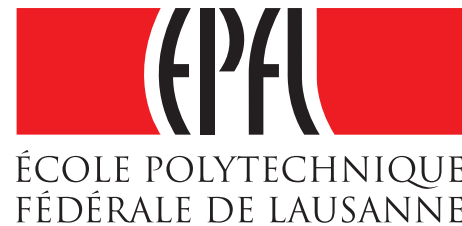


Investigation for photon polarisation in $B \rightarrow K\pi\pi\gamma$ decays at LHCb

BELLE Violaine (EPFL)

PAIS P., PUIG A., SCHNEIDER O.,
TRABELSI K., VENEZIANO G.

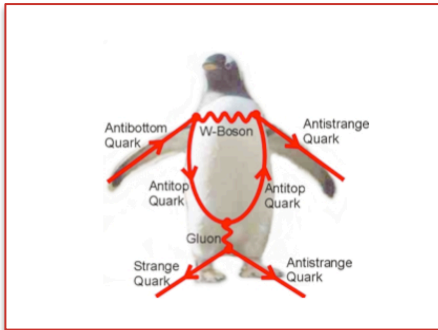
SPS Annual Meeting
25/08/2016 ~ *LUGANO*



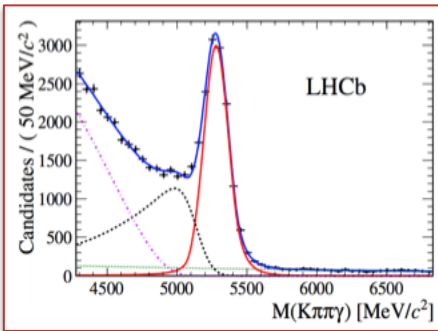
ÉCOLE POLYTECHNIQUE
FÉDÉRALE DE LAUSANNE



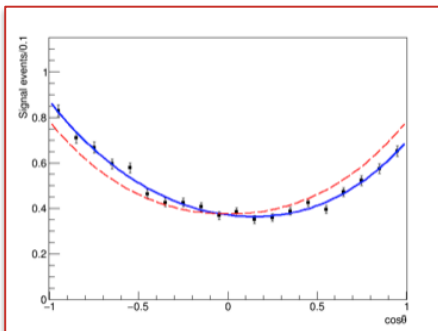
Outline



Photon polarisation in radiative B decays

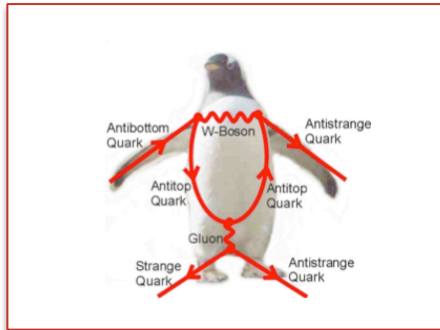


Observation of non-zero photon polarisation in $B \rightarrow \text{K}\pi\pi\gamma$ decays
Amplitude analysis of the $\text{K}\pi\pi$ system

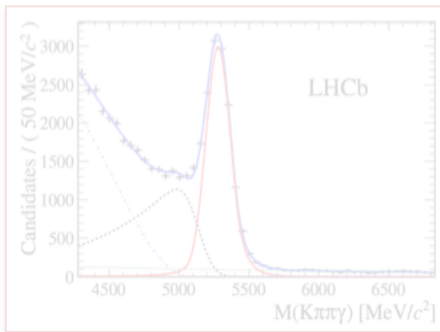


Generating amplitudes in 5D
Studies of the up-down asymmetry

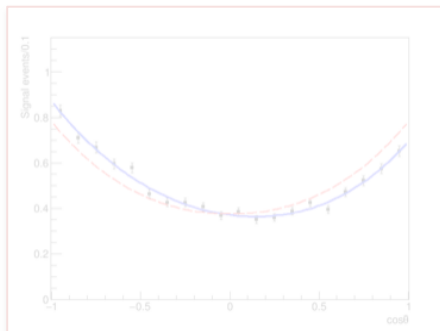
Photon polarisation in $b \rightarrow s\gamma$ transitions



Photon polarisation in radiative B decays



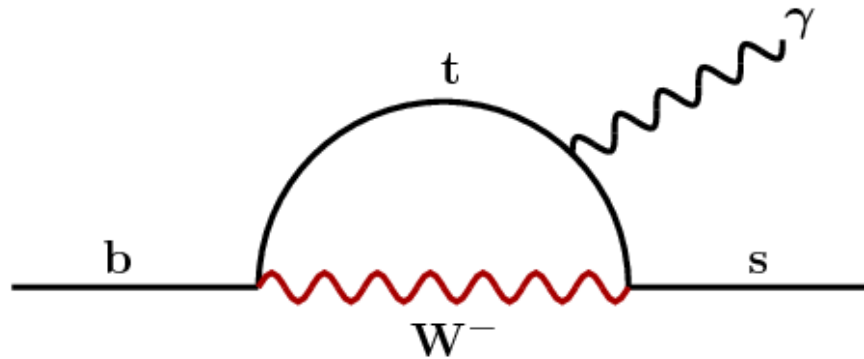
Observation of non-zero photon polarisation in $B \rightarrow K\pi\pi\gamma$ decays
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Generating amplitudes in 5D
Studies of the up-down asymmetry

Radiative B decays

- FCNC with a final state photon
- The $b \rightarrow sy$ transition occurs through a penguin loop



- In the SM, the photon in $b \rightarrow sy$ transitions is mostly left-handed

Photon polarisation in SM $b \rightarrow s\gamma$ transitions

- The decay amplitude for $b \rightarrow s\gamma$ transitions is proportional to:

$$\langle f | \mathcal{H}_{\text{eff}} | i \rangle = -4 \frac{G_F}{\sqrt{2}} V_{tb} V_{ts}^* [C_7^{\text{eff}}(m_b) \langle f | \mathcal{O}_7(m_b) | i \rangle + C_7^{\prime \text{eff}}(m_b) \langle f | \mathcal{O}_7'(m_b) | i \rangle]$$

Coupling to left-handed photon

Coupling to right-handed photon

[arXiv:1206.1502]

- In SM, Wilson coefficients C_7 and C_7' are such that:

$$C_7'/C_7 \cong m_s/m_b \cong 0.02$$

- The photon polarisation parameter λ_γ is defined as the asymmetry between the right and left components:

$$\lambda_\gamma = \frac{|C_7'|^2 - |C_7|^2}{|C_7'|^2 + |C_7|^2}$$

In SM, $\lambda_\gamma \cong -1$ (with corrections of $O(m_s^2/m_b^2)$) for decays of a b quark

Photon polarisation: a probe for NP

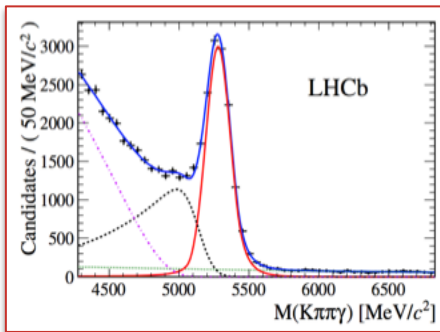
- NP processes could introduce right handed currents, hence modifying the photon polarisation
- The photon polarisation is a test for SM but it has **never been measured**
- Maybe some new penguins around !



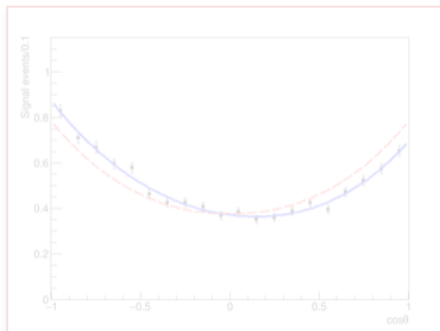
Up-Down asymmetry in $B \rightarrow K\pi\pi\gamma$ decays



Photon polarisation in radiative B decays



Observation of non-zero photon polarisation in $B \rightarrow K\pi\pi\gamma$ decays
Amplitude analysis of the $K\pi\pi$ system

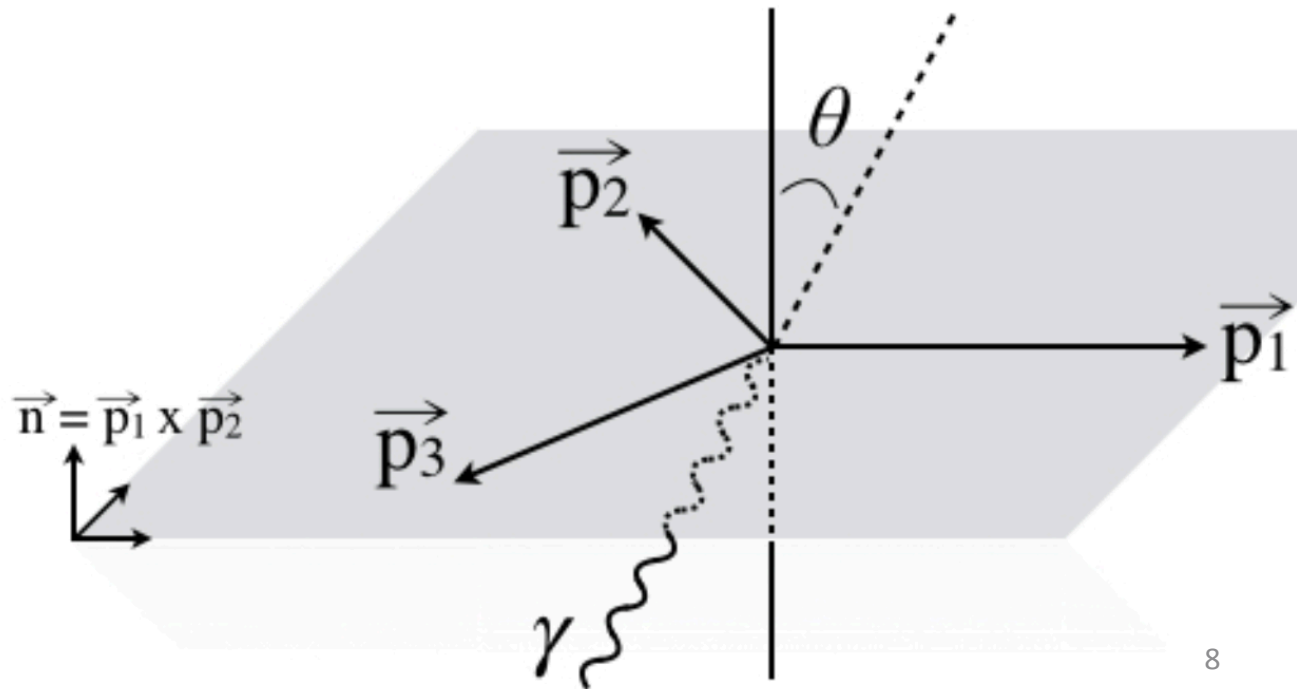


Generating amplitudes in 5D
Studies of the up-down asymmetry

Why do we need 3 hadrons in the final state?

Minimum number of tracks needed to build a P-odd quantity proportional to the photon polarisation using the final state momenta

$$\vec{p}_\gamma \cdot (\vec{p}_1 \times \vec{p}_2)$$



$B^+ \rightarrow K^+_{\text{res}}(K\pi\pi)\gamma$ decay rate

- Decay rate for the $B^+ \rightarrow K^+_{\text{res}} (\rightarrow K^+\pi^-\pi^+) \gamma$ decay:

[Gronau *et al*, PRD66 (2002) 054008]

$$d\Gamma(B \rightarrow K\pi\pi\gamma) = \left| \sum_k \frac{c_{k,R}^{\text{weak}} \times A_{k,R}^{\text{strong}}}{m_{K\pi\pi}^2 - m_k^2 - im_k\Gamma_k} \right|^2 + \left| \sum_k \frac{c_{k,L}^{\text{weak}} \times A_{k,L}^{\text{strong}}}{m_{K\pi\pi}^2 - m_k^2 - im_k\Gamma_k} \right|^2$$

$B^+ \rightarrow K^+_{res} (K\pi\pi)\gamma$ decay rate

- Decay rate for the $B^+ \rightarrow K^+_{res} (\rightarrow K^+\pi^-\pi^+) \gamma$ decay:

[Gronau *et al*, PRD66 (2002) 054008]

$$d\Gamma(B \rightarrow K\pi\pi\gamma) = \left| \sum_k \frac{c_{k,R}^{\text{weak}} \times A_{k,R}^{\text{strong}}}{m_{K\pi\pi}^2 - m_k^2 - im_k\Gamma_k} \right|^2 + \left| \sum_k \frac{c_{k,L}^{\text{weak}} \times A_{k,L}^{\text{strong}}}{m_{K\pi\pi}^2 - m_k^2 - im_k\Gamma_k} \right|^2$$

$$\lambda_\gamma = \frac{|c_R^{\text{weak}}|^2 - |c_L^{\text{weak}}|^2}{|c_R^{\text{weak}}|^2 + |c_L^{\text{weak}}|^2}$$

Photon polarisation parameter

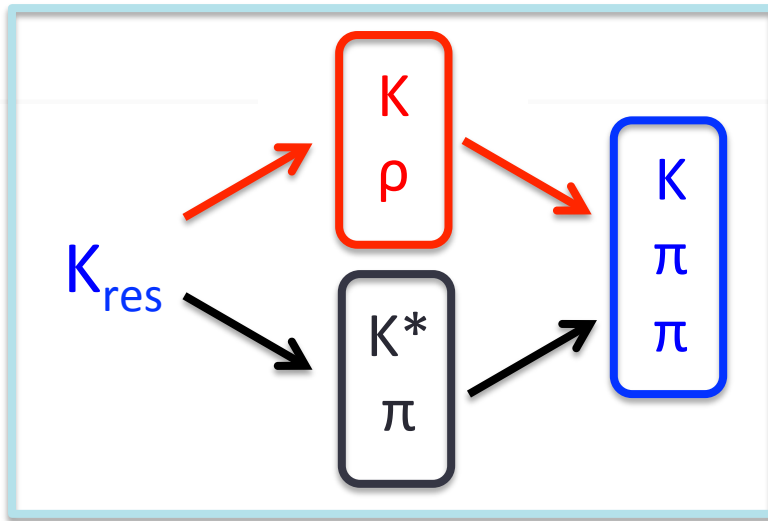
$$\lambda_\gamma = \frac{|C'_7|^2 - |C_7|^2}{|C'_7|^2 + |C_7|^2}$$

How to access λ_γ in $B^+ \rightarrow K^+_{res} (K\pi\pi)\gamma$ decays ?

In the case of a single 1^+ resonance:

[Gronau *et al*, PRD66 (2002) 054008]

$$\frac{d\Gamma(B \rightarrow K\pi\pi\gamma)}{ds ds_{13} ds_{23} d\cos\theta} \propto \frac{1}{2} |\vec{\mathcal{J}}|^2 (1 + \cos^2\theta) + \lambda_\gamma \cos\theta \text{Im}[\vec{n} \cdot (\vec{\mathcal{J}} \times \vec{\mathcal{J}}^*)]$$



interference !

invariant mass dependencies

where s stands for $m^2(K\pi\pi)$, s_{13} for $m^2(K\pi)$ and s_{23} for $m^2(\pi\pi)$

Adding resonances is not so simple

Adding more resonances (1^+ , 2^+ , 1^-), the formula gets complex:

$$\begin{aligned} \frac{d\Gamma}{ds_{13}ds_{23}d\cos\theta} &= |A|^2 \left\{ \frac{1}{4} |\vec{J}|^2 (1 + \cos^2 \theta) + \frac{1}{2} \lambda_\gamma \text{Im}[\vec{n} \cdot (\vec{J} \times \vec{J}^*)] \cos \theta \right\} \\ + |B|^2 &\left\{ \frac{1}{4} |\vec{K}|^2 (\cos^2 \theta + \cos^2 2\theta) + \frac{1}{2} \lambda_\gamma \text{Im}[\vec{n} \cdot (\vec{K} \times \vec{K}^*)] \cos \theta \cos 2\theta \right\} + |C|^2 \frac{1}{2} \sin^2 \theta \\ + &\left\{ \frac{1}{2} (3 \cos^2 \theta - 1) \text{Im}[AB^* \vec{n} \cdot (\vec{J} \times \vec{K}^*)] + \lambda_\gamma \text{Re}[AB^* (\vec{J} \cdot \vec{K}^*)] \cos^3 \theta \right\} . \end{aligned}$$

[Gronau *et al*, PRD66 (2002) 054008]

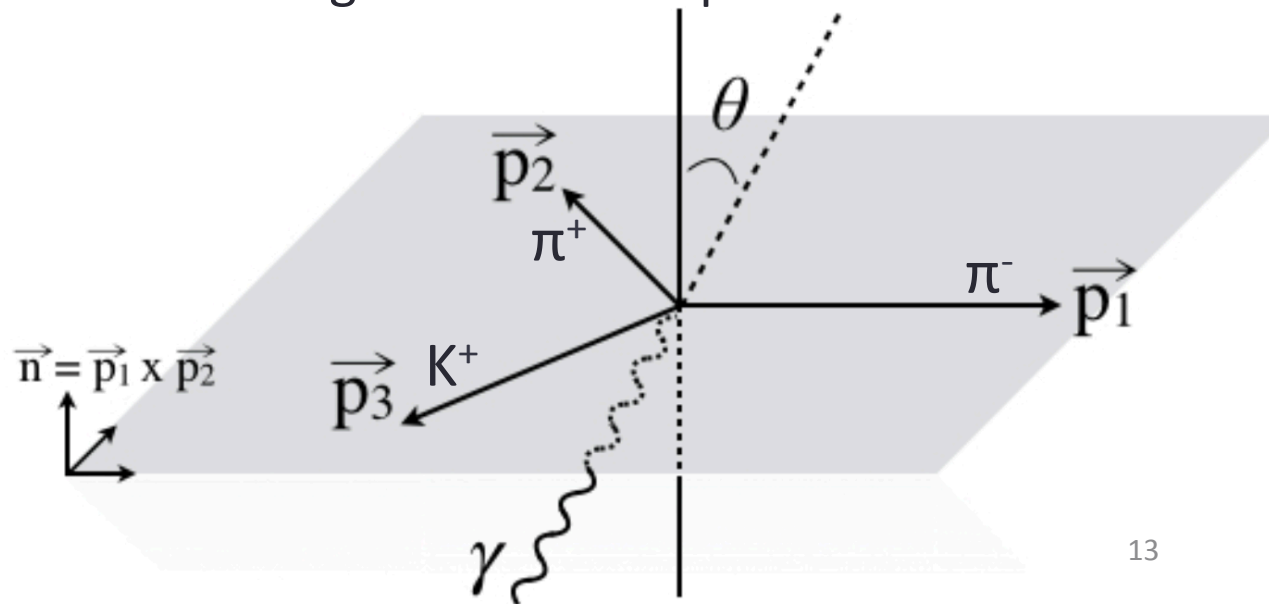
As the $K^+\pi^-\pi^+$ system is not known, **simplification is needed !**

How to access λ_γ in $B^+ \rightarrow K^+_{res} (K\pi\pi)\gamma$ decays ?

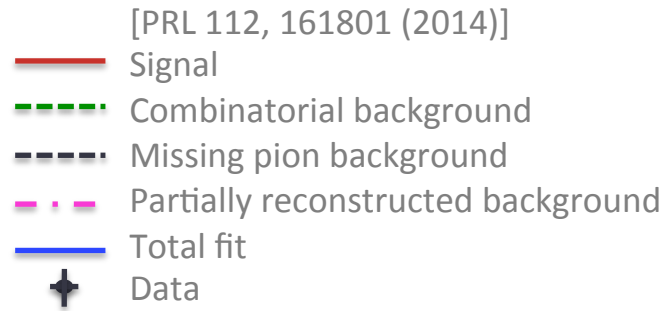
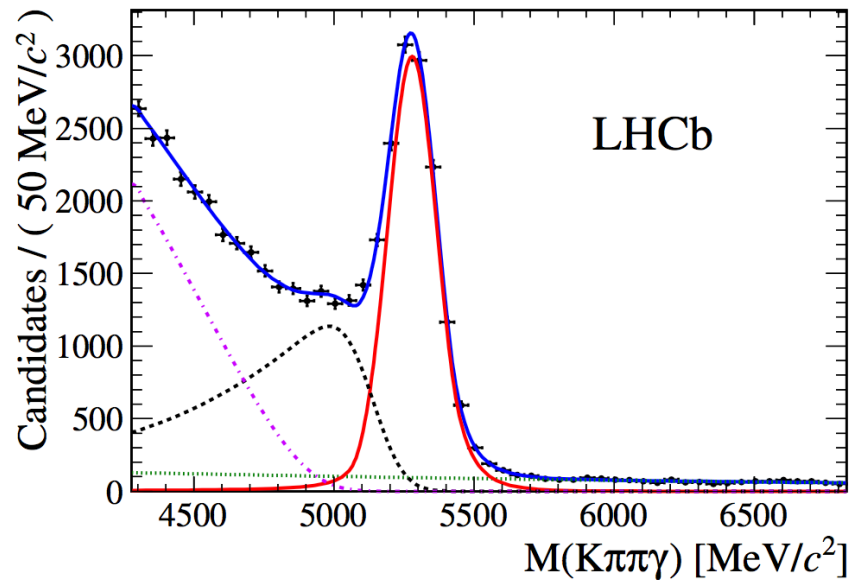
The idea is to **integrate over the Dalitz plot and the angular distribution** to obtain the up-down asymmetry:

$$\mathcal{A}_{ud} \equiv \frac{\int_0^1 d\cos\theta \frac{d\Gamma}{d\cos\theta} - \int_{-1}^0 d\cos\theta \frac{d\Gamma}{d\cos\theta}}{\int_{-1}^1 d\cos\theta \frac{d\Gamma}{d\cos\theta}} = C\lambda_\gamma$$

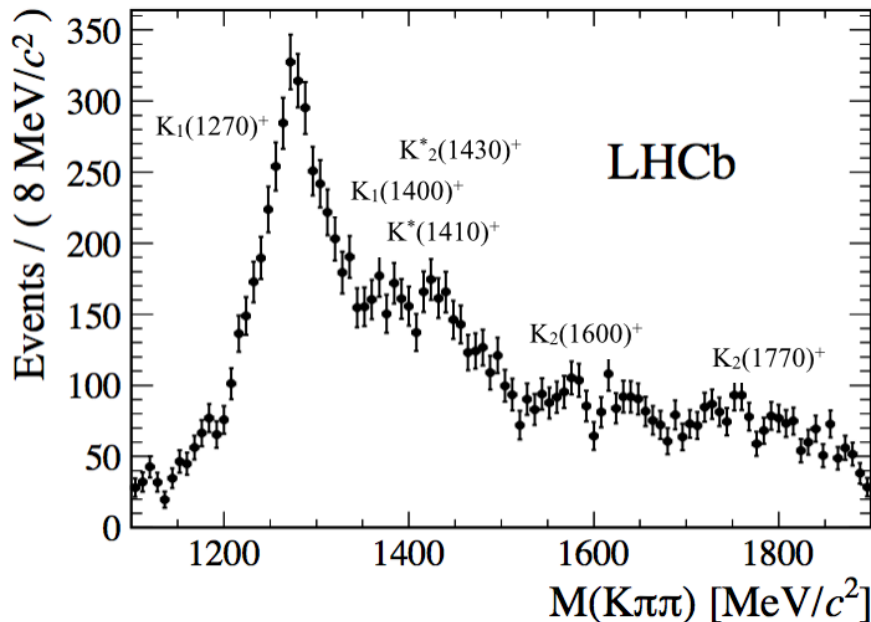
Where C takes into account the integrations and depends on the $K\pi\pi$ system content



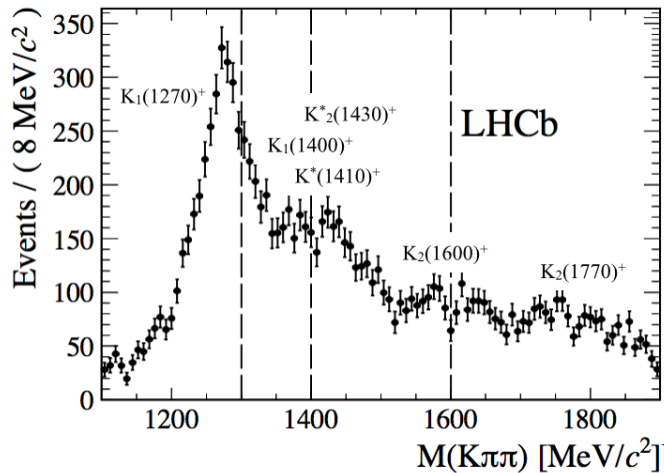
Selection of the $B \rightarrow K\pi\pi\gamma$ events



- Almost 14,000 signal events are reconstructed and selected in the full 3 fb^{-1} LHCb Run 1 data sample
- The background-subtracted $K\pi\pi$ mass spectrum is obtained

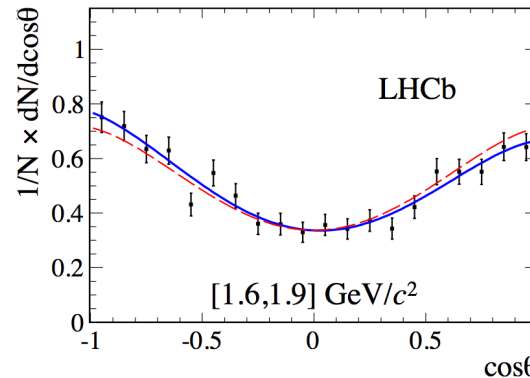
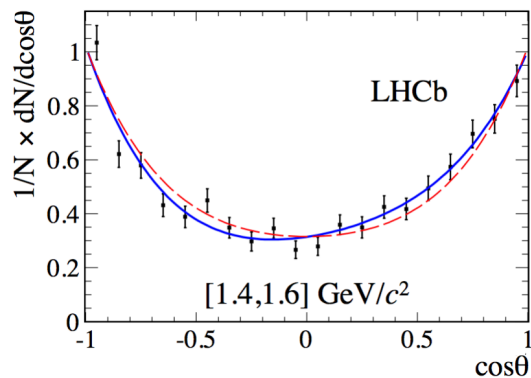
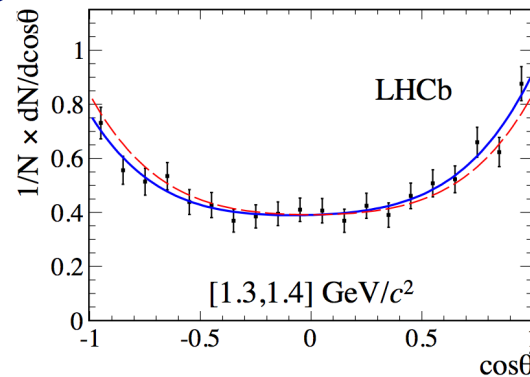
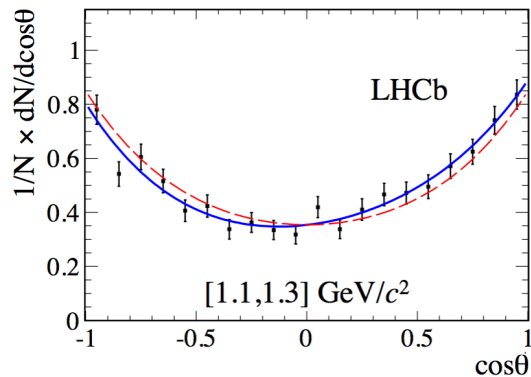


Observation of λ_γ



- The $K\pi\pi$ mass spectrum is obtained and divided in bins of $m(K\pi\pi)$
- In each bin, the $\cos\theta$ distribution is fitted

[PRL 112, 161801 (2014)]

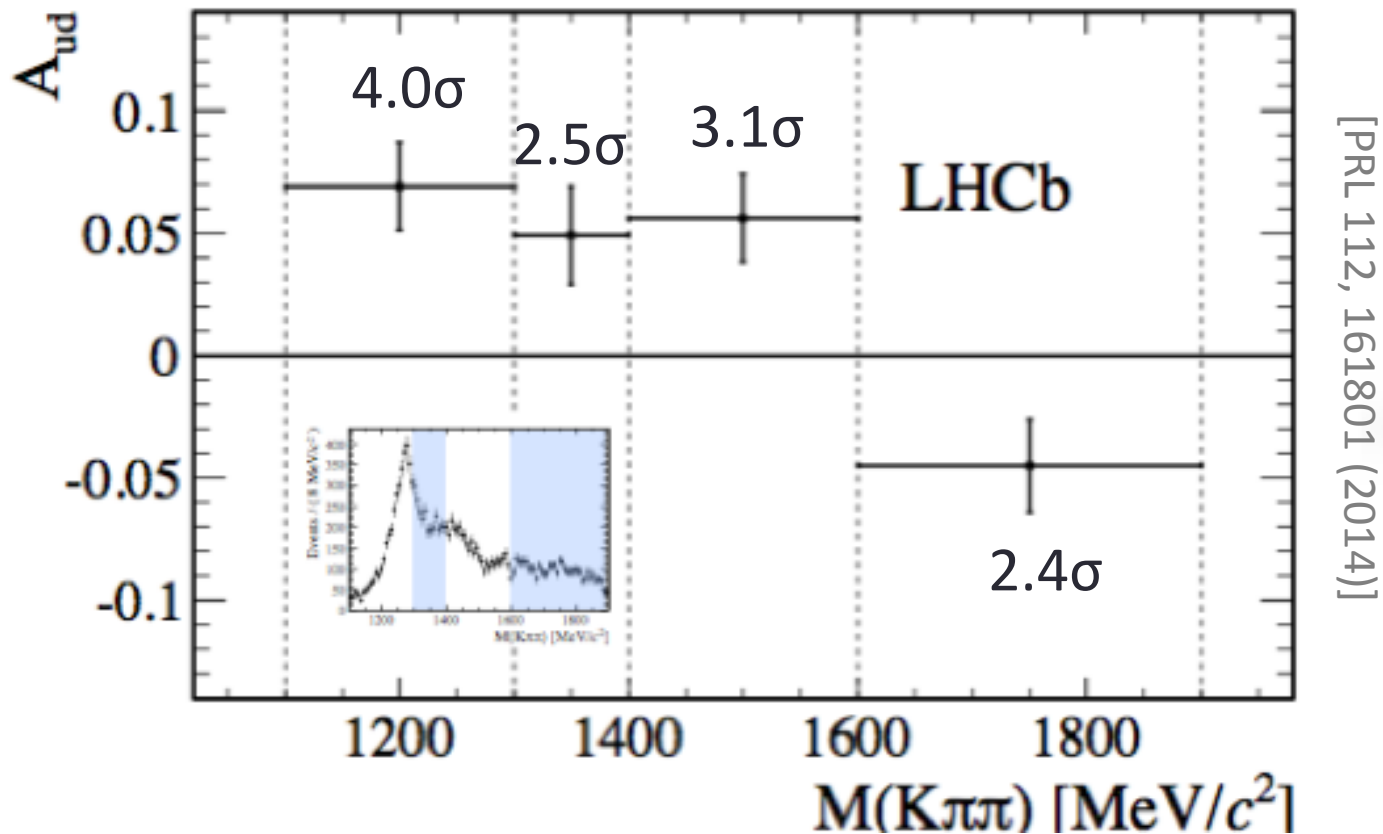


[LHCb collaboration, PRL 112, 161801 (2014)]

— Fit
 - - - Fit without asymmetry
 + Data

Observation of λ_γ

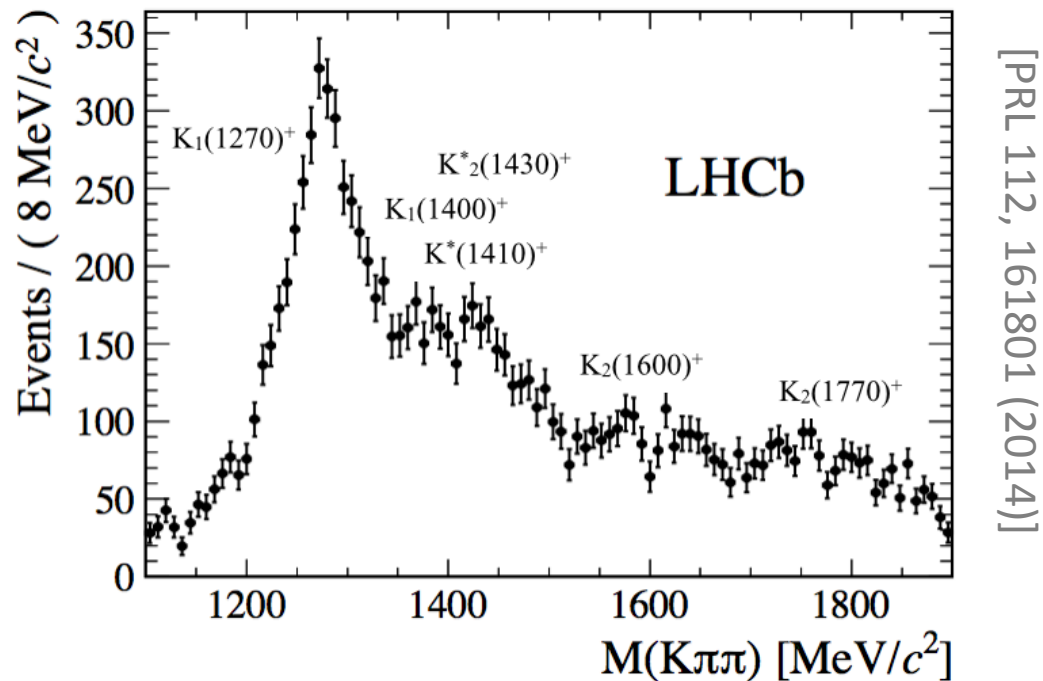
- Up-down asymmetry expected to be proportional to λ_γ



- **Observation of a non-zero photon polarisation with A_{ud} different from 0 at 5.2σ .**

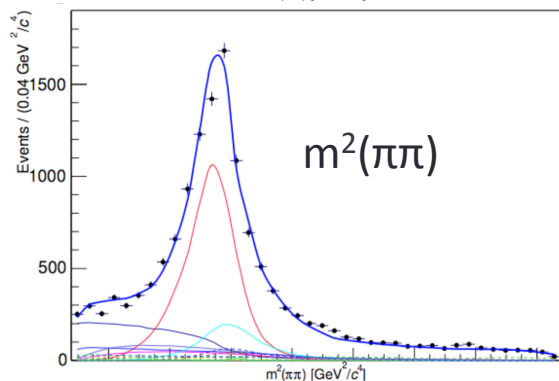
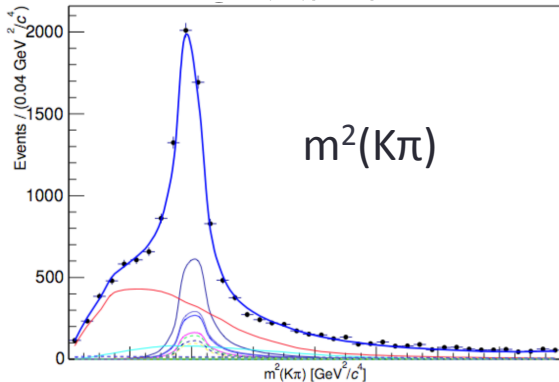
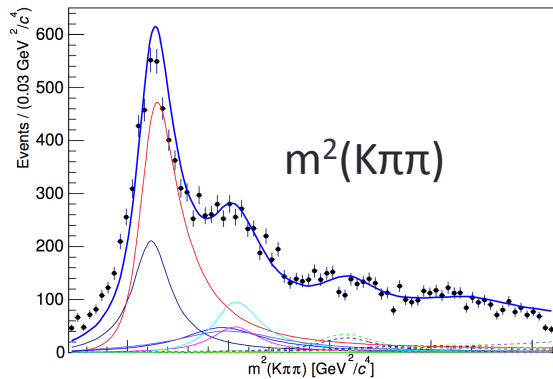
Towards a measurement of λ_γ

- To measure the value of λ_γ , we need to know the proportionality coefficient C between A_{ud} and λ_γ .
- This coefficient depends on the content of the $K\pi\pi$ system



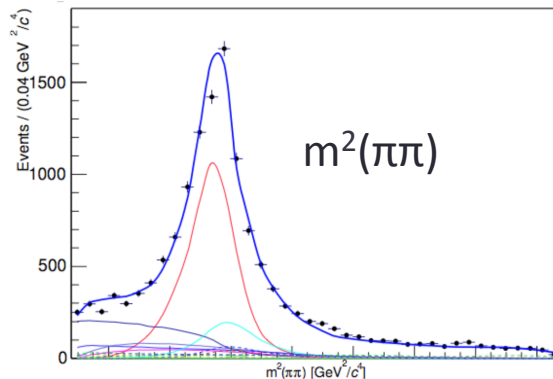
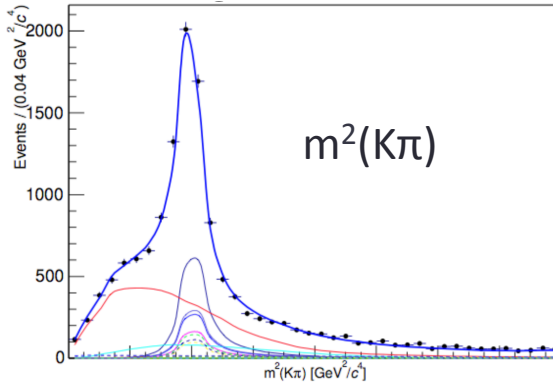
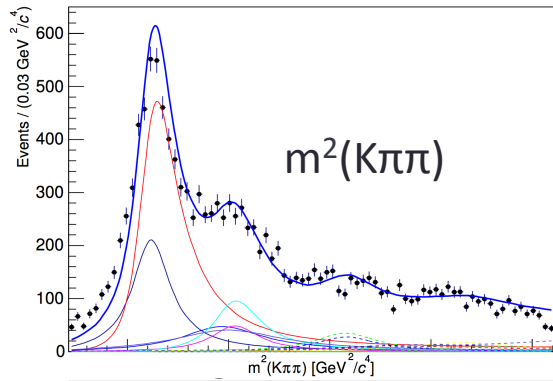
- Missing knowledge of the $K\pi\pi$ system to make a measurement

Amplitude analysis of the $K\pi\pi$ system



- Fit of the three invariant masses squared $m^2(K\pi\pi)$, $m^2(K\pi)$ and $m^2(\pi\pi)$, integrating on the angles
- Using 18 amplitudes

Amplitude analysis of the $K\pi\pi$ system



1^+

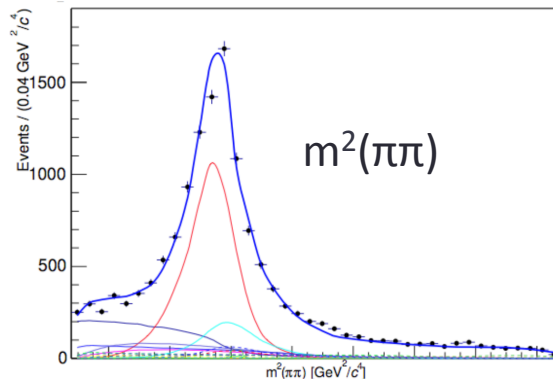
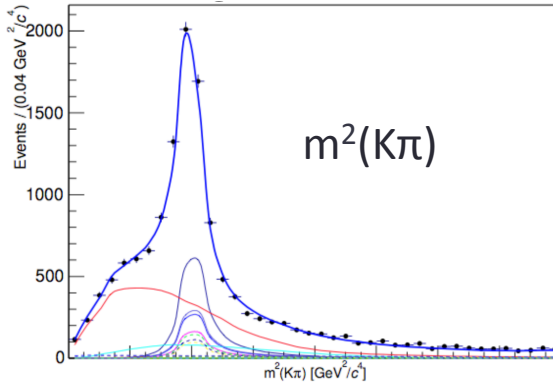
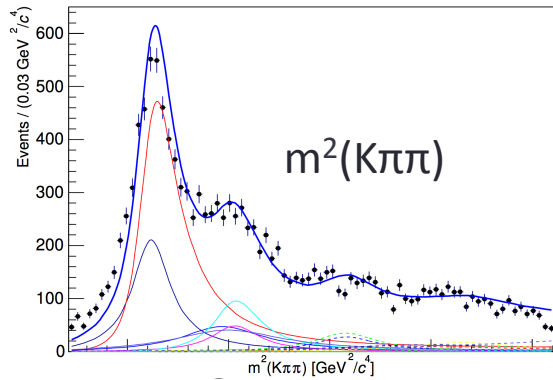
1^-

2^+

2^-

Decay channel k	Modulus	Phase [rad]	FF_k (10^{-2})
$K_1(1270)^+ \rightarrow K^*(892)^0\pi^+$	1 (fixed)	0 (fixed)	16.8 ± 0.9
$K_1(1270)^+ \rightarrow K^+\rho(770)$	1.747 ± 0.049	-0.91 ± 0.028	$39.9^{+0.6}_{-0.7}$
$K_1(1270)^+ \rightarrow K^+\omega(782)$	0.302 ± 0.085	0.302 ± 0.27	$0.068^{+0.028}_{-0.180}$
$K_1(1270)^+ \rightarrow K^*(1430)^0\pi^+$	0.382 ± 0.055	-1.637 ± 0.162	$0.69^{+0.22}_{-0.20}$
$K_1(1400)^+ \rightarrow K^*(892)^0\pi^+$	0.42 ± 0.022	-0.755 ± 0.053	7.8 ± 0.8
$K^*(1410)^+ \rightarrow K^*(892)^0\pi^+$	0.479 ± 0.042	0 (fixed)	$8.4^{+2.8}_{-3.3}$
$K^*(1680)^+ \rightarrow K^*(892)^0\pi^+$	0.219 ± 0.022	0.443 ± 0.124	$3.5^{+1.7}_{-2.1}$
$K^*(1680)^+ \rightarrow K^+\rho(770)$	0.112 ± 0.01	1.403 ± 0.221	2.4 ± 0.4
$K_2^*(1430)^+ \rightarrow K^*(892)^0\pi^+$	0.509 ± 0.034	0 (fixed)	4.8 ± 1.0
$K_2^*(1430)^+ \rightarrow K^+\rho(770)$	0.511 ± 0.026	1.798 ± 0.09	9.0 ± 0.8
$K_2^*(1430)^+ \rightarrow K^+\omega(782)$	0.332 ± 0.078	-2.353 ± 0.236	$0.30^{+0.13}_{-0.26}$
$K_2(1600)^+ \rightarrow K^*(892)^0\pi^+$	0.172 ± 0.01	2.883 ± 0.119	$4.4^{+0.9}_{-1.0}$
$K_2(1600)^+ \rightarrow K^+\rho(770)$	0.095 ± 0.012	2.442 ± 0.129	$3.33^{+0.34}_{-0.50}$
$K_2(1770)^+ \rightarrow K^*(892)^0\pi^+$	0.107 ± 0.008	0 (fixed)	$3.0^{+0.6}_{-0.8}$
$K_2(1770)^+ \rightarrow K^+\rho(770)$	0.018 ± 0.005	2.527 ± 0.266	$0.23^{+0.08}_{-0.32}$
$K_2(1770)^+ \rightarrow K_2^*(1430)^0\pi^+$	0.087 ± 0.009	-2.06 ± 0.128	$0.67^{+0.10}_{-0.09}$
$K_2(1770)^+ \rightarrow K^+f_2(1270)^0$	0.17 ± 0.007	-0.174 ± 0.088	$1.30^{+0.15}_{-0.16}$
Non resonant	0.051 ± 0.002	0 (fixed)	4.1 ± 0.5

Amplitude analysis of the $K\pi\pi$ system

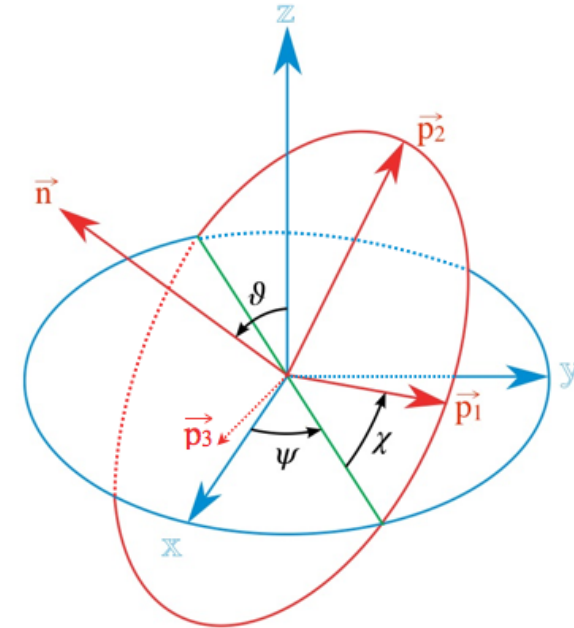


Decay channel k	Modulus	Phase [rad]	FF_k (10^{-2})
$K_1(1270)^+ \rightarrow K^*(892)^0\pi^+$	1 (fixed)	0 (fixed)	16.8 ± 0.9
$K_1(1270)^+ \rightarrow K^+\rho(770)$	1.747 ± 0.049	-0.91 ± 0.028	$39.9^{+0.6}_{-0.7}$

- The 2 main amplitudes are **$K(1270) \rightarrow K^*\pi$** and **$K(1270) \rightarrow K\rho$** with a ratio of modulus of **1.78** and a phase difference of **-0.91 rad**

Limits of the amplitude analysis

- The previous study brought **knowledge of the $K\pi\pi$ system**
- **Main limitation: use of 3 dimensions**
($m^2(K\pi\pi)$, $m^2(K\pi)$, $m^2(\pi\pi)$)
→ Not sensitive to the photon polarisation that appears in the angular variables
- **To measure the photon polarisation, a fit in 5 dimensions is necessary**
→ Use of the squared masses + θ (polar angle) and χ (azimuthal angle)



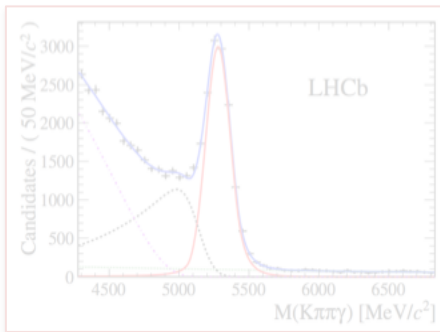
A tool to generate and fit in 5D is needed:

MINT [arXiv:1201.5716]

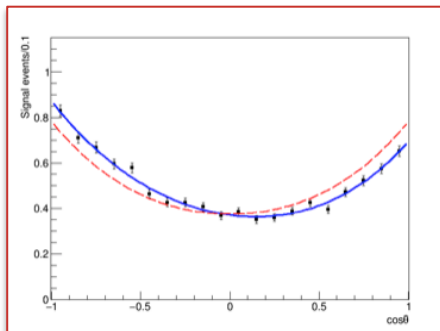
Relating A_{ud} to the interferences and λ_γ



Photon polarisation in radiative B decays



Observation of non-zero photon polarisation in $B \rightarrow K\pi\pi\gamma$ decays
Amplitude analysis of the $K\pi\pi$ system



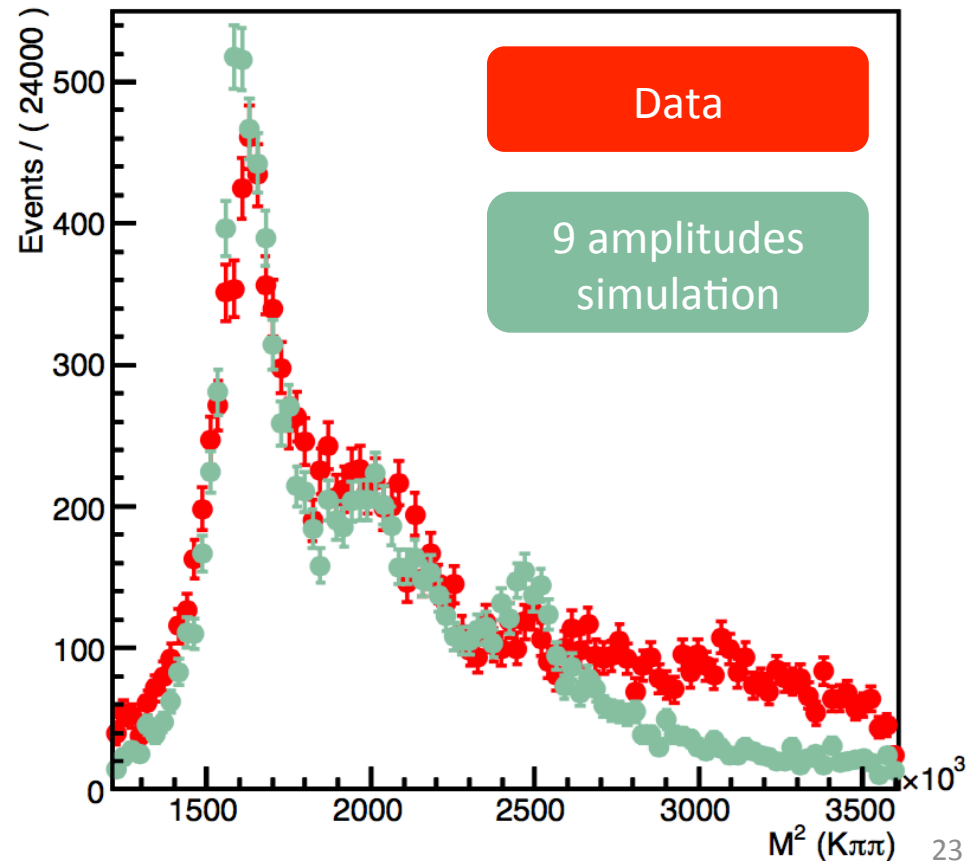
Generating amplitudes in 5D
Studies of the up-down asymmetry

Modelling the $K\pi\pi$ system in 5D

- Using our knowledge of the $K\pi\pi$ system, we can generate in 5D simulated samples close to the data
- A model with 9 amplitudes reproduces well the lower part of the $m^2(K\pi\pi)$ spectrum

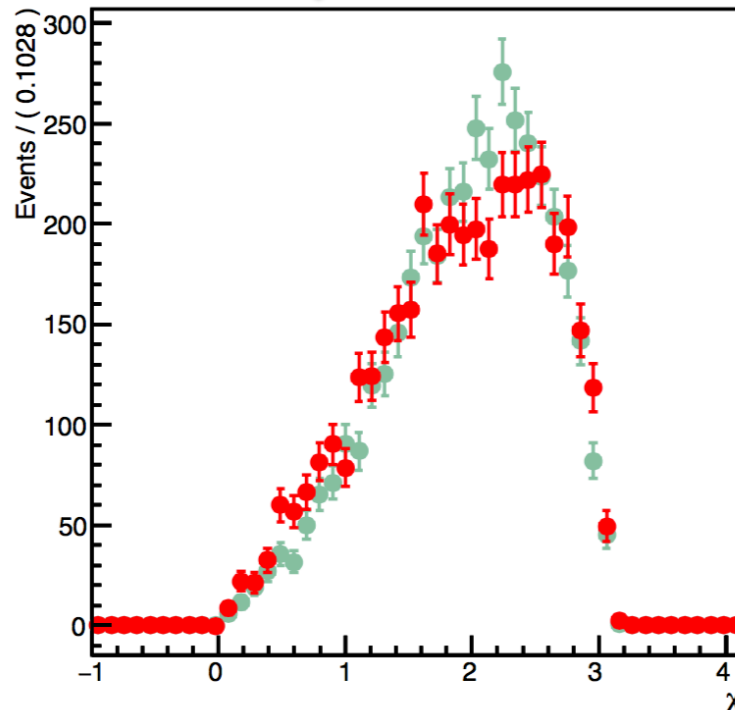
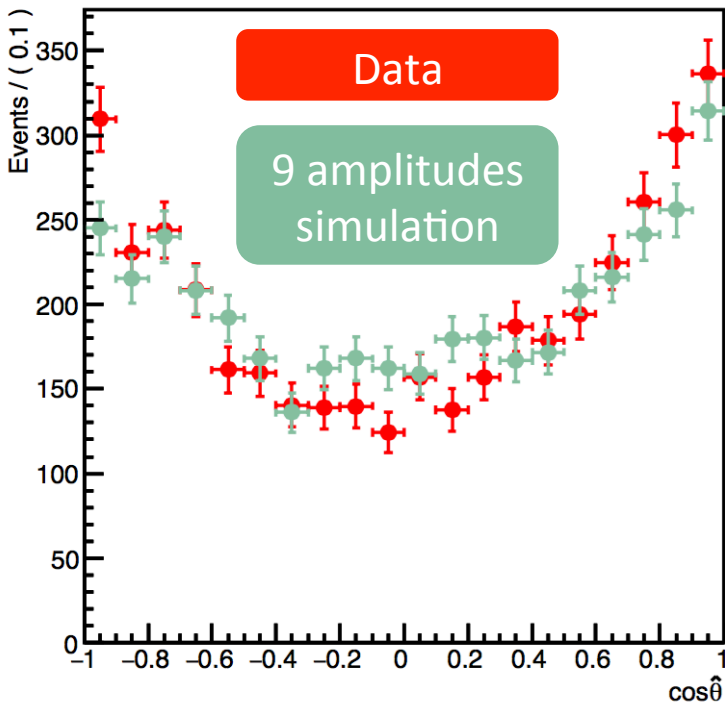
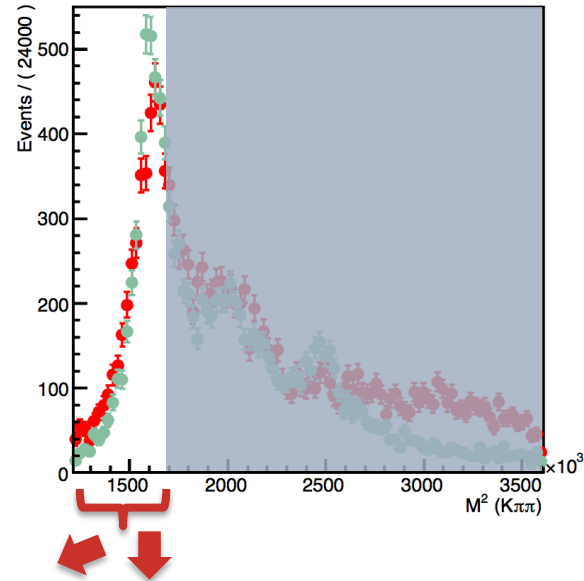
9 amplitudes:

- $K(1270) \rightarrow K^*\pi$ [S]
 - $K(1270) \rightarrow K^*\pi$ [D]
 - $K(1270) \rightarrow K\rho$
 - $K(1400) \rightarrow K^*\pi$
 - $K(1410) \rightarrow K^*\pi$
 - $K_2(1430) \rightarrow K^*\pi$
 - $K_2(1430) \rightarrow K\rho$
 - $K_2(1580) \rightarrow K^*\pi$
 - $K_2(1580) \rightarrow K\rho$
- 1+
1-
2+
2-



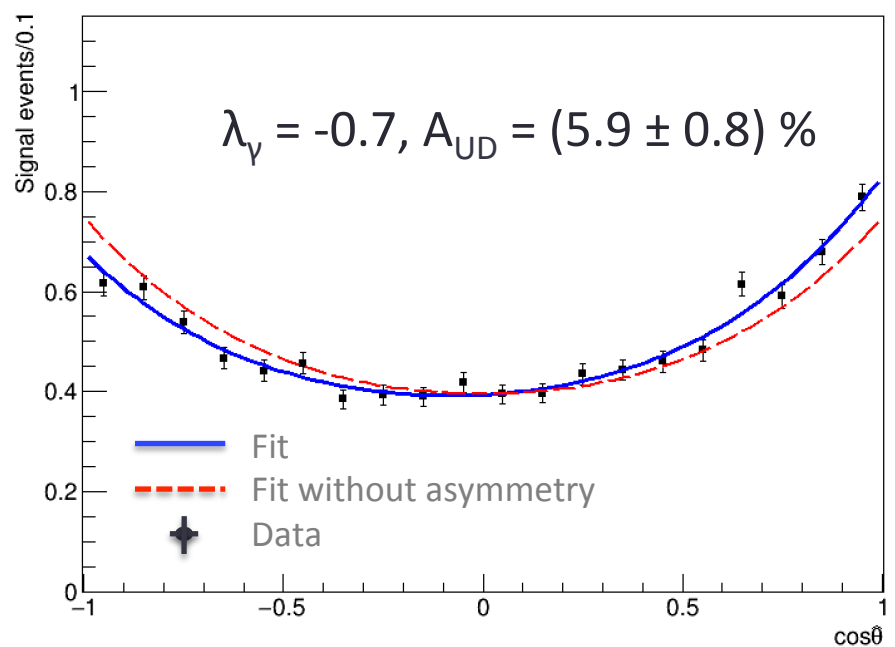
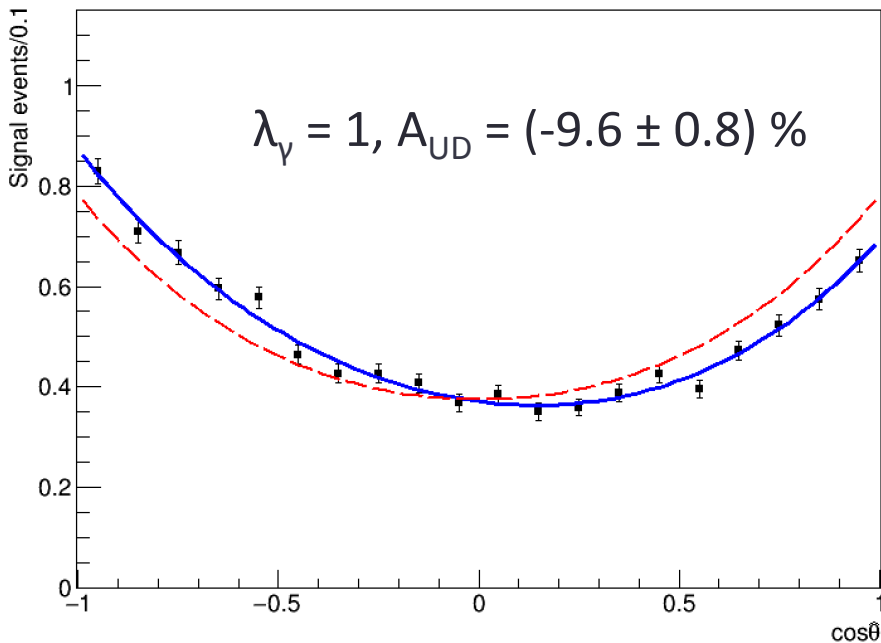
Modelling the $K\pi\pi$ system in 5D

- The angular distributions (that were not considered in the 3D fit) are reasonably well reproduced in the lower part of the $m^2(K\pi\pi)$ spectrum



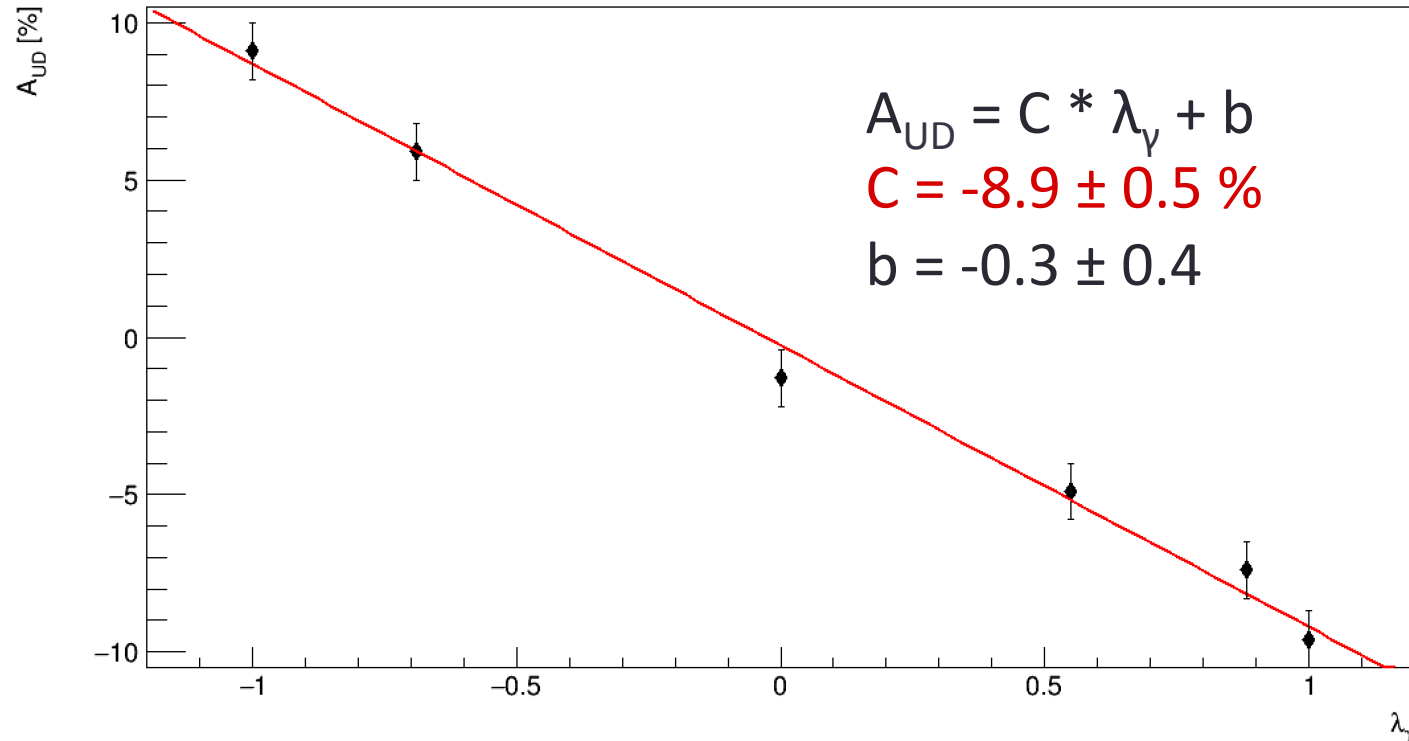
Relating A_{UD} and λ_γ

- The λ_γ parameter is first set to 1
- Simulated events are generated using a simple model with 2 amplitudes: $\mathbf{K}(1270) \rightarrow \mathbf{K}\rho$ and $\mathbf{K}(1270) \rightarrow \mathbf{K}^*\pi$ with the complex ratio observed in data
- The up-down asymmetry is computed by fitting the $\cos \theta$ distribution



Relating A_{UD} and λ_γ

- The dependance of A_{UD} and λ_γ is obtained and is linear
- From theory: $A_{UD} = C \cdot \lambda_\gamma$



A_{UD} as a function of λ_γ for toy samples of 10k events generated with $K(1270) \rightarrow K\rho$ and $K(1270) \rightarrow K^*\pi$ amplitudes (complex ratio corresponding to data)

Conclusion

- The study of the up-down asymmetry in $B \rightarrow K\pi\pi\gamma$ decays has led to **the observation of a non-zero photon polarisation**
- The **knowledge of the $K\pi\pi$ system** has been dramatically improved thanks to the **3D fit**
- A **5D simulation of the $B \rightarrow K\pi\pi\gamma$ events** has been obtained and has allowed to compute the **proportionality coefficient between A_{UD} and λ_γ** in a simplified case
- Next step: **Fitting in 5D !**

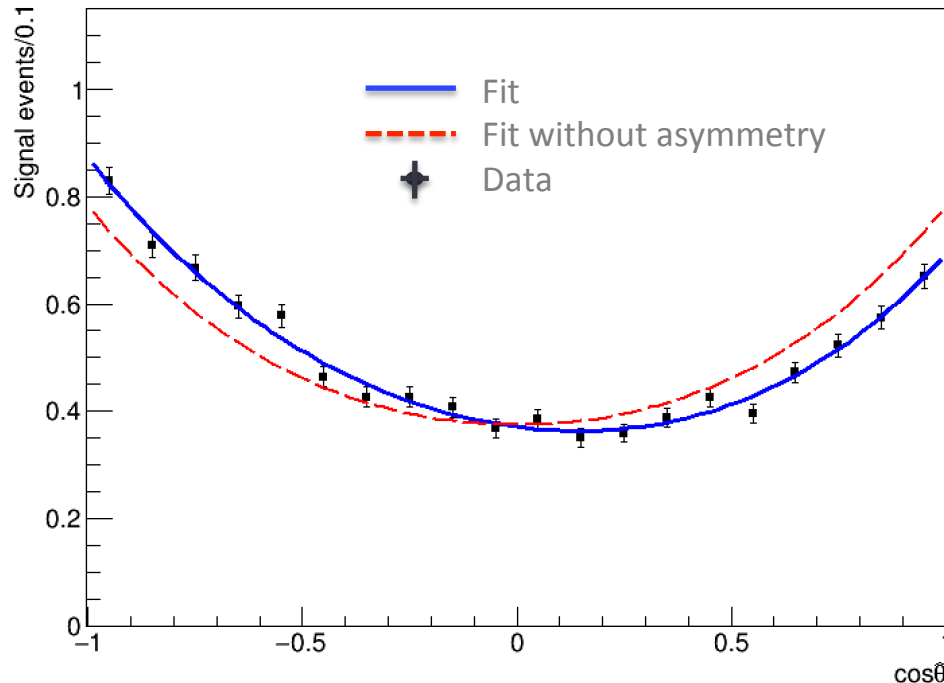


Thanks for your attention



Relating A_{UD} with the interference pattern

- The up-down asymmetry can also be computed as a function of the complex ratio of the two amplitudes considered
- Several sets of simulated events are generated varying the ratio of modulus and the phase difference between amplitudes



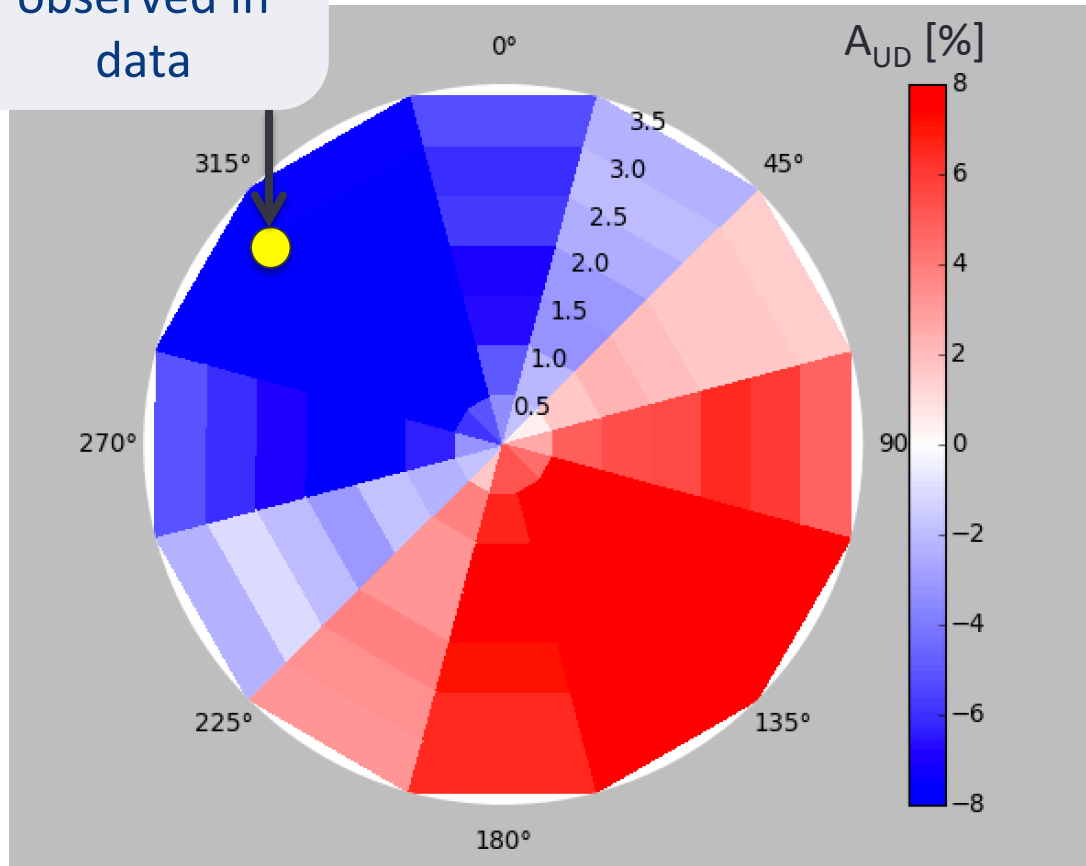
● $A_{UD} = (-9.6 \pm 0.8) \%$

Example: Up-down asymmetry for the complex ratio of amplitudes seen in data ($\lambda_\gamma = 1$)

Relating A_{UD} with the interference pattern

- In some regions of the interference pattern, A_{UD} is zero
- > A_{UD} is not always sensitive to λ_γ

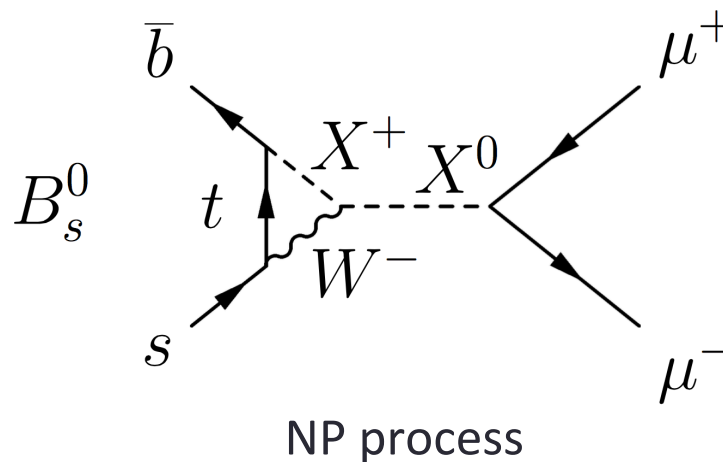
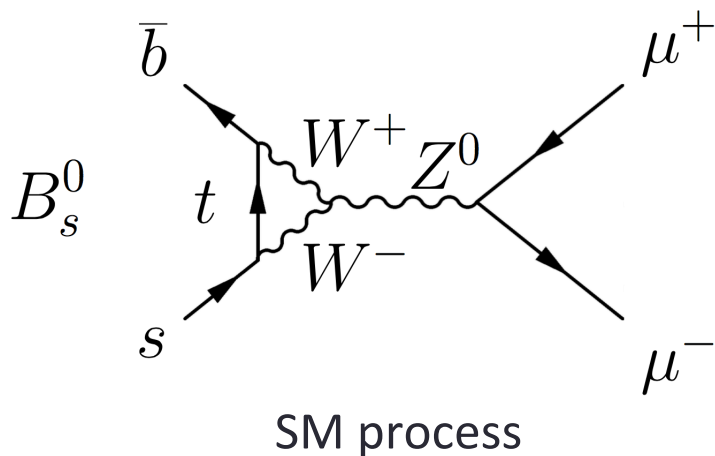
Ratio of amplitudes observed in data



Up-Down asymmetry as a function of the complex ratio between the $K(1270) \rightarrow K\rho$ and $K(1270) \rightarrow K^*\pi$ amplitudes for $\lambda_\gamma = 1$

Rare B decays

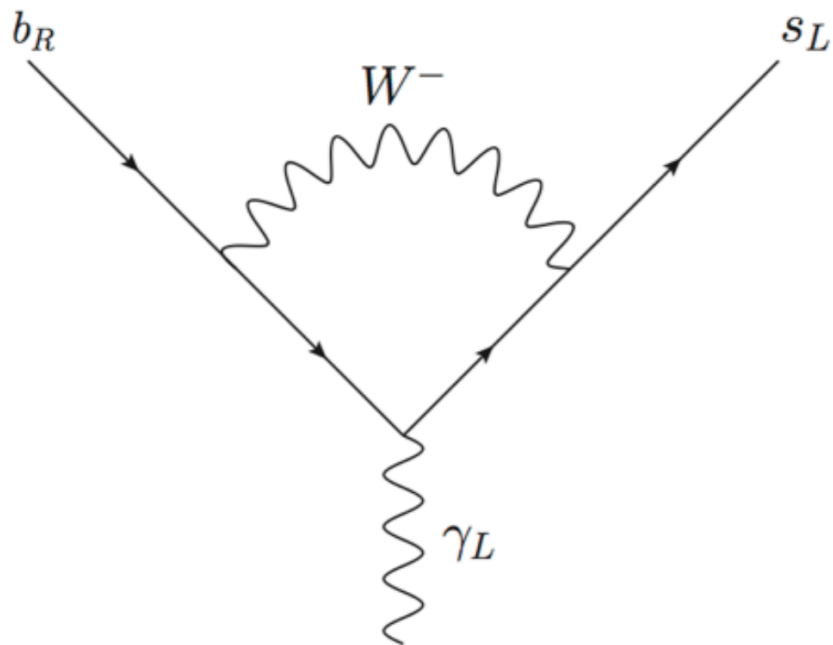
- **Flavour changing neutral currents** with $\Delta F = 1$ are forbidden at tree level in the SM
- Occur through loops (boxes, penguin loops) which may receive contributions from new physics (NP)
- For e.g., for $B_s^0 \rightarrow \mu^+ \mu^-$:



- Rare decays are used for indirect searches of NP:
 - Suppressed in the SM
 - Sensitive to NP

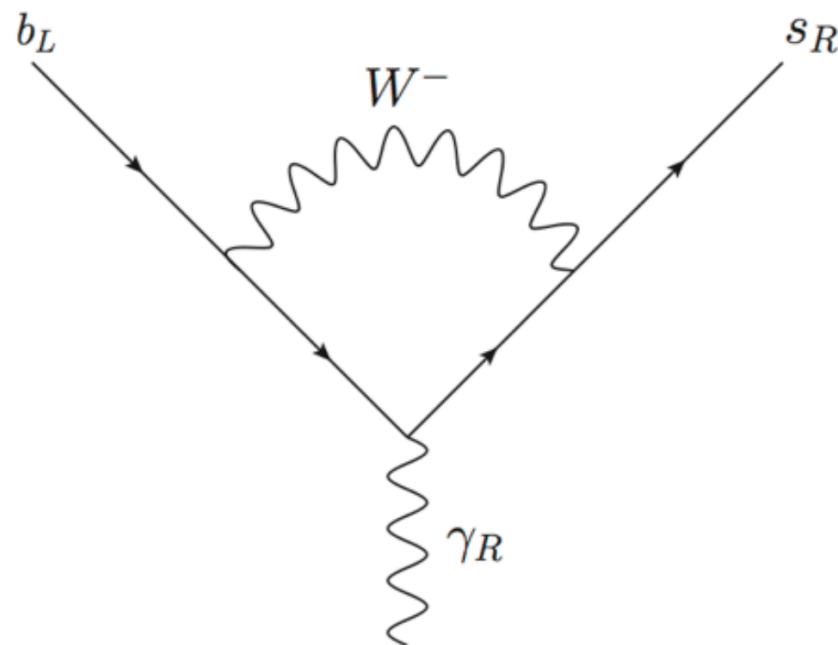
Photon polarisation in $b \rightarrow sy$ transitions

- In the SM, the photon in $b \rightarrow sy$ transitions is mostly **left handed** because the W couples left-handedly



Leading contribution

$$m_b \bar{s}_L \sigma_{\mu\nu} q^\nu b_R$$

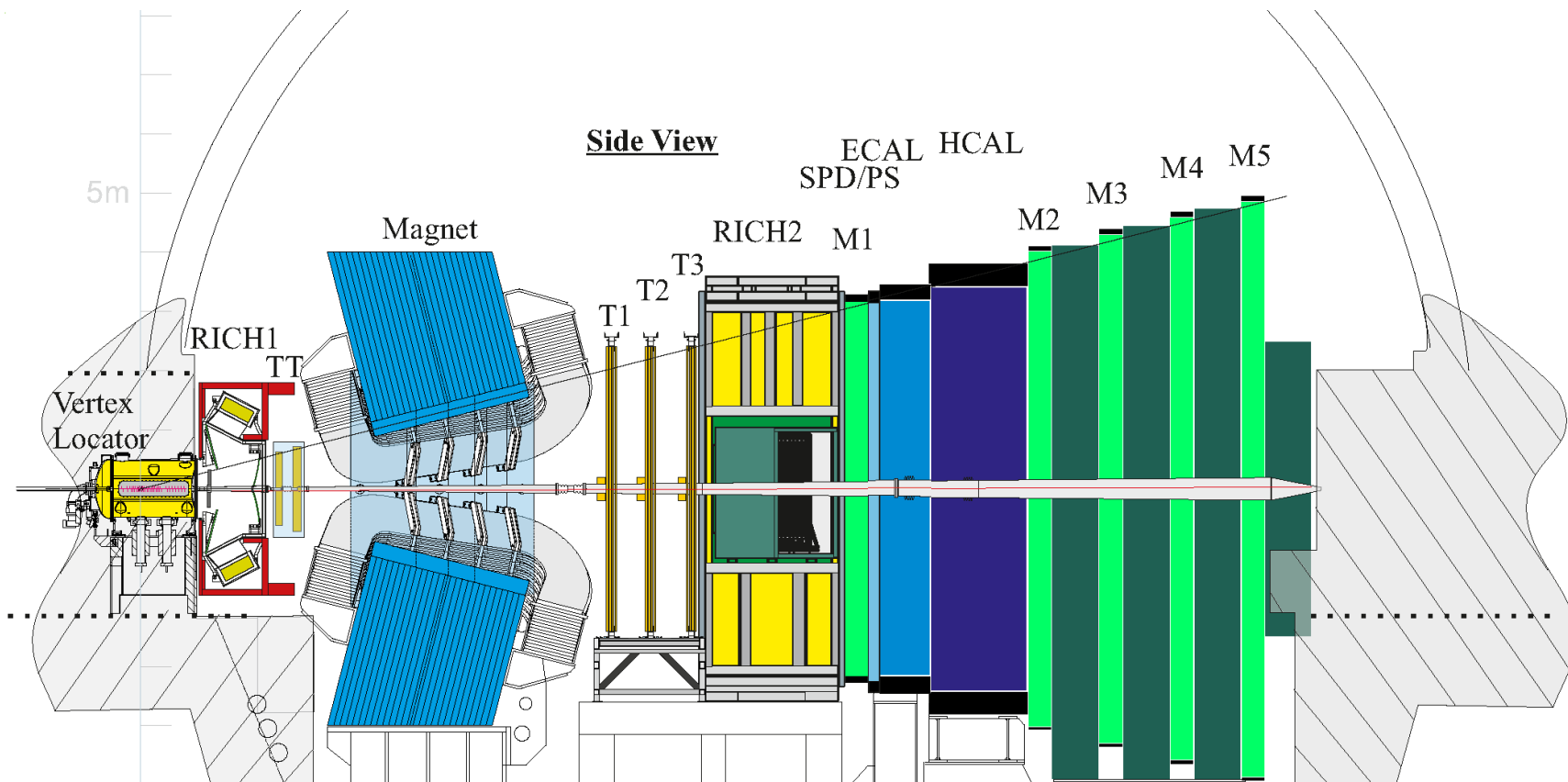


Contribution with chirality flip

$$m_s \bar{s}_R \sigma_{\mu\nu} q^\nu b_L$$

The LHCb detector

- Forward spectrometer dedicated to **heavy flavour physics** at LHC



Tracking:

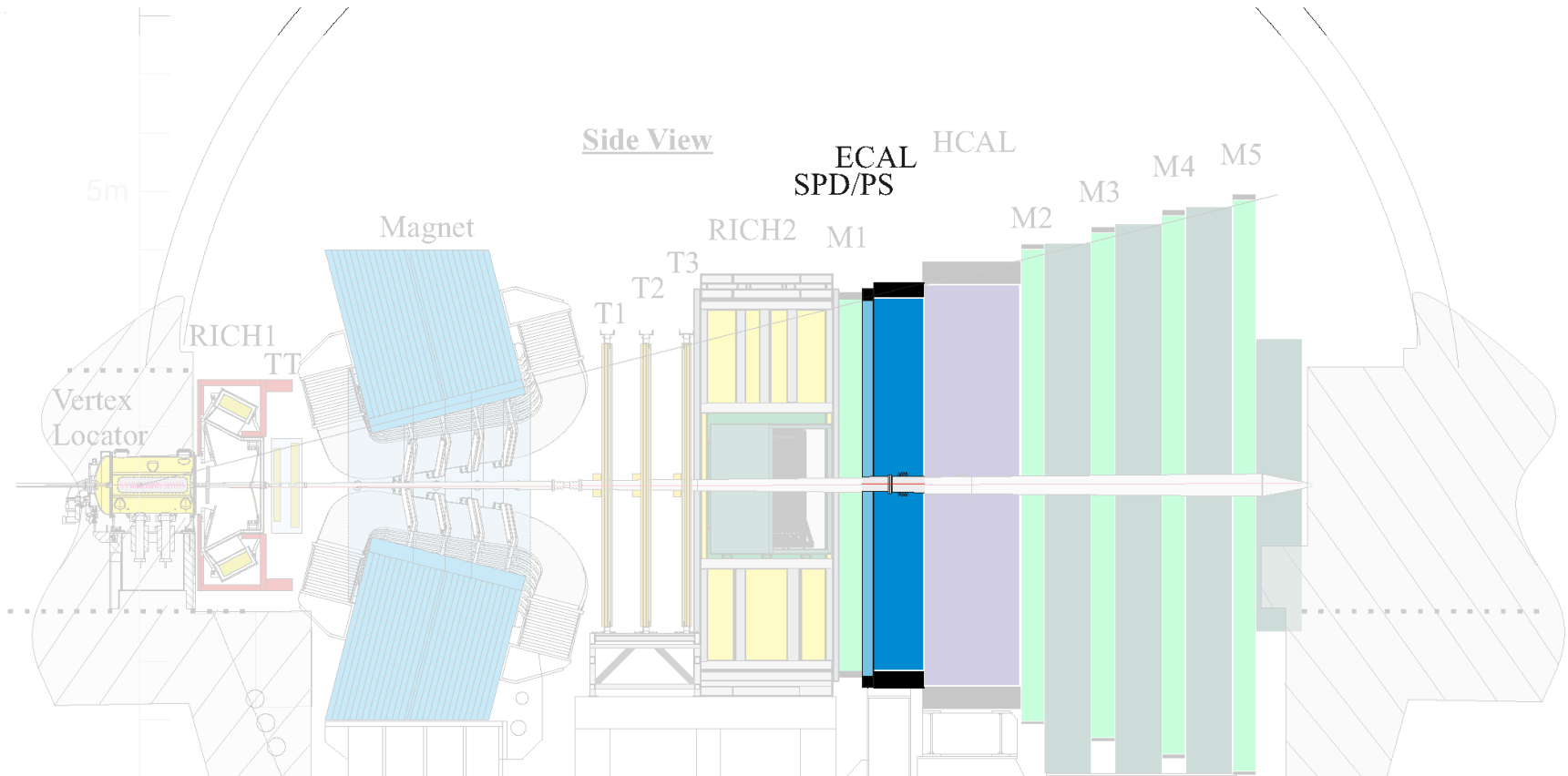
- ✓ $\Delta p/p \approx 0.5-1\%$ in the full momentum range
- ✓ $\sigma_{\tau} \approx 45$ fs and $\sigma_{IP} \approx 20$ μm for high- p_T tracks

Particle identification: K/ π separation over 2-100 GeV/c ($\epsilon_K \approx 90\%$ and $p_{\text{mis-id } \pi \rightarrow K} \approx 5\%$)

Calorimeter system: $\sigma_E/E \approx 10\%/\sqrt{E} + 1\%$

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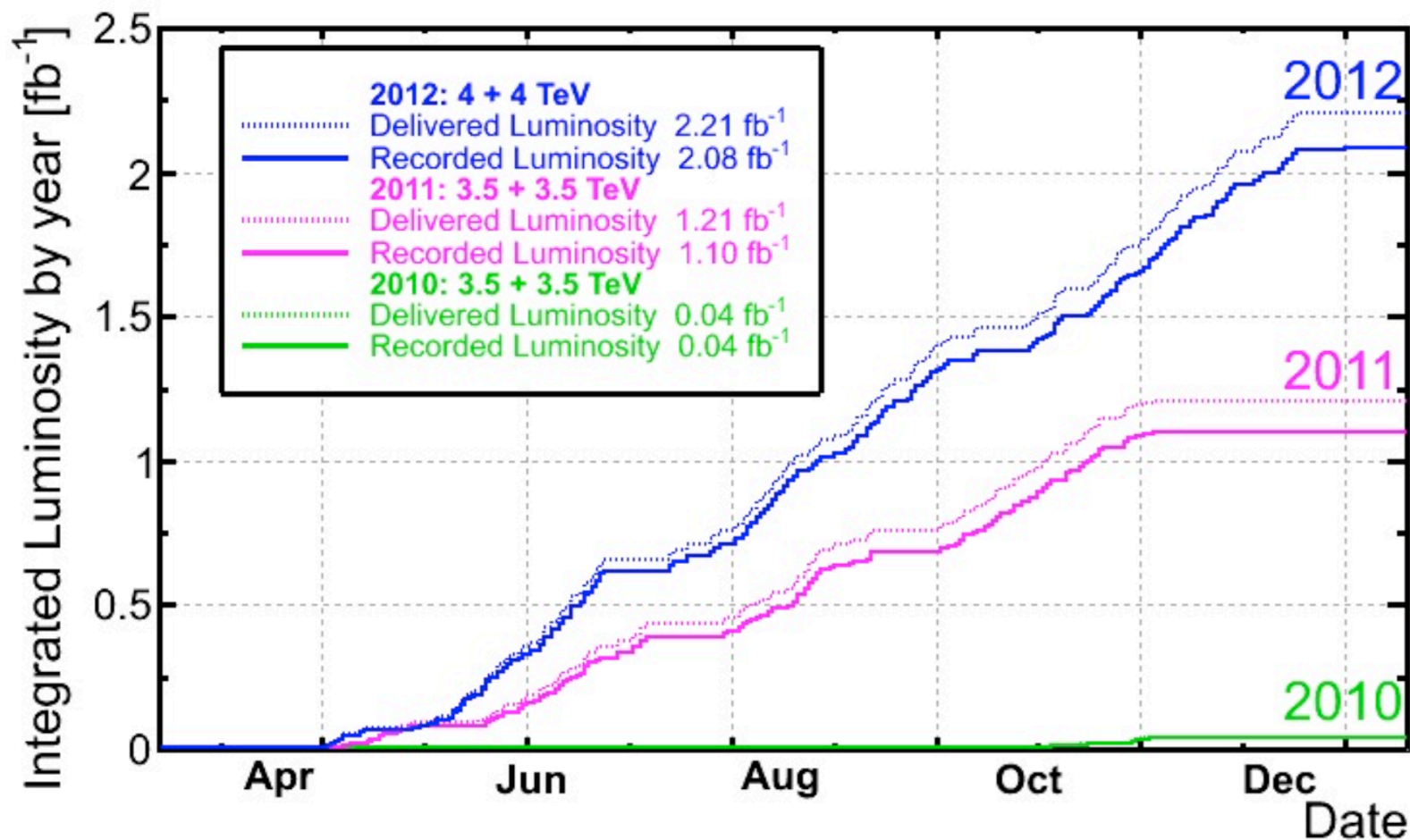
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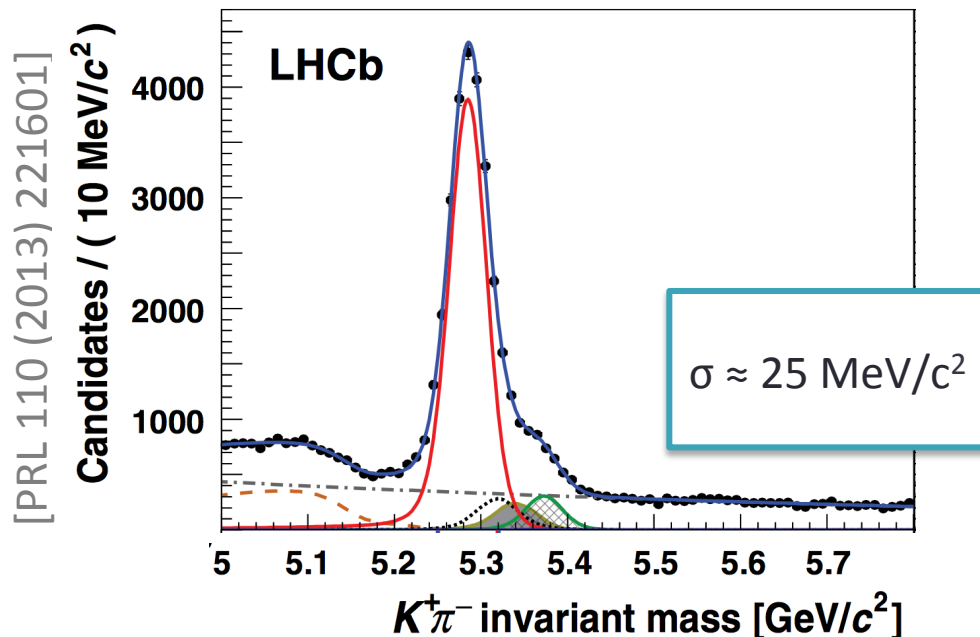
Operations

- Data taking: 2009 – 2012 (Run 1, integrated luminosity of 3fb^{-1})
- Now in Run 2: 2015 – 2018

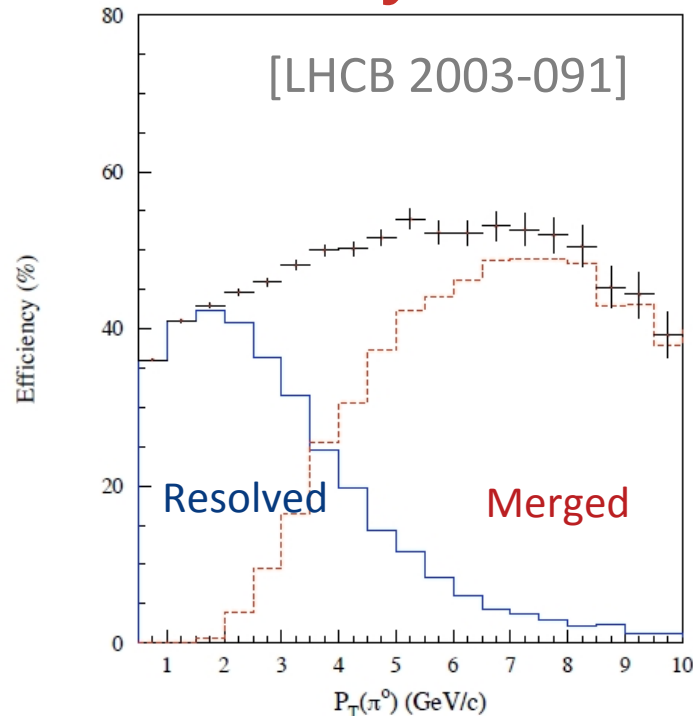
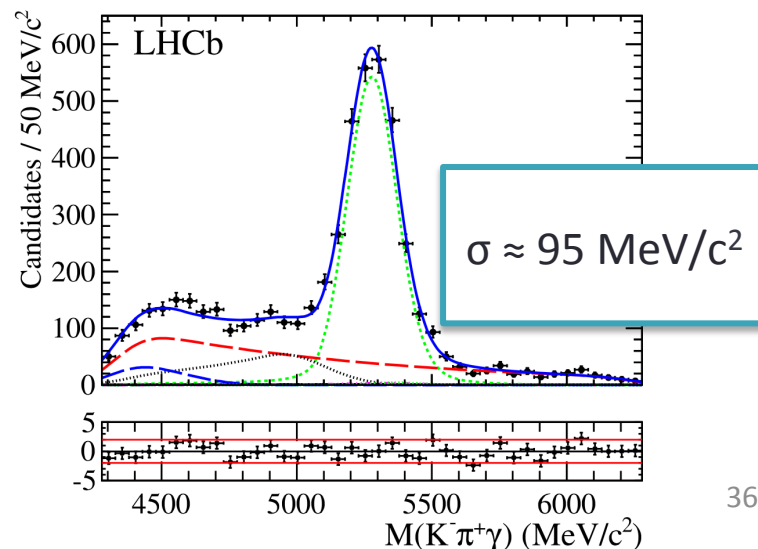


Main challenges for radiative decays

- High level of **background in pp collisions**
- For energies above 4 GeV the two clusters from $\pi^0 \rightarrow \gamma\gamma$ are reconstructed as a single cluster in the calorimeter
- Mass **resolution dominated by photon reconstruction**



[Nucl. Phys. B 867 (2012)]



Adding resonances is not so simple

The amplitude for a system containing $(1^+, 2^+, 1^-)$ is:

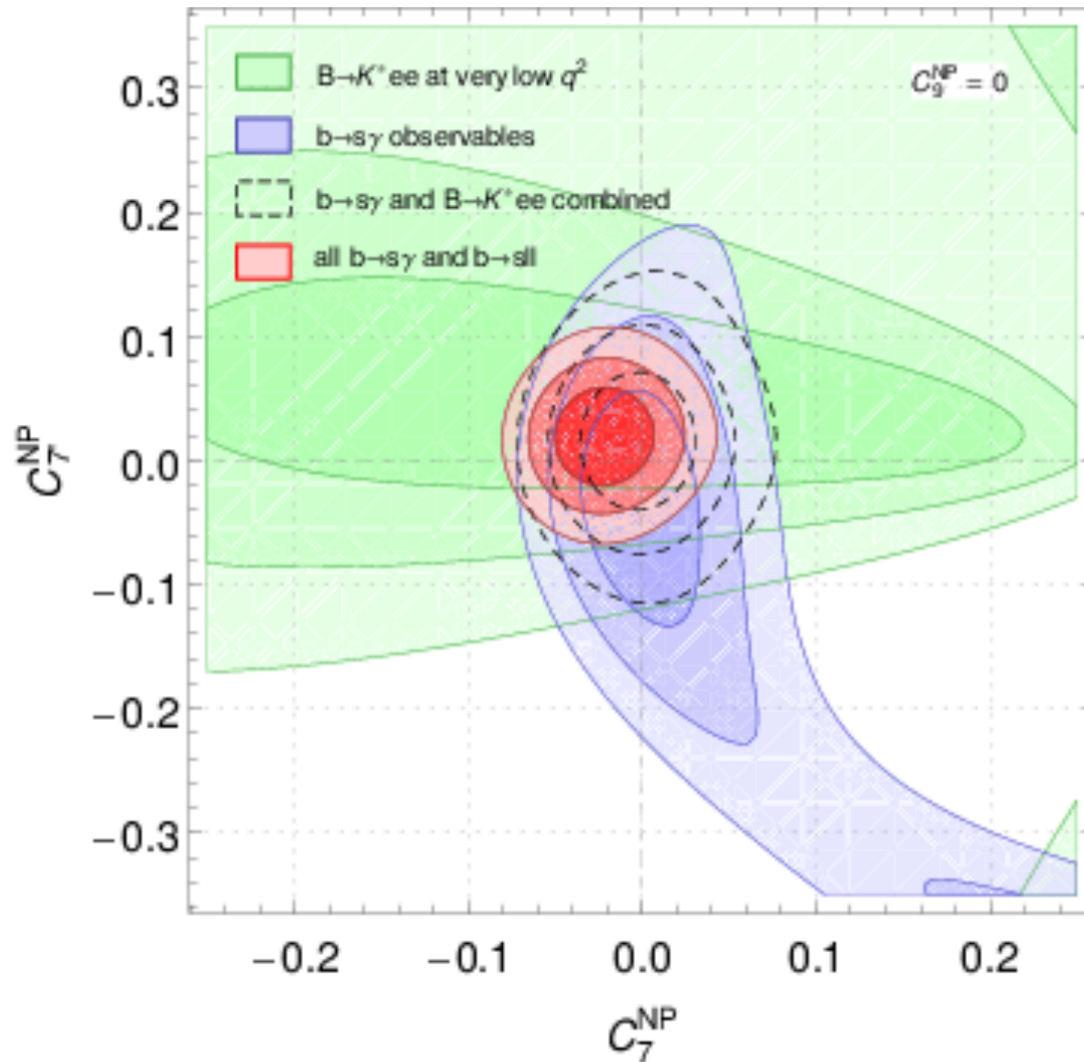
$$\mathcal{A}_{R,L}(\bar{B} \rightarrow \bar{K}\pi\pi\gamma_{R,L}) = A(\vec{\varepsilon}_{\pm} \cdot \vec{J}) \pm B \left((\vec{\varepsilon}_{\pm} \cdot \vec{n})(\vec{\varepsilon}_0 \cdot \vec{K}) + (\vec{\varepsilon}_{\pm} \cdot \vec{K})(\vec{\varepsilon}_0 \cdot \vec{n}) \right) \pm C(\vec{\varepsilon}_{\pm} \cdot \vec{n})$$

where A, B and C are constant obtained after factorizing out the terms depending on the polarisation and the final state momenta in the respective expressions of the amplitudes of the 1^+ , 2^+ and 1^- resonances.

The vectors are defined using the final state momenta:

$$\begin{aligned} \vec{J} &= C_1 \vec{p}_1 - C_2 \vec{p}_2 \quad , \\ \vec{K} &= |\vec{p}_1 \times \vec{p}_2| \{ \vec{p}_1 [B_{K^*}(s_{23}) + \kappa_{\rho} B_{\rho}(s_{12})] + \vec{p}_2 [B_{K^*}(s_{13}) + \kappa_{\rho} B_{\rho}(s_{12})] \} \end{aligned}$$

Constraints on C_7 and C_7^{NP}



[arXiv: 1510.04239]

Likelihood function

- Function to be minimized

$$\mathcal{L} = -2 \cdot \ln \left(\prod_{data} \mathcal{L}_i \right)$$

$$\mathcal{L}_i = \frac{y_i \cdot \left(\left| \sum_{amp} c_j \cdot A_j \right|^2 \right)}{\sum_{m \in MC} \frac{y_m}{N_{MC}} \sum_{m \in MC} \frac{w_m \cdot \left| \sum_{amp} c_j \cdot A_{jm} \right|^2}{\sum_{k \in MC} w_k}}$$

Amplitudes

- Amplitudes are products of spin factor and Breit-Wigner distributions

$$A_{K(1270) \rightarrow K\rho} = \mathcal{S}_f(B \rightarrow V_0 A(\rightarrow P_1 V(\rightarrow P_2 P_3)) BW(K(1270) \rightarrow K\rho) BW(\rho \rightarrow \pi\pi)$$

$$A_{K(1270) \rightarrow K^*\pi} = \mathcal{S}_f(B \rightarrow V_0 A(\rightarrow P_1 V(\rightarrow P_2 P_3)) BW(K(1270) \rightarrow K^*\pi) BW(K^* \rightarrow K\pi)$$

Spin-factors

- Spin factors:

$$\mathcal{S}_f(B \rightarrow V_0 A(\rightarrow P_1 V(\rightarrow P_2 P_3))) = \epsilon_\alpha^*(V_0) P_{(1)}^{\alpha\beta}(A) L_{(1)\beta}(V)$$

Conjugated polarisation vector

Spin1 projection tensor

Encodes the state of pure angular momentum 1 of the 2-particles system coming from V

Spin-factors

- where:

$$\epsilon^\mu(p, m = \mp 1) = \frac{\pm 1}{\sqrt{2}} \begin{pmatrix} 0 \\ \pm \frac{p_z}{E} + (1 - \frac{p_z}{E}) \left(\frac{p_y}{p_x^2 + p_y^2} \right) (\pm p_y - ip_x) \\ -i \frac{p_z}{E} + (1 - \frac{p_z}{E}) \left(\frac{p_x}{p_x^2 + p_y^2} \right) (\pm p_y - ip_x) \\ \frac{\sin(\arccos \frac{p_z}{E})}{p_x^2 + p_y^2} (\mp p_x + ip_y) \end{pmatrix}$$

$$P_{(1)}^{\mu\nu}(p) = \sum_m \epsilon^\mu(p, m) \epsilon^{*\nu}(p, m) = -g^{\mu\nu} + \frac{p^\mu p^\nu}{M^2}$$

$$L_{(1)}^\mu(p_R, q_R) = -P_{(1)\mu\nu}(p_R) q_R^\nu$$