4-jet production with kt-factorization plus parton showers

Mirko Serino

Institute of Nuclear Physics, Polish Academy of Sciences,
Cracow, Poland

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Work in collaboration with
Marcin Bury, Andreas van Hameren, Hannes Jung and Krzysztof Kutak

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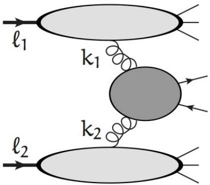
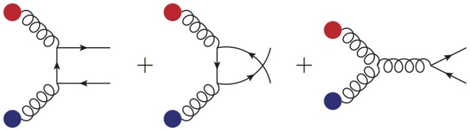
High-Energy-factorisation

High-Energy-factorisation *(Catani,Ciafaloni,Hautmann, 1991 / Collins,Ellis, 1991)*





$$\sigma_{h_1,h_2\to q\bar{q}} = \int d^2k_{1\perp} d^2k_{2\perp} \frac{dx_1}{x_1}\frac{dx_2}{x_2}\mathcal{F}_g(x_1,k_{1\perp})\mathcal{F}_g(x_2,k_{2\perp})\hat{\sigma}_{gg}\left(\frac{m^2}{x_1x_2s},\frac{k_{1\perp}}{m},\frac{k_{2\perp}}{m}\right)$$

where the $\mathcal{F}_g$'s are the gluon densities, obeying BFKL, BK, CCFM evolution equations, and $\hat{\sigma}$ the **gauge invariant** parton cross section (!!!)

Non negligible transverse momentum $\Leftrightarrow$ small-x physics.

Momentum parameterisation:

$$k_1^\mu = x_1\, p_1^\mu + k_{1\perp}^\mu \quad , \quad k_2^\mu = x_2\, p_2^\mu + k_{2\perp}^\mu \quad \text{for} \quad p_i\cdot k_i = 0 \quad k_i^2 = -k_{i\perp}^2 \quad i = 1,2$$

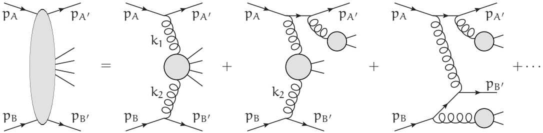
Gauge invariant off-shell amplitudes

Problem: general partonic processes must be described by gauge invariant amplitudes
$\Rightarrow$ ordinary Feynman rules are not enough !

ONE IDEA:

on-shell amplitudes are gauge invariant, so off-shell gauge-invariant amplitudes could be got by embedding them into on-shell processes...

...first result...: 1) For off-shell gluons: represent g^* as coming from a $\bar{q}qg$ vertex, with the quarks taken to be on-shell



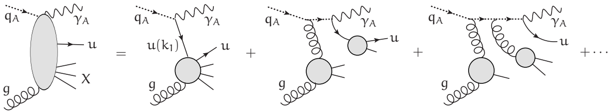
- embed the scattering of the off-shell gluons in the scattering of two quark pairs carrying momenta $p_A^\mu = k_1^\mu$, $p_B^\mu = k_2^\mu$, $p_{A'}^\mu = 0$, $p_{B'}^\mu = 0$
- Assign the spinors $|p_1\rangle, |p_1]$ to the A-quark and the propagator $\frac{i\,\not{p}_1}{p_1 \cdot k}$ instead of $\frac{i\not{k}}{k^2}$ to the propagators of the A-quark carrying momentum k; same thing for the B-quark line.
- ordinary Feynman elsewhere and factor $x_1\sqrt{-k_\perp^2/2}$ to match to the collinear limit

K. Kutak, P. Kotko, A. van Hameren, JHEP 1301 (2013) 078

Prescription for off-shell quarks

... and second result:

2) for off-shell quarks: represent q^* as coming from a $\gamma\bar{q}q$ vertex, with a 0 momentum and $\bar{q}$ on shell (and vice-versa)



- embed the scattering of the quark with whatever set of particles in the scattering of an auxiliary quark-photon pair, q_A and γ_A carrying momenta $p^\mu_{q_A} = k^\mu_1$, $p^\mu_{\gamma_A} = 0$
- Let q_A-propagators of momentum k be $\frac{i\,\not{p}_1}{p_1\cdot k}$ and assign the spinors $|p_1\rangle, |p_1]$ to the A-quark.
- Assign the polarization vectors $\epsilon^\mu_+ = \frac{\langle q|\gamma^\mu|p_1]}{\sqrt{2}\langle p_1 q\rangle}$, $\epsilon^\mu_- = \frac{\langle p_1|\gamma^\mu|q]}{\sqrt{2}[p_1 q]}$ to the auxiliary photon, with q a light-like auxiliary momentum.
- Multiply the amplitude by $x_1\sqrt{-k^2_{1\perp}/2}$ and use ordinary Feynman rules everywhere else.

K. Kutak, T. Salwa, A. van Hameren, Phys.Lett. B727 (2013) 226-233

One left issue: huge slowness for many legs

The diagrammatic approach is too slow to allow for the computation of amplitudes containing more than 4 particles in a reasonable time.

Computing scattering amplitudes in Yang-Mills theories via ordinary Feynman diagrams: soon overwhelming !

Number of Feynman diagrams at tree level on-shell:

# of gluons	4	5	6	7	8	9	10
# of diagrams	4	25	220	2485	34300	559405	10525900

And there are even more with the proposed method for amplitudes with off-shell particles due to the gauge-restoring terms.

A method to efficiently compute helicity amplitudes: **BCFW recursion relation**

Britto, Cachazo, Feng, Nucl.Phys. B715 (2005) 499-522
Britto, Cachazo, Feng, Witten, Phys.Rev.Lett. 94 (2005) 181602

BCFW recursion relation

Two very simple ideas for tree level amplitudes:

1. Cauchy's residue theorem: if the amplitude is formally treated as a function of a complex variable z and if it is rational and vanishes for $z \to \infty$, then the integral extended to an infinite contour enclosing all poles vanishes

$$\lim_{z \to \infty} \mathcal{A}(z) = 0 \Rightarrow \frac{1}{2\pi i} \oint dz \, \frac{\mathcal{A}(z)}{z} = 0$$

implying that the value at $z = 0$ (physical amplitude) can be determined as a sum of the residues at the poles:

$$\mathcal{A}(0) = -\sum_i \frac{\lim_{z \to z_i} [(z - z_i)\, f(z)]}{z_i}$$

where z_i is the location of the i-th pole

2. Unitarity: Poles in Yang-Mills tree level amplitudes can only be due to gluon propagators dividing the n-point amplitude into two on-shell sub-amplitudes with $k+1$ and $n-k+1$ gluons $\Rightarrow$ it is all about finding the proper way to "complexify" an amplitude.

To properly "complexify" $\mathcal{A}$: for helicities $(h_1, h_n) = (-, +)$ (no loss of generality...)

$$\begin{aligned} |1] &\rightarrow |\hat{1}] \equiv |1] - z\,|n] \Rightarrow p_1 \rightarrow \hat{p}_1 = |1]\langle 1| - z|1]\langle n| \\ |n\rangle &\rightarrow |\hat{n}\rangle \equiv |n\rangle + z|1\rangle \Rightarrow p_n \rightarrow \hat{p}_n = |n]\langle n| + z|1]\langle n| \end{aligned}$$

With such a choice

- On-shellness, gauge invariance and momentum conservation preserved throughout.
- the most serious issue is the behaviour for $z \rightarrow \infty$, but either a result derived with twistor methods (*Cachazo,Svrcek and Witten JHEP 0409 (2004) 006*) or a smart choice of reference lines always allow to overcome the problem, so that $\lim_{z\rightarrow\infty} \mathcal{A}(z) = 0$ holds

BCFW applies to* color-ordered partial amplitudes*, for which the kinematics and gauge structure are factorised like

$$\mathcal{M}_n = g^{n-2} \sum_{\sigma \in S_n/Z_n} \mathrm{Tr}(T_{\sigma(1)} \dots T_{\sigma(n)})\, \mathcal{A}(g_{\sigma(1)}, \dots, g_{\sigma(n)})$$

Amazingly simple recursive relation:

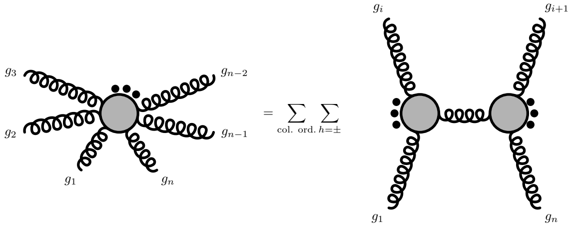
any tree-level color-ordered amplitude is the sum of residues of the poles it develops when it is made dependent on a complex variable as above.

Such residues are simply products of color-ordered lower-point amplitudes evaluated at the pole times an intermediate propagator.

Shifted particles are always on opposite sides of the propagator.

$$\mathcal{A}(g_1,\dots,g_n) = \sum_{i=2}^{n-2}\sum_{h=+,-} \mathcal{A}(g_1,\dots,g_i,\hat{P}^h)\,\frac{1}{(p_1+\dots+p_i)^2}\,\mathcal{A}(-\hat{P}^{-h},g_{i+1},\dots,g_n)$$

$$z_i = \frac{(p_1+\dots+p_i)^2}{[1|p_1+\dots+p_i|n\rangle}$$ location of the pole corresponding for the "i-th" partition



General outline of our results

- It is necessary to understand which shifts are legitimate in the off-shell case, i.e. for which choices $\lim_{z\to\infty}\mathcal{A}(z)=0$. We provided a full classification of the possibilities, in three steps.
- In ***A. van Hameren, JHEP 1407 (2014) 138*** first generalisation of the BCFW recursion relation, only for gluons
- In ***A. van Hameren, M.S. JHEP 1507 (2015) 010*** extension to include fermion pairs
- In ***K. Kutak, A. van Hameren, M.S. arXiv:1611.04380*** extension to two off-shell legs and arbitrary number of fermion pairs, including general recursibility proof for general QCD amplitudes.
- Numerical cross-checks are always successful. They were performed cross checked with a program implementing Berends-Giele recursion relation, *A. van Hameren, M. Bury, Comput.Phys.Commun. 196 (2015) 592-598*
- Wilson line approach:
  P. Kotko, JHEP 1407 (2014) 128
  P. Kotko, M.S., A.M. Stasto JHEP 1608 (2016) 026

Introducing Double Parton Scattering

DPS $\equiv$ the simultaneous occurrence of two partonic hard scatterings in the same proton-proton collision

For a review of DPS and more formal approach:
Diehl, Ostermeier, Schafer, JHEP 1203 (2012) 089

$$\sigma^D = \mathcal{S} \sum_{i,j,k,l} \int \Gamma_{ij}(x_1, x_2, b; t_1, t_2)\, \Gamma_{kl}(x'_1, x'_2, b; t_1, t_2)\, \hat{\sigma}(x_1, x'_1)\, \hat{\sigma}(x_2, x'_2)\, dx_1 dx_2 dx'_1 dx'_2 d^2 b$$

Usual assumption: separation of longitudinal and transverse DOFs:

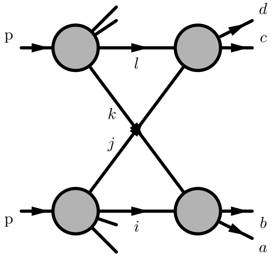
$$\Gamma_{ij}(x_1, x_2, b; t_1, t_2) = D_h^{ij}(x_1, x_2; t_1, t_2)\, F^{ij}(b) = D_h^{ij}(x_1, x_2; t_1, t_2)\, F(b)$$

- Longitudinal correlations, most often ignored or assumed to be negligible, especially at small x : $D_h^{ij}(x_1, x_2; t_1, t_2) = D^i(x_1; t_1)\, D^j(x_2; t_2)$
- Transverse correlation, assumed to be independent of the parton species, taken into account via $\sigma_{eff}^{-1} = \int d^2 b\, F(b)^2 \approx (15mb)^{-1}$ (CDF, D0, ATLAS and CMS)

Usual final kind-of-crafty formula:

$$\sigma^D = \frac{\mathcal{S}}{\sigma_{eff}} \sum_{i_1, j_1, k_1, l_1; i_2, j_2, k_2, l_2} \sigma(i_1 j_1 \to k_1 l_1) \times \sigma(i_2 j_2 \to k_2 l_2)$$

Introducing Double Parton Scattering: the 4 jet case



$$
\begin{aligned}
k^\mu_{i/k} &= x_{i/k}\, l^\mu_{i/k} + k^\mu_{i/k\perp} \\
k^\mu_{l/j} &= x_{l/j}\, l^\mu_{l/j} + k^\mu_{l/j\perp} \\
\sqrt{s} &= 7/8\,TeV \quad \text{or} \quad 13/14\,TeV
\end{aligned}
$$

Double Parton Scattering LHC was already known as a potentially mischievous child for other reasons:

- For two hard-enough scattering to take place, x's or the transferred momentum must be large...
- ...which implies that the c.o.m. energy should be large enough...
- ...which implies, considering the known PDFs behaviour, that two **very high energy** scatterings (transverse momentum cuts higher tan $40 - 50 GeV$ in the final state) are going to miss it.

Technical framework

KaTie (A. van Hameren) : https://bitbucket.org/hameren/KaTie, arXiv:1611.00680

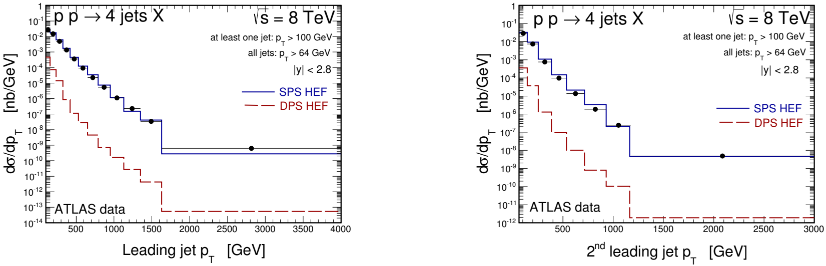
- complete Monte Carlo program for tree-level calculations of any process within the Standard Model; any initial-state partons on-shell or off-shell; numerical Dyson-Schwinger recursion to calculate helicity amplitudes
- CASCADE-2.4.07: DGLAP or CCFM initial and final state parton showers (Hannes Jung et al.)
- Only u and d initial state quarks, final states with all the $N_f = 5$ lightest flavours.
- KMR PDFs and **running** α_s from the MSTWnlo68cl PDF sets
- **Massless quarks approximation** $E_{cm} = 7/8\,TeV \Rightarrow m_{q/\bar{q}} = 0.$
- **Scale** $\mu_R = \mu_F \equiv \mu = \frac{H_T}{2} \equiv \frac{1}{2} \sum_i p_T^i$, (sum over final state particles)

We don't take into account correlations in DPS: $D(x_1, x_2, \mu) = f(x_1, \mu)\, f(x_2, \mu)$.
There are attempts to go beyond this approximation:
Golec-Biernat, Lewandowska, Snyder, M.S., Stasto, Phys.Lett. B750 (2015) 559-564
Golec-Biernat, Stasto: arXiv:1611.02033, **WITH k_T dependence**
Work by Matteo Rinaldi and collaborators in the Perugia group.

Framework validation with hard jets

Most recent ATLAS paper on 4-jet production in proton-proton collision:
ATLAS, JHEP 1512 (2015) 105

$$p_T(1) \geq 100\,\text{GeV}\,, \quad p_T(2,3,4) \geq 64\,\text{GeV}\,, \quad |\eta| \leq 2.8\,, \quad R = 0.4$$

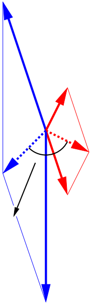


- All channels included and running α_s @ NLO
- Good agreement with data and all LO results by BlackHat and NLOJet++ reproduced
- DPS effects are manifestly too small for such hard cuts: this could be expected from elementary first principles considerations.

Softer cuts: do we see DPS in CMS data ?

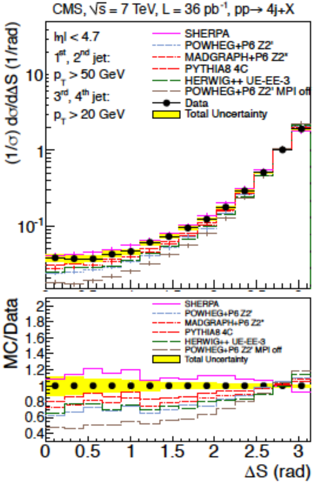
$$p_T(1,2) \geq 50\,\text{GeV}\,, \quad p_T(3,4) \geq 20\,\text{GeV}$$
$$|\eta| \leq 4.7\,, \quad R = 0.5$$

A potential smoking gun for DPS:

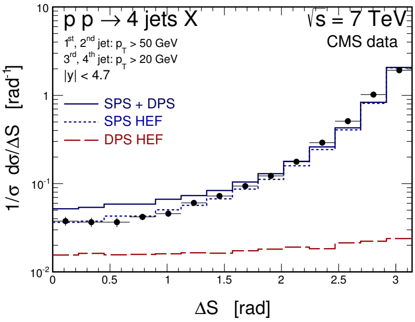


Angle between the soft and the hard jet pair: expected to be flat for DPS. No collinear MonteCarlo describes the data over the whole range. What can k_T-factorization say ?

CMS, Phys.Rev. D89 (2014) no.9, 092010



ΔS: the only-matrix-element prediction



- We roughly describe the data via pQCD effects within our HEF approach which are (equally partially) described by parton-showers and soft MPIs by CMS. K. Kutak, R. Maciula, M.S., A. Szczurek, A. van Hameren, JHEP 1604 (2016) 175
- We seem to overshoot the data when adding DPS
- Natural to ask what happens when we include initial and final state radiation $\Rightarrow$ we need to match parton-level k_T-factorization with parton showers.

Adding parton showers to k_T factorization

Matching the hard off-shell matrix elements with parton showers:

- Generate the hard matrix element in full High Energy Factorization: KaTie
- Add final state CCFM or DGLAP parton showers: CASCADE
- Perform backward evolution in order to have the transverse momentum in the hard matrix element unfolded to initial state radiation: CASCADE
- Reconstruct jets with anti-k_T algorithm: FastJet

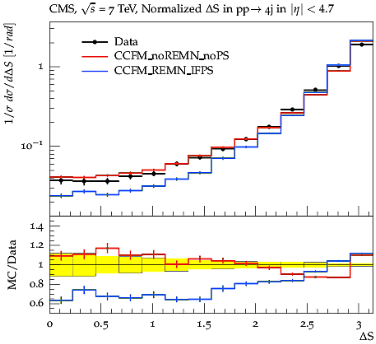
Difference with respect to the collinear generators (MadGraph, Pythia, etc.):

We do not need to perform boosts and rotations of the hard matrix element in order to accomodate for the transverse momentum. Exact kinematics from the very beginning.

This is because this comes directly from the matrix element in a fully gauge invariant way. So, with respect to the fully collinear case, we include the additional hard dynamics coming form transverse momentum.

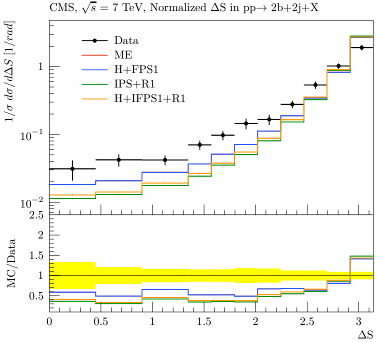
ΔS: k_T factorization plus CCFM parton showers for four jets

- We generate matrix elements without the restriction $k_{T,i}^2 < \mu^2$.
- Jets equally hard or harder than those from the hard matrix element can come from the showering.
- The predictions without parton showers roughly agrees with the data
- Once we include showers and full remnant treatment, we see that we recover a similar result as in the collinear case.
- We conclude that, in this ME+PS scenario, High energy Factorization suggests the need for **MPIs (!?)**
  $\Rightarrow$ see Robert Ciesielski's talk



ΔS: k_T factorization plus CCFM parton showers with two b-tagged jets

- We generate matrix elements without the restriction $k_{T,i}^2 < \mu^2$.
- Jets equally hard or harder than those from the hard matrix element can come from the showering.
- The predictions without parton showers now do not agree with the data
- Once we include showers and full remnant treatment, we are even more off.
- We conclude that, in this ME+PS scenario, High energy Factorization suggests the need for **MPIs (!?)**
  $\Rightarrow$ see CMS, arXiv:1609.03489
  Robert Ciesielski's talk
  Paolo Gunnellini's talk MPI@LHC2016



DPS effects in collinear and HEF: the problem of asymmetric cuts

Inspired by Maciula, Szczurek, Phys.Lett. B749 (2015) 57-62
DPS effects are expected to become significant for lower cuts on the final state transverse momenta, like the ones of the CMS collaboration, Phys.Rev. D89 (2014) no.9, 092010

$$p_T(1,2) \geq 50\,\text{GeV}\,, \quad p_T(3,4) \geq 20\,\text{GeV}\,, \quad |\eta| \leq 4.7\,, \quad R = 0.5$$

$$\begin{aligned}
\text{CMS collaboration :} \quad & \sigma_{tot} = 330 \pm 5\,(\text{stat.}) \pm 45\,(\text{syst.})\, nb \\
\text{LO collinear factorization :} \quad & \sigma_{SPS} = 697\, nb\,, \quad \sigma_{DPS} = \mathbf{125\,nb}\,, \quad \sigma_{tot} = 822\, nb \\
\text{LO HEF } k_T\text{-factorization :} \quad & \sigma_{SPS} = 548\, nb\,, \quad \sigma_{DPS} = \mathbf{33\,nb}\,, \quad \sigma_{tot} = 581\, nb
\end{aligned}$$

In HE factorization DPS gets suppressed and does not dominate at low p_T

Counterintuitive result from well-tested perturbative framework
$\Rightarrow$ phase space effect ?

Higher order corrections to 2-jet production

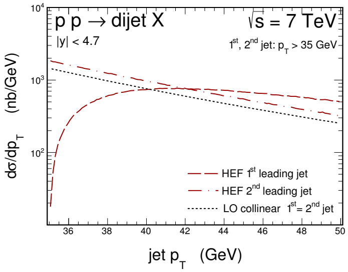


Figure: The transverse momentum distribution of the leading (long dashed line) and subleading (long dashed-dotted line) jet for the dijet production in HEF.

NLO corrections to 2-jet production suffer from instability problem when using symmetric cuts: Frixione, Ridolfi, Nucl.Phys. B507 (1997) 315-333

Symmetric cuts rule out from integration final states in which the momentum imbalance due to the initial state non vanishing transverse momenta gives to one of the jets a lower transverse momentum than the threshold.

ATLAS data vs. theory (nb) @ LHC7 for 2,3,4 jets. Cuts are defined in Eur.Phys.J. C71 (2011) 1763; theoretical predictions from Phys.Rev.Lett. 109 (2012) 042001

#jets	ATLAS	LO	NLO
2	$620 \pm 1.3^{+110}_{-66} \pm 24$	$958(1)^{+316}_{-221}$	$1193(3)^{+130}_{-135}$
3	$43 \pm 0.13^{+12}_{-6.2} \pm 1.7$	$93.4(0.1)^{+50.4}_{-30.3}$	$54.5(0.5)^{+2.2}_{-19.9}$
4	$4.3 \pm 0.04^{+1.4}_{-0.79} \pm 0.24$	$9.98(0.01)^{+7.40}_{-3.95}$	$5.54(0.12)^{+0.08}_{-2.44}$

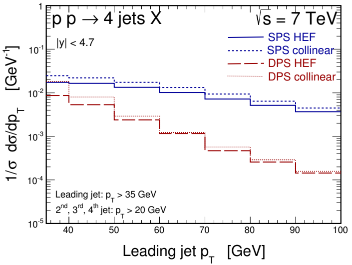
Reconciling HE and collinear factorisation: asymmetric p_T cuts

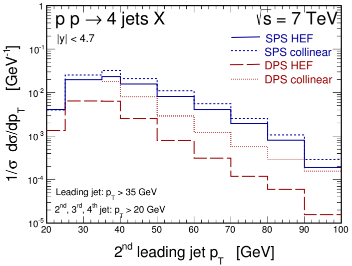
In order to open up wider region of soft final states and thereof expected that the DPS contribution increases

$$p_T(1) \geq 35\text{GeV}\,, \quad p_T(2,3,4) \geq 20\text{ GeV}\,, |\eta| < 4.7\,, \quad \Delta R > 0.5$$

LO collinear factorization : $\sigma_{SPS} = 1969\,nb\,, \quad \sigma_{DPS} = 514\,nb\,, \quad \sigma_{tot} = 2309\,nb$

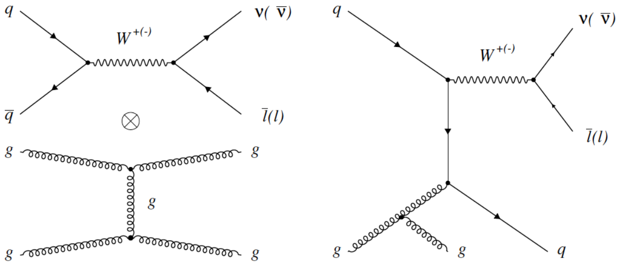
LO HEF k_T-factorization : $\sigma_{SPS} = 1506\,nb\,, \quad \sigma_{DPS} = 297\,nb\,, \quad \sigma_{tot} = 1803\,nb$





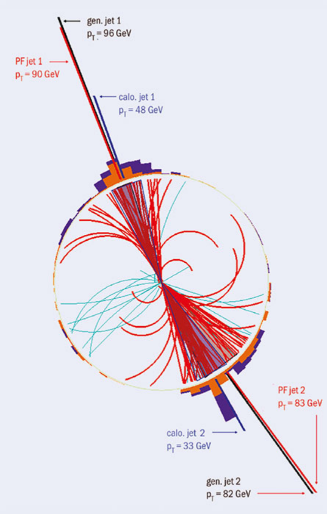
DPS dominance pushed to even lower p_T but restored in HE factorization as well
Next natural step: fully asymmetric cuts !

Why do we want it so bad ?



- Clean channel to see DPS (well, so far...) : $p\,p \Rightarrow W\,j\,j$
- Measurment of σ_{eff} very clean: just reject all events with $\#j > 2$: accepted jets around back-to-back ! $f_{DPS} = 0.055 \pm 0.002(\text{stat.}) \pm 0.014(\text{syst.})$
- No chance that we can do anything equally clean in four-jet production with symmetric cuts. See the take-home statement of CMS, arXiv:1609.03489: **CMS sees MPIs** .
- But **we want specifically DPS !** Pure ME level and $\sigma_{eff} = 15mb$ with asymmetric cuts we get $f_{DPS} \simeq 0.19$!!!

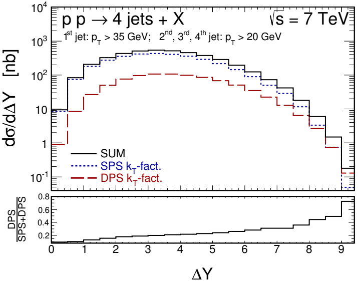
Why the experiment disfavors fully asymmetric cuts to look for DPS

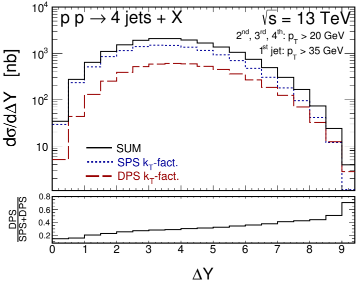


- We want a measurement backed-up by a firm and specific theory prediction of DPS.
- The best way to track jets in CMS is the particle flow algorithm.
- CMS particle-flow jets calibrated up to 20 GeV.
- With $p_T > 35/40 GeV$ we lose too much DPS-favorable phase space
- Recent result from the ATLAS collaboration measurement at 7 TeV (JHEP 1611 (2016) 110) is encouraging:
  $p_T(1) > 42.5 GeV$
  $p_T(2,3,4) > 20 GeV$
  $|\eta| < 4.4$
  $\Rightarrow f_{DPS} = 0.8$.
- Planned analysis by CMS with asymmetric cuts !

Pinning down double parton scattering with new variables: large rapidity separation

K. Kutak, R. Maciula, M.S., A. Szczurek, A. van Hameren
Phys.Rev. D94 (2016) no.1, 014019

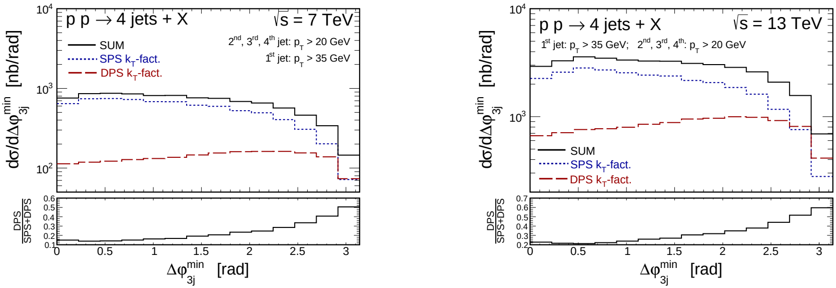




- It is interesting to look for kinematic variables which could make DPS apparent.
- The maximum rapidity separation in the four jet sample is one such variable, especially at 13 GeV.
- for $\Delta Y > 6$ the total cross section is dominated by DPS.

Pinning down double parton scattering with new variables: $\Delta\phi_3^{min}$ - azimuthal separation

K. Kutak, R. Maciula, M.S., A. Szczurek, A. van Hameren
Phys.Rev. D94 (2016) no.1, 014019



- Definition: $\Delta\phi_3^{min} = min_{i,j,k[1,4]} \left(|\phi_i - \phi_j| + |\phi_j - \phi_k|\right)$, $\quad i \neq j \neq k$
- Nicely discriminates between two back-to-back jet pairs and 1vs.3 configurations. High values favor configurations closer to 2 back-to-back pairs, i.e. DPS
- For $\Delta\phi_3^{min} \geq \pi/2$ the total cross section is dominated by DPS

Summary and perspectives

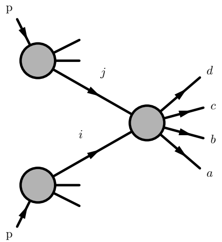
- We have a complete framework for the evaluation of cross sections from amplitudes with off-shell quarks and TMDs. The results form the KaTie are matched to the Monte Carlo CASCADE parton shower event generators
- ΔS variables, potential DPS smoking gun, does not really well without DPS with PS. With PS + remnant: hardest k_T not always coming from the hard matrix element.
- We will scan various PDFs, in order to gauge the dependence of these results on them and we will consistently perform an initial state evolution with KMR PDFs, improving the preliminary results showcased here.
- It will be interesting to explain the ATLAs data and have another experimental analysis with *asymmetric and soft cuts*, in order to enhance DPS. We are going to produce predictions with parton showers also for such configurations.
- **Perspective 1**: improving the pocket-formula framework integrating the more formal developments. First step: include the effect of DPDFs $\Rightarrow$ Golec Biernat and Staśto working on the numerical implementation of arXiv:1611.02033 .
- **Perspective 2**: I hope to be working soon on NLO k_T-factorization.

Conjectured formulas for 2 and 4 jets production:

$$\begin{aligned}
\sigma_{2-jets} &= \sum_{i,j} \int \frac{dx_1}{x_1}\frac{dx_2}{x_2}\, d^2k_{T1} d^2k_{T2}\, \mathcal{F}_i(x_1, k_{T1}, \mu_F)\, \mathcal{F}_j(x_2, k_{T2}, \mu_F) \\
&\quad \times \frac{1}{2\hat{s}} \prod_{l=i}^{2} \frac{d^3k_l}{(2\pi)^3 2E_l} \Theta_{2-jet}\, (2\pi)^4\, \delta\left(P - \sum_{l=1}^{2} k_i\right) \overline{|\mathcal{M}(i^*, j^* \to 2\,\text{part.})|^2} \\
\sigma_{4-jets} &= \sum_{i,j} \int \frac{dx_1}{x_1}\frac{dx_2}{x_2}\, d^2k_{T1} d^2k_{T2}\, \mathcal{F}_i(x_1, k_{T1}, \mu_F)\, \mathcal{F}_j(x_2, k_{T2}, \mu_F) \\
&\quad \times \frac{1}{2\hat{s}} \prod_{l=i}^{4} \frac{d^3k_l}{(2\pi)^3 2E_l} \Theta_{4-jet}\, (2\pi)^4\, \delta\left(P - \sum_{l=1}^{4} k_i\right) \overline{|\mathcal{M}(i^*, j^* \to 4\,\text{part.})|^2}
\end{aligned}$$

- PDFs and matrix elements well defined.
- No rigorous factorization proof around (proving gauge invariance at loop order could help with factorization proofs in the TMD case : see Tuesday's morning session discussion)
- Reasonable description of data justifies this formula *a posteriori*

4-jet production: Single Parton Scattering (SPS)



We take into account all the (according to our conventions) 20 channels.

Here q and q' stand for different quark flavours in the initial (final) state.

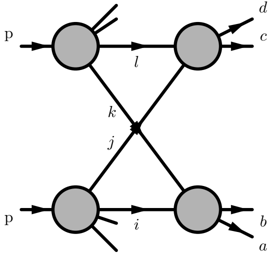
We do not introduce K factors, amplitudes@LO.

$\sim$ 95 % of the total cross section

There are 19 different channels contributing to the cross section at the parton-level:

$$gg \to 4g\,,\, gg \to q\bar{q}\,2g\,,\, qg \to q\,3g\,,\, q\bar{q} \to q\bar{q}\,2g\,,\, qq \to qq\,2g\,,\, qq' \to qq'\,2g\,,$$
$$gg \to q\bar{q}q\bar{q}\,,\, gg \to q\bar{q}q'\bar{q}'\,,\, qg \to qgq\bar{q}\,,\, qg \to qgq'\bar{q}'\,,$$
$$q\bar{q} \to 4g\,,\, q\bar{q} \to q'\bar{q}'\,2g\,,\, q\bar{q} \to q\bar{q}q\bar{q}\,,\, q\bar{q} \to q\bar{q}q'\bar{q}'\,,\, q\bar{q} \to q'\bar{q}'q'\bar{q}'\,,$$
$$q\bar{q} \to q'\bar{q}'q''\bar{q}''\,,\, qq \to qqq\bar{q}\,,\, qq \to qqq'\bar{q}'\,,\, qq' \to qq'q\bar{q}\,,$$

4-jet production: Double parton scattering (DPS)



$$\sigma = \sum_{i,j,a,b;k,l,c,d} \frac{\mathcal{S}}{\sigma_{eff}} \sigma(i,j \to a,b)\,\sigma(k,l \to c,d)$$

$$\mathcal{S} = \begin{cases} 1/2 & \text{if} \quad i\,j = k\,l \quad \text{and} \quad a\,b = c\,d \\ 1 & \text{if} \quad i\,j \neq k\,l \quad \text{or} \quad a\,b \neq c\,d \end{cases}$$

$$\sigma_{eff} = 15\,mb\,, \text{(CDF, D0 and LHCb collaborations)}\,,$$

Experimental data may hint at different values of σ_{eff}; main conclusions not affected

In our conventions, 9 channels from $2 \to 2$ SPS events,

$$\begin{aligned} \#1 &= gg \to gg\,, & \#6 = u\bar{u} \to d\bar{d} \\ \#2 &= gg \to u\bar{u}\,, & \#7 = u\bar{u} \to gg \\ \#3 &= ug \to ug\,, & \#8 = uu \to uu \\ \#4 &= gu \to ug\,, & \#9 = ud \to ud \\ \#5 &= u\bar{u} \to u\bar{u} \end{aligned}$$

$\Rightarrow$ 45 channels for the DPS; only 14 contribute to $\geq 95\%$ of the cross section :

$$(1,1),(1,2),(1,3),(1,4),(1,8),(1,9),(3,3)$$
$$(3,4),(3,8),(3,9),(4,4),(4,8),(4,9),(9,9)$$