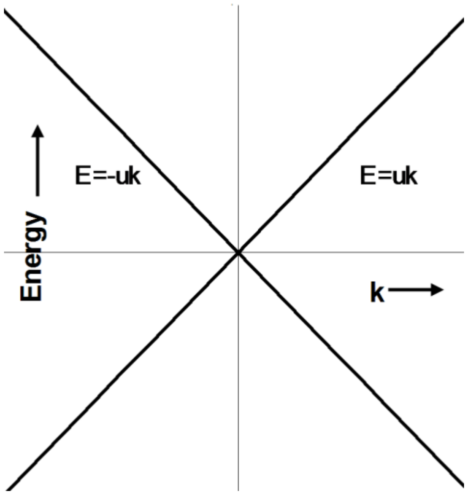
frequency $\hbar\omega \approx \varepsilon_g$ amplitude $\lambda_Z \approx 20\,\mathrm{\mathring{A}}$



Band dispersion E(k) in graphene from ARPES

Graphene

Hamiltonian $$H_G = u\begin{pmatrix} 0 & p_x - ip_y \\ p_x + ip_y & 0 \end{pmatrix}$$



where u≈1x10^{8}cm/s $\varepsilon = \pm u\hbar k$ no gap!

$$\frac{1}{m^*} = \frac{1}{\hbar^2 k}\frac{d\varepsilon}{dk} \qquad m^* = \frac{\varepsilon}{u^2} \qquad \varepsilon = m^* u^2$$

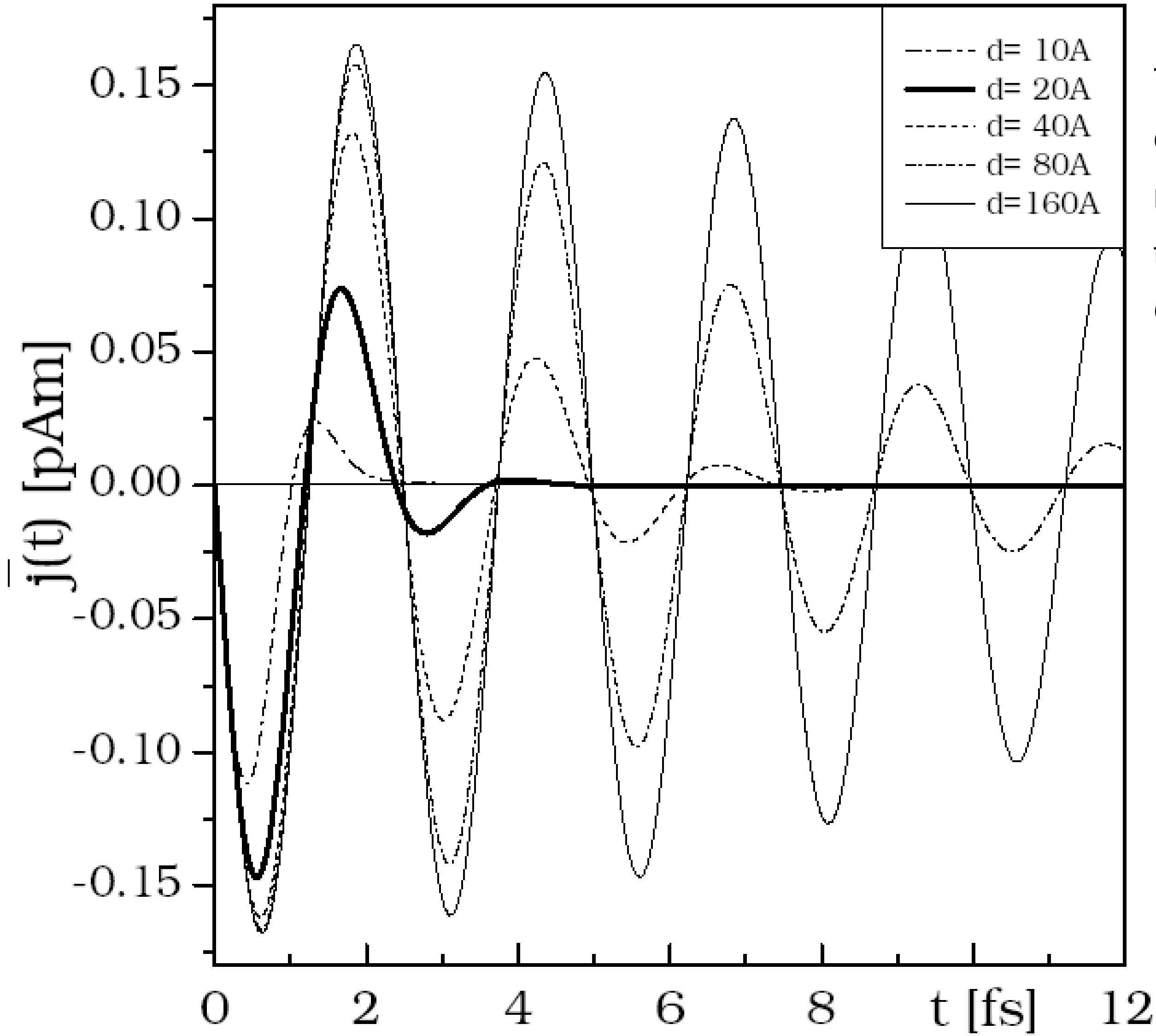
ZB with T. M. Rusin

velocity $$\mathrm{v}_i = \frac{\partial H_G}{\partial p_i} \qquad \mathbf{v}(t) = e^{iHt/\hbar}\mathbf{v}e^{-iHt/\hbar}$$

$$\mathrm{v}_x^{(11)}(t) = u\frac{k_y}{k}\sin(2ukt)$$

Frequency $\hbar\omega_Z = 2u\hbar k$

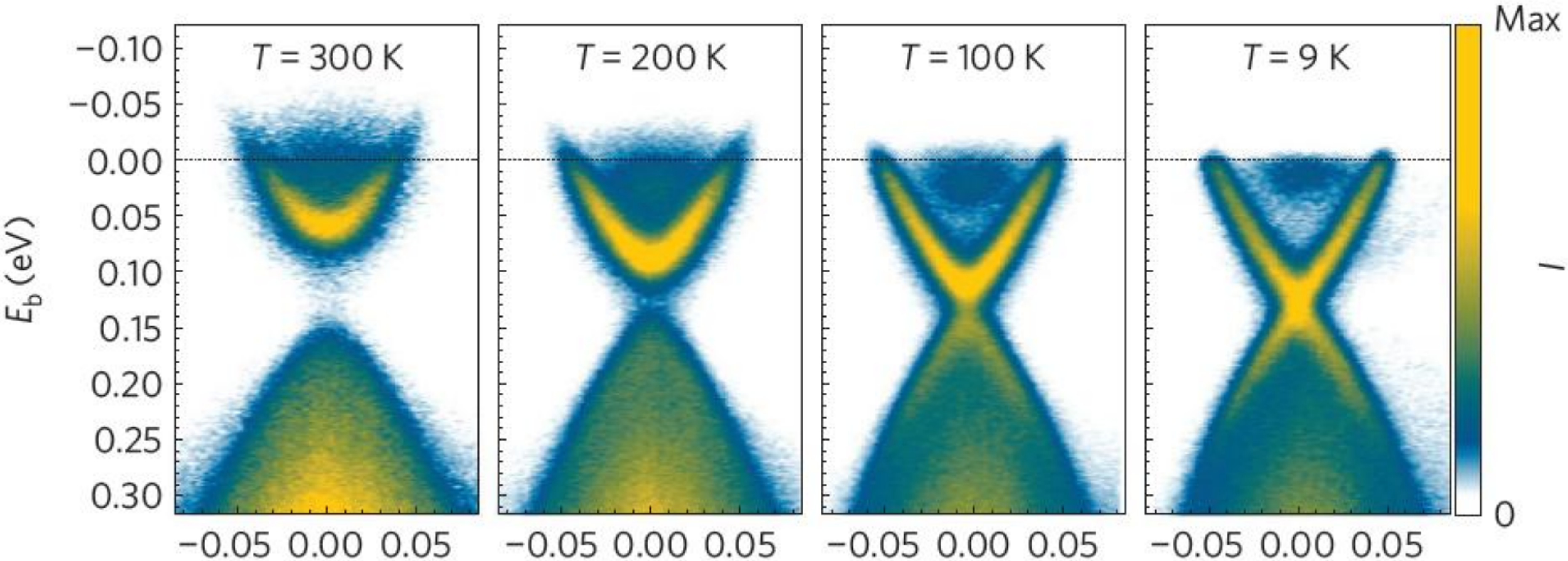
Graphene



Transient oscillatory electric current caused by the ZB in monolayer graphene *versus* time, calculated for a Gaussian wave packet

ω_Z nearly constant, amplitude depends on the packet's width *d*

Topological insulators



Band dispersions E(k) in PbSnSe versus T (ARPES Dziawa 2012)