SAT1000
Searching for footprints of saturation in proton-proton collisions

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Introduction

- Did we already see (onset of) saturation?
  - Low-$x$, low-$Q^2$ DIS (GBW and many others)
    There are alternative approaches.
  - High-energy, low-$p_t$ particle production (McLarran-Praszalowicz)
    - $pA$ processes.

- Where to search for saturation effects?
Contents

1. Low mass Drell-Yan production.
2. Exclusive $pp \rightarrow ppJ/\psi$.
4. Inclusive forward production of $J/\psi$. 
The presented work is based on our recent analyses:

W. Schäfer and A. S.,


A. Cisek and A. S., a paper in preparation
Drell-Yan at low dilepton invariant masses

Where the dipole formulae apply?
Do we observe a sign of saturation?
Introduction to Drell-Yan

1. The Drell-Yan process is one of important sources of the partonic structure.

2. Drell-Yan production of low invariant masses in forward direction could be a good place in searching for onset of (gluon) saturation because small-x region
   (Brodsky et al., Gelis and Jalilian-Marian, ...)

3. Color dipole model with parametrized dipole-nucleon cross section in the so-called mixed representation inspired by saturation is often used.
   Only some observables available, not full kinematics, no experimental cuts on individual kinematical variables of leptons available.

4. \(k_t\)-factorization with unintegrated quark/antiquark distribution functions (Szczurek-Slipek, Nefedov-Nikolaev-Saleev, Baranov-Lipatov-Zotov) was also used at mid rapidities.

5. At forward/backward directions (low-x) gluon distributions have to be taken into account (formally higher order).

6. A hybrid (collinear, \(k_t\)-factorization) approach is proposed, using UGDEs, with control of momenta of individual leptons.
Dipole approach to the Drell-Yan process

- Basso, Goncalves, Nemchik, Pasechnik, Sumbera, Phys. Rev. D93, 034023 (2016) (also for $Z$ boson production)
All in the so-called mixed (transverse position space) representation, use saturation inspired parametrizations of dipole-nucleon cross section.

Approximate kinematics - only variables of the diphoton explicit ($\alpha$ and $p_{t,\text{pair}}$), associated jet not included in calculating $x$.

Only one-side contribution included

Not sufficient when cuts on individual leptons required.

So far applied to rather low energies and midrapidities where the formalism does not apply.
Forward and backward production of dilepton pairs

Figure: The diagrams relevant for forward and backward production of dilepton pairs.

Similar diagrams as in the dipole approach.
$k_T$-factorization form

The full dilepton cross section is then

$$\frac{d\sigma(pp \to l^+ l^- X)}{dy_+ dy_- d^2 k_+ d^2 k_-} = \frac{d\sigma(pp \to l^+ l^- X)}{dx_+ dx_- d^2 k_+ d^2 k_-} = \frac{\alpha^2_{\text{em}}}{8\pi^2 N_c M^2} \sum_f e_f^2 \int_{x_F}^1 dx_1 \left[ x_1 q_f(x_1, \mu^2) + x_1 \bar{q}_f(x_1, \mu^2) \right]$$

$$\int \frac{d^2 \kappa_2}{\pi \kappa_4^4} F(x_2, \kappa_2^2) \alpha_S(\bar{q}^2)$$

$$\left\{ \begin{array}{l} \frac{x_F}{x_1} l_T \left( \frac{x_F}{x_1}, q, \kappa_2 \right) D_T \left( \frac{x_+}{x_F} \right) \\ + \frac{x_F}{x_1} l_L \left( \frac{x_F}{x_1}, q, \kappa_2 \right) D_L \left( \frac{x_+}{x_F} \right) \\ + \frac{x_F}{x_1} l_\Delta \left( \frac{x_F}{x_1}, q, \kappa_2 \right) D_\Delta \left( \frac{x_+}{x_F} \right) \left( \frac{l}{|l|} \cdot \frac{q}{|q|} \right) \\ + \frac{x_F}{x_1} l_{\Delta\Delta} \left( \frac{x_F}{x_1}, q, \kappa_2 \right) D_{\Delta\Delta} \left( \frac{x_+}{x_F} \right) \left( 2 \left( \frac{l}{|l|} \cdot \frac{q}{|q|} \right)^2 - 1 \right) \end{array} \right\}.$$
**$k_T$-factorization form**

If we also want to include the recoiling jet - insert

$$\text{d}x_J \delta(x_J + x_F - x_1) \, d^2k_J \, \delta^{(2)}(k_2 - q - k_J).$$  \hspace{1cm} (1)

This gives us the fully differential distribution

$$\frac{d\sigma(pp \rightarrow l^+ l^- X)}{dy_+ dy_- dy_J d^2k^+ d^2k^- d^2k_J} = \frac{\alpha^2_{\text{em}}}{8\pi^3 N_c M^2} \frac{x_F x_J}{x_F + x_J}$$

$$\times \sum_f e_f^2 \left[ q_f(x_F + x_J, \mu^2) + \bar{q}_f(x_F + x_J, \mu^2) \right] \times \frac{\alpha_S(q^2) F(x_2, q + k_J)}{(q + k_J)^4}$$

$$\times \left\{ I_T^f \left( \frac{x_F}{x_F + x_J}, q, q + k_J \right) D_T \left( \frac{x_+}{x_F} \right) + I_L^f \left( \frac{x_F}{x_F + x_J}, q, q + k_J \right) D_L \left( \frac{x_+}{x_F} \right) + I_{\Delta}^f \left( \frac{x_F}{x_F + x_J}, q, q + k_J \right) D_{\Delta} \left( \frac{x_+}{x_F} \right) \left( \frac{l}{|l|} \cdot \frac{q}{|q|} \right) + I_{\Delta\Delta}^f \left( \frac{x_F}{x_F + x_J}, q, q + k_J \right) D_{\Delta\Delta} \left( \frac{x_+}{x_F} \right) \left( 2 \left( \frac{l}{|l|} \cdot \frac{q}{|q|} \right)^2 - 1 \right) \right\}. \hspace{1cm} (2)$$
**Kinematics**

Rapidities are obtained as:

\[ y_i = \log \left( \frac{x_i \sqrt{S}}{\sqrt{k^2_i}} \right) \iff x_i = \sqrt{\frac{k^2_i}{S}} \cdot e^{y_i}, \quad i = +, -, J. \quad (3) \]

The longitudinal momentum fractions \( x_1, x_2 \) entering the quark and gluon distributions are

\[
\begin{align*}
x_1 &= \sqrt{\frac{k^2_+}{S}} e^{y_+} + \sqrt{\frac{k^2_-}{S}} e^{y_-} + \sqrt{\frac{k^2_J}{S}} e^{y_J}, \\
x_2 &= \sqrt{\frac{k^2_+}{S}} e^{-y_+} + \sqrt{\frac{k^2_-}{S}} e^{-y_-} + \sqrt{\frac{k^2_J}{S}} e^{-y_J}. \quad (4)
\end{align*}
\]

The invariant mass of the dilepton system is

\[ M^2 = m_{\perp+}^2 + m_{\perp-}^2 + 2m_{\perp+} m_{\perp-} \cosh(y_+ - y_-) - q^2, \quad m_{\perp\pm} = \sqrt{k^2_{\pm} + m^2_{\pm}}. \quad (5) \]
Fourier transform of dipole - nucleon cross section

The dipole cross section is related to the unintegrated glue as

$$\sigma(x, r) = \frac{4\pi}{N_c} \int \frac{d^2 \kappa}{\kappa^4} \alpha_s \mathcal{F}(x, \kappa) \left\{ 1 - \exp(i\kappa r) \right\}.$$  \hspace{1cm} (6)

The parametrizations of Albacete et al. are presented in the form

$$\sigma(x, r) = \sigma_0 \cdot N(x, r),$$  \hspace{1cm} (7)

with $N(x, r) \to 1$ at large $r$. We can therefore easily obtain, that

$$\frac{\alpha_s \mathcal{F}(x, \kappa)}{\kappa^4} = \frac{\sigma_0 N_c}{4\pi} \int \frac{d^2 r}{(2\pi)^2} \exp(-i\kappa r) \left[ 1 - N(x, r) \right],$$  \hspace{1cm} (8)

or

$$\mathcal{F}(x, \kappa) = \frac{\sigma_0 N_c}{8\pi^2} \frac{\kappa^2}{\alpha_s(\kappa^2)} \int_0^\infty rdr J_0(\kappa r) \left[ 1 - N(x, r) \right],$$  \hspace{1cm} (9)

where $J_0(x)$ is the Bessel function. The Fourier-Bessel (or Hankel-) transform (9) can pose severe numerical problems, if values at large $\kappa^2$ are required. For the evaluations of these integrals we use therefore a dedicated code FFTLog.
We use the following UGDFs:

- **Kimber-Martin-Ryskin** UGDF,
  transverse momentum in the last step of evolution

- **Kutak-Staśto** UGDF,
  includes nonlinear effects

- **Albacete, Armesto, Milhano, Salgado**
  - solving BK evolution equation and Fourier transform.

- **Golec-Biernat** UGDF,
  saturation inspired parametrization of photon-nucleon cross section.
Parton distributions

In the present calculations we use MSTW08 distributions to generate the KMR unintegrated gluon distributions. Here we use numerical implementation by Maciuła and Szczurek used e.g. in the production of charm and double charm. For the quark and antiquark distributions we use MSTW08 leading-order distributions. For most of the calculations we used $M^2_{\parallel}$ both as a factorization and renormalization scales. We have also tried:

$$\mu^2_R = \max \left( \kappa^2_\perp, q^2_\perp + \varepsilon^2 \right),$$
$$\mu^2_F = q^2_\perp + \varepsilon^2.$$  

(10)

The corresponding results turned out to be almost identical.
Full rapidity range

The rapidities of both leptons are strongly correlated i.e. $y_+ \approx y_-$. 

Figure: Two-dimensional $(y_+, y_-)$ distribution for $\sqrt{s} = 7$ TeV and $k_{T+}, k_{T-} > 3$ GeV for MSTW08 PDF and KMR (left) and KS (right) UGDFs.
Full rapidity range

Figure: Distribution in rapidity of the dileptons for $\sqrt{s} = 7$ TeV and $k_T^+, k_T^- > 3$ GeV for MSTW08 PDF and different UGDFs: KMR (solid), KS (dashed), AAMS (dotted) and GBW (dash-dotted).
Full rapidity range

Figure: Two-dimensional \((y_*, q_T)\) distribution for \(\sqrt{s} = 7\) TeV and \(k_{T+}, k_{T-} > 3\) GeV for MSTW08 PDF and KMR (left) and KS (right) UGDF.
Figure: Distribution in $y_*$ for exact (solid) and approximate (dashed) formula for calculating $x_1$ and $x_2$ for $\sqrt{s} = 7$ TeV and $k_{T+}, k_{T-} > 3$ GeV for MSTW08 PDF and KMR UGDF. In the right panel we show the ratio of the two distributions.
Full rapidity range

Figure: Distribution in rapidity of the dileptons for $\sqrt{s} = 7$ TeV and $k_T^+, k_T^- > 3$ GeV for MSTW08 valence quark distributions and KMR UGDF. The dashed line is the contribution from valence quarks only.
Forward region - LHCb

**Figure:** Invariant mass distribution (only the dominant component) for the LHCb cuts: $2 < y_+, y_- < 4.5$, $k_{T+}, k_{T-} > 3$ GeV for different UGDFs: KMR (solid), Kutak-Stasto (dashed), AAMS (dotted) and GBW (dash-dotted).
Forward region - LHCb

Figure: Two-dimensional \((k_{T+}, k_{T-})\) distribution for \(\sqrt{s} = 7\) TeV and \(k_{T+}, k_{T-} > 3\) GeV for MSTW08 PDF and KMR (left), KS (middle) and AAMS (right) UGDFs.
Forward region - LHCb

Figure: Dilepton transverse momentum distribution (only the dominant component) for the LHCb cuts: $2 < y_+, y_- < 4.5$, $k_{T+}, k_{T-} > 3$ GeV for different UGDFs: KMR (solid), Kutak-Stasto (dashed), AAMS (dotted) and GBW (dash-dotted).
Forward region - LHCb

Figure: The $T$ and $L$ contributions to the dilepton invariant mass distribution for the LHCb kinematics: $2 < y_+, y_- < 4.5$, $k_{T+}, k_{T-} > 3$ GeV. KMR UGDF was used here.
Figure: The $R_{int}$ as a function of $M_\parallel$ for the LHCb kinematics: $2 < y_+, y_- < 4.5$, $k_{T+}, k_{T-} > 3$ GeV. KMR UGDF was used here. The fluctuations are due to insufficient statistics of our Monte Carlo calculation.

$$R_{int} = \frac{d\sigma_{all} - d\sigma_{T+L}}{d\sigma_{all}}.$$
Forward region - LHCb

Figure: Contributions of the second-side component for the LHCb kinematics: $2 < y_+, y_- < 4.5$, $k_{T+}, k_{T-} > 3$ GeV. KMR UGDF was used here.
Midrapidity region - ATLAS

**Figure:** Invariant dilepton mass distribution for the ATLAS kinematics: $-2.4 < y_+ , y_- < 2.4, k_{T+}, k_{T-} > 6$ GeV. Here both $gq/\bar{q}$ and $q/\bar{q}g$ contributions have been included.
Figure: Lepton transverse momentum correlations for the ATLAS kinematics: $-2.4 < y_+, y_- < 2.4$, $k_{T+}, k_{T-} > 6$ GeV. The left panel is for the KMR UGDF, the middle panel for the KS UGDF and the right panel for the AAMS UGDF.
Midrapidity region - ATLAS

Figure: Transverse momentum distribution of dileptons for the ATLAS kinematics: $-2.4 < y_+, y_- < 2.4$, $k_{T+}, k_{T-} > 6$ GeV for MWST08 PDF and for different UGDFs: KMR (solid), KS (dashed), AAMS (dotted) and GBW (dash-dotted).
\[ pp \rightarrow ppJ/\psi \]

The interference term vanishes for rapidity distributions in Born approximation
Imaginary part of the forward $\gamma p \to J/\psi p$ amplitude

$$\Im m \mathcal{M}_T(W, \Delta^2 = 0, Q^2 = 0) = W^2 \frac{c_v \sqrt{4\pi \alpha_{em}}}{4\pi^2} 2 \int_0^1 \frac{dz}{z(1-z)} \int_0^\infty \pi dk^2 \psi_V(z, k^2)$$

$$\int_0^\infty \frac{\pi d\kappa^2}{\kappa^4} \alpha_S(q^2) \mathcal{F}(x_{\text{eff}}, \kappa^2) \left( A_0(z, k^2) \, W_0(k^2, \kappa^2) + A_1(z, k^2) \, W_1(k^2, \kappa^2) \right)$$

dependence on the meson wave function and UGDF
No wave functions in collinear calculations (Jones, Martin, Ryskin)

The full amplitude, at finite momentum transfer parametrized:

$$\mathcal{M}(W, \Delta^2) = (i + \rho) \Im m \mathcal{M}(W, \Delta^2 = 0, Q^2 = 0) \exp(-B(W)\Delta^2/2),$$
pp → ppJ/ψ

Deviations from exponential t dependence
In the Born approximation:

\[
\mathcal{M}_{h_1 h_2 \to h_1 h_2 V}^{\lambda_1 \lambda_2 \to \lambda_1' \lambda_2' \lambda V} (s, s_1, s_2, t_1, t_2) = \mathcal{M}_{\gamma p} + \mathcal{M}_{p \gamma}
\]

\[
= \langle p_1', \lambda_1' | J_\mu | p_1, \lambda_1 \rangle \epsilon^*_\mu (q_1, \lambda V) \frac{\sqrt{4\pi \alpha_{em}}}{t_1} \mathcal{M}_{\gamma^* h_2 \to V h_2}^{\lambda_1 \lambda_2 \to \lambda V \lambda_2} (s_2, t_2, Q_1^2) + \langle p_2', \lambda_2' | J_\mu | p_2, \lambda_2 \rangle \epsilon^*_\mu (q_2, \lambda V) \frac{\sqrt{4\pi \alpha_{em}}}{t_2} \mathcal{M}_{\gamma^* h_1 \to V h_1}^{\lambda_1 \lambda_2 \to \lambda V \lambda_1} (s_1, t_1, Q_2^2).
\]
Then, the amplitude of Eq. (14) for the emission of a photon of transverse polarization $\lambda_V$, and transverse momentum $q_1 = -p_1$ can be written as:

$$
\langle p', \lambda'_1 | J_\mu | p_1, \lambda_1 \rangle \epsilon^*_\mu(q_1, \lambda_V) = \left( e^*(\lambda_V) q_1 \right) \frac{2}{\sqrt{1 - z_1}} \chi_{\lambda'} \left\{ F_1(Q_1^2) - \frac{i \kappa_p F_2(Q_1^2)}{2m_p} (\sigma_1 \cdot [q_1, n]) \right\} \chi_{\lambda}. \tag{15}
$$

$F_1$ - Dirac em ff

$F_2$ - Pauli em ff (new)
$pp \to ppJ/\psi$

HERA data at $W \sim 100\text{-}200$ GeV
LHCb quasi-data at $W \sim 1$ TeV
$pp \rightarrow pp J/\psi$
$pp \rightarrow ppJ/\psi$
$pp \rightarrow ppJ/\psi$

Survival factor depends on the phase space point!
$pp \rightarrow ppJ/\psi$

with absorption

similar for $\psi'$
$pp \rightarrow ppJ/\psi$

with absorption

Ivanov-Nikolaev

Kutak-Stasto linear

Kutak-Stasto nonlinear

similar for $\psi'$

$W = 7$ TeV
$\psi'$ to $J/\psi$ ratio

Gauss WF much better than Coulomb WF
Semiexclusive production, electromagnetic excitation

One-side excitation only
A. Cisek, W. Schäfer and A. S., arXiv:1611.08210
Electromagnetic excitation

The cross section for such processes can be written as:

$$\frac{d\sigma(pp \rightarrow XVp; s)}{dyd^2pdM_X^2} = \int \frac{d^2q}{\pi q^2} F_{\gamma/p}^{(\text{inel})}(z_+, q^2, M_X^2) \frac{1}{\pi} \frac{d\sigma^{\gamma^* p \rightarrow Vp}}{dt}(z_+s, t = -(q - p)^2)$$

where $z_\pm = e^{\pm y} \sqrt{(p^2 + m_V^2)/s}$.

$$F_{\gamma/p}^{(\text{inel})}(z, q^2, M_X^2) = \frac{\alpha_{\text{em}}}{\pi} (1 - z) \theta(M_X^2 - M_{thr}^2) \frac{F_2(x_{Bj}, Q^2)}{M_X^2 + Q^2 - m_p^2} \left[ \frac{q^2}{q^2 + z(M_X^2 - m_p^2)} \right]$$

where

$$Q^2 = \frac{1}{1 - z} \left[ q^2 + z(M_X^2 - m_p^2) + z^2 m_p^2 \right]$$

$$x_{Bj} = \frac{Q^2}{Q^2 + M_X^2 - m_p^2}. \quad (18)$$
Semiexclusive production, resonance diffractive excitation

One-side excitation only
Diffractive resonance contribution

We consider Jenkovszky et al. dual Regge model. The contribution of three positive-parity baryon resonances on the nucleon trajectory are taken into account:

1. $N(1680)$, $J = \frac{5}{2}$
2. $N(2220)$, $J = \frac{9}{2}$
3. $N(2700)$, $J = \frac{13}{2}$

Their contribution is accounted for by

$$\Im m A(M_X^2, t) = \sum_{n=1,3} [f(t)]^{2(n+1)} \cdot \frac{\Im m \alpha(M_X^2)}{(J_n - \Re e \alpha(M_X^2))^2 + (\Im m \alpha(M_X^2))^2}.$$ (19)

Here $J_n$ is the spin of the $n$th resonance, and the explicit form of the complex Regge trajectory $\alpha(M_X^2)$ as well as the form factor $f(t)$ are from Jenkovszky et al.

We can now compute the contribution from diffractive excitation of small masses from:

$$\frac{d\sigma(pp \rightarrow XVp; s)}{dyd^2pdM_X^2} = \int \frac{d^2q}{\pi q^2} F^{(el)}_{\gamma/p}(z_+, q^2) \frac{1}{\pi} \frac{d\sigma(\gamma p \rightarrow VX)}{dtdM_X^2}(z_+s) + (z_+ \leftrightarrow z_-).$$ (20)
Semiexclusive production, partonic diffractive excitation

One-side excitation only
\( \mathbf{P} \) means here two-gluon exchange
Diffractive partonic excitation

We neglect transverse momenta of the photon and initial quark/antiquark. In this approximation:

\[
\frac{d\sigma_{\text{diff,partonic}}^{pp\rightarrow Vj}}{dy_V dy_j d^2 p_t} = \frac{1}{16\pi^2 \hat{s}^2} x_1 q_{\text{eff}}(x_1, \mu_F^2) x_2 \gamma_{\text{el}}(x_2) |M_{q\gamma\rightarrow Vq}|^2 
\]

\[+ \frac{1}{16\pi^2 \hat{s}^2} x_1 \gamma_{\text{el}}(x_1) x_2 q_{\text{eff}}(x_2, \mu_F^2) |M_{q\gamma\rightarrow Vq}|^2 . (21)\]

The effective “quark” distribution reads as

\[q_{\text{eff}}(x, \mu_F^2) = \frac{81}{16} g(x, \mu_F^2) + \sum_f \left[ q_f(x, \mu_F^2) + \bar{q}_f(x, \mu_F^2) \right] . (22)\]

We take \(\mu_F^2 = m_V^2 + |t|\) for the factorization scale. The matrix element for the partonic subprocess is related to the corresponding \(t\)-dependence as

\[d\sigma_{\gamma q\rightarrow Vq}^{\gamma q\rightarrow Vq} = \frac{1}{16\pi \hat{s}^2} |M_{\gamma q\rightarrow Vq}|^2 . (23)\]

In the following we shall use a simple formula for two-gluon exchange (Ivanov et al.) where
EM vs diffractive excitation

EM excitation contribution bigger than diffractive excitation contribution
Missing mass distribution

\[ \frac{d\sigma(J/\Psi)}{dM_x} \text{ (nb/GeV)} \]

- **EM ALLM**
- **EM Fiore**
- **Diff. resonance**
- **Diff. partonic**

**M_x (GeV)**

**full phase space**
Transverse momentum distribution for full phase space

full phase space
Transverse momentum distribution for LHCb

LHCb rapidity acceptance
Semi-exclusive to exclusive ratio

All dissociation processes
The ratio strongly depends on the range of missing masses
Inclusive $J/\psi$ production
Inclusive $J/\psi$ production

\[
\frac{d\sigma(pp \rightarrow J/\psi gX)}{dy_{J/\psi} dy_g d^2 p_{J/\psi,t} d^2 p_{g,t}} = \frac{1}{16\pi^2 \hat{s}^2} \int \frac{d^2 q_{1t}}{\pi} \frac{d^2 q_{2t}}{\pi} |M_{\text{off-shell}}^{g^* g^* \rightarrow J/\psi g}|^2 
\times \delta^2 (\vec{q}_{1t} + \vec{q}_{2t} - \vec{p}_{H,t} - \vec{p}_{g,t}) F_g(x_1, q_{1t}^2, \mu_F^2) F_g(x_2, q_{2t}^2, \mu_F^2) .
\]

The corresponding matrix element squared for the $gg \rightarrow J/\psi g$ is

\[
|M_{gg \rightarrow J/\psi g}|^2 \propto \alpha_s^3 |R(0)|^2 .
\]

We use Baranov matrix elements
In the $k_t$-factorization approach the leading-order cross section for the $\chi_c$ meson production can be written as:

$$\sigma_{pp \rightarrow \chi_c} = \int dy d^2 p_t d^2 q_t \frac{1}{s x_1 x_2} \frac{1}{m_{t, \chi_c}^2} |\mathcal{M}_{g^* g^* \rightarrow \chi_c}|^2$$

$$\mathcal{F}_g(x_1, q_{1t}^2, \mu_F^2) \mathcal{F}_g(x_2, q_{2t}^2, \mu_F^2)/4 ,$$

(27)

which can also be used to calculate rapidity and transverse momentum distributions of the $\chi_c$ mesons. The matrix element squared for the $gg \rightarrow \chi_c$ subprocess is

$$|\mathcal{M}_{gg \rightarrow \chi_c}|^2 \propto \alpha_s^2 |R'(0)|^2 .$$

(28)

We used the matrix element taken from the Kniehl, Vasin and Saleev paper.
χc contribution to inclusive $J/ψ$

Both KMR and nonlinear KS acceptable
$\chi_c$ contribution to inclusive $J/\psi$

Only nonlinear KS acceptable
$\chi_c$ contribution to inclusive $J/\psi$

Only nonlinear KS acceptable
Conclusions, Drell-Yan

- Forward production of low-mass dileptons has been discussed.
- Corresponding momentum space formula has been derived and presented.
- In contrast to the dipole formalism correct treatment of kinematics and can be applied to the analysis of real experiment.
- We have obtained four terms instead of two terms in the traditional dipole approach.
- A program which includes $2 \rightarrow 3$ subprocesses has been written.
- Correlation between a jet and leptons can be calculated.
- Calculation of differential cross section for experimental conditions have been performed.
- General tendencies have been shown.
- The results have been compared with LHCb and ATLAS data.
- Critical remarks to dipole model applications have been made.
Conclusions, Drell-Yan

- The saturation inspired UGDFs fail to describe LHCb and ATLAS data.
- KMR UGDF (relation to DGLAP approach) is much better.
- Some strength at large $M_{ll}$ is missing (lack of meson cloud).
- Both side contributions have to be included (even for LHCb) in contrast to calculations in dipole model in the literature.
- Contribution of different terms (L, T, etc) strongly depends on kinematics.
- New interference terms rather small.
- Meson cloud effects missing here as well as in dipole approach.
- No clear hints of saturation at small $M_{ll}$. 
Conclusion, $pp \rightarrow ppJ/\psi$

There is some model dependent indication of nonlinear effects

Open problems:

- The present experiments are not exclusive.
- So far proton dissociation "extracted" in a model dependent way assuming some functional form in $p_t$.
- We have now better knowledge about diffractive dissociation (HERA).
- Compared to HERA there is also photon dissociation. The contribution of electromagnetic dissociation bigger than that for diffractive dissociations.
- Interference effects due to the two diagrams were predicted. It would be nice to see modulation in $\phi_{pp}$ due to interference effects between the two contributing diagrams.
- CMS+TOTEM and ATLAS+ALFA could measure purely exclusive reaction and study dependences on many more variables.
Conclusion, inclusive production of $J/\psi$

- There seems to be a signature of saturation at large energies (13 TeV) and large rapidities. Already $\chi_c$ component seems to be in conflict with the LHCb data. For comparison at low energies (2.76 GeV) more standard approach agrees with the data.

- More detailed studies are necessary to draw definite conclusions.
There seems to be a signature of saturation at large energies (13 TeV) and large rapidities. Already $\chi_c$ component seems to be in conflict with the LHCb data. For comparison at low energies (2.76 GeV) more standard approach agrees with the data.

More detailed studies are necessary to draw definite conclusions.