Is the QGP created in heavy ion collisions in local thermal equilibrium?

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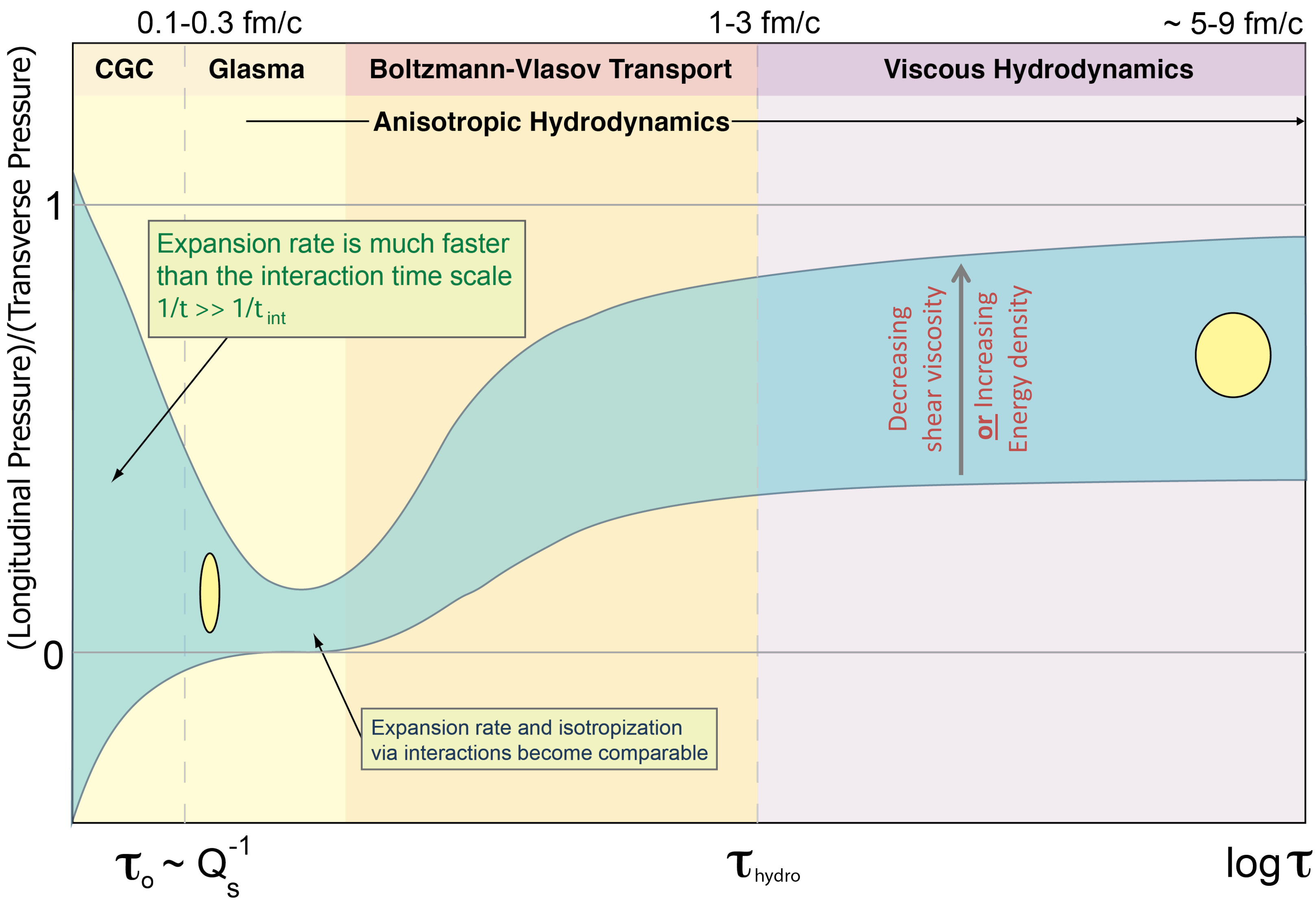
No.

Discussion

- Of course, no system is ever in perfect equilibrium, so this is a somewhat silly question.
- We should instead ask how close is the system to equilibrium using some measure.
- For example, we could compare the diagonal components of the energy-momentum tensor in the local rest frame of the matter.
- If they are the same, then the system is isotropic in momentum space. This occurs if the system is in equilibrium and hence is a required for thermalization (necessary but not sufficient).

QGP momentum anisotropy cartoon

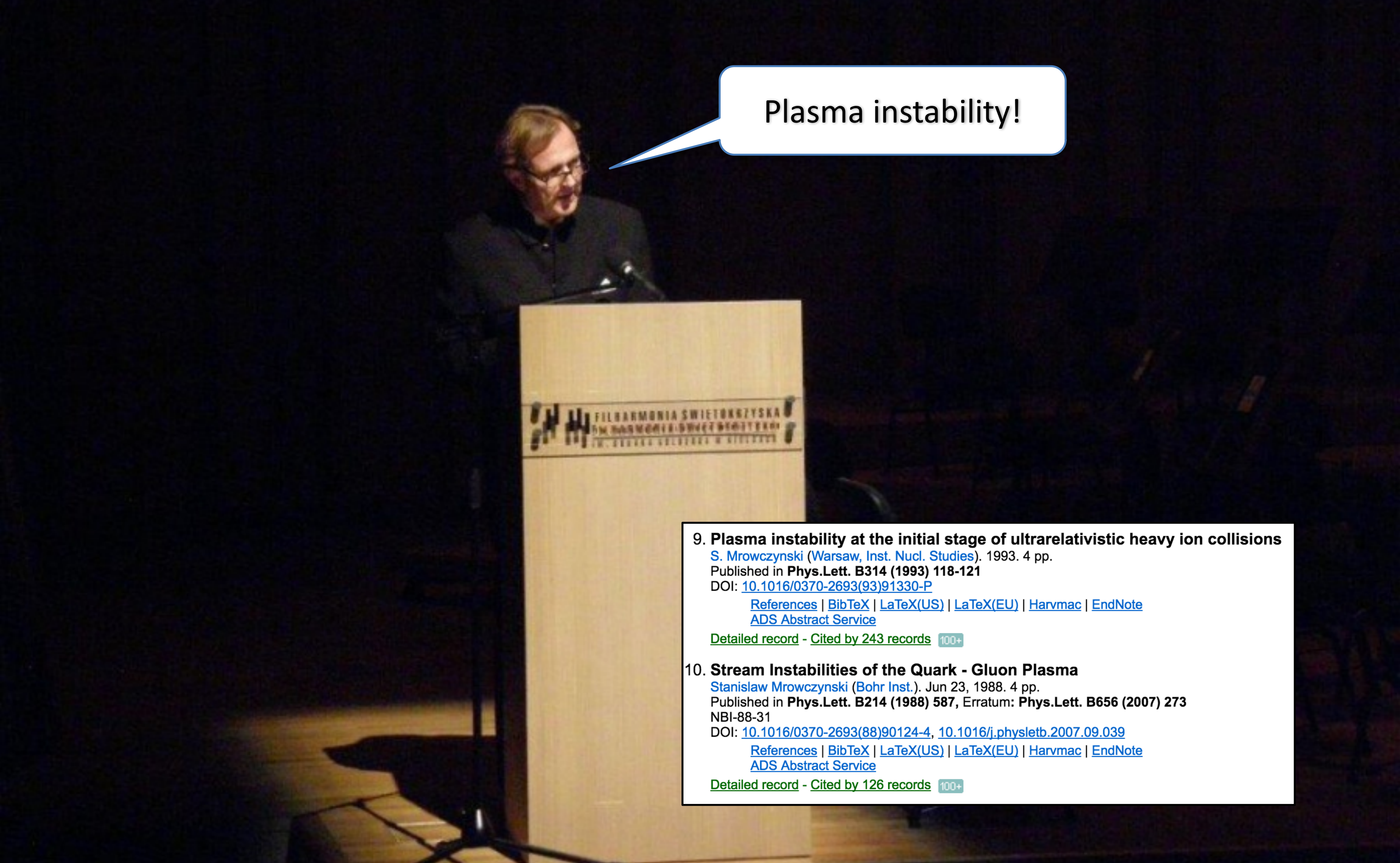


What are the relevant EQ mechanisms?

- Based on both weak and strong coupling calculations, the initial particle distribution functions are expected to anisotropic in momentum-space. How do they relax to EQ?
- In the high temperature (weak coupling) limit, one can approach this question systematically using perturbative QCD.
- In 2000 Baier, Mueller, Schiff, and Son calculated the thermalization time scale ("bottom up thermalization").
- They included the effects of gluon saturation in the initial state, elastic collisions, hard and soft particle radiation, etc.
- Estimated time for system to be perfectly isotropic with the particles having a thermal momentum distribution.

$$\tau_{\rm therm} \sim \frac{3}{2}\,\alpha_s^{-13/5} Q_s^{-1} \longrightarrow \begin{array}{c} \alpha_s \sim 0.3 \\ Q_s \sim 2\ \text{GeV} \end{array} \longrightarrow \tau_{\rm therm} \sim 3.4\ \text{fm/c}$$

But they missed something…



9. **Plasma instability at the initial stage of ultrarelativistic heavy ion collisions**
S. Mrowczynski (Warsaw, Inst. Nucl. Studies). 1993. 4 pp.
Published in **Phys.Lett. B314 (1993) 118-121**
DOI: 10.1016/0370-2693(93)91330-P
References | BibTeX | LaTeX(US) | LaTeX(EU) | Harvmac | EndNote
ADS Abstract Service
Detailed record - Cited by 243 records 100+

10. **Stream Instabilities of the Quark - Gluon Plasma**
Stanislaw Mrowczynski (Bohr Inst.). Jun 23, 1988. 4 pp.
Published in **Phys.Lett. B214 (1988) 587,** Erratum**: Phys.Lett. B656 (2007) 273**
NBI-88-31
DOI: 10.1016/0370-2693(88)90124-4, 10.1016/j.physletb.2007.09.039
References | BibTeX | LaTeX(US) | LaTeX(EU) | Harvmac | EndNote
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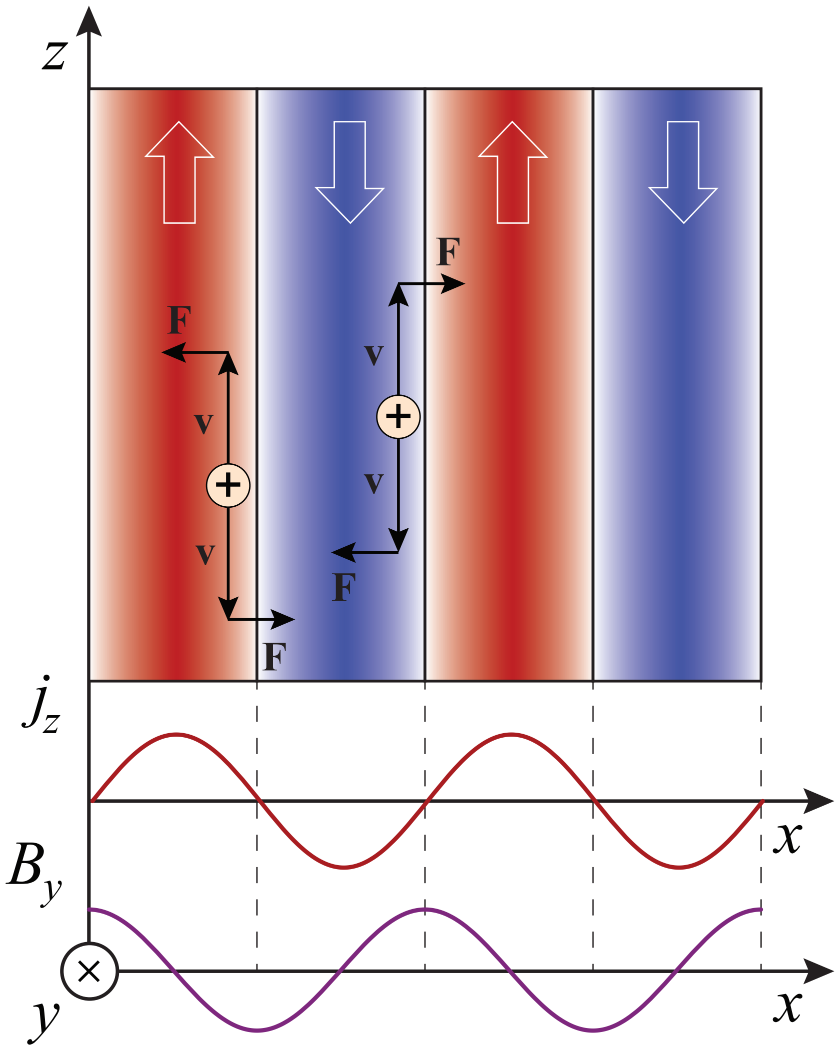
Detailed record - Cited by 126 records 100+

The Weibel Instability



- Consider an anisotropic ensemble of electrically charged particles.
- And then imagine that there is a super small **fluctuation in the background magnetic field.**
- In an anisotropic plasma, this causes **filamentation** of the currents.
- The filamented currents generate an induced magnetic field that adds to background field making it stronger.
- This then leads to stronger filamentation → **exponential growth of the magnetic field**.

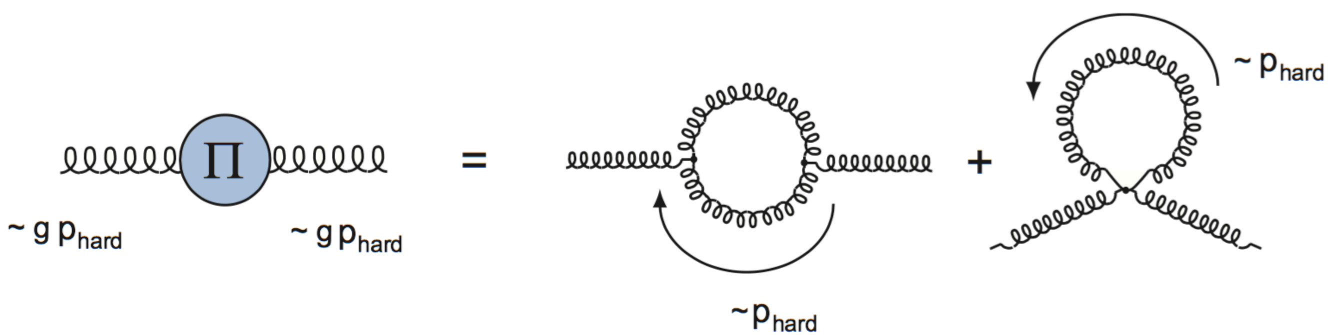
The Chromo-Weibel Instability

The high-energy medium gluon polarization tensor can be obtained by linearizing collisionless transport theory : $f(p,x) \to f(\mathbf{p}) + \delta f(p,x)$

$$[v \cdot D_x, \delta f(p,x)] + g v_\mu F^{\mu\nu} \partial_\nu^{(p)} f(\mathbf{p}) = 0$$

$$D_\mu F^{\mu\nu} = J^\nu = g \int_p v^\nu \delta f(p,x)$$

or diagrammatically using "hard-loop" perturbation theory

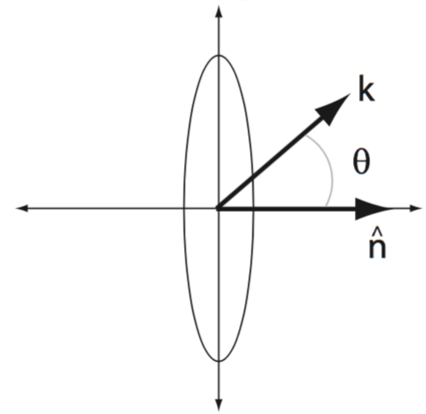


$$\Pi_{ab}^{ij}(\omega,\mathbf{k}) = -g^2 \delta_{ab} \int_{\mathbf{p}} v^i \frac{\partial f(\mathbf{p})}{\partial p^l} \left(\delta^{jl} - \frac{v^j k^l}{\omega - \mathbf{v}\cdot\mathbf{k} + i\epsilon} \right)$$

[Mrowczynski and Thoma, hep-ph/0001164]

The Chromo-Weibel Instability

For simplicity assume that the anisotropic distribution function can be obtained from an arbitrary isotropic distribution function by a change of its argument. [Romatschke and MS, hep-ph/0304092]



$$f(p^2) \rightarrow f(p^2 + \xi(\mathbf{p}\cdot\mathbf{n})^2)$$

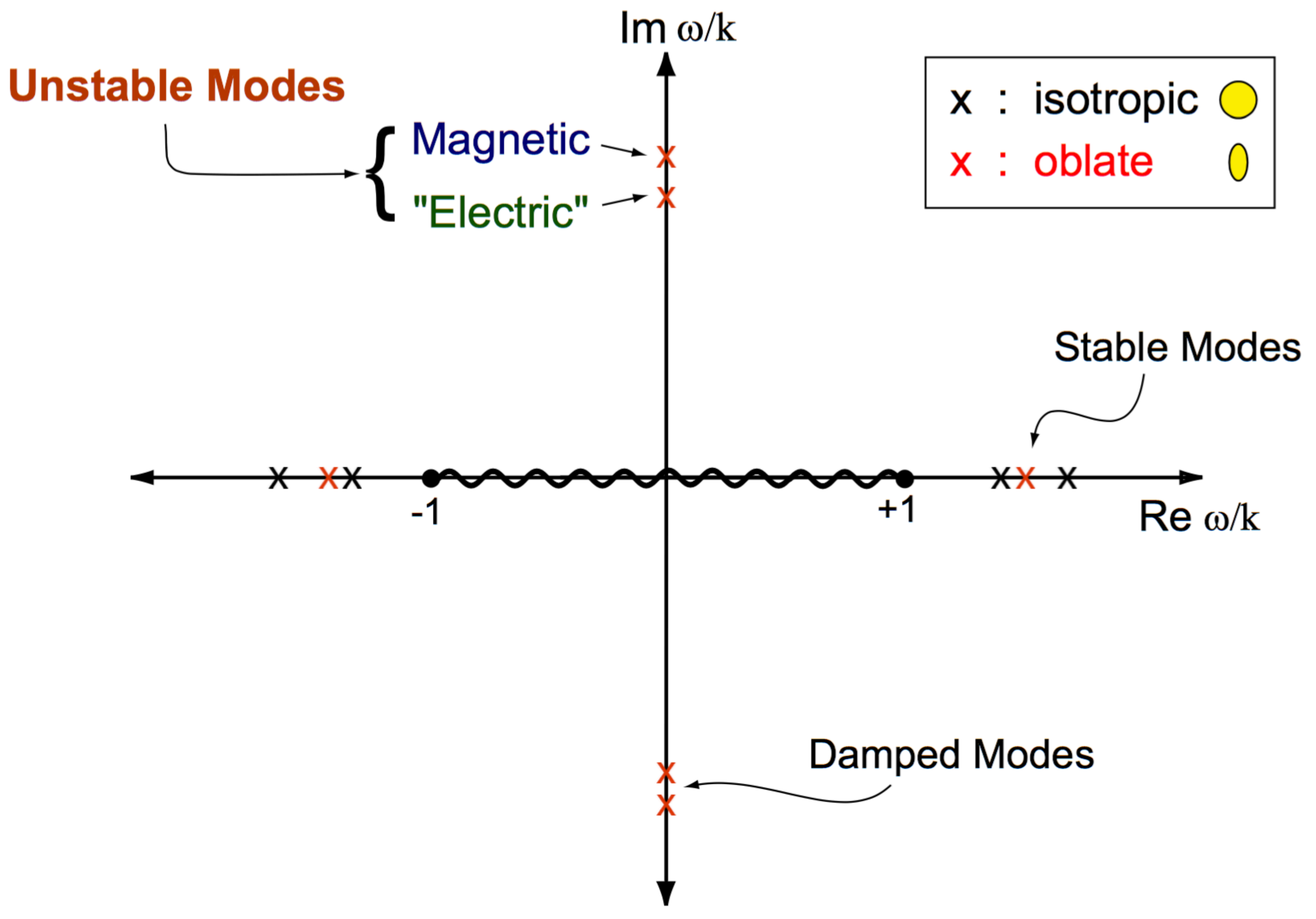
The polarization tensor can then be written as

$$\Pi^{ij}_{ab}(\omega,k) = m_D^2\,\delta_{ab}\int \frac{d\Omega}{4\pi} v^i \frac{v^l + \xi(\mathbf{v}\cdot\mathbf{n})n^l}{(1+\xi(\mathbf{v}\cdot\mathbf{n})^2)^2}\left(\delta^{jl} - \frac{v^j k^l}{\omega - \mathbf{v}\cdot\mathbf{k} + i\epsilon}\right)$$

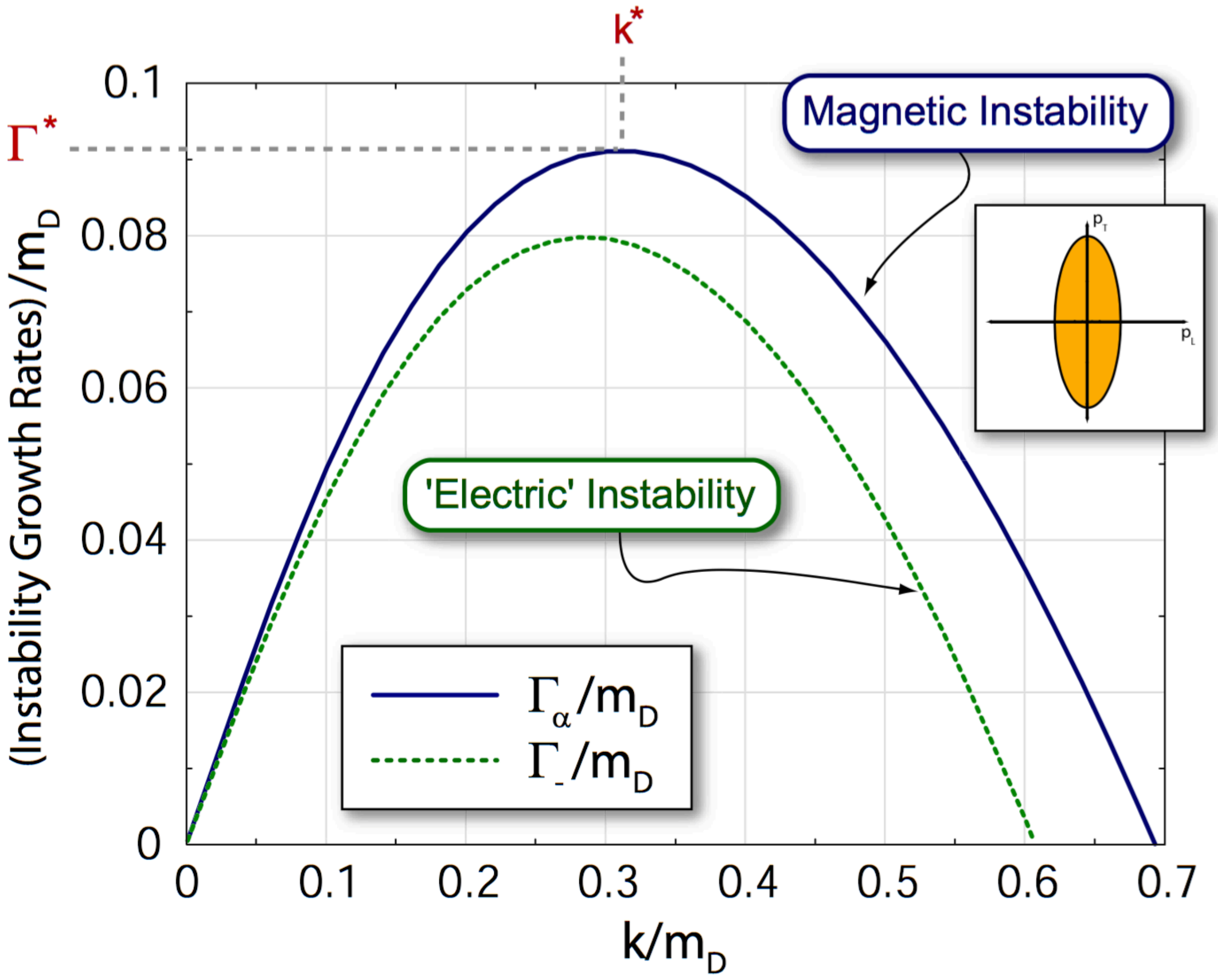
where m_D is the *isotropic* Debye mass

$$m_D^2 = -\frac{g^2}{2\pi^2}\int_0^\infty dp\, p^2 \frac{df(p^2)}{dp} \sim g^2 p_{\rm hard}^2$$

Anisotropic Gluonic Collective Modes (ξ > 0)



Anisotropic Gluonic Collective Modes (ξ > 0)



Instability growth rates as a function of momentum for $\langle p_T^2 \rangle / 2 \langle p_L^2 \rangle \simeq 10$ and $\theta_{\rm glue} = \pi/8$ with respect to the beamline.

Using $\alpha_s = 0.3$ and $Q_s \sim 1.5 - 2\ \mathrm{GeV}$

$$m_D = g\,Q_s \rightarrow 3 - 4\ \mathrm{GeV}$$

$$m_D \sim \sqrt{\frac{g^2 N_c n_g}{Q_s}} \sim \sqrt{\frac{12\left(\frac{100}{\mathrm{fm}^3}\right)}{1.5 - 2\ \mathrm{GeV}}} \rightarrow 2 - 3\ \mathrm{GeV}$$

$$\Gamma \sim 0.5 - 2.4\ \mathrm{GeV}$$

e-Folding time
0.1 - 0.4 fm/c

Hard loop effective action

- Having seen that there is an instability, the natural follow-up question is for how long will it grow and how will it regulate itself.
- To answer this question we need the full effective action for an anisotropic QGP.

$$S_{\rm aniso} = -\frac{g^2}{2}\int_x \int_{\mathbf{p}} \left\{ \frac{f(\mathbf{p})}{|\mathbf{p}|}\, F^a_{\mu\nu}(x) \left(\frac{p^\nu p^\rho}{(p\cdot D)^2} \right)_{ab} F_\rho{}^{b\,\mu}(x) + i\frac{C_F}{2}\frac{\tilde{f}(\mathbf{p})}{|\mathbf{p}|}\, \bar{\Psi}(x)\frac{p\cdot\gamma}{p\cdot D}\Psi(x) \right\} .$$

[Mrowczynski, Rebhan, and MS, hep-ph/0403256]

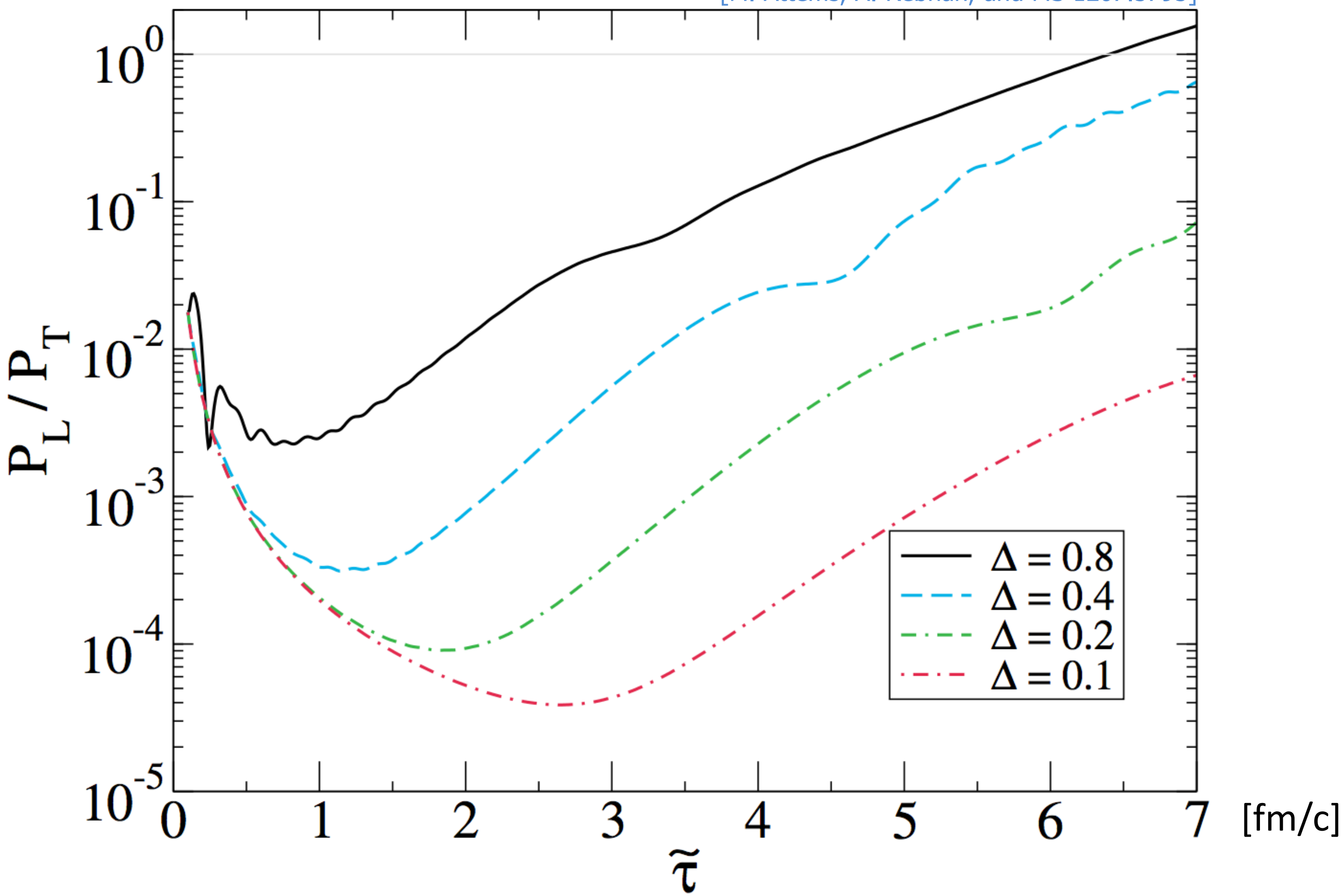
- With this, one can simulate instability evolution in a non-Abelian plasma using real-time lattice gauge theory methods.

Results from 3+1d simulations

- I will skip the discussion of static box simulations and jump straight to the result obtained from simulations which included the dynamical longitudinal expansion of the system → HEL simulations (hard expanding loops).
- In this case, the hard particle momentum-space anisotropy evolves in time.
- For the results I will show here we assumed that the hard particles were free streaming → $\xi(\tau) = \tau^2/\tau_0^2 - 1.$

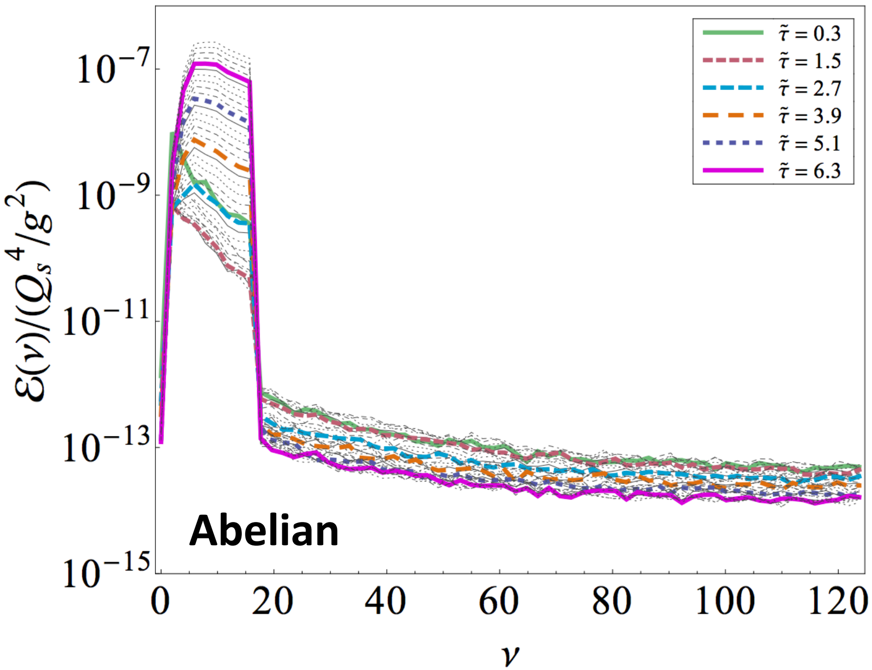
Pressure anisotropy

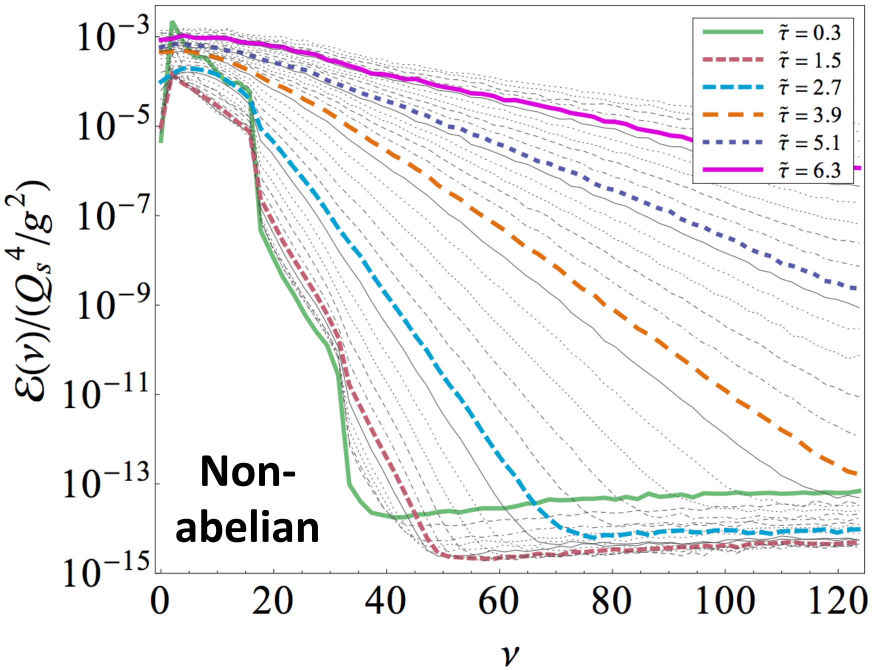
[M. Attems, A. Rebhan, and MS 1207.5795]



Gluon spectra

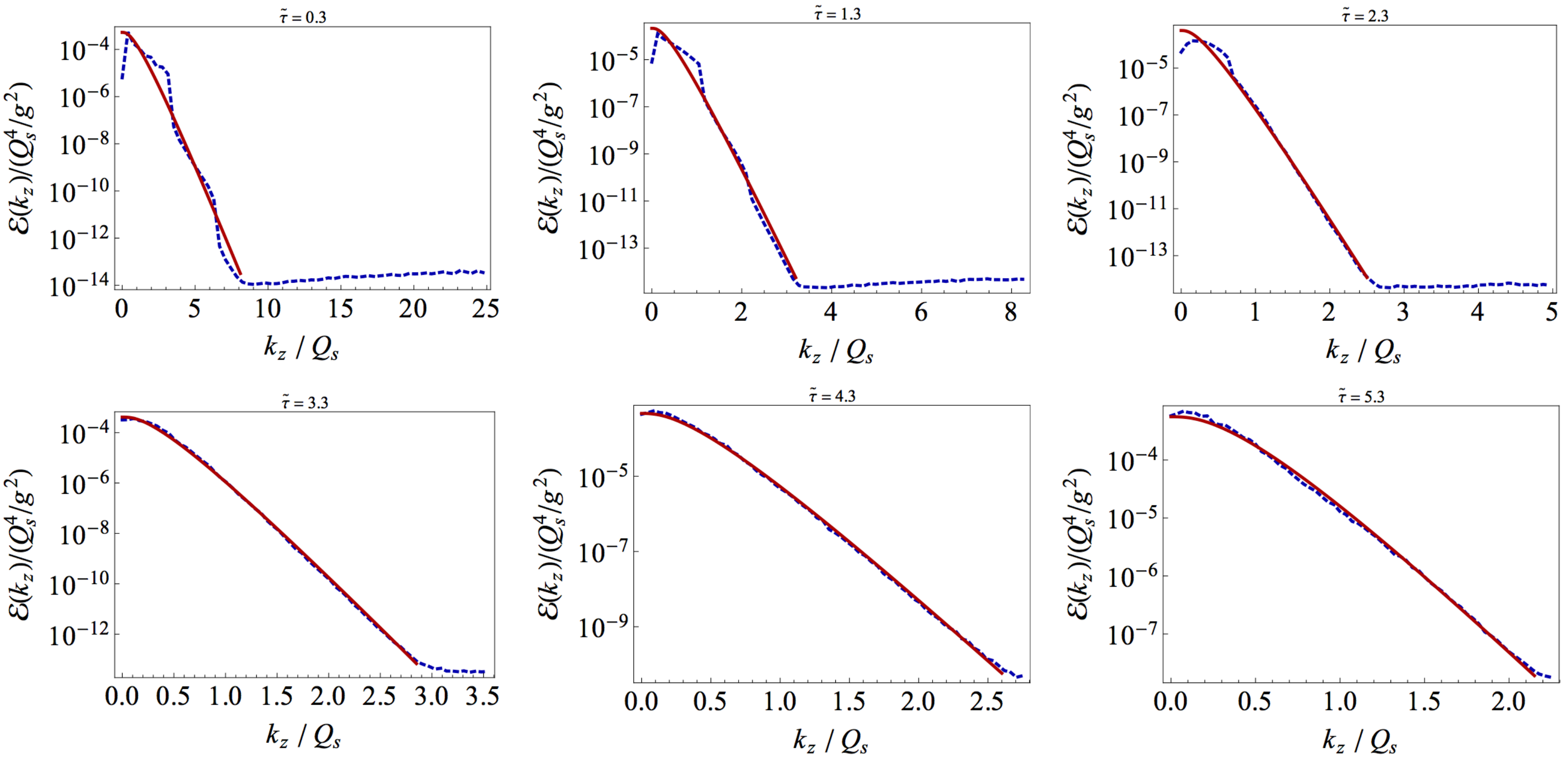
[Attems, Rebhan, and MS 1207.5795]





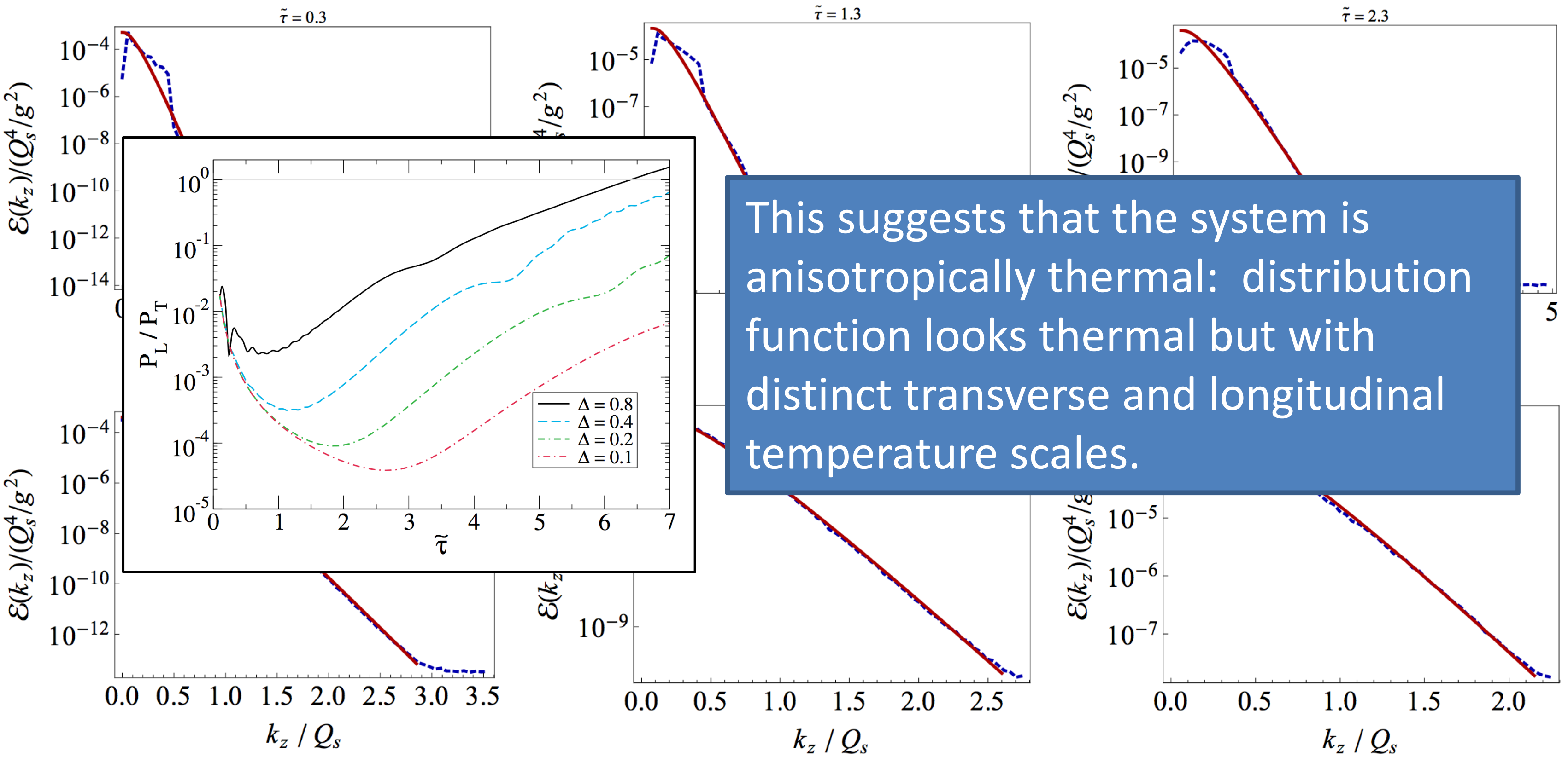
In a non-abelian gauge theory, unstable modes trigger a rapid UV cascade due to the non-linearities, e.g. three gluon vertex.

Gluon spectra



We see the very rapid development of a thermal distribution even though the initial condition was far from thermal (step function in momentum in this case). [Attems, Rebhan, and MS 1207.5795]

Gluon spectra



This suggests that the system is anisotropically thermal: distribution function looks thermal but with distinct transverse and longitudinal temperature scales.

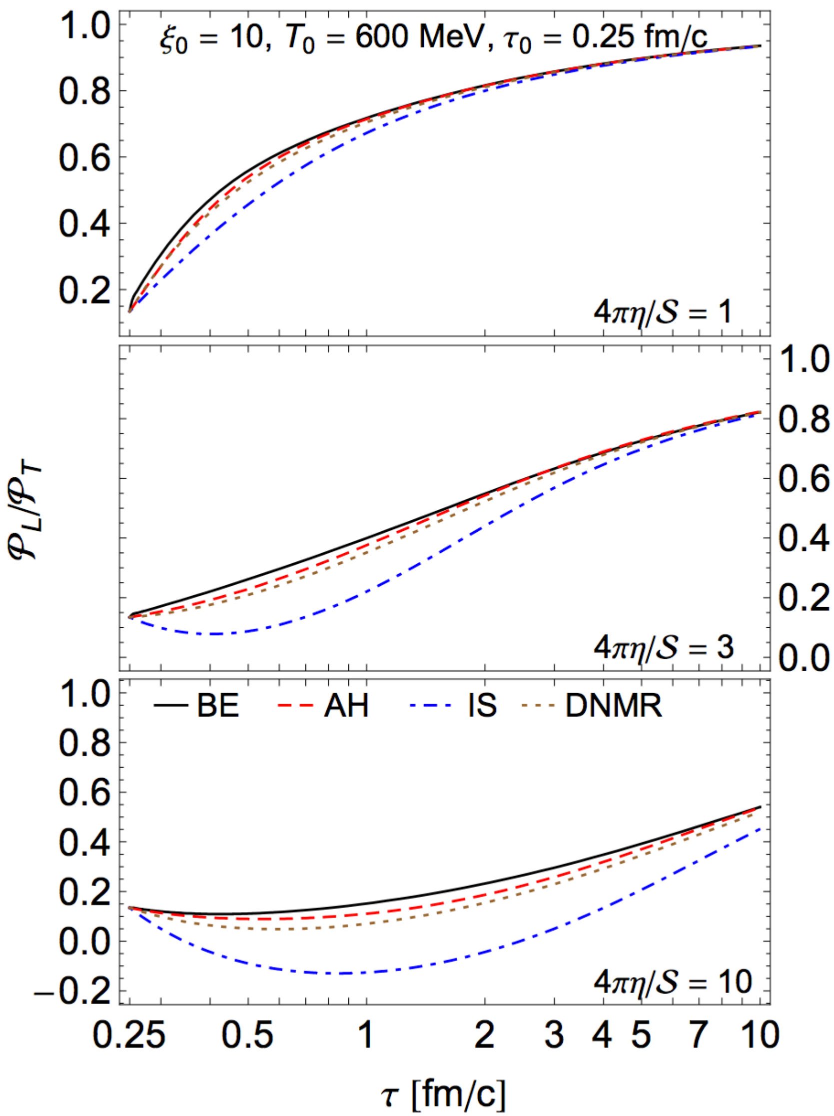
We see the very rapid development of a thermal distribution even though the initial condition was far from thermal (step function in momentum in this case). [Attems, Rebhan, and MS 1207.5795]

The unreasonable effectiveness of viscous hydrodynamics

- At this point you might be asking yourself: If the system is not in a near-equilibrium state, **why does viscous hydrodynamics work so well to describe the data?**
- As it turns out, viscous hydrodynamics can work reasonably well to describe the dynamics of the QGP even when the system is quite anisotropic in momentum space (next slide).
- In the language of viscous hydrodynamics the momentum-space anisotropy of the QGP is encode in the shear-stress correction to the ideal $T^{\mu\nu}$.

Comparison to exact solutions

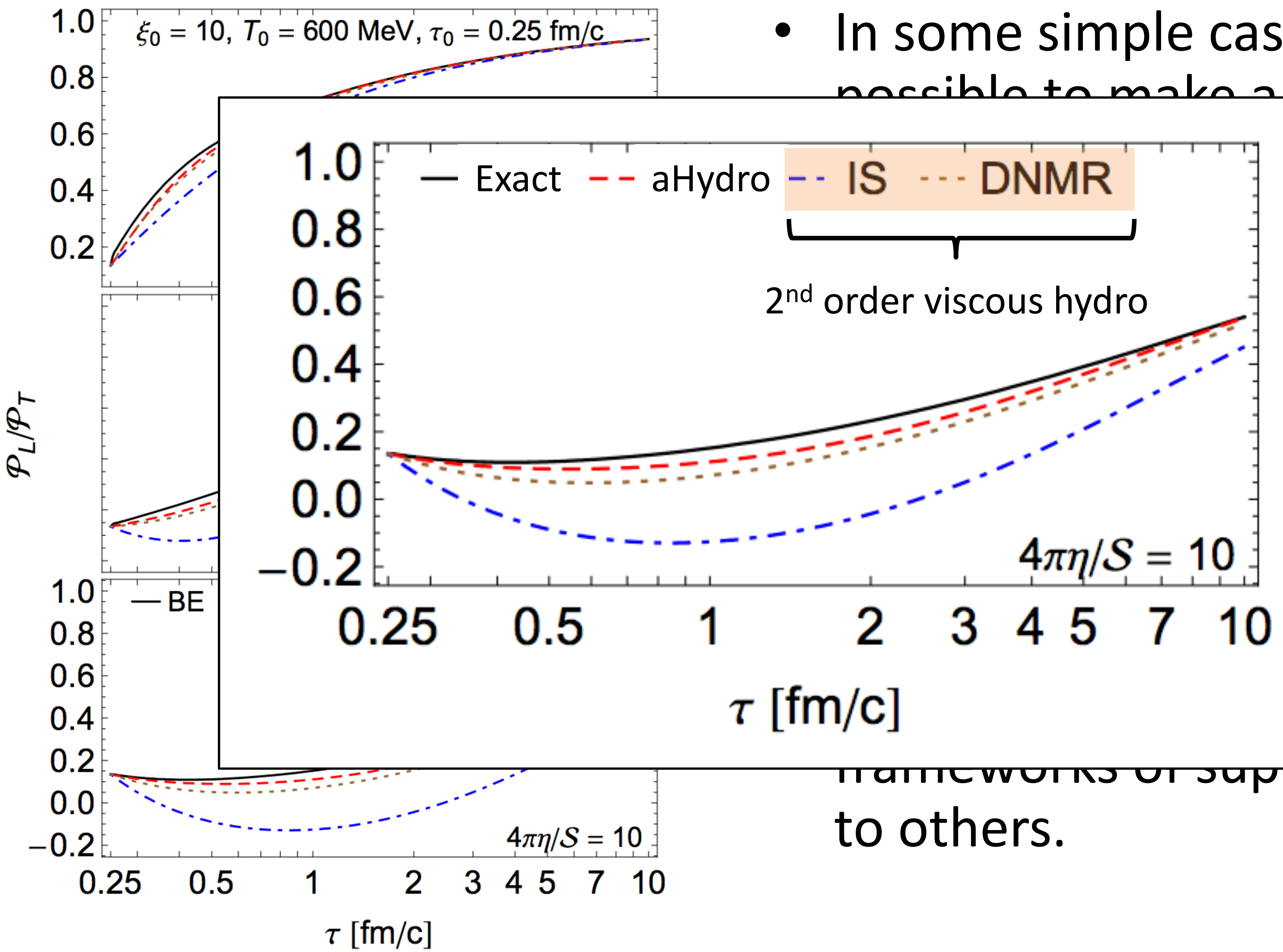
[Florkowski, Ryblewski, and MS 1304.0665]



- In some simple cases, it is possible to make an exact solution of the Boltzmann equation
- One finds that viscous hydrodynamics works reasonably well even in situations you might expect it not to
- Some viscous hydrodynamical frameworks of superior to others.

Comparison to exact solutions

[Florkowski, Ryblewski, and MS 1304.0665]



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  to others.

Anisotropic Hydrodynamics

Viscous Hydrodynamics Expansion

See e.g.
- W. Florkowski and R. Ryblewski, 1007.0130
- M. Martinez and MS, 1007.0889
- D. Bazow, U. Heinz, and MS, 1311.6720
- D. Bazow, U. Heinz, and M. Martinez, 1503.07443
- E. Molnar, H. Niemi, and D. Rischke, 1602.00573; 1606.09019

$$f(\tau, \mathbf{x}, \mathbf{p}) = \underbrace{f_{\rm eq}(\mathbf{p}, T(\tau, \mathbf{x}))}_{\text{Isotropic in momentum space}} + \delta f$$

Isotropic in momentum space

Anisotropic Hydrodynamics (aHydro) Expansion

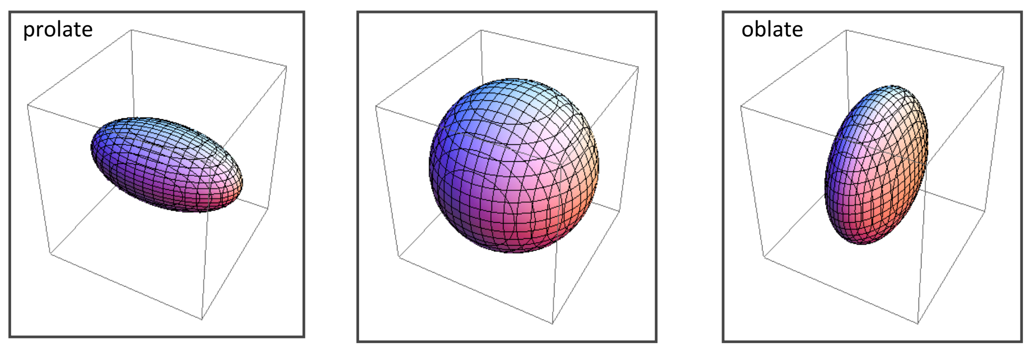
$$f(\tau, \mathbf{x}, \mathbf{p}) = f_{\rm aniso}(\mathbf{p}, \underbrace{\Lambda(\tau, \mathbf{x})}_{T_\perp}, \underbrace{\xi(\tau, \mathbf{x})}_{\text{anisotropy}}\ ) + \delta \tilde{f}$$

Treat this term perturbatively
→ "NLO aHydro"

→ "Romatschke-Strickland" form in LRF

$$f^{LRF}_{\rm aniso} = f_{\rm iso}\left(\frac{\sqrt{\mathbf{p}^2 + \xi(\mathbf{x},\tau) p_z^2}}{\Lambda(\mathbf{x},\tau)}\right)$$

$$\xi = \frac{\langle p_T^2 \rangle}{2\langle p_L^2 \rangle} - 1$$



$-1 < \xi < 0$ $\quad$ $\xi = 0$ $\quad$ $\xi > 0$

Generalized aHydro formalism

In generalized aHydro, so far one assumes that the distribution function is of the form

$$f(x,p) = f_{\rm eq}\left(\frac{\sqrt{p^\mu \Xi_{\mu\nu}(x) p^\nu}}{\lambda(x)}, \frac{\mu(x)}{\lambda(x)}\right) + \delta\tilde{f}(x,p)$$

$$\Xi^{\mu\nu} = \underbrace{u^\mu u^\nu}_{\text{LRF four velocity}} + \underbrace{\xi^{\mu\nu}}_{\text{Traceless symmetric anisotropy tensor}} - \underbrace{\Delta^{\mu\nu}}_{\text{Transverse projector}} \underbrace{\Phi}_{\text{"Bulk"}}$$

$$u^\mu u_\mu = 1$$
$$\xi^\mu{}_\mu = 0$$
$$\Delta^\mu{}_\mu = 3$$
$$u_\mu \xi^{\mu\nu} = u_\mu \Delta^{\mu\nu} = 0$$

- 3 degrees of freedom in u^μ
- 5 degrees of freedom in $\xi^{\mu\nu}$
- 1 degree of freedom in Φ
- 1 degree of freedom in λ
- 1 degree of freedom in μ

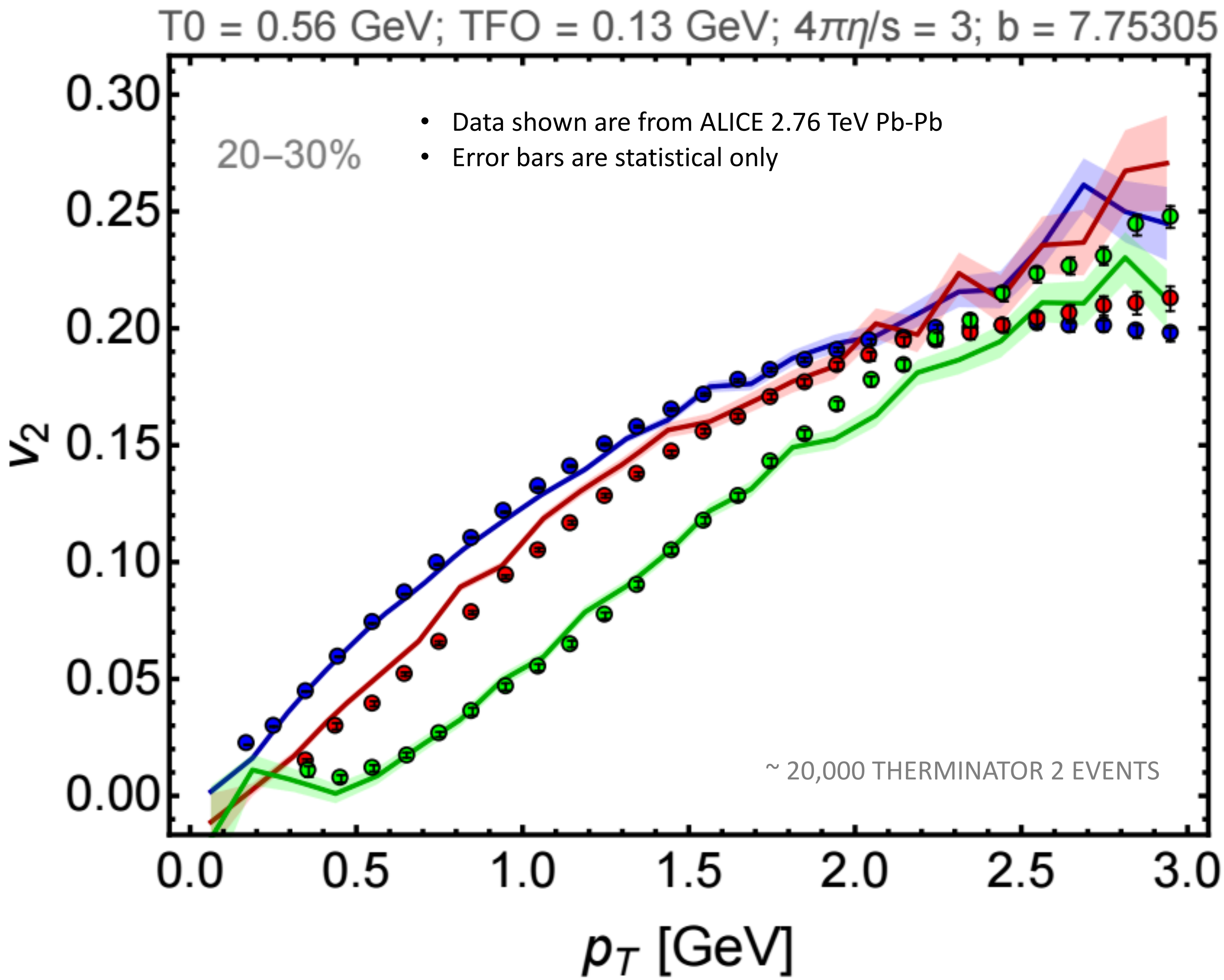
→ 11 DOFs

See e.g.

- M. Martinez, R. Ryblewski, and MS, 1204.1473
- L. Tinti and W. Florkowski, 1312.6614
- M. Nopoush, R. Ryblewski, and MS, 1405.1355

Anisotropic hydrodynamics vs data

[Nopoush, Ryblewski, and MS 1610.10055]



Conclusions and Outlook

- The QGP is not in isotropic thermal equilibrium.
- But, do not lose hope because viscous hydrodynamics seems to reasonably well describe QGP evolution even when the system is momentum-space anisotropic.
- Anisotropic hydrodynamics attempts to further improve the description of a momentum-space anisotropic QGP by including anisotropies at LO.
- Even in the weak-coupling limit we can achieve “hydrodynamization”; strongly coupled framework not required

If you want to learn more about instabilities

1603.08946

Color Instabilities in the Quark-Gluon Plasma

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(Dated: March 29, 2016)

- 104 pages
- 584 equations (!)
- 49 figures
- 8 years to complete (!)

Happy Birthday Stan!