

Testing the running coupling k_T -factorization formula for the inclusive gluon production

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based on work done with
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Physical Review D 94, 054023 (2016)
(arXiv:1607.02082)



5th Workshop on QCD and diffraction
Dec. 5th-7th, 2016. Krakow – Poland

Outline:

- ◆ Motivation
- ◆ The k_T -factorization approach
- ◆ Ingredients of our calculation
- ◆ Results for dN/dy vs y : p+p, p(d)+A and A+A
- ◆ Results for $dN/d\eta|_{\eta=0}$ vs \sqrt{s} : p+p, p(d)+A and A+A
- ◆ Conclusions

Motivation

The CGC effective theory has entered in the next-to-leading order (NLO) era
→ several processes calculated at this level (and their numerical implementation are underway).

- **Regarding the Balitsky-Kovchegov (BK) evolution equation:**

Running coupling corrections to the kernel of the BK equation with the solution being able to describe several observables @ HERA, RHIC and LHC;

Balitsky, PRD 75, 014001 (2007); Balitsky and Chirilli, PRD 77, 014019 (2008)
Kovchegov and Weigert, NPA 784, 188 (2007), NPA 789, 260 (2007);
Kovchegov, Kuokkanen, Rummukainen and Weigert, NPA 823, 47 (2009).

Large single and double transverse logarithms have been resummed to all orders in the NLO BK equation

Iancu, Madrigal, Mueller, Soyez and Triantafyllopoulos, PLB 750, 643 (2015);
Lappi and Mäntysaari, PRD 91, no. 7, 074016 (2015), PRD 93, no. 9, 094004 (2016).

- **JIMWLK evolution equation @ NLO**

Kovner, Lublinsky and Mulian, PRD 89, no. 6, 061704 (2014), JHEP 1408, 114 (2014); Lublinsky and Mulian, arXiv:1610.03453.

Motivation

- **Regarding the “hybrid formalism”:**

NLO corrections calculated and implemented numerically → better agreement with experimental data @ RHIC/LHC energies for forward hadron production;

Chirilli, Xiao and Yuan, PRL 108, 122301 (2012);

Stasto, Xiao and Zaslavsky, PRL 112, no. 1, 012302 (2014);

Altinoluk, Armesto, Beuf, Kovner and Lublinsky, PRD 91, no. 9, 094016 (2015);

Watanabe, Xiao, Yuan and Zaslavsky, PRD 92, no. 3, 034026 (2015).

- **Regarding the k_T – factorization:**

Generalization to higher orders in α_s of the k_T – factorization formalism for inclusive gluon production conjectured few years ago

Horowitz and Kovchegov, NPA 849, 72 (2011)

No studies about the impact of the high-order corrections in this formalism on observables so far!

“Versions” of the
 k_T -factorization
formula for inclusive
gluon production

The fixed coupling k_T -factorization formula


$$\frac{d^3\sigma}{d^2k_T dy} = \frac{2}{C_F} \frac{1}{\mathbf{k}^2} \int d^2q \alpha_s \phi_{h_1}(\mathbf{q}, y) \phi_{h_2}(\mathbf{k} - \mathbf{q}, Y - y)$$

Gribov, Levin and Ryskin, Phys. Rept. 100, 1 (1983)
Braun, PLB 483, 105 (2000)
Kovchegov and Tuchin, PRD 65, 074026 (2002)

$$\mathbf{k} = (k_x, k_y)$$

$$\phi_{h_i}(\mathbf{k}, y) = \frac{C_F}{\alpha_s (2\pi)^3} \int d^2b d^2r e^{-i\mathbf{k}\cdot\mathbf{r}} \nabla_r^2 \mathcal{N}_{h_i}^G(\mathbf{r}, \mathbf{b}, y)$$

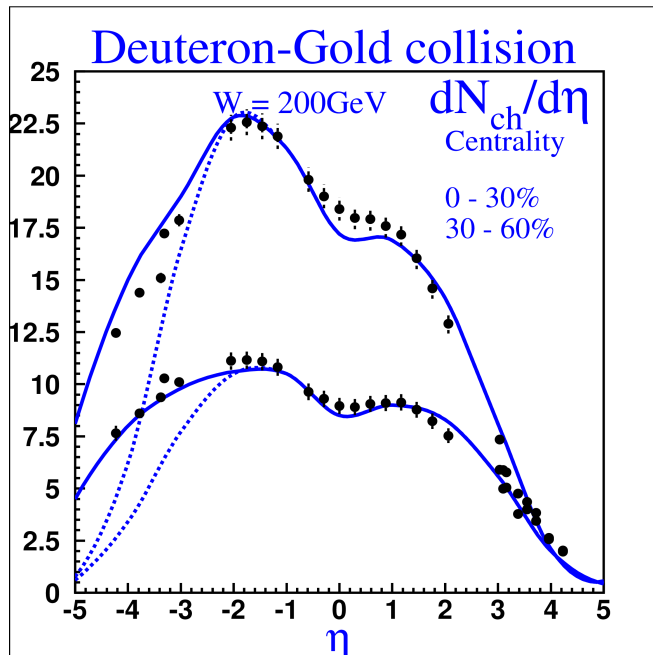
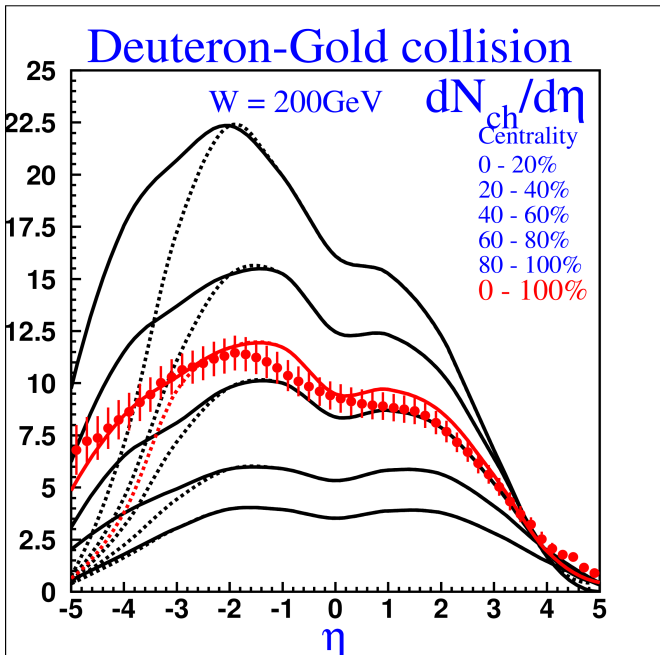
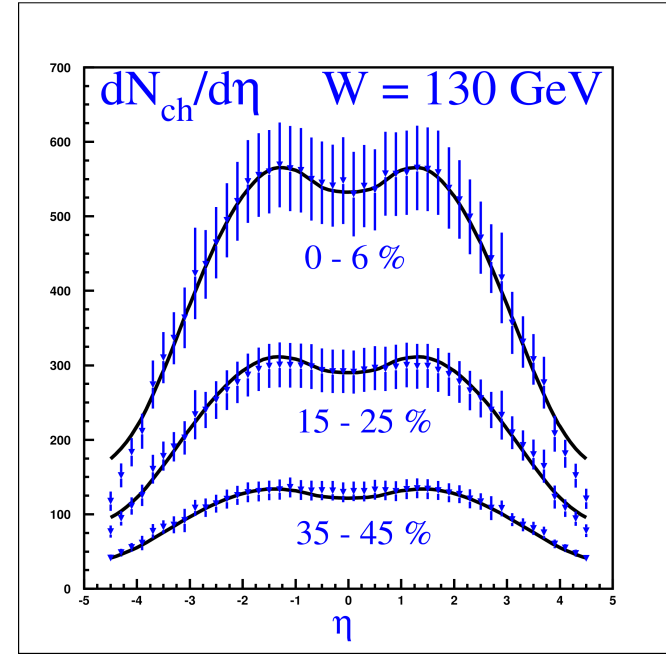
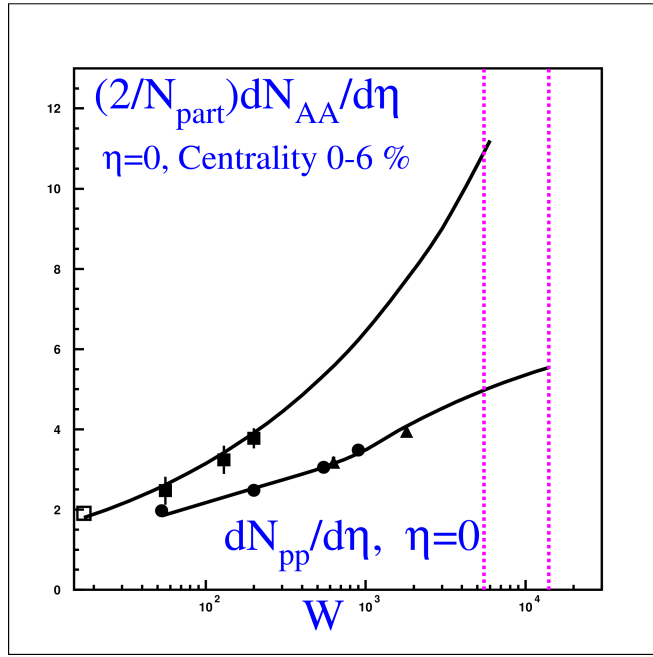
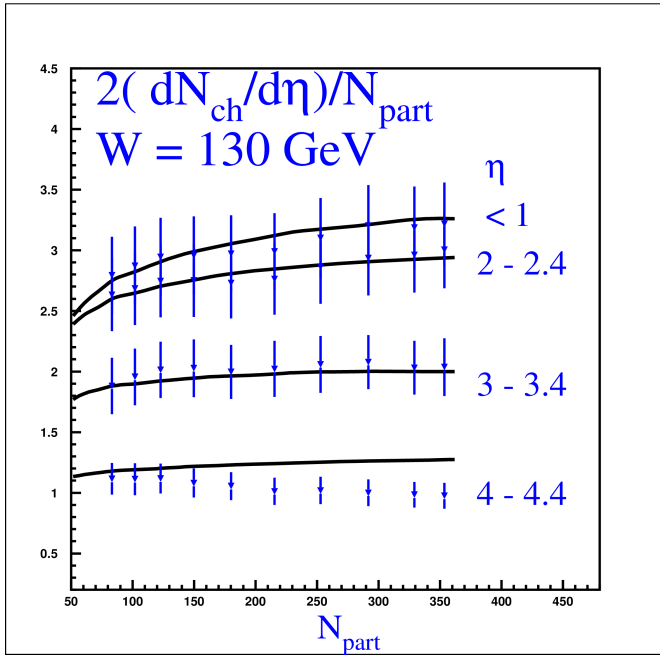
 Unintegrated gluon
distribution (ugd)

 Dipole-hadron forward
scattering amplitude for a gluon
dipole of transverse size \mathbf{r} and a
given impact parameter \mathbf{b}

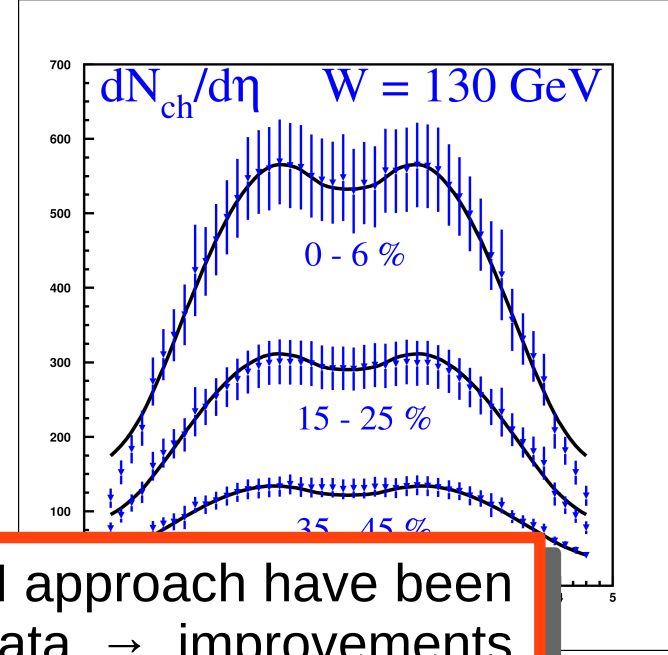
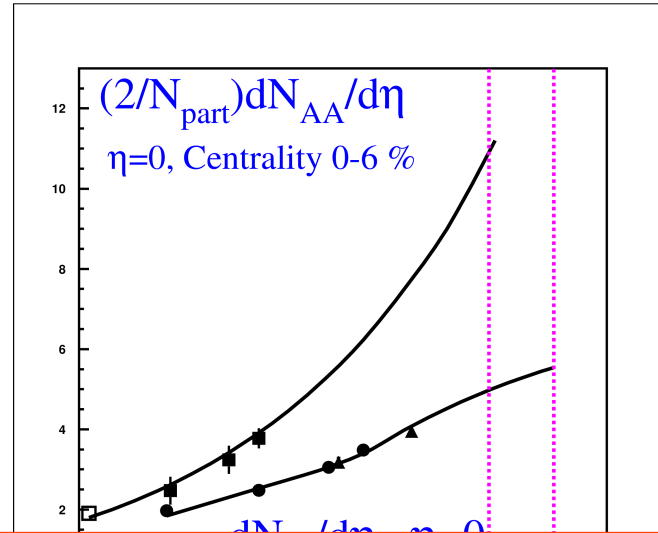
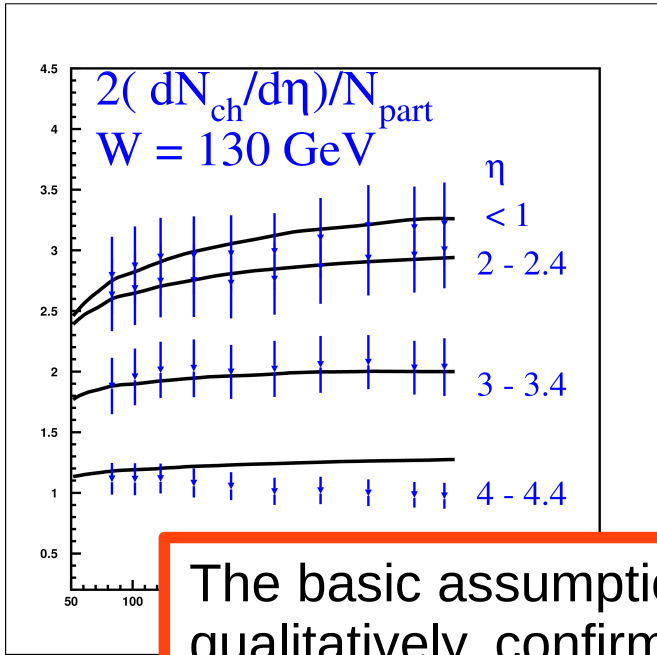
Widely used in phenomenological studies @ RHIC energies.

e.g. Kharzeev-Levin-Nardi (KLN) papers: [PLB 523 \(2001\) 79](#); [PRC 71 \(2005\) 054903](#); [PLB 561 \(2003\) 93](#);
[NPA 730 \(2004\) 448](#); [NPA 743 \(2004\) 329](#); [NPA 747 \(2005\) 609](#).

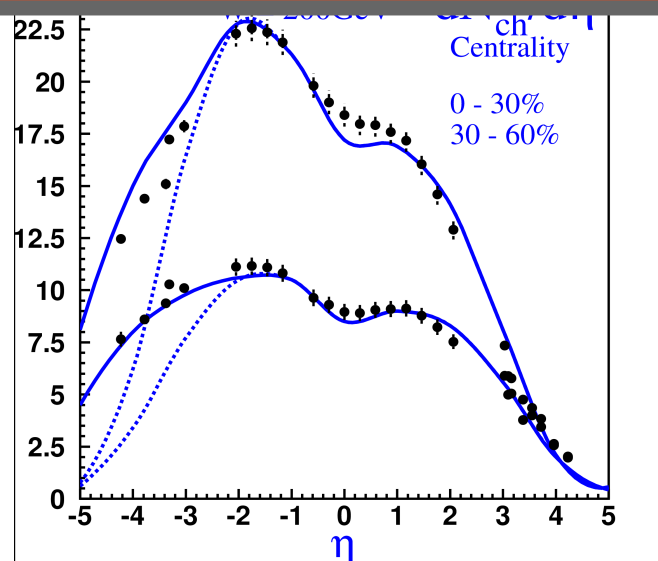
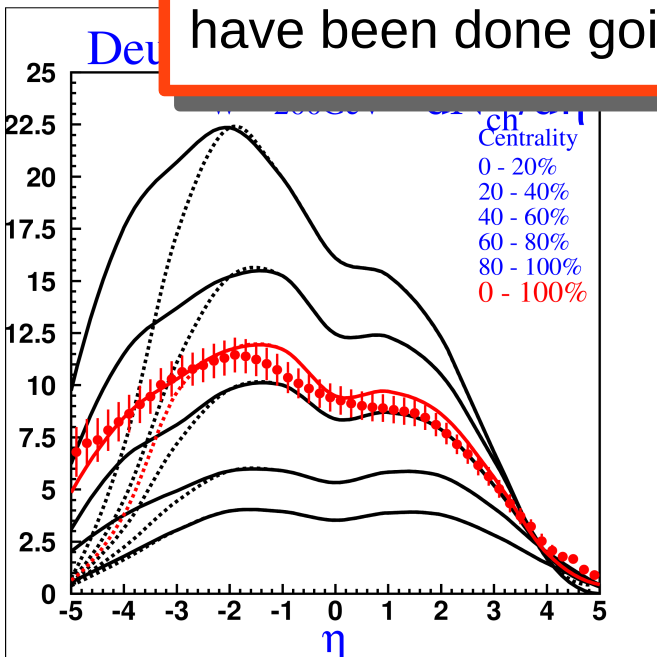
The fixed coupling k_T -factorization formula



The fixed coupling k_T -factorization formula



The basic assumptions and predictions of the KLN approach have been qualitatively confirmed by the RHIC and LHC data → improvements have been done going towards to the quantitative direction.



The ("naive") rc k_T -factorization formula

I) **Assumes** that the factorization from the LO expression is preserved after the inclusion of the running coupling corrections;

$$\text{II) } \alpha_s \rightarrow \alpha_s(Q^2) = \frac{12\pi}{\beta_0 \ln(Q^2/\Lambda_{QCD}^2)} ; \quad \beta_0 = 33 - 2n_f$$

$$\frac{d^3\sigma}{d^2k_T dy} = \frac{2}{C_F} \frac{1}{\mathbf{k}^2} \int d^2q \alpha_s(Q^2) \phi_{h_1}(\mathbf{q}, y) \phi_{h_2}(\mathbf{k} - \mathbf{q}, Y - y)$$

$$\phi_{h_i}(\mathbf{k}, y) = \frac{C_F}{\alpha_s(Q^2) (2\pi)^3} \int d^2b d^2r e^{-i\mathbf{k}\cdot\mathbf{r}} \nabla_r^2 \mathcal{N}_{h_i}^G(\mathbf{r}, \mathbf{b}, y)$$

Widely used in phenomenological studies @ RHIC / LHC energies;

Levin and Rezaeian, PRD 82, 014022 (2010), PRD 82, 054003 (2010), PRD 83, 114001 (2011)

Albacete and Dumitru, arXiv:1011.5161

Albacete, Dumitru, Fujii and Nara, NPA 897, 1 (2013)

Dumitru, Kharzeev, Levin and Nara, PRC 85, 044920 (2012)

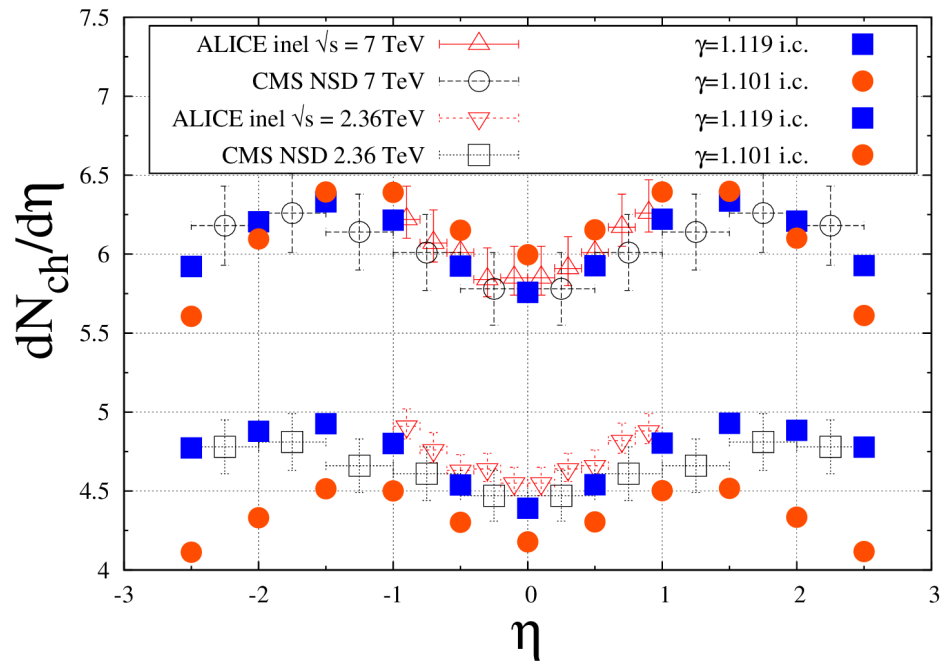
Tribedy and Venugopalan, NPA 850, 136 (2011), NPA 859, 185 (2011), PLB 710, 125 (2012)

Schenke, Tribedy and Venugopalan, PRC 89, no. 2, 024901 (2014)

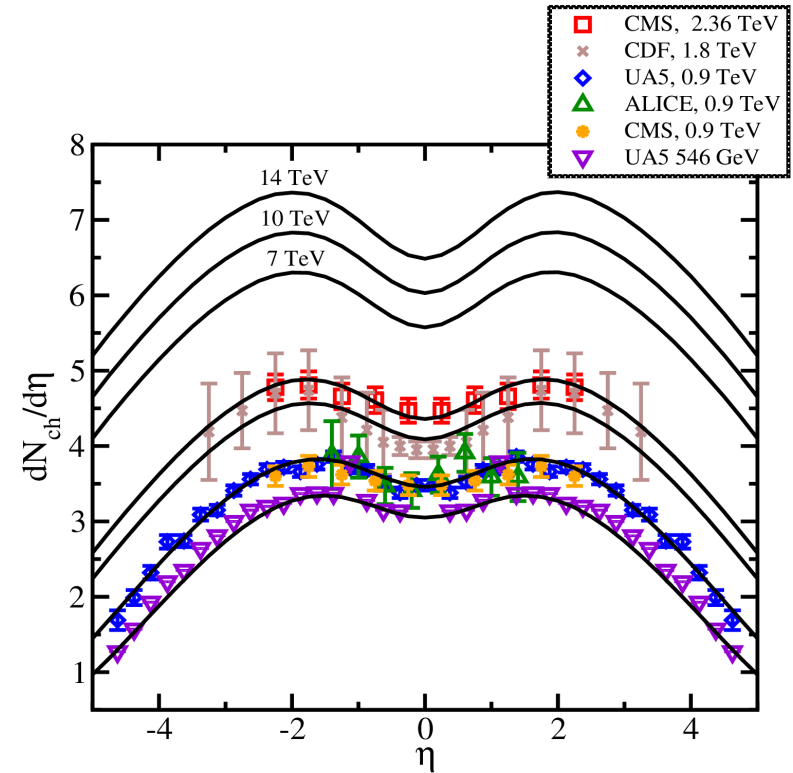
The ("naive") rc k_T -factorization formula

Considering the running of the coupling constant has led to an improvement of the agreement between theory and experimental data...

Albacete, Dumitru, Fujii and Nara, NPA 897, 1 (2013)



Levin and Rezaeian, PRD 82, 014022 (2010)

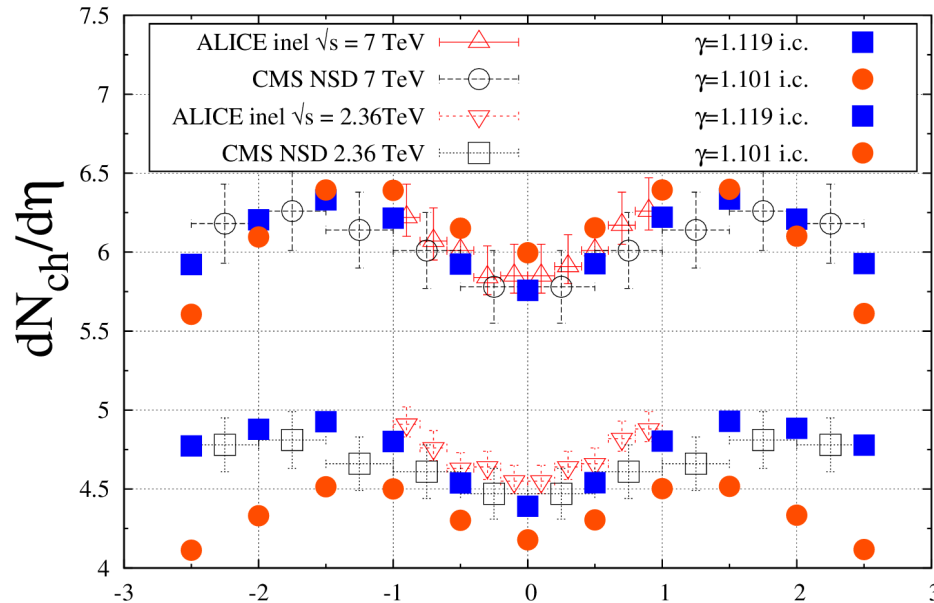


The ("naive") rc k_T -factorization formula

Considering the running of the coupling constant has led to an improvement of the agreement between theory and experimental data...

Albacete, Dumitru, Fujii and Nara, NPA 897, 1 (2013)

$$Q^2 = \max\{|\mathbf{q} + \mathbf{k}|^2/4, |\mathbf{q} - \mathbf{k}|^2/4\}$$



$$\phi_{h_1} \propto 1/\alpha_s(\mathbf{q}) \quad \eta$$

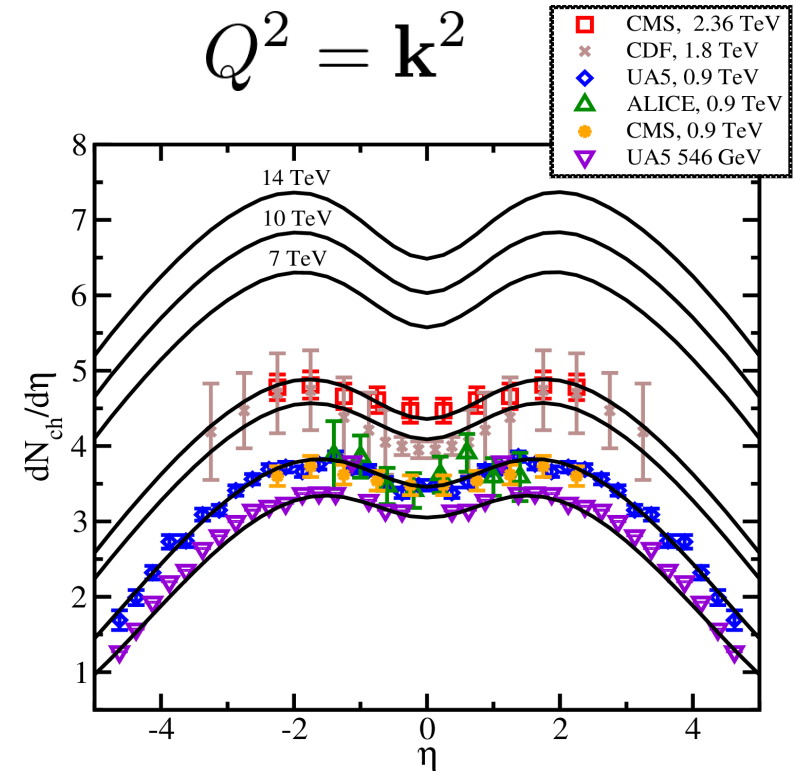
$$\phi_{h_2} \propto 1/\alpha_s(\mathbf{k} - \mathbf{q})$$

... however the scale Q^2 is not known in this expression and it has been chosen by hand!

A formal inclusion of the running coupling effects is mandatory to fix Q^2

Levin and Rezaeian, PRD 82, 014022 (2010)

$$Q^2 = \mathbf{k}^2$$



$$\phi_{h_i} \propto 1/\alpha_s(Q_s(x_i))$$

The α_s corrected k_T -factorization formula

Horowitz and Kovchegov, NPA 849, 72 (2011)

$$\frac{d^3\sigma}{d^2k_T dy} = \frac{2C_F}{\pi^2} \frac{1}{\mathbf{k}^2} \int d^2q \bar{\phi}_{h_1}(\mathbf{q}, 0) \bar{\phi}_{h_2}(\mathbf{k} - \mathbf{q}, 0) \frac{\alpha_s (\Lambda_{\text{coll}}^2 e^{-5/3})}{\alpha_s (Q^2 e^{-5/3}) \alpha_s (Q^{*2} e^{-5/3})}$$

$$\bar{\phi}_{h_i}(\mathbf{k}, y) = \alpha_s \phi_{h_i}(\mathbf{k}, y) = \frac{C_F}{(2\pi)^3} \int d^2b d^2r e^{-i\mathbf{k}\cdot\mathbf{r}} \nabla_r^2 \mathcal{N}_{h_i}(\mathbf{r}, \mathbf{b}, y)$$

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Λ_{coll}^2 is a collinear infrared cutoff for the case where the would-be produced gluon splits into a collinear gluon-gluon or quark-antiquark pair (whose invariant mass must be less than this quantity)

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Q^2 is given by:



Now it is fully determined from a formal calculation!

$$\begin{aligned} \ln \frac{Q^2}{\mu_{\text{MS}}^2} = & \frac{1}{2} \ln \frac{q^2 (\mathbf{k} - \mathbf{q})^2}{\mu_{\text{MS}}^4} - \frac{1}{4q^2 (\mathbf{k} - \mathbf{q})^2 [(\mathbf{k} - \mathbf{q})^2 - q^2]^6} \left\{ \mathbf{k}^2 [(\mathbf{k} - \mathbf{q})^2 - q^2]^3 \right. \\ & \times \left\{ [[(\mathbf{k} - \mathbf{q})^2]^2 - (q^2)^2] [(\mathbf{k}^2)^2 + ((\mathbf{k} - \mathbf{q})^2 - q^2)^2] + 2\mathbf{k}^2 [(q^2)^3 - [(\mathbf{k} - \mathbf{q})^2]^3] \right. \\ & \left. \left. - q^2 (\mathbf{k} - \mathbf{q})^2 [2(\mathbf{k}^2)^2 + 3[(\mathbf{k} - \mathbf{q})^2 - q^2]^2 - 3\mathbf{k}^2 [(\mathbf{k} - \mathbf{q})^2 + q^2]] \ln \left(\frac{(\mathbf{k} - \mathbf{q})^2}{q^2} \right) \right\} \right. \\ & + i [(\mathbf{k} - \mathbf{q})^2 - q^2]^3 \left\{ \mathbf{k}^2 [(\mathbf{k} - \mathbf{q})^2 - q^2] \left[\mathbf{k}^2 [(\mathbf{k} - \mathbf{q})^2 + q^2] - (q^2)^2 - [(\mathbf{k} - \mathbf{q})^2]^2 \right] \right. \\ & \left. \left. + q^2 (\mathbf{k} - \mathbf{q})^2 \left(\mathbf{k}^2 [(\mathbf{k} - \mathbf{q})^2 + q^2] - 2(\mathbf{k}^2)^2 - 2[(\mathbf{k} - \mathbf{q})^2 - q^2]^2 \right) \ln \left(\frac{(\mathbf{k} - \mathbf{q})^2}{q^2} \right) \right\} \right. \\ & \left. \times \sqrt{2q^2 (\mathbf{k} - \mathbf{q})^2 + 2\mathbf{k}^2 (\mathbf{k} - \mathbf{q})^2 + 2q^2 \mathbf{k}^2 - (\mathbf{k}^2)^2 - (q^2)^2 - [(\mathbf{k} - \mathbf{q})^2]^2} \right\}, \end{aligned}$$

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$$\frac{d^3\sigma}{d^2k_T dy} = \frac{2C_F}{\pi^2} \frac{1}{\mathbf{k}^2} \int d^2q \bar{\phi}_{h_1}(\mathbf{q}, 0) \bar{\phi}_{h_2}(\mathbf{k} - \mathbf{q}, 0) \frac{\alpha_s (\Lambda_{\text{coll}}^2 e^{-5/3})}{\alpha_s (Q^2 e^{-5/3}) \alpha_s (Q^{*2} e^{-5/3})}$$

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Q^2 is given by:



Now it is fully determined from a formal calculation!

But this result is valid only for $y = 0$



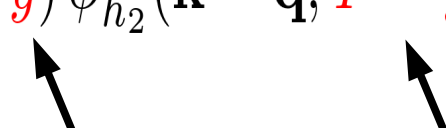
$$\begin{aligned} \ln \frac{Q^2}{\mu_{\text{MS}}^2} = & \frac{1}{2} \ln \frac{q^2 (\mathbf{k} - \mathbf{q})^2}{\mu_{\text{MS}}^4} - \frac{1}{4q^2 (\mathbf{k} - \mathbf{q})^2 [(\mathbf{k} - \mathbf{q})^2 - q^2]^6} \left\{ k^2 [(\mathbf{k} - \mathbf{q})^2 - q^2]^3 \right. \\ & \times \left\{ [[(\mathbf{k} - \mathbf{q})^2]^2 - (q^2)^2] [(k^2)^2 + ((\mathbf{k} - \mathbf{q})^2 - q^2)^2] + 2k^2 [(q^2)^3 - [(\mathbf{k} - \mathbf{q})^2]^3] \right. \\ & \left. \left. - q^2 (\mathbf{k} - \mathbf{q})^2 [2 (k^2)^2 + 3 [(\mathbf{k} - \mathbf{q})^2 - q^2]^2 - 3k^2 [(\mathbf{k} - \mathbf{q})^2 + q^2]] \ln \left(\frac{(\mathbf{k} - \mathbf{q})^2}{q^2} \right) \right\} \right. \\ & + i [(\mathbf{k} - \mathbf{q})^2 - q^2]^3 \left\{ k^2 [(\mathbf{k} - \mathbf{q})^2 - q^2] [k^2 [(\mathbf{k} - \mathbf{q})^2 + q^2] - (q^2)^2 - [(\mathbf{k} - \mathbf{q})^2]^2] \right. \\ & \left. \left. + q^2 (\mathbf{k} - \mathbf{q})^2 \left(k^2 [(\mathbf{k} - \mathbf{q})^2 + q^2] - 2 (k^2)^2 - 2 [(\mathbf{k} - \mathbf{q})^2 - q^2]^2 \right) \ln \left(\frac{(\mathbf{k} - \mathbf{q})^2}{q^2} \right) \right\} \right. \\ & \left. - [(\mathbf{k} - \mathbf{q})^2]^2 \right\}, \end{aligned}$$

What about the small-x (rapidity) evolution?

Conjecture for

Horowitz and Kovchegov, NPA 849, 72 (2011)

the α_s corrected k_T -factorization formula

$$\frac{d^3\sigma}{d^2k_T dy} = \frac{2C_F}{\pi^2} \frac{1}{\mathbf{k}^2} \int d^2q \bar{\phi}_{h_1}(\mathbf{q}, y) \bar{\phi}_{h_2}(\mathbf{k} - \mathbf{q}, Y - y) \frac{\alpha_s (\Lambda_{\text{coll}}^2 e^{-5/3})}{\alpha_s (Q^2 e^{-5/3}) \alpha_s (Q^{*2} e^{-5/3})}$$


$$\bar{\phi}_{h_i}(\mathbf{k}, y) = \alpha_s \phi_{h_i}(\mathbf{k}, y)$$

$$\begin{aligned} \ln \frac{Q^2}{\mu_{\overline{\text{MS}}}^2} &= \frac{1}{2} \ln \frac{q^2 (\mathbf{k} - \mathbf{q})^2}{\mu_{\overline{\text{MS}}}^4} - \frac{1}{4q^2 (\mathbf{k} - \mathbf{q})^2 [(\mathbf{k} - \mathbf{q})^2 - q^2]^6} \left\{ k^2 [(\mathbf{k} - \mathbf{q})^2 - q^2]^3 \right. \\ &\quad \times \left\{ [(\mathbf{k} - \mathbf{q})^2]^2 - (q^2)^2 \right\} [(\mathbf{k}^2)^2 + ((\mathbf{k} - \mathbf{q})^2 - q^2)^2] + 2k^2 [(q^2)^3 - [(\mathbf{k} - \mathbf{q})^2]^3] \\ &\quad \left. - q^2 (\mathbf{k} - \mathbf{q})^2 \left[2(\mathbf{k}^2)^2 + 3[(\mathbf{k} - \mathbf{q})^2 - q^2]^2 - 3k^2 [(\mathbf{k} - \mathbf{q})^2 + q^2] \right] \ln \left(\frac{(\mathbf{k} - \mathbf{q})^2}{q^2} \right) \right\} \\ &\quad + i [(\mathbf{k} - \mathbf{q})^2 - q^2]^3 \left\{ k^2 [(\mathbf{k} - \mathbf{q})^2 - q^2] \left[k^2 [(\mathbf{k} - \mathbf{q})^2 + q^2] - (q^2)^2 - [(\mathbf{k} - \mathbf{q})^2]^2 \right] \right. \\ &\quad \left. + q^2 (\mathbf{k} - \mathbf{q})^2 \left(k^2 [(\mathbf{k} - \mathbf{q})^2 + q^2] - 2(\mathbf{k}^2)^2 - 2[(\mathbf{k} - \mathbf{q})^2 - q^2]^2 \right) \ln \left(\frac{(\mathbf{k} - \mathbf{q})^2}{q^2} \right) \right\} \\ &\quad \times \sqrt{2q^2 (\mathbf{k} - \mathbf{q})^2 + 2k^2 (\mathbf{k} - \mathbf{q})^2 + 2q^2 k^2 - (\mathbf{k}^2)^2 - (q^2)^2 - [(\mathbf{k} - \mathbf{q})^2]^2} \left. \right\}, \end{aligned}$$

What is the impact of the α_s corrections in the conjectured k_T -factorization formula?

Re: assume a simple setup and calculate some observables considering each “version” of the k_T -factorization formula previously mentioned and compare the results.

F. O. Durães, A.V.G., V. P. B. Gonçalves and F. S. Navarra

Physical Review D 94, 054023 (2016)

In light of the above “answer” this is a qualitative work!

Ingredients of our calculation

$$\begin{aligned}\text{KLN ugd: } \phi_{KLN}(k, y) &= \frac{2C_F}{3\pi^2}, \quad k \leq Q_s \\ &= \frac{2C_F}{3\pi^2} \frac{Q_s^2}{k^2}, \quad k > Q_s\end{aligned}$$

$$Q_s^2 = A_{\text{eff}}^{1/3} \left(\frac{3 \times 10^{-4}}{x} \right)^{0.288}$$

$$A_{\text{eff}}^{\text{Au}} = 18.5, \quad A_{\text{eff}}^{\text{Pb}} = 20$$

Local Parton Hadron Duality: meaning rapidity distribution of partons and hadrons only change by a numerical factor.

Introduces an effective mass that incorporates nonperturbative effects:

$$m_h = 0.350 \text{ GeV} \quad \text{for all cases}$$

Simplified nuclear geometry: no Monte Carlo fluctuation on the nucleon's position for nuclei.

Bottom line: simplest setup possible! Motivation: isolate the impact of the α_s corrections in the k_T -factorization formula.

Ingredients of our calculation

Fixed coupling (FC): $\alpha_s = 0.25$

("Naive") running coupling: two different prescription for Q^2

RC1: scales fixed as in Albacete, Dumitru, Fujii and Nara, NPA 897, 1 (2013)

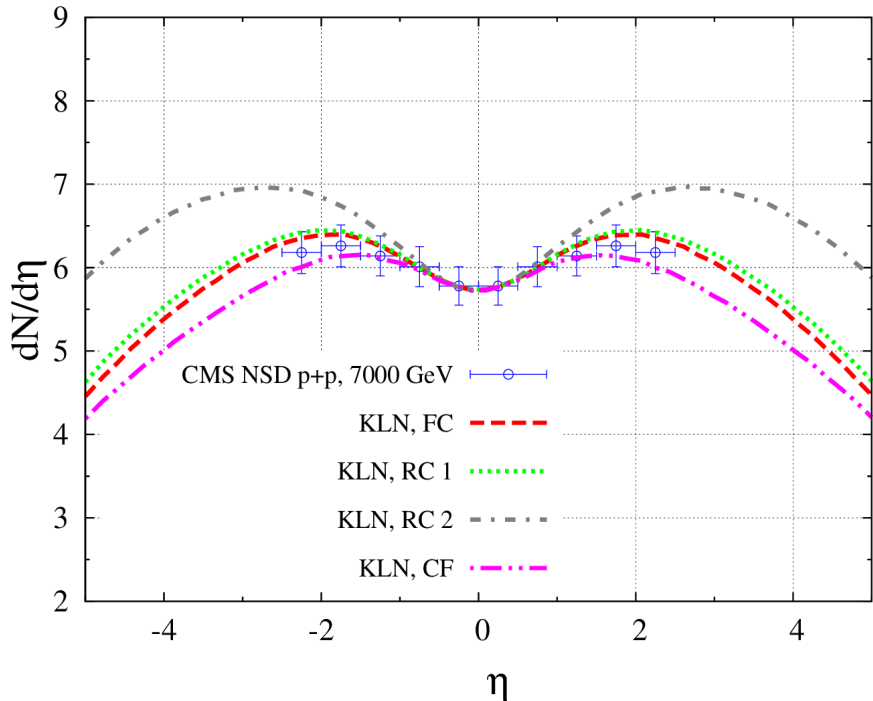
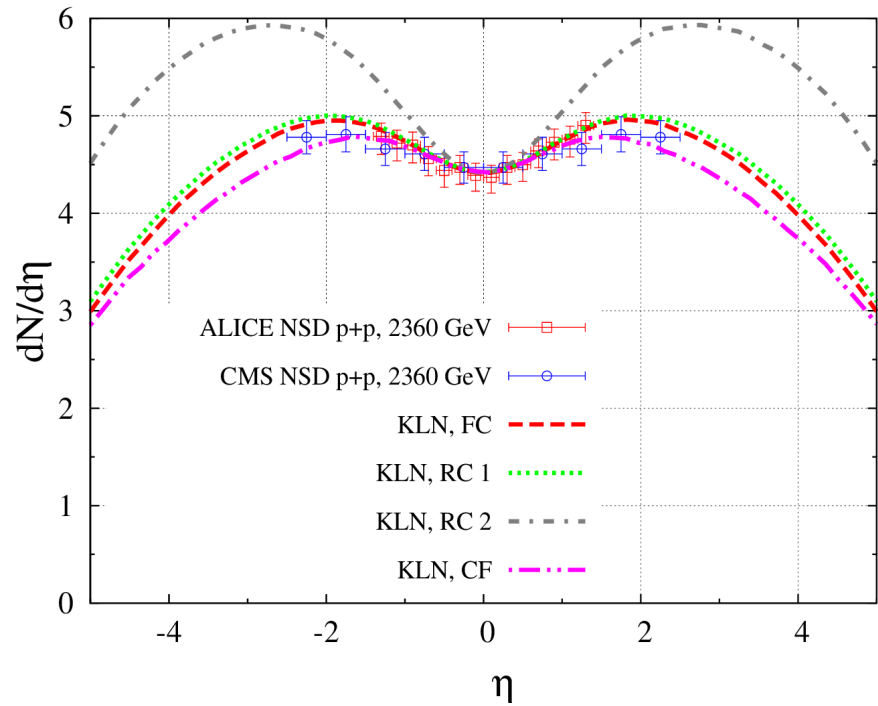
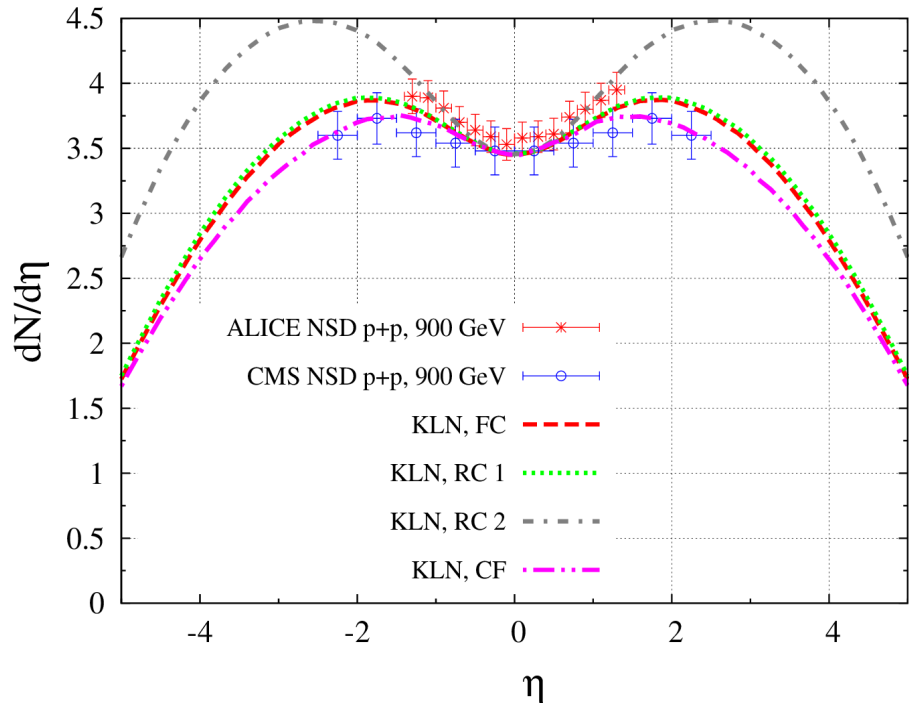
$$Q^2 = \max\{|\mathbf{q} + \mathbf{k}|^2/4, |\mathbf{q} - \mathbf{k}|^2/4\} ; \phi_{h_1} \propto 1/\alpha_s(\mathbf{q}) ; \phi_{h_2} \propto 1/\alpha_s(\mathbf{k} - \mathbf{q})$$

RC2: scales fixed as in Levin and Rezaeian, PRD 82, 014022 (2010)

$$Q^2 = \mathbf{k}^2 \quad \phi_{h_i} \propto 1/\alpha_s(Q_s(x_i))$$

α_s corrected formula (CF): $\alpha_s(\Lambda_{\text{coll}}^2 e^{-5/3}) = 0.25$

Results for dN/dy vs y : p+p collisions



Change in the normalization from 0.9 TeV to 7 TeV:

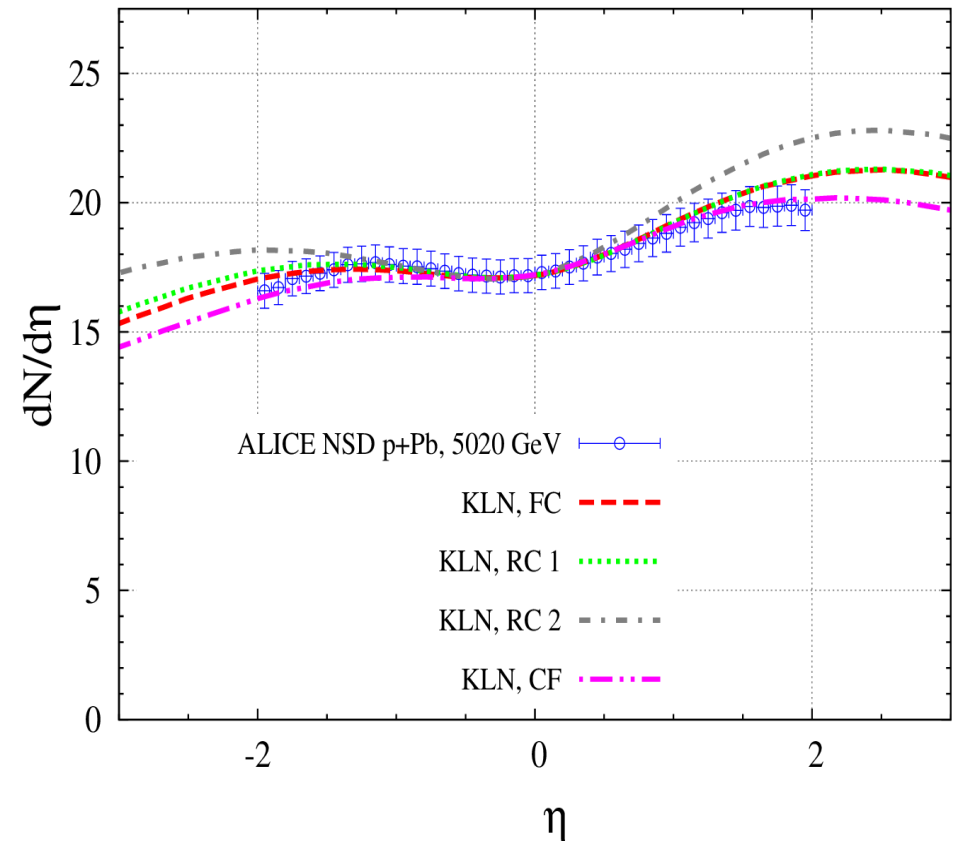
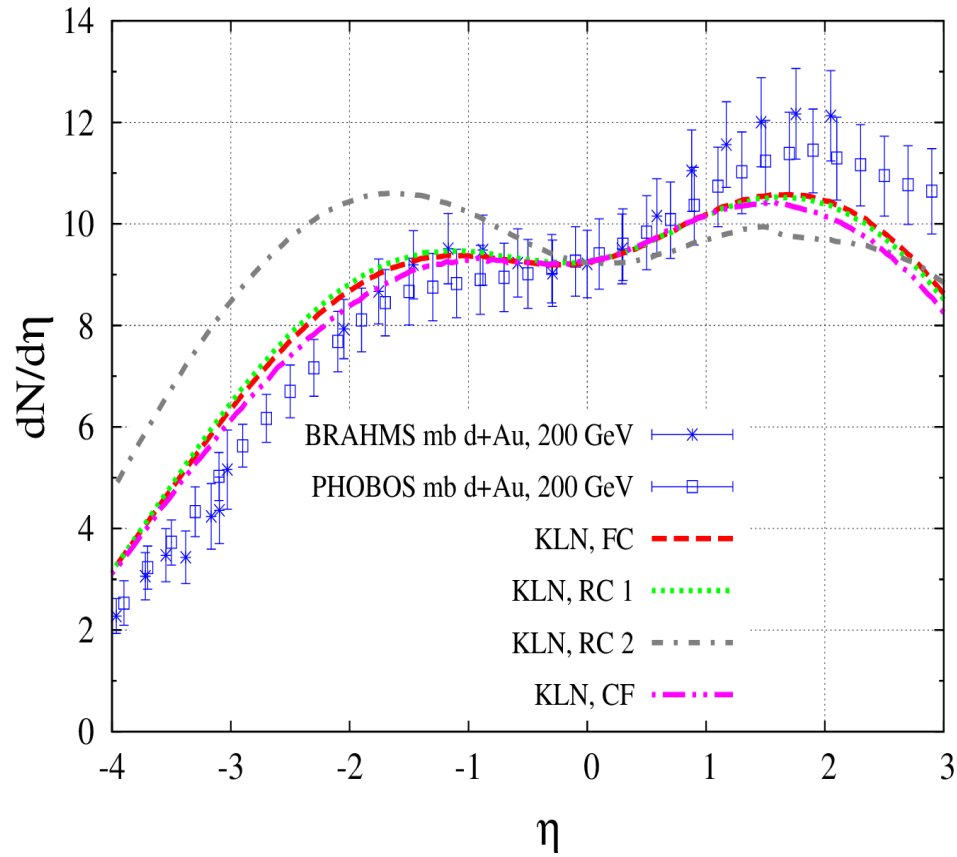
FC ~ 20.0%

RC1 ~ 26.0%

RC2 ~ 35.0%

CF ~ 0.94%

Results for dN/dy vs y : p(d)+A collisions



Change in the normalization from 0.2 TeV to 5.02 TeV:

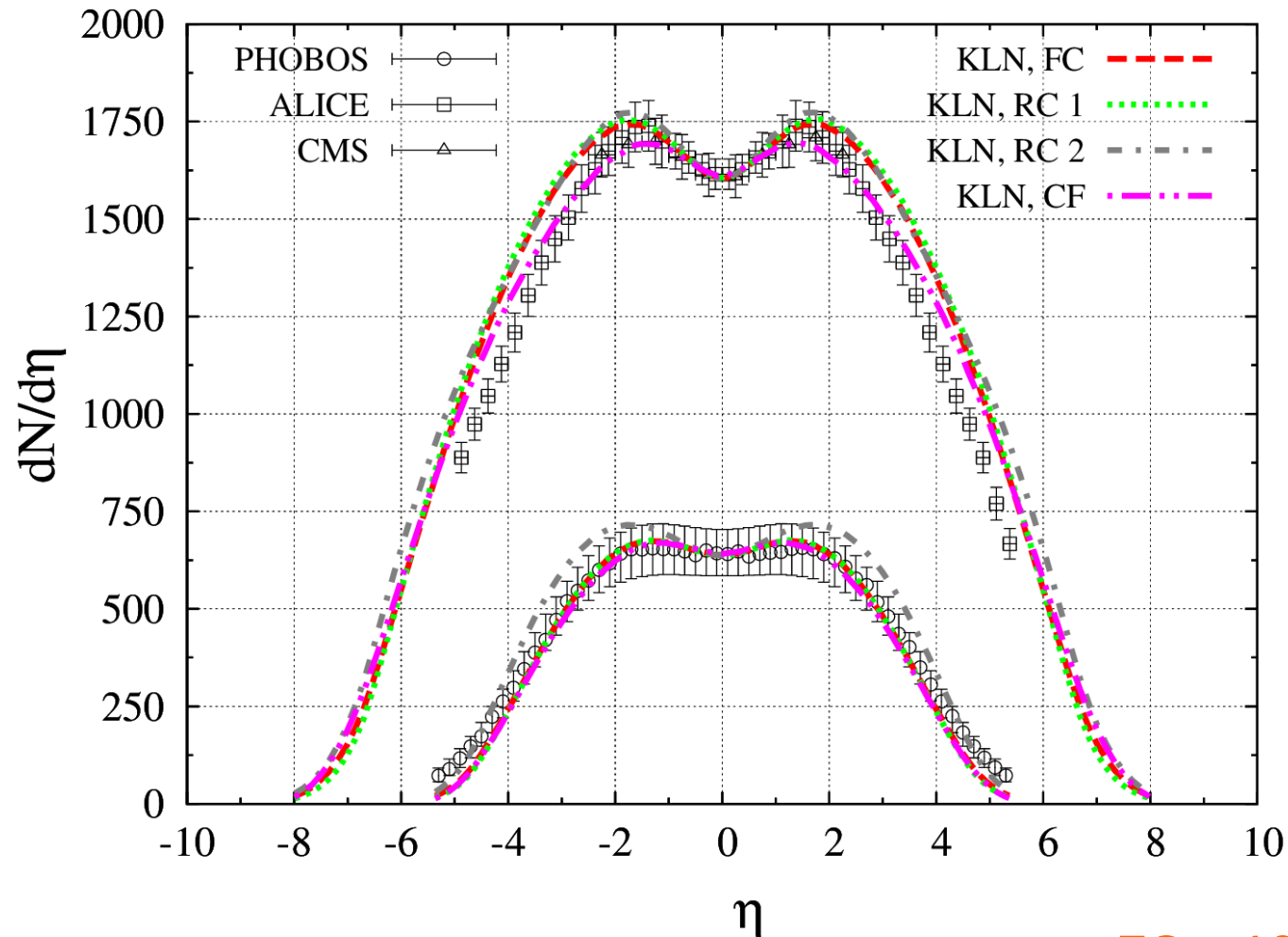
FC ~ 41.0%

RC1 ~ 48.42%

RC2 ~ 55.52%

CF ~ 17.42%

Results for dN/dy vs y : A+A collisions



Change in the normalization from 0.13 TeV to 2.76 TeV:

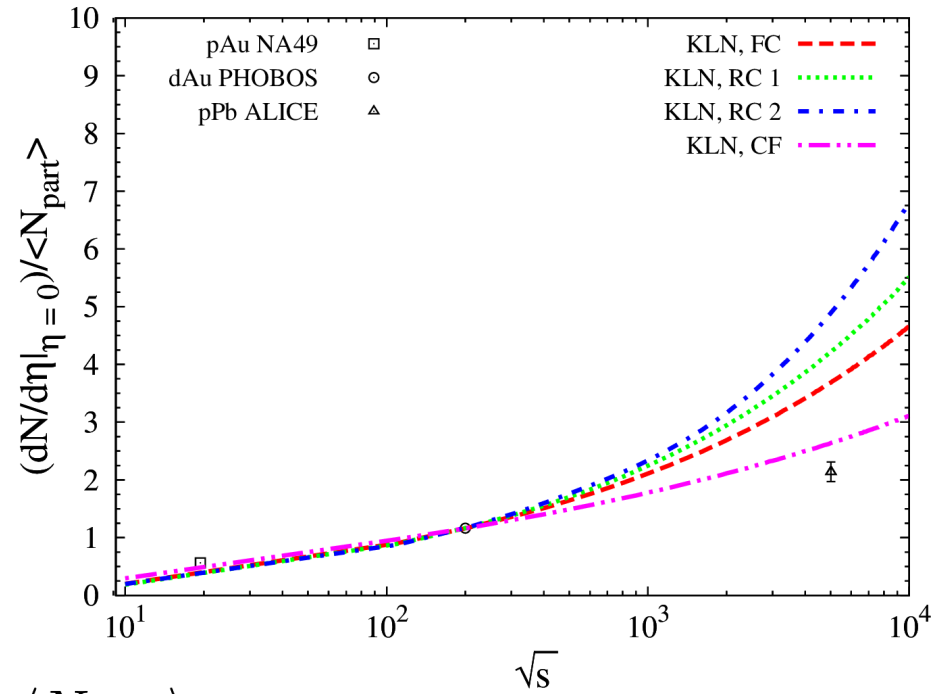
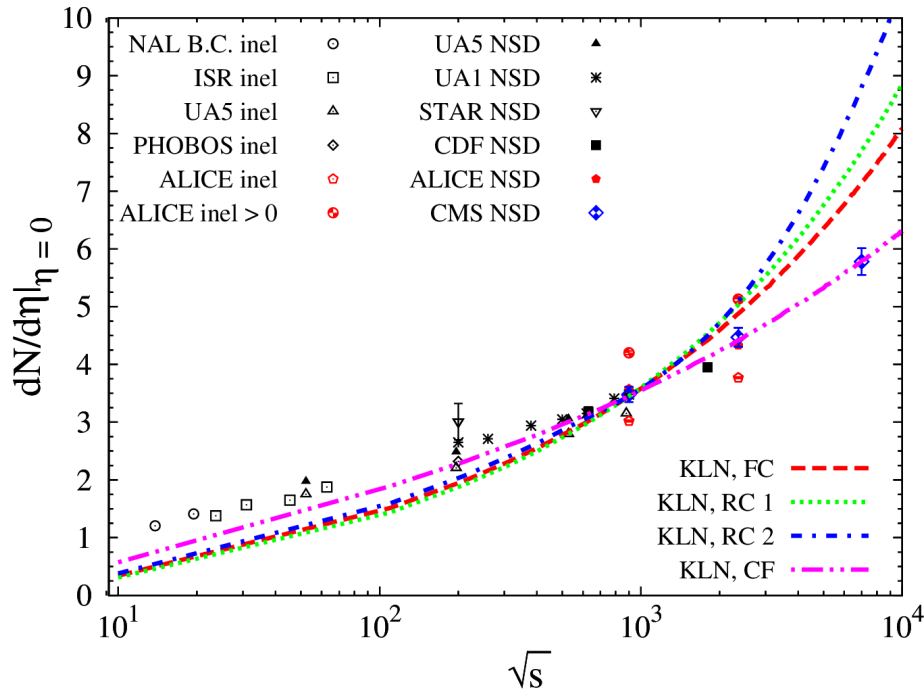
FC ~ 10.0%

RC1 ~ 22.0%

RC2 ~ 43.0%

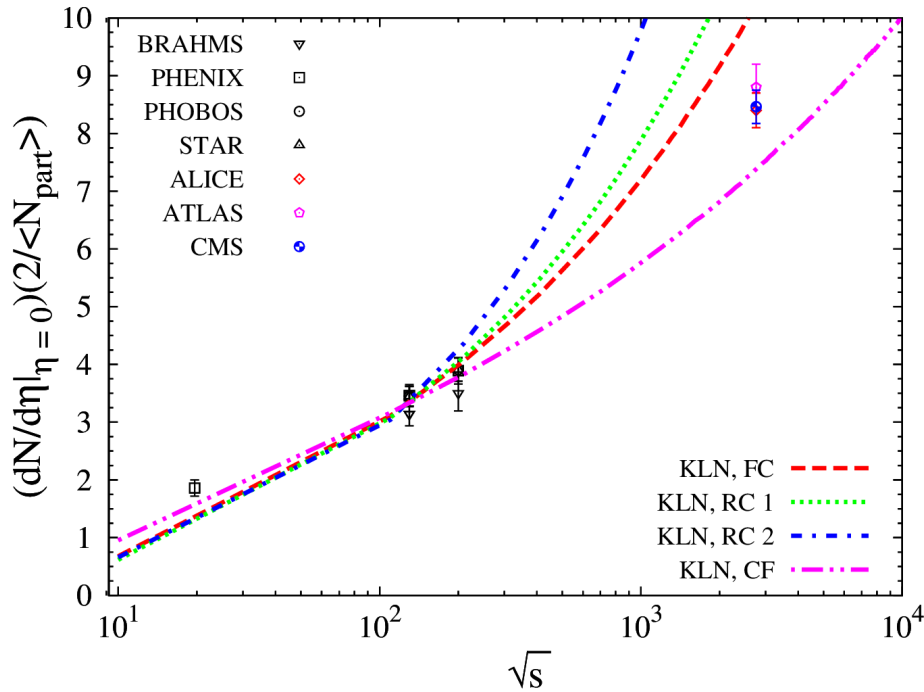
CF ~ 25.0%

Results for $dN/d\eta|_{\eta=0}$ vs \sqrt{s} : all coll. systems



$\langle N_{\text{part}} \rangle$ from minimum bias pPb@LHC

ALICE Collab. PRL 110, 032301



$\langle N_{\text{part}} \rangle$ from 3% most central AuAu collisions @ 130 GeV

Phobos Collab. PRC 65, 061901(R) (2002)

Conclusions and remarks

Using a simple setup, KLN ugd + Local Parton Hadron Duality + simplified model for nuclear geometry we showed:

- the impact of α_s corrections on the observables is small \rightarrow predictions of the distinct approaches for the pseudorapidity distributions and charged hadron multiplicities being similar;
- the biggest difference was found in the energy dependence of the observables, with the corrected formula predicting a weaker energy dependence;
- Our results motivate a more robust calculation, employing better ugd models and a realistic model for nuclear geometry;

Conclusions and remarks

- From rapidity distributions: the corrected k_T - factorization formula seems to encode important energy dependent corrections improving the energy behavior of the observables.

 need further studies for confirmation!

Backup slides

