Proton-Proton Hollowness From Inverse Scattering

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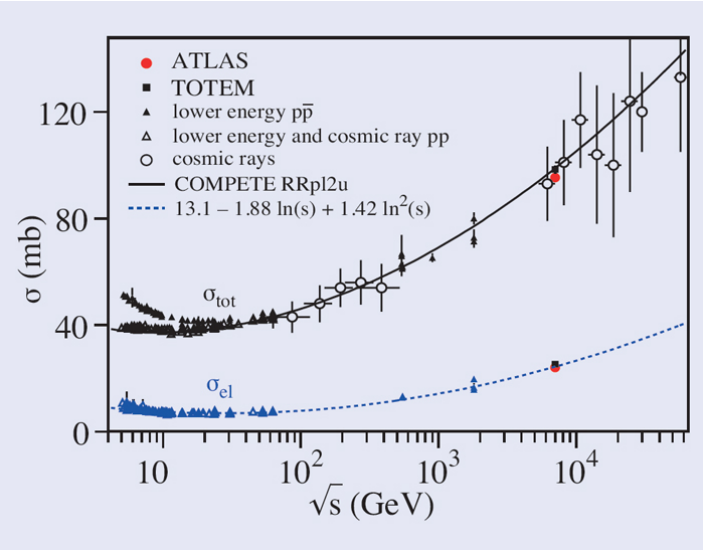
QCD and Diffraction - Saturation 1000+
Krakow, 5-7 Decembre 2016

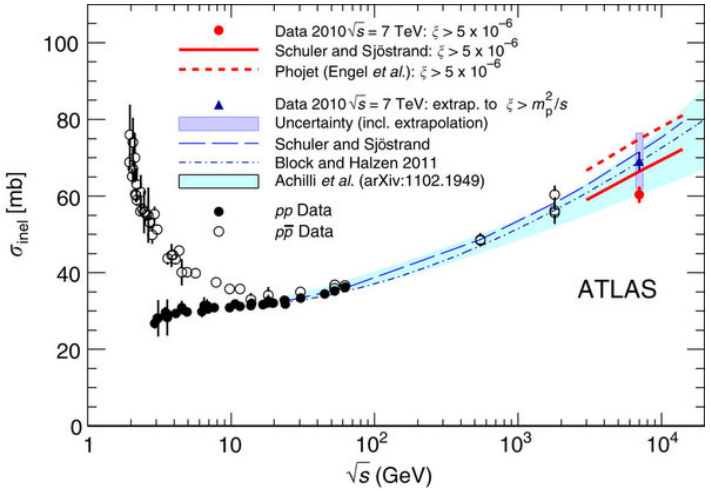
Alhambra-Wawel collaboration: 50+ papers , 800+ citations

Research with **Wojcieh Broniowski**

Based on [arXiv:1609.05597]

- Low energy nuclear reactions
- High energy NN scattering



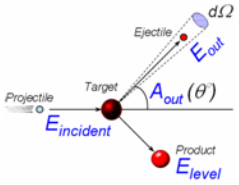


Inelastic transition → Regge Transition → Hollowness transition

Larger , Blacker, Edgier

1000 eV+

Neutron-nucleus scattering



- Partial wave expansion elastic scattering amplitude

$$f(\theta) = \sum_{l=0}^{pR} (2l+1) P_l(\cos\theta) \frac{S_l - 1}{2ip}$$

- Differential elastic cross section

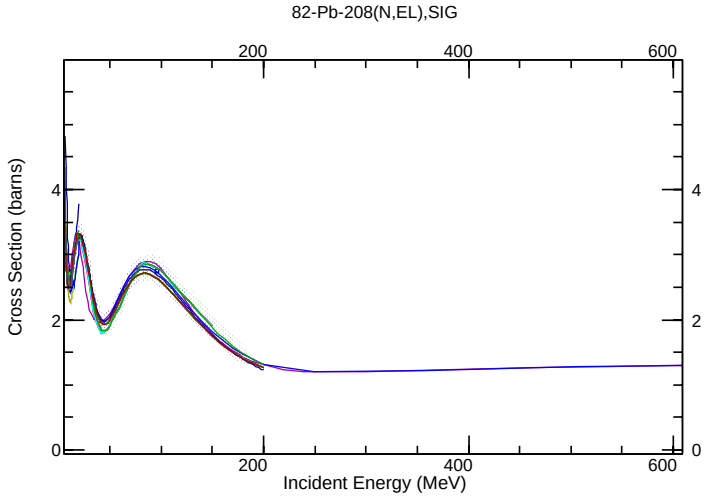
$$\frac{d\sigma_{el}}{d\Omega} = |f(\theta)|^2 \quad \rightarrow \sigma_{el} = \frac{4\pi}{p^2} \sum_{l=0}^{\infty} (2l+1)|1 - S_l|^2$$

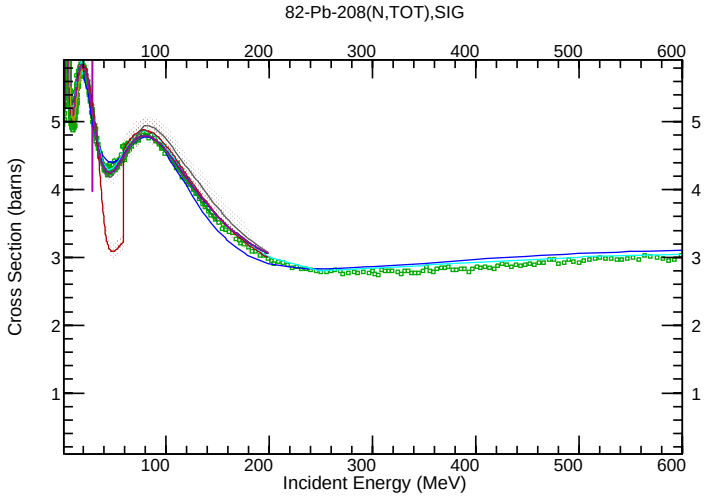
- Total Cross section and Optical theorem

$$\sigma_T = \frac{4\pi}{p^2} \sum_{l=0}^{\infty} (2l+1)(2 - 2\mathrm{Re} S_l) = \frac{4\pi}{p} \mathrm{Im} f(\theta = 0)$$

- Reaction (inelastic) cross section

Neutron-Nucleus scattering





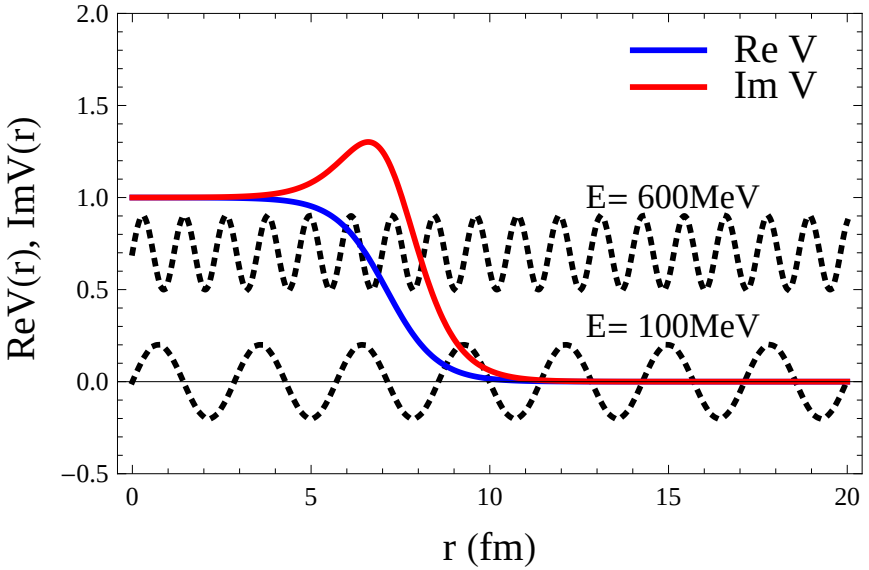
- Black disk $S_l = 0$ for $l \leq pR$ and $S_l = 1$ for $l > pR$

$$\sigma_T = \frac{4\pi}{p^2} \sum_{l=0}^{pR} (2l+1)(2 - 2\mathrm{Re}S_l) = 2\pi(R + 1/p)^2 = 2\sigma_{\mathrm{el}}$$

- Using $R = r_0 A^{\frac{1}{3}} = 1.2(208)^{\frac{1}{3}}\,\mathrm{fm} = 7\mathrm{fm}$

$$\sigma_T = 2\pi R^2 = 3\mathrm{barn} \qquad \sigma_{\mathrm{el}} = \pi R^2 = 1.5\mathrm{barn}$$

Optical model and Optical Potential



- Optical model (Energy dependent complex potential)

$$V(r,E) = \mathrm{Re}V(r,E) + i\mathrm{Im}V(r,E)$$

- Woods-Saxon forms

$$F(r) = \frac{1}{1+e^{(r-R)/a}} \qquad R = r_0 A^{\frac{1}{3}} \qquad a = 0.7\mathrm{fm}$$

- Real and imaginary potentials

$$\mathrm{Re}V(r) \sim F(r) \qquad \mathrm{Im}V(r) \sim F'(r)$$

1000 KeV+

Proton-Proton scattering

- First inelastic threshold (1π)

$$s = 4(p^2 + M_N^2) \qquad \sqrt{s_{\rm in}} = 2M_N + m_\pi \to E_L = 300{\rm MeV}$$

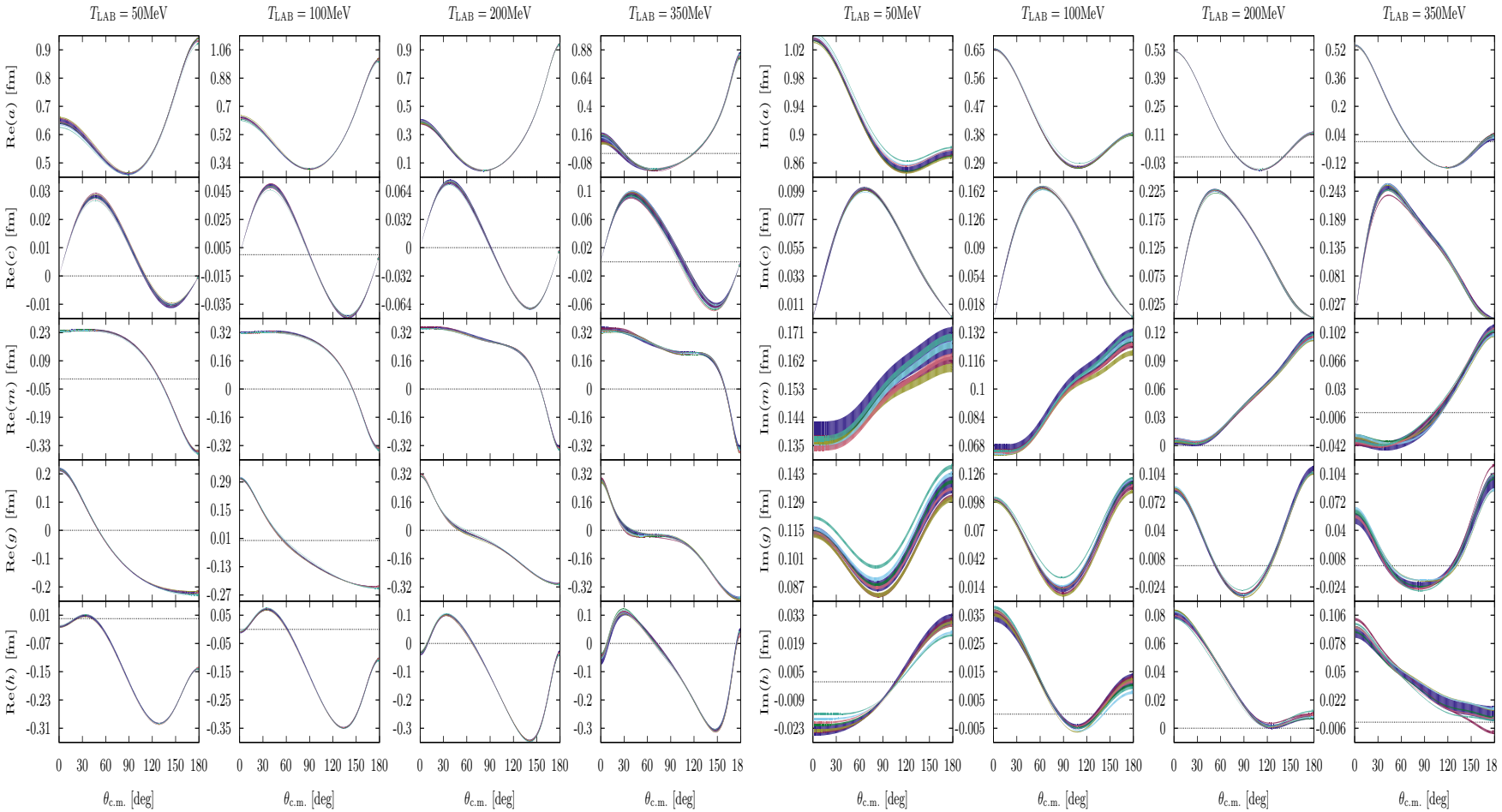
- Scattering amplitude

$$\begin{aligned} M &= a + m(\sigma_1 \cdot \mathbf{n})(\sigma_2 \cdot \mathbf{n}) + (g-h)(\sigma_1 \cdot \mathbf{m})(\sigma_2 \cdot \mathbf{m}) \\ &+ (g+h)(\sigma_1 \cdot \mathbf{l})(\sigma_2 \cdot \mathbf{l}) + c(\sigma_1 + \sigma_2) \cdot \mathbf{n} \end{aligned}$$

- Five complex a, m, g, h, c depend on energy and angle
- 24 measurable cross-sections and polarization asymmetries

NN scattering below inelastic threshold

- Fit to 8000 np and pp scattering data from 1950-2013 below 350 MeV
- $\chi^2/\mathrm{DOF} = 1.04 \rightarrow$ NN potential with errors [Navarro,Amaro, E.R.A. 2013]



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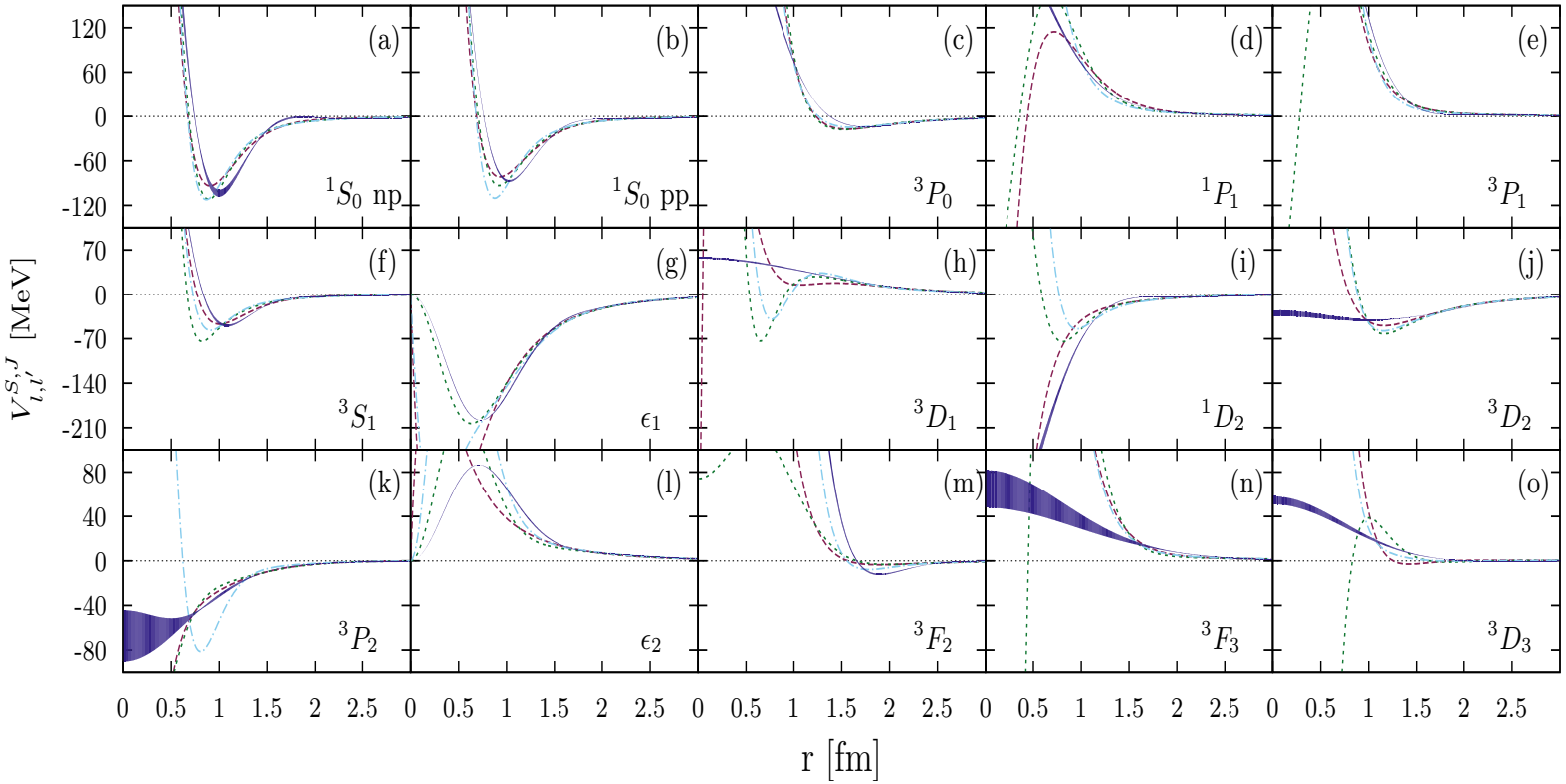


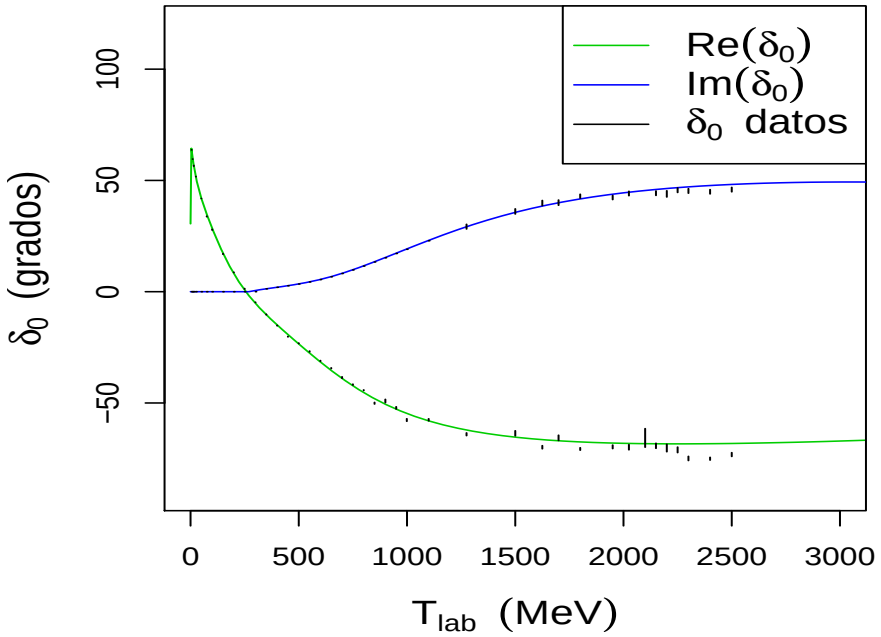
1000 MeV+

Inelastic resonance transition

- The inelastic transition at $\sqrt{s} = M_N + M_\Delta$

$$NN \to \Delta N \to \pi NN \qquad NN \to \Delta\Delta \to 2\pi NN \qquad \ldots$$

- Partial wave analysis stops at 3 GeV



- We need a Complete set of measurements at higher energies !!!

Regge transition

- From Pion Exchange to Pomeron Exchange
- Motivated by Regge theory (spin neglected)

[Barger,Phillips PLB (1972)]

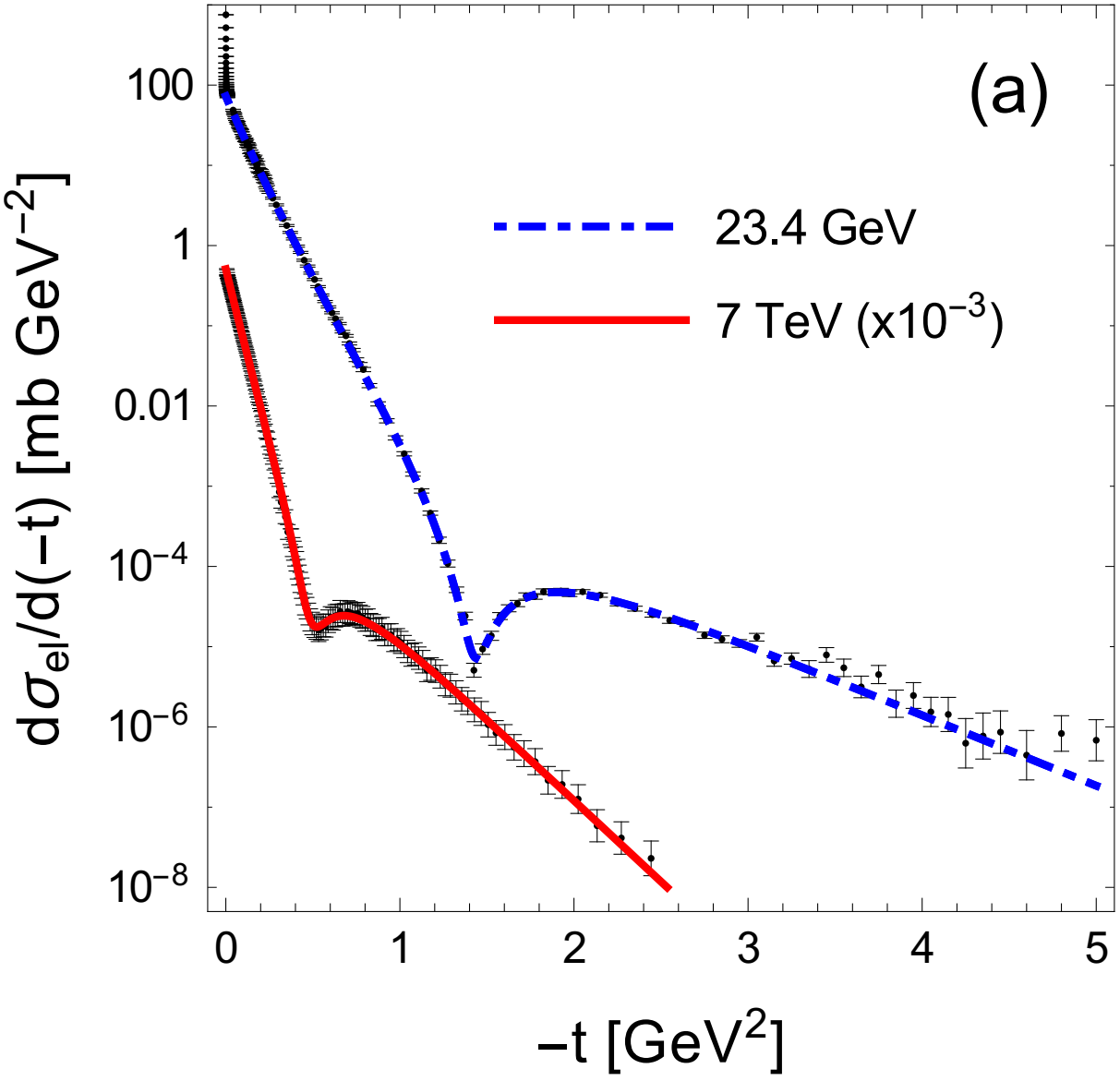
$$\mathcal{A}(s,t) \sim \sum_n A_n s^{\alpha_n(t)} \qquad \alpha_n(t) = \alpha_n(0) + \alpha_n'(0)t$$

- Two terms and a relative phase

$$A(s,t) = i\left(\sqrt{A}e^{Bt/2} + \sqrt{C}e^{Dt/2+i\phi}\right)$$

1000 GeV+

From ISR to TOTEM



Parametrization of the scattering amplitude

S Parametrization by [Fagundes 2013], based on [Barger-Phillips 1974], motivated by the Regge asymptotics:

$$\frac{f(s,t)}{p} = \sum_n c_n(s) F_n(t) s^{\alpha_n(t)} = \frac{i\sqrt{A}e^{\frac{Bt}{2}}}{\left(1-\frac{t}{t_0}\right)^4} + i\sqrt{C}e^{\frac{Dt}{2}+i\phi}$$

s-dependent (real) parameters are fitted separately to all known differential pp cross sections for $\sqrt{s} = 23.4$, 30.5, 44.6, 52.8, 62.0, and 7000 GeV with $\chi^2/\mathrm{d.o.f} \sim 1.2 - 1.7$

$$\frac{d\sigma_{\mathrm{el}}}{dt} = \frac{\pi}{p^2}|f(s,t)|^2, \quad \sigma_T = \frac{4\pi}{p}\mathrm{Im} f(s,0)$$

Eikonal approximation

$$f(s,t) = \sum_{l=0}^{\infty}(2l+1)f_l(p)P_l(\cos\theta)$$
$$= \frac{p^2}{\pi}\int d^2b\, h(\vec{b},s)\, e^{i\vec{q}\cdot\vec{b}} = 2p^2\int_0^{\infty} bdbJ_0(bq)h(b,s)$$

$t = -\vec{q}^2$, $q = 2p\sin(\theta/2)$, $bp = l + 1/2 + \mathcal{O}(s^{-1})$, $P_l(\cos\theta) \to J_0(qb)$, hence the amplitude in the impact-parameter representation becomes

$$h(b,s) = \frac{i}{2p}\left[1 - e^{i\chi(b)}\right] = f_l(p) + \mathcal{O}(s^{-1})$$

The eikonal approximation works well for $b < 2$ fm and $\sqrt{s} > 20$ GeV

Procedure: $f(s,t) \to h(b,s) \to \chi(b)$...

Eikonal approximation 2

The standard formulas for the total, elastic, and total inelastic cross sections read

$$\begin{aligned}
\sigma_T &= \frac{4\pi}{p}\mathrm{Im} f(s,0) = 4p\int d^2b \mathrm{Im} h(\vec{b},s) = 2\int d^2b\left[1-\mathrm{Re}\, e^{i\chi(b)}\right] \\
\sigma_{\mathrm{el}} &= \int d\Omega |f(s,t)|^2 = 4p^2\int d^2b|h(\vec{b},s)|^2 = \int d^2b|1-e^{i\chi(b)}|^2 \\
\sigma_{\mathrm{in}} &\equiv \sigma_T - \sigma_{\mathrm{el}} = \int d^2b n_{\mathrm{in}}(b) = \int d^2b\left[1-e^{-2\mathrm{Im}\chi(b)}\right]
\end{aligned}$$

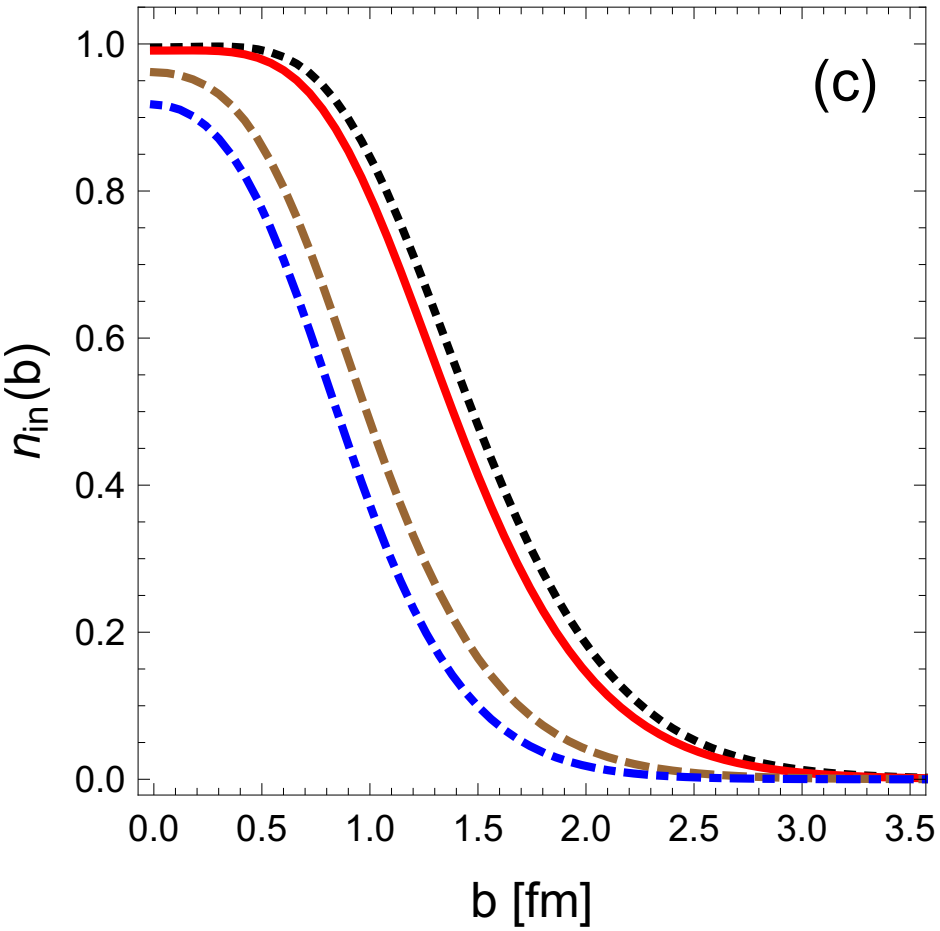
The inelasticity profile

$$n_{\mathrm{in}}(b) = 4p\mathrm{Im} h(b,s) - 4p^2|h(b,s)|^2$$

satisfies $n_{\mathrm{in}}(b) \leq 1$ (unitarity)

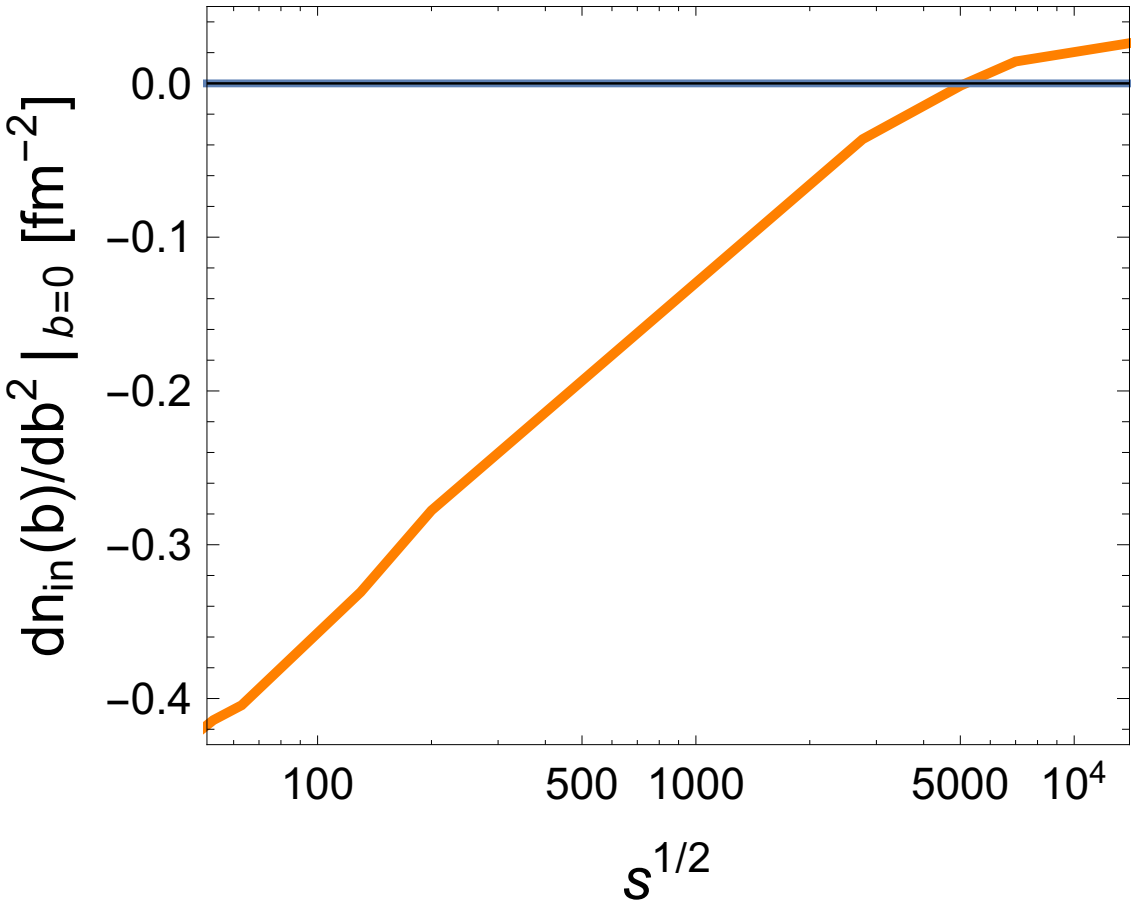
Profiles

Dip in the inelasticity profile



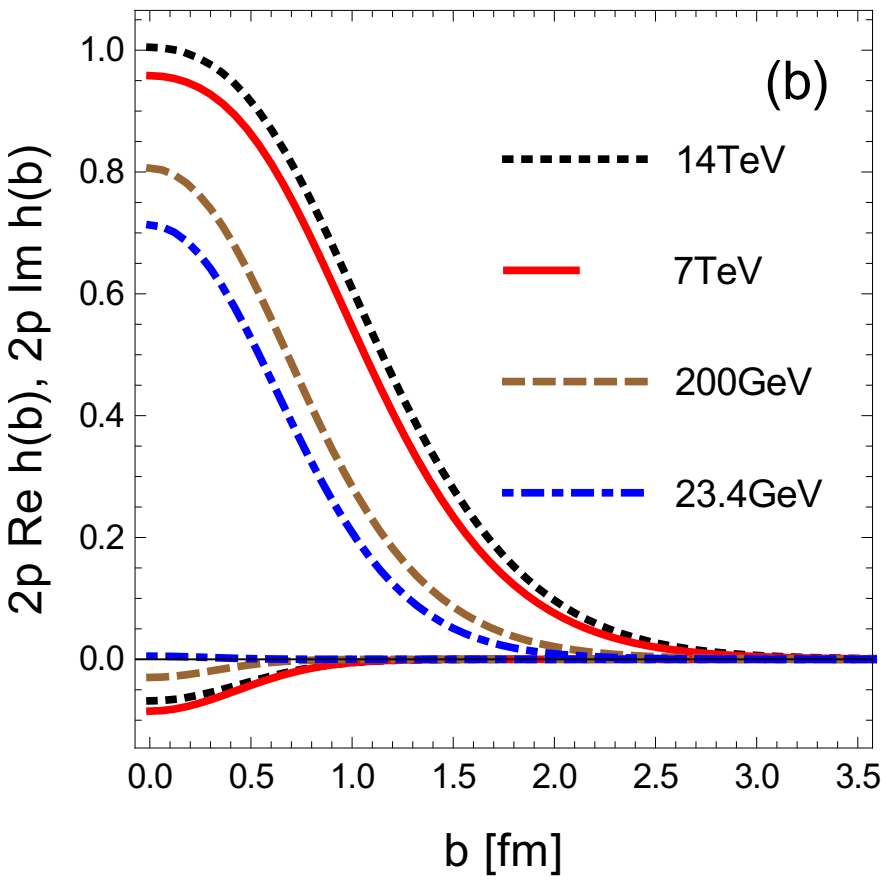
From top to bottom: $\sqrt{s} = 14000, 7000, 200, 23.4$ GeV

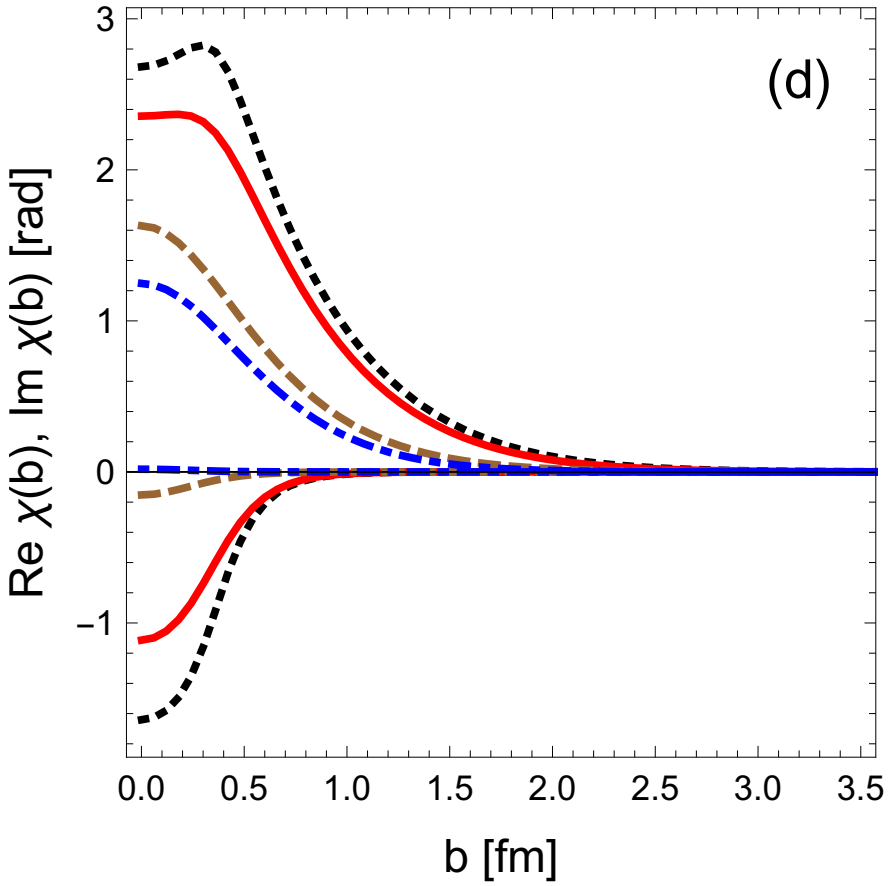
Slope of the inelasticity profile



Transition around $\sqrt{s} = 5$ TeV

Amplitude and eikonal phase





$$2p\, h(b) = i\left[1 - e^{i\chi(b)}\right]$$

(top curves - Im, bottom - Re)

Potentials

The field theory approach

- Multichannel Bethe-Salpeter equation (linear, off-shell)

$$T = V + VG_0T$$

- The Low equation for the elastic amplitude (non-linear, on-shell)

[Namyslowsi, PR 160 (1967)1522]

$$T_{\rm el} = W + T_{\rm el}G_0T_{\rm el}^\dagger$$

- In partial waves

$$f_l(s) = w_l(s) + \frac{1}{\pi}\int_{s_0}^{\infty}\frac{f_l(s')p(s')f_l^\dagger(s')}{s'-s-i0^+} \quad , \quad p(s) = \sqrt{s/4 - M_N^2}$$

- Unitarity (right cut)

$$\mathrm{Im} f_l(s) - p|f_l(s)|^2 = \mathrm{Im} w_l(s)$$

- Causality

$$w_l(s) = \frac{1}{\pi}\int_{s_0}^{\infty}\frac{\mathrm{Im} w_l(s')}{s'-s-i0^+}ds'$$

- Particle exchange (left cut)

The on-shell optical potential

- Locality

$$w_l(s) = -\frac{1}{p}\int_0^\infty r^2 dr\,[j_l(pr)]^2\,W(r,s)$$

- Mandelstam representation (Superposition of Yukawas)

[Cornwall,Ruderman, PR (1965)]

$$W(r,s) = \int_0^\infty d\mu\rho(\mu,s)\frac{e^{-\mu r}}{r}$$

- Dispersion relation

$$\mathrm{Re}W(r,s) =$$

- Inelastic cross section (On shell optical potential)

$$\sigma_{\rm in} \equiv \sigma_T - \sigma_{\rm el} = -\frac{1}{p}\int d^3x\,\mathrm{Im}\,W(\vec{x},s)$$

The equivalent potential: Invariant mass approach

- Invariant mass method (free particles)

$$\mathcal{M}^2 = P^\mu P_\mu = (p_1 + p_2) \stackrel{\text{CM}}{=} 4(p^2 + M_N^2)$$

- Invariant mass method: Interaction

[Allen,Payne,Polyzou PRD (2000)]

$$\mathcal{M}^2 = P^\mu P_\mu + \mathcal{V} \stackrel{\text{CM}}{=} 4(p^2 + M_N^2) + \mathcal{V}(\vec{x}) \to 4(-\nabla^2 + M_N^2) + \mathcal{V}(\vec{x})$$

- Non-relativistic limit $\mathcal{V}(x) = 4U(x) = 4M_N V(\vec{x})$
- Equivalent Schrödinger equation

$$\left[-\nabla^2 + U(\vec{x})\right] \Psi(\vec{x}) = (s/4 - M_N^2)\Psi(\vec{x}) \qquad s = 4(p^2 + M_N^2)$$

- Optical potential

$$U(\vec{x}) \to U(\vec{x}, s) = \mathrm{Re}U(\vec{x}, s) + i\mathrm{Im}U(\vec{x}, s)$$

The equivalent potential approach: Flux

- Scattering boundary condition

$$\Psi(\vec{x}) \to e^{i\vec{k}\cdot\vec{x}} + f(\hat{x})\frac{e^{ik}}{r}$$

- Flux balance

$$\int d\Omega \lim_{r\to\infty} r^2 \left[\Psi^* \partial_r \Psi - \partial_r \Psi^* \Psi\right]_{\vec{x}=r\hat{x}} = \int d^3x \mathrm{Im} U(\vec{x}) |\Psi(\vec{x})|^2$$

- Inelastic cross section

$$\sigma_T - \sigma_{\mathrm{el}} \equiv \sigma_{\mathrm{in}} = -\frac{1}{p}\int d^3x \mathrm{Im}\, U(\vec{x}, s)|\psi(\vec{x})|^2$$

- We can identify the On-shell Optical Potential with the "Standard" one

$$\mathrm{Im}\, W(\vec{x}, s) = \mathrm{Im}\, U(\vec{x}, s)|\psi(\vec{x})|^2$$

The OS-OP has no dependence on the wave function !

The equivalent potential: Eikonal approximation

- In the eikonal approximation

$$\psi(\vec{x}) = \exp\left[ipz - \frac{i}{2p}\int_{-\infty}^{z} U(b,z')dz'\right]$$

and thus

$$\frac{1}{p}\mathrm{Im}\, U(\vec{x},s)|\psi(\vec{x})|^2 = \partial_z \exp\left[\frac{1}{p}\int_{-\infty}^{z} \mathrm{Im}U(b,z')dz'\right]$$

- The total

$$-\frac{1}{p}\int d^3x \mathrm{Im}\, W(\vec{x},s) \stackrel{\text{eikonal}}{\longrightarrow} \int d^2b\left(1-e^{-\mathrm{Im}\chi(b)}\right)$$

- Transverse probability form the On-shell Optical Potential (no eikonal approximation !)

$$-\frac{1}{p}\int_{-\infty}^{\infty} dz \mathrm{Im}\, W(b,z) = n_{\mathrm{in}}(b) \quad , \qquad \int d^2b n_{\mathrm{in}}(b) = \sigma_{\mathrm{in}}$$

Inversion

Inverse scattering and optical potential

In the eikonal approximation one has

$$\Psi(\vec{x}) = \exp\left[ipz - \frac{i}{2p}\int_{-\infty}^{z} U(\vec{b}, z')dz'\right]$$

$$\chi(b) = -\frac{1}{2p}\int_{-\infty}^{\infty} U(\sqrt{b^2+z^2})dz = -\frac{1}{p}\int_{b}^{\infty}\frac{rU(r)\,dr}{\sqrt{r^2-b^2}}$$

is the (complex) eikonal phase [Glauber 1959]. This Abel-type equation can be inverted:

$$U(r) = M_N V(r) = \frac{2p}{\pi}\int_{r}^{\infty} db\frac{\chi'(b)}{\sqrt{b^2-r^2}}$$

On-shell optical potential

From the definition of the inelastic cross section

$$\sigma_{\rm in} = -\frac{1}{p}\int d^3x\, {\rm Im}\, U(\vec{x})|\Psi(\vec{x})|^2$$

$\rightarrow$ density of inelasticity is proportional to the absorptive part of the optical potential times the square of the modulus of the wave function. One can identify the *on-shell optical potential* as

$$\mathrm{Im}\, W(\vec{x}) = \mathrm{Im}\, U(\vec{x})|\Psi(\vec{x})|^2$$

Upon z integration,

$$-\frac{1}{p}\int dz \mathrm{Im}\, W(\vec{b}, z) = n_{in}(b)$$

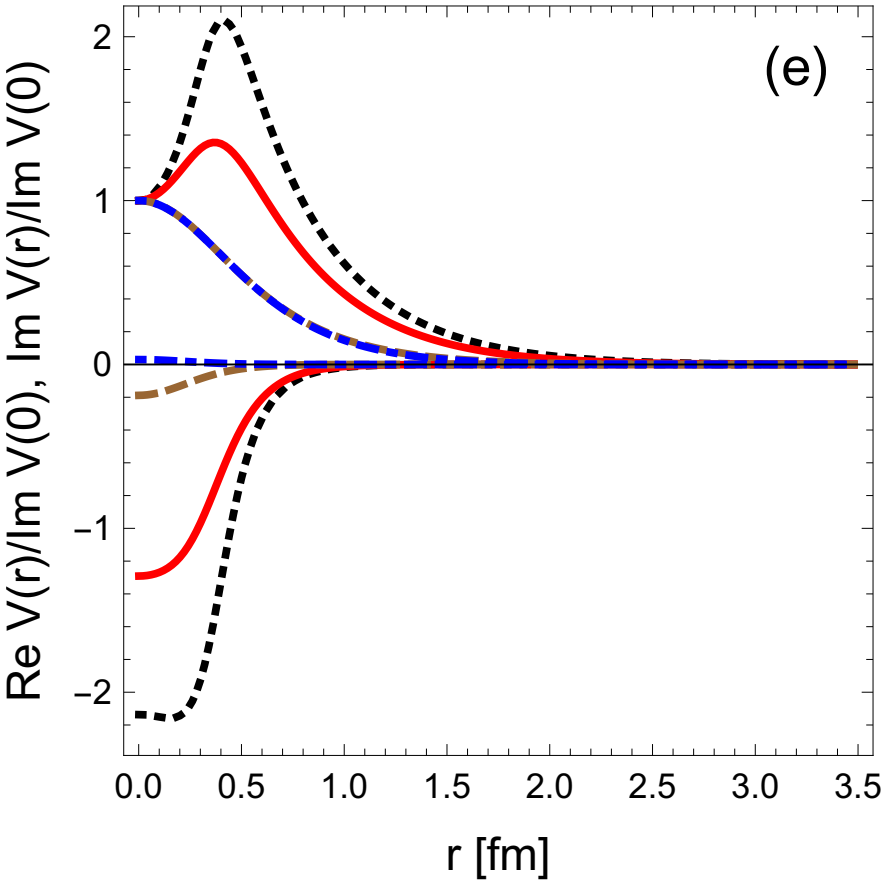
Inversion yields

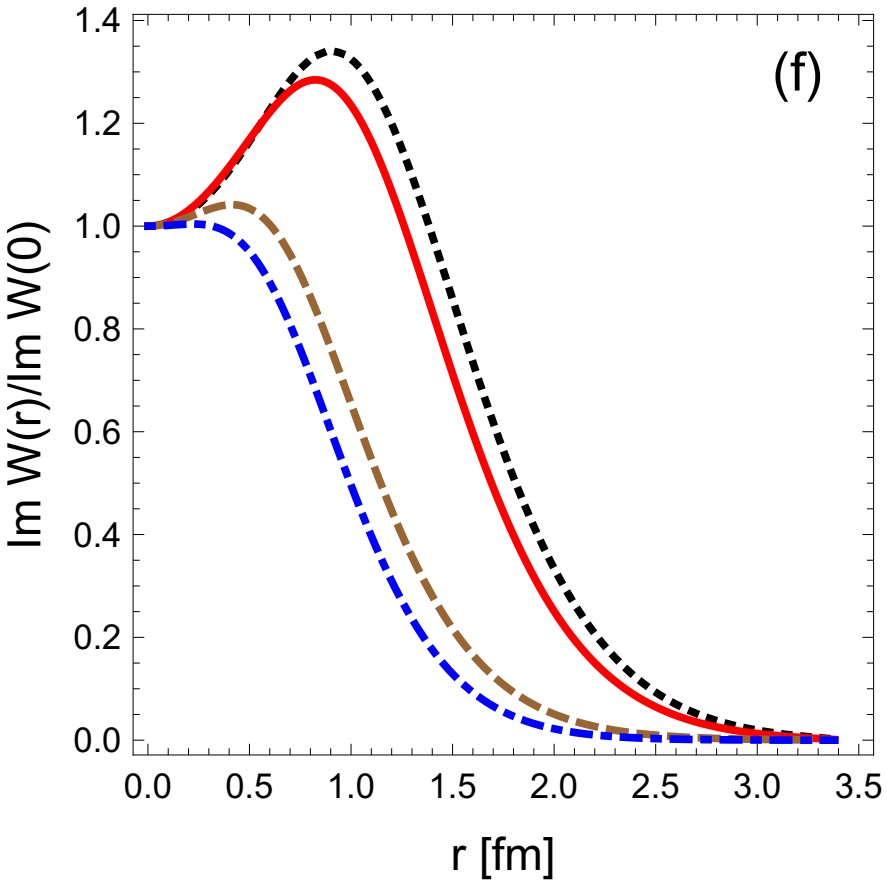
$$\mathrm{Im} W(r) = \frac{2p}{\pi}\int_r^\infty db \frac{n'(b)}{\sqrt{b^2 - r^2}}$$

The hollowness transition

exp. amplitude $\rightarrow$ eikonal phase $\rightarrow U(r) = M_N V(r)$

exp. amplitude $\rightarrow$ inelasticity profile $\rightarrow W(r)$





From top to bottom: $\sqrt{s} = 14000, 7000, 200, 23.4$ GeV

Large dip in the absorptive parts, in $W(r)$ starts already at RHIC!

No classical folding of absorptive parts

The hollowness effect cannot be reproduced by by folding of uncorrelated proton structures. We would then get, small r

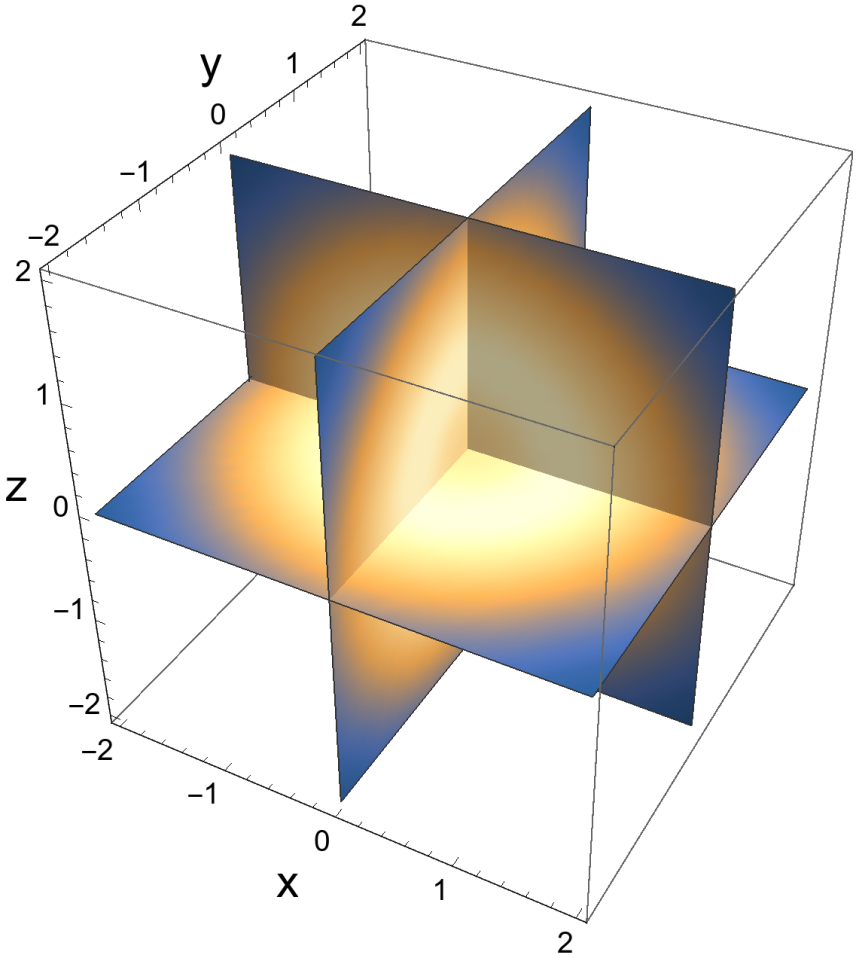
$$\begin{aligned} W(r) &= \int d^3y \rho(\vec{y}+\vec{r}/2)\rho(\vec{y}-\vec{r}/2) \\ &= \int d^3y \rho(\vec{y})^2 - \frac{1}{4}\int d^3y [\vec{r}\cdot\nabla\rho(\vec{y})]^2 + \ldots \end{aligned}$$

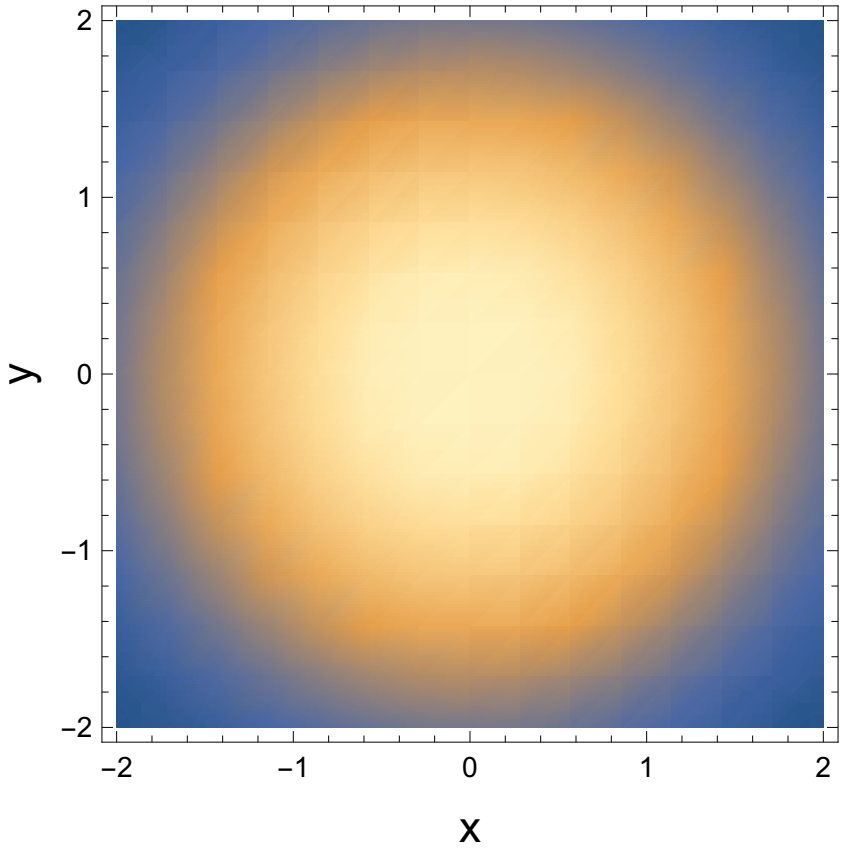
$\rightarrow$ $W(r)$ would necessarily have a local maximum at $r=0$, in contrast to the phenomenological result

$\rightarrow$ not possible to obtain hollowness classically by folding the absorptive parts from uncorrelated constituents

2D vs 3D opacity

Projection of 3D on 2D covers up the hollow: $f(x, y, z)$ vs $\int_{-\infty}^{\infty} dz f(x, y, z)$



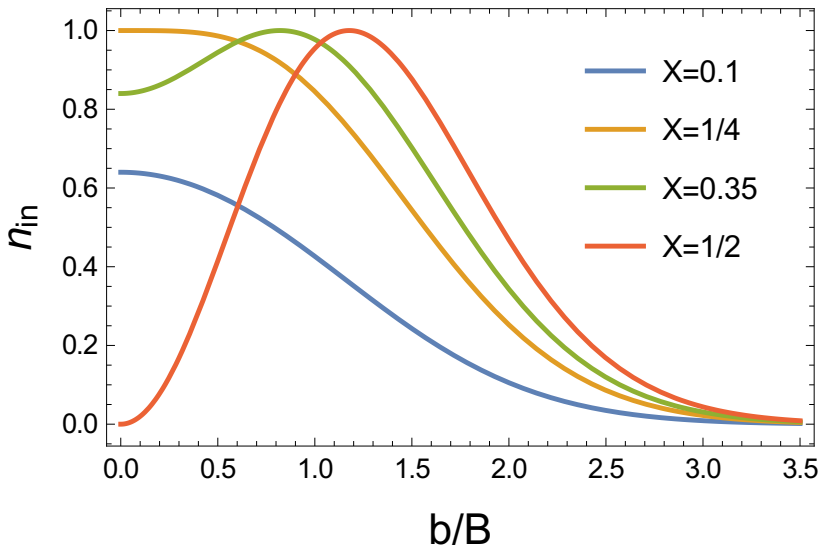


The hollow is covered up

Aspects of unitarity: model of [Dremin 2014]

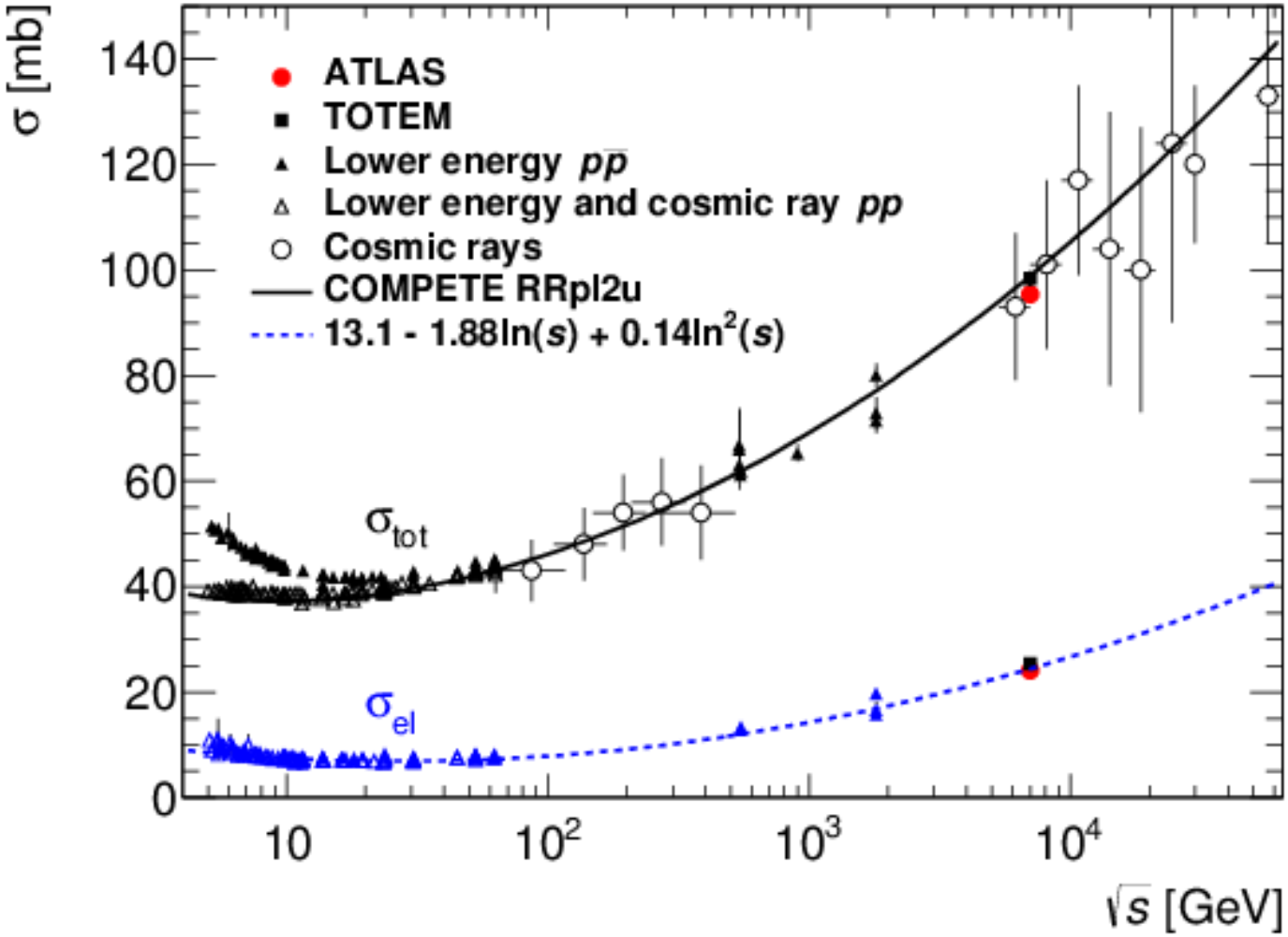
$$2p\mathrm{Im}h(b) \equiv k(b) = 4Xe^{-b^2/(2B)}, \;\; \mathrm{Re}h(b) = 0, \;\; X = \sigma_{el}/\sigma_T$$

$$n_{in}(b) = 2k(b) - k(b)^2 = 8Xe^{-b^2/(2B^2)} - 16X^2e^{-b^2/B^2}$$



- $X > 1/4$: $n_{in}(b)$ has a maximum at $b_0 = \sqrt{2}B\log(4X) > 0$, with $k(b_0) = 1$
- $X = 1/2$: black disk limit
- $W(r)$ develops a dip when $X > \sqrt{2}/8 = 0.177$

Cross sections



Ratio goes above 1/4 as energy increases!

Aspects of unitarity 2

If $2ph(b) = k(b)$ is not necessarily Gaussian but purely imaginary, then

$$n_{in}(b) = 2k(b) - k(b)^2$$

$$\frac{dn_{in}(b)}{db^2} = 2\frac{dk(b)}{db^2}[1 - k(b)]$$

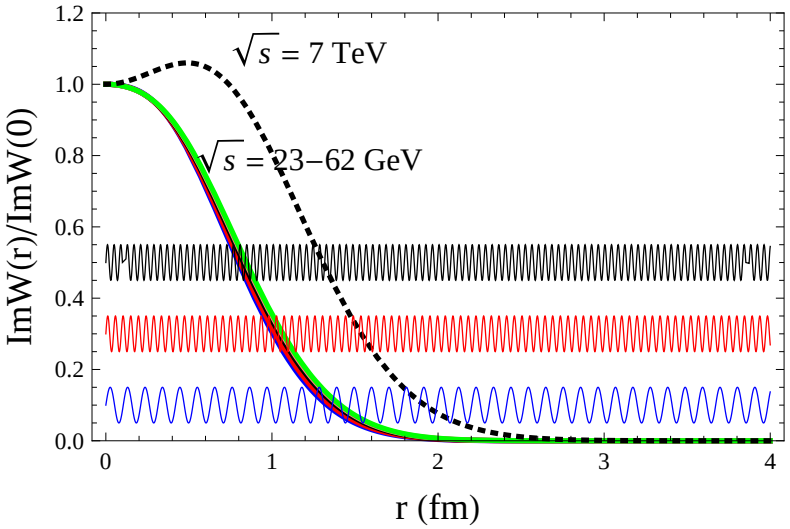
hence the minimum of $n(b)$ moves away from the origin when $k(0) > 1$

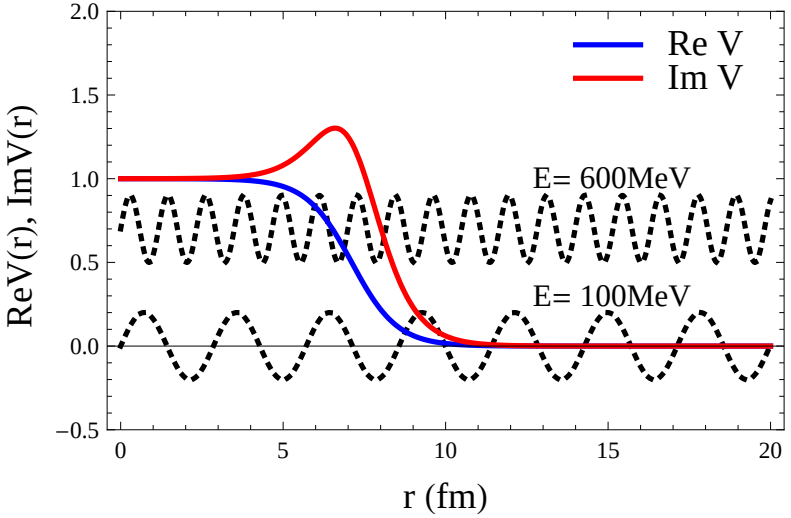
The real part of the amplitude, which is $\sim 10\%$, brings in corrections at the level of 1%

From 1000+GeV to 1000+eV

Epilogue

- Are low energy nuclear reactions and high NN interactions that different ?





Conclusions

- There is a hollowness transition at 1000+ GeV
- Quantum effect, rise of $2p\mathrm{Im}h(b)$ above 1
- Not possible to obtain classically by folding the absorptive parts from uncorrelated constituents
- 2D $\rightarrow$ 3D magnifies the effect (flat in 2D means hollow in 3D)
- Microscopic/dynamical explanations open [Alba Soto, Albacete 2016]
- Similar hollowness effect in low-energy n-A scattering