Proton-Proton Hollowness From Inverse Scattering

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QCD and Diffraction - Saturation 1000+
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Alhambra-Wawel collaboration: 50+ papers, 800+ citations
Research with Wojcieh Broniowski

Based on [arXiv:1609.05597]
- Low energy nuclear reactions
- High energy NN scattering

Inelastic transition $\rightarrow$ Regge Transition $\rightarrow$ Hollowness transition
Larger, Blacker, Edgier
1000 eV+
Neutron-nucleus scattering

- Partial wave expansion elastic scattering amplitude

\[ f(\theta) = pR \sum_{l=0}^{\infty} (2l + 1) P_l(\cos \theta) \frac{S_l - 1}{2ip} \]

- Differential elastic cross section

\[ \frac{d\sigma_{el}}{d\Omega} = |f(\theta)|^2 \quad \rightarrow \quad \sigma_{el} = \frac{4\pi}{p^2} \sum_{l=0}^{\infty} (2l + 1)|1 - S_l|^2 \]

- Total Cross section and Optical theorem

\[ \sigma_T = \frac{4\pi}{p^2} \sum_{l=0}^{\infty} (2l + 1)(2 - 2\text{Re}S_l) = \frac{4\pi}{p} \text{Im}f(\theta = 0) \]

- Reaction (inelastic) cross section
Black disk $S_l = 0$ for $l \leq pR$ and $S_l = 1$ for $l > pR$

$$\sigma_T = \frac{4\pi}{p^2} \sum_{l=0}^{pR} (2l + 1)(2 - 2\text{Re}S_l) = 2\pi(R + 1/p)^2 = 2\sigma_{el}$$

Using $R = r_0 A^{1/3} = 1.2(208)^{1/3}$fm = 7fm

$$\sigma_T = 2\pi R^2 = 3\text{barn} \quad \sigma_{el} = \pi R^2 = 1.5\text{barn}$$
Optical model and Optical Potential

- Optical model (Energy dependent complex potential)
  \[ V(r, E) = \text{Re}V(r, E) + i\text{Im}V(r, E) \]

- Woods-Saxon forms
  \[ F(r) = \frac{1}{1 + e^{(r-R)/a}} \quad R = r_0A^{\frac{1}{3}} \quad a = 0.7\text{fm} \]

- Real and imaginary potentials
  \[ \text{Re}V(r) \sim F(r) \quad \text{Im}V(r) \sim F'(r) \]
1000 KeV+
Proton-Proton scattering

- First inelastic threshold ($1\pi$)

$$s = 4(p^2 + M_N^2) \quad \sqrt{s_{\text{in}}} = 2M_N + m_\pi \rightarrow E_L = 300\text{MeV}$$

- Scattering amplitude

$$M = a + m(\sigma_1 \cdot n)(\sigma_2 \cdot n) + (g - h)(\sigma_1 \cdot m)(\sigma_2 \cdot m)$$
$$+ (g + h)(\sigma_1 \cdot l)(\sigma_2 \cdot l) + c(\sigma_1 + \sigma_2) \cdot n$$

- Five complex $a, m, g, h, c$ depend on energy and angle
- 24 measurable cross-sections and polarization asymmetries
NN scattering below inelastic threshold

- Fit to 8000 np and pp scattering data from 1950-2013 below 350 MeV
- $\chi^2$/DOF = 1.04 $\rightarrow$ NN potential with errors [Navarro, Amaro, E.R.A. 2013]
**NN scattering below inelastic threshold**

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1000 MeV+
Inelastic resonance transition

- The inelastic transition at $\sqrt{s} = M_N + M_\Delta$

\[ NN \rightarrow \Delta N \rightarrow \pi NN \quad NN \rightarrow \Delta\Delta \rightarrow 2\pi NN \quad \ldots \]

- Partial wave analysis stops at 3 GeV

- We need a Complete set of measurements at higher energies !!!
Regge transition

- From Pion Exchange to Pomeron Exchange
- Motivated by Regge theory (spin neglected)

\[ A(s, t) \sim \sum_n A_n s^{\alpha_n(t)} \]
\[ \alpha_n(t) = \alpha_n(0) + \alpha'_n(0)t \]

- Two terms and a relative phase

\[ A(s, t) = i \left( \sqrt{A} e^{Bt/2} + \sqrt{C} e^{Dt/2} + i\phi \right) \]
1000 GeV+
From ISR to TOTEM

\[ \frac{d\sigma_{\text{el}}}{d(-t)} [\text{mb GeV}^{-2}] \]

- 23.4 GeV
- 7 TeV (x10^{-3})

E. Ruiz Arriola (U. Granada)
Parametrization of the scattering amplitude

S Parametrization by [Fagundes 2013], based on [Barger-Phillips 1974], motivated by the Regge asymptotics:

\[
\frac{f(s, t)}{p} = \sum_n c_n(s) F_n(t) s^{\alpha_n(t)} = \frac{i\sqrt{A} e^{\frac{Bt}{2}}}{(1 - \frac{t}{t_0})^4} + i\sqrt{C} e^{\frac{Dt}{2} + i\phi}
\]

s-dependent (real) parameters are fitted separately to all known differential pp cross sections for \(\sqrt{s} = 23.4, 30.5, 44.6, 52.8, 62.0, \text{and } 7000 \text{ GeV} \) with \(\chi^2/d.o.f \sim 1.2 - 1.7\)

\[
\frac{d\sigma_{el}}{dt} = \frac{\pi}{p^2} |f(s, t)|^2, \quad \sigma_T = \frac{4\pi}{p} \text{Im} f(s, 0)
\]
Eikonal approximation

\[ f(s, t) = \sum_{l=0}^{\infty} (2l + 1) f_l(p) P_l(\cos \theta) \]

\[ = \frac{p^2}{\pi} \int d^2 b h(\vec{b}, s) e^{i\vec{q} \cdot \vec{b}} = 2p^2 \int_0^\infty bdb J_0(bq) h(b, s) \]

\[ t = -q^2, \quad q = 2p \sin(\theta/2), \quad bp = l + 1/2 + O(s^{-1}), \quad P_l(\cos \theta) \rightarrow J_0(qb), \]

hence the amplitude in the impact-parameter representation becomes

\[ h(b, s) = \frac{i}{2p} \left[ 1 - e^{i\chi(b)} \right] = f_l(p) + O(s^{-1}) \]

The eikonal approximation works well for \( b < 2 \text{ fm} \) and \( \sqrt{s} > 20 \text{ GeV} \)

Procedure: \( f(s, t) \rightarrow h(b, s) \rightarrow \chi(b) \ldots \)
Eikonal approximation 2

The standard formulas for the total, elastic, and total inelastic cross sections read

$$\sigma_T = \frac{4\pi}{p} \text{Im} f(s, 0) = 4p \int d^2b \text{Im} h(b, s) = 2 \int d^2b \left[ 1 - \text{Re} e^{i\chi(b)} \right]$$

$$\sigma_{el} = \int d\Omega |f(s, t)|^2 = 4p^2 \int d^2b |h(b, s)|^2 = \int d^2b |1 - e^{i\chi(b)}|^2$$

$$\sigma_{in} \equiv \sigma_T - \sigma_{el} = \int d^2b n_{in}(b) = \int d^2b \left[ 1 - e^{-2\text{Im}\chi(b)} \right]$$

The inelasticity profile

$$n_{in}(b) = 4p\text{Im} h(b, s) - 4p^2|h(b, s)|^2$$

satisfies $n_{in}(b) \leq 1$ (unitarity)
Profiles
Dip in the inelasticity profile

From top to bottom: $\sqrt{s} = 14000, 7000, 200, 23.4$ GeV
Slope of the inelasticity profile

\[ \frac{d n_{\text{in}}(b)}{db^2} \mid_{b=0} \text{ [fm}^{-2}] \]

Transition around \( \sqrt{s} = 5 \text{ TeV} \)
Amplitude and eikonal phase

\[ 2p h(b) = i \left[ 1 - e^{i\chi(b)} \right] \]
Potentials
The field theory approach

- Multichannel Bethe-Salpeter equation (linear, off-shell)
  \[ T = V + VG_0T \]

- The Low equation for the elastic amplitude (non-linear, on-shell)
  \[ T_{el} = W + T_{el}G_0T_{el}^\dagger \]

- In partial waves
  \[ f_l(s) = w_l(s) + \frac{1}{\pi} \int_{s_0}^{\infty} \frac{f_l(s')p(s')f_l^\dagger(s')}{s' - s - i0^+} , \quad p(s) = \sqrt{s/4 - M_N^2} \]

- Unitarity (right cut)
  \[ \text{Im}f_l(s) - p|f_l(s)|^2 = \text{Im}w_l(s) \]

- Causality
  \[ w_l(s) = \frac{1}{\pi} \int_{s_0}^{\infty} \frac{\text{Im}w_l(s')}{s' - s - i0^+} ds' \]

- Particle exchange (left cut)
The on-shell optical potential

- Locality
  \[ w_l(s) = -\frac{1}{p} \int_0^\infty r^2 dr \left[ j_l(pr) \right]^2 W(r, s) \]

- Mandelstam representation (Superposition of Yukawas)
  \[ W(r, s) = \int_0^\infty d\mu \rho(\mu, s) \frac{e^{-\mu r}}{r} \]
  [Cornwall, Ruderman, PR (1965)]

- Dispersion relation
  \[ \text{Re}W(r, s) = \]

- Inelastic cross section (On shell optical potential)
  \[ \sigma_{\text{in}} \equiv \sigma_T - \sigma_{\text{el}} = -\frac{1}{p} \int d^3x \text{Im} W(\vec{x}, s) \]
The equivalent potential: Invariant mass approach

- Invariant mass method (free particles)

\[ \mathcal{M}^2 = P^\mu P_\mu = (p_1 + p_2)^{\text{CM}} = 4(p^2 + M_N^2) \]

- Invariant mass method: Interaction

\[ \mathcal{M}^2 = P^\mu P_\mu + V^{\text{CM}} = 4(p^2 + M_N^2) + V(\vec{x}) \rightarrow 4(-\nabla^2 + M_N^2) + V(\vec{x}) \]

- Non-relativistic limit \( V(x) = 4U(x) = 4M_N V(\vec{x}) \)

- Equivalent Schrödinger equation

\[ \left[ -\nabla^2 + U(\vec{x}) \right] \Psi(\vec{x}) = \left( s/4 - M_N^2 \right) \Psi(\vec{x}) \quad s = 4(p^2 + M_N^2) \]

- Optical potential

\[ U(\vec{x}) \rightarrow U(\vec{x}, s) = \text{Re}U(\vec{x}, s) + i\text{Im}U(\vec{x}, s) \]
The equivalent potential approach: Flux

- Scattering boundary condition

\[ \Psi(\vec{x}) \to e^{i\vec{k} \cdot \vec{x}} + f(\hat{x}) \frac{e^{ik}}{r} \]

- Flux balance

\[ \int d\Omega \lim_{r \to \infty} r^2 [\Psi^* \partial_r \Psi - \partial_r \Psi^* \Psi]_{\vec{x} = r \hat{x}} = \int d^3 x \text{Im} U(\vec{x}) |\Psi(\vec{x})|^2 \]

- Inelastic cross section

\[ \sigma_T - \sigma_{el} \equiv \sigma_{in} = -\frac{1}{p} \int d^3 x \text{Im} U(\vec{x}, s) |\psi(\vec{x})|^2 \]

- We can identify the On-shell Optical Potential with the “Standard” one

\[ \text{Im} W(\vec{x}, s) = \text{Im} U(\vec{x}, s) |\psi(\vec{x})|^2 \]

The OS-OP has no dependence on the wave function!
The equivalent potential: Eikonal approximation

- In the eikonal approximation

\[ \psi(\vec{x}) = \exp \left[ ipz - \frac{i}{2p} \int_{-\infty}^{z} U(b, z')dz' \right] \]

and thus

\[ \frac{1}{p} \text{Im } U(\vec{x}, s) |\psi(\vec{x})|^2 = \partial_z \exp \left[ \frac{1}{p} \int_{-\infty}^{z} \text{Im } U(b, z')dz' \right] \]

- The total

\[ -\frac{1}{p} \int d^3x \text{Im } W(\vec{x}, s) \xrightarrow{\text{eikonal}} \int d^2b \left( 1 - e^{-\text{Im } \chi(b)} \right) \]

- Transverse probability form the On-shell Optical Potential (no eikonal approximation !)

\[ -\frac{1}{p} \int_{-\infty}^{\infty} dz \text{Im } W(b, z) = n_{\text{in}}(b) \quad , \quad \int d^2b n_{\text{in}}(b) = \sigma_{\text{in}} \]
Inversion
Inverse scattering and optical potential

In the eikonal approximation one has

\[ \Psi(\vec{x}) = \exp \left[ ipz - \frac{i}{2p} \int_{-\infty}^{z} U(\vec{b}, z') dz' \right] \]

\[ \chi(b) = -\frac{1}{2p} \int_{-\infty}^{\infty} U(\sqrt{b^2 + z^2}) dz = -\frac{1}{p} \int_{b}^{\infty} \frac{rU(r)dr}{\sqrt{r^2 - b^2}} \]

is the (complex) eikonal phase [Glauber 1959]. This Abel-type equation can be inverted:

\[ U(r) = M_N V(r) = \frac{2p}{\pi} \int_{r}^{\infty} db \frac{\chi'(b)}{\sqrt{b^2 - r^2}} \]
On-shell optical potential

From the definition of the inelastic cross section

\[
\sigma_{\text{in}} = -\frac{1}{p} \int d^3x \ \text{Im} \ U(\vec{x})|\Psi(\vec{x})|^2
\]

→ density of inelasticity is proportional to the absorptive part of the optical potential times the square of the modulus of the wave function. One can identify the on-shell optical potential as

\[
\text{Im} \ W(\vec{x}) = \text{Im} \ U(\vec{x})|\Psi(\vec{x})|^2
\]

Upon \(z\) integration,

\[
-\frac{1}{p} \int dz \text{Im} \ W(\vec{b}, z) = n_{\text{in}}(b)
\]

Inversion yields

\[
\text{Im} W(r) = \frac{2p}{\pi} \int_{r}^{\infty} db \frac{n'(b)}{\sqrt{b^2 - r^2}}
\]
The hollowness transition

exp. amplitude $\rightarrow$ eikonal phase $\rightarrow$ $U(r) = M_N V(r)$
exp. amplitude $\rightarrow$ inelasticity profile $\rightarrow$ $W(r)$

From top to bottom: $\sqrt{s} = 14000, 7000, 200, 23.4$ GeV
Large dip in the absorptive parts, in $W(r)$ starts already at RHIC!
No classical folding of absorptive parts

The hollowness effect cannot be reproduced by by folding of uncorrelated proton structures. We would then get, small $r$

$$W(r) = \int d^3y \rho(\vec{y} + \vec{r}/2) \rho(\vec{y} - \vec{r}/2)$$

$$= \int d^3y \rho(\vec{y})^2 - \frac{1}{4} \int d^3y [\vec{r} \cdot \nabla \rho(\vec{y})]^2 + \ldots$$

$\rightarrow W(r)$ would necessarily have a local maximum at $r = 0$, in contrast to the phenomenological result

$\rightarrow$ not possible to obtain hollowness classically by folding the absorptive parts from uncorrelated constituents
2D vs 3D opacity

Projection of 3D on 2D covers up the hollow: \( f(x, y, z) \) vs 
\[ \int_{-\infty}^{\infty} dz f(x, y, z) \]

The hollow is covered up
Aspects of unitarity: model of [Dremin 2014]

\[ 2p\text{Im}h(b) \equiv k(b) = 4X e^{-b^2/(2B)}, \quad \text{Re}h(b) = 0, \quad X = \sigma_{el}/\sigma_T \]

\[ n_{in}(b) = 2k(b) - k(b)^2 = 8X e^{-b^2/(2B^2)} - 16X^2 e^{-b^2/B^2} \]

\[ \bullet \quad X > 1/4: \, n_{in}(b) \text{ has a maximum at } b_0 = \sqrt{2}B \log(4X) > 0, \text{ with } k(b_0) = 1 \]

\[ \bullet \quad X = 1/2: \, \text{black disk limit} \]

\[ \bullet \quad W(r) \text{ develops a dip when } X > \sqrt{2}/8 = 0.177 \]
Cross sections

Ratio goes above 1/4 as energy increases!
Aspects of unitarity 2

If \( 2p \chi(b) = \chi(b) \) is not necessarily Gaussian but purely imaginary, then

\[
n_{in}(b) = 2\chi(b) - \chi(b)^2
\]

\[
\frac{dn_{in}(b)}{db^2} = 2\frac{d\chi(b)}{db^2} [1 - \chi(b)]
\]

hence the minimum of \( n(b) \) moves away from the origin when \( \chi(0) > 1 \)

The real part of the amplitude, which is \( \sim 10\% \), brings in corrections at the level of \( 1\% \)
From $1000\text{+GeV}$ to $1000\text{+eV}$
Epilogue

- Are low energy nuclear reactions and high NN interactions that different?

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$\sqrt{s} = 7 \text{ TeV}$

$\sqrt{s} = 23-62 \text{ GeV}$

$E = 600 \text{ MeV}$

$E = 100 \text{ MeV}$
Conclusions

- There is a **hollowness transition** at $1000+\text{ GeV}$
- Quantum effect, rise of $2p\text{Im}h(b)$ above 1
- Not possible to obtain classically by folding the absorptive parts from uncorrelated constituents
- $2\text{D} \rightarrow 3\text{D}$ magnifies the effect (flat in 2D means hollow in 3D)
- Microscopic/dynamical explanations open [Alba Soto, Albacete 2016]
- Similar hollowness effect in low-energy n-A scattering