Testing the running coupling $k_T$-factorization formula for the inclusive gluon production

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based on work done with
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Outline:

- Motivation
- The $k_T$-factorization approach
- Ingredients of our calculation
- Results for $dN/dy$ vs $y$: $p+p$, $p(d)+A$ and $A+A$
- Results for $dN/d\eta|_{\eta=0}$ vs $\sqrt{s}$: $p+p$, $p(d)+A$ and $A+A$
- Conclusions
Motivation

The CGC effective theory has entered in the next-to-leading order (NLO) era → several processes calculated at this level (and their numerical implementation are underway).

● Regarding the Balitsky-Kovchegov (BK) evolution equation:

Running coupling corrections to the kernel of the BK equation with the solution being able to describe several observables @ HERA, RHIC and LHC;

Balitsky, PRD 75, 014001 (2007); Balitsky and Chirilli, PRD 77, 014019 (2008)
Kovchegov and Weigert, NPA 784, 188 (2007), NPA 789, 260 (2007);

Large single and double transverse logarithms have been resummed to all orders in the NLO BK equation

Iancu, Madrigal, Mueller, Soyez and Triantafyllopoulos, PLB 750, 643 (2015);
Lappi and Määntysaari, PRD 91, no. 7, 074016 (2015), PRD 93, no. 9, 094004 (2016).

● JIMWLK evolution equation @ NLO

**Motivation**

- **Regarding the “hybrid formalism”:**

  NLO corrections calculated and implemented numerically → better agreement with experimental data @ RHIC/LHC energies for forward hadron production;

  Chirilli, Xiao and Yuan, PRL 108, 122301 (2012);
  Stasto, Xiao and Zaslavsky, PRL 112, no. 1, 012302 (2014);
  Altinoluk, Armesto, Beuf, Kovner and Lublinsky, PRD 91, no. 9, 094016 (2015);
  Watanabe, Xiao, Yuan and Zaslavsky, PRD 92, no. 3, 034026 (2015).

- **Regarding the $k_T$ – factorization:**

  Generalization to higher orders in $\alpha_s$ of the $k_T$– factorization formalism for inclusive gluon production conjectured few years ago

  Horowitz and Kovchegov, NPA 849, 72 (2011)

No studies about the impact of the high-order corrections in this formalism on observables so far!
"Versions" of the $k_T$-factorization formula for inclusive gluon production
The fixed coupling $k_T$-factorization formula

$$\frac{d^3 \sigma}{d^2 k_T dy} = \frac{2}{C_F} \frac{1}{k^2} \int d^2 q \, \alpha_s \, \phi_{h_1} (q, y) \, \phi_{h_2} (k - q, Y - y)$$

$k = (k_x, k_y)$

$$\phi_{h_i} (k, y) = \frac{C_F}{\alpha_s (2\pi)^3} \int d^2 b \, d^2 r \, e^{-ik \cdot r} \, \nabla_r^2 \mathcal{N}_{h_i}^G (r, b, y)$$

Unintegrated gluon distribution (ugd)

Dipole-hadron forward scattering amplitude for a gluon dipole of transverse size $r$ and a given impact parameter $b$

Widely used in phenomenological studies @ RHIC energies.

The fixed coupling $k_T$-factorization formula
The basic assumptions and predictions of the KLN approach have been qualitatively confirmed by the RHIC and LHC data → improvements have been done going towards to the quantitative direction.
The ("naive") rc $k_T$-factorization formula

I) **Assumes** that the factorization from the LO expression is preserved after the inclusion of the running coupling corrections;

\[ \alpha_s \rightarrow \alpha_s(Q^2) = \frac{12\pi}{\beta_0 \ln(Q^2/\Lambda_{QCD}^2)}; \quad \beta_0 = 33 - 2n_f \]

\[ \frac{d^3\sigma}{d^2k_T\,dy} = \frac{2}{C_F} \frac{1}{k^2} \int d^2q \, \alpha_s(Q^2) \, \phi_{h_1}(q, y) \, \phi_{h_2}(k - q, Y - y) \]

\[ \phi_{h_i}(k, y) = \frac{C_F}{\alpha_s(Q^2)(2\pi)^3} \int d^2b \, d^2r \, e^{-i\mathbf{k} \cdot \mathbf{r}} \, \nabla_{\mathbf{r}}^2 \mathcal{N}^G_{h_i}(\mathbf{r}, \mathbf{b}, y) \]

Widely used in phenomenological studies @ RHIC / LHC energies;

Levin and Rezaeian, PRD 82, 014022 (2010), PRD 82, 054003 (2010), PRD 83, 114001 (2011)
Albacete and Dumitru, arXiv:1011.5161
Albacete, Dumitru, Fujii and Nara, NPA 897, 1 (2013)
Dumitru, Kharzeev, Levin and Nara, PRC 85, 044920 (2012)
Schenke, Tribedy and Venugopalan, PRC 89, no. 2, 024901 (2014)
The (“naive”) $rc \ k_T$-factorization formula

Considering the running of the coupling constant has led to an improvement of the agreement between theory and experimental data...

Albacete, Dumitru, Fujii and Nara, NPA 897, 1 (2013)
Levin and Rezaeian, PRD 82, 014022 (2010)
The ("naive") $rc \ k_T$-factorization formula

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Levin and Rezaeian, PRD 82, 014022 (2010)

$$Q^2 = \max\{|q + k|^2/4, |q - k|^2/4\}$$

\[ \phi_{h_1} \propto \frac{1}{\alpha_s(q)} \eta \]
\[ \phi_{h_2} \propto \frac{1}{\alpha_s(k - q)} \]

... however the scale $Q^2$ is not known in this expression and it has been chosen by hand!

A formal inclusion of the running coupling effects is mandatory to fix $Q^2$
The $\alpha_s$ corrected $k_T$-factorization formula

Horowitz and Kovchegov, NPA 849, 72 (2011)

\[
\frac{d^3 \sigma}{d^2 k_T \, dy} = \frac{2 C_F}{\pi^2} \frac{1}{k^2} \int d^2 q \, \overline{\phi}_{h_1}(q, 0) \overline{\phi}_{h_2}(k - q, 0) \frac{\alpha_s \left( \Lambda^2_{\text{coll}} e^{-5/3} \right)}{\alpha_s \left( Q^2 e^{-5/3} \right) \alpha_s \left( Q^*^2 e^{-5/3} \right)}
\]

\[
\overline{\phi}_{h_i}(k, y) = \alpha_s \phi_{h_i}(k, y) = \frac{C_F}{(2\pi)^3} \int d^2 b \, d^2 r \, e^{-i k \cdot r} \, \nabla^2_r N_{h_i}(r, b, y)
\]
The corrected $k_T$-factorization formula

Horowitz and Kovchegov, NPA 849, 72 (2011)

\[
\frac{d^3\sigma}{d^2 k_T \, dy} = \frac{2 \, C_F}{\pi^2} \frac{1}{k^2} \int d^2 q \, \overline{\phi_{h_1}}(q, 0) \overline{\phi_{h_2}}(k - q, 0) \frac{\alpha_s \left( \Lambda_{\text{coll}}^2 e^{-5/3} \right)}{\alpha_s \left( Q^2 e^{-5/3} \right) \alpha_s \left( Q^*^2 e^{-5/3} \right)}
\]

\[
\overline{\phi_{h_i}}(k, y) = \alpha_s \phi_{h_i}(k, y) = \frac{C_F}{(2\pi)^3} \int d^2 b \, d^2 r \, e^{-i k \cdot r} \, \nabla_r^2 \, N_{h_i}(r, b, y)
\]

$\Lambda_{\text{coll}}^2$ is a collinear infrared cutoff for the case where the would-be produced gluon splits into a collinear gluon-gluon or quark-antiquark pair (whose invariant mass must be less than this quantity)
The $\alpha_s$ corrected $k_T$-factorization formula

Horowitz and Kovchegov, NPA 849, 72 (2011)

$$\frac{d^3\sigma}{d^2k_T dy} = \frac{2 C_F}{\pi^2} \frac{1}{k^2} \int d^2q \, \bar{\phi}_{h_1}(q, 0) \bar{\phi}_{h_2}(k - q, 0) \alpha_s \left( \frac{\Lambda_{\text{coll}}^2 e^{-5/3}}{Q^2 e^{-5/3}} \right) \alpha_s \left( \frac{Q^*^2 e^{-5/3}}{Q^2 e^{-5/3}} \right)$$

$$\bar{\phi}_{h_i}(k, y) = \alpha_s \phi_{h_i}(k, y) = \frac{C_F}{(2\pi)^3} \int d^2b \, d^2r \, e^{-i b \cdot r} \nabla_r^2 N_{h_i}(r, b, y)$$

$Q^2$ is given by:

$$\ln \frac{Q^2}{\mu^2_{\text{MS}}} = \frac{1}{2} \ln \frac{q^2 (k - q)^2}{\mu^4_{\text{MS}}} - \frac{1}{4 q^2 (k - q)^2} \left\{ k^2 \left[ (k - q)^2 - q^2 \right]^3 \right. \\
\times \left\{ ((k - q)^2 - q^2)^2 \right. \left[ (k^2)^2 + ((k - q)^2 - q^2)^2 \right] + 2 k^2 \left[ (q^2)^2 - (k - q)^2 \right]^3 \\
- q^2 (k - q)^2 \left[ 2 (k^2)^2 + 3 [(k - q)^2 - q^2]^2 - 3 k^2 [(k - q)^2 + q^2] \right] \ln \left( \frac{(k - q)^2}{q^2} \right) \} \\
+ i [(k - q)^2 - q^2]^3 \left\{ k^2 \left[ (k - q)^2 - q^2 \right] \left[ k^2 \left[ (k - q)^2 + q^2 \right] - (q^2)^2 - [(k - q)^2]^2 \right] \\
+ q^2 (k - q)^2 \left( k^2 \left[ (k - q)^2 + q^2 \right] - 2 (k^2)^2 - 2 [(k - q)^2 - q^2]^2 \right) \ln \left( \frac{(k - q)^2}{q^2} \right) \} \\
\times \sqrt{2 q^2 (k - q)^2 + 2 k^2 (k - q)^2 + 2 q^2 k^2 - (k^2)^2 - (q^2)^2 - [(k - q)^2]^2}$$

Now it is fully determined from a formal calculation!
The $\alpha_s$ corrected $k_T$-factorization formula

Horowitz and Kovchegov, NPA 849, 72 (2011)

$$\frac{d^3\sigma}{d^2k_T dy} = \frac{2C_F}{\pi^2} \frac{1}{k^2} \int d^2q \ \bar{\phi}_{h_1}(q,0) \bar{\phi}_{h_2}(k-q,0) \frac{\alpha_s}{\alpha_s} \frac{\Lambda_{\text{coll}}^2 e^{-5/3}}{\alpha_s (Q^2 e^{-5/3})} \frac{\alpha_s}{\alpha_s} \frac{Q^2 e^{-5/3}}{(Q^*)^2 e^{-5/3}}$$

$$\bar{\phi}_{h_i}(k,y) = \alpha_s \phi_{h_i}(k,y) = \frac{C_F}{(2\pi)^3} \int d^2b d^2r e^{-i k \cdot r} \nabla_r^2 N_{h_i}(r,b,y)$$

$Q^2$ is given by:

$$\ln \frac{Q^2}{\mu_{\text{MS}}^2} = \frac{1}{2} \ln \frac{q^2 (k-q)^2}{\mu_{\text{MS}}^4} - \frac{1}{4 q^2 (k-q)^2} \frac{1}{[(k-q)^2 - q^2]^6} \left\{ k^2 \left[ (k-q)^2 - q^2 \right]^3 \times \left\{ \left[ (k-q)^2 - q^2 \right]^2 \left[ (k-q)^2 + (k-q)^2 - q^2 \right] + 2 k^2 \left[ (q^2)^3 - (k-q)^2 \right]^3 \right. \right.$$  
$$- q^2 (k-q)^2 \left[ 2 (k-q)^2 + 3 [(k-q)^2 - q^2]^2 - 3 k^2 \left[(k-q)^2 + q^2 \right] \right] \ln \left( \frac{(k-q)^2}{q^2} \right) \right\}$$  
$$+ i \left[ (k-q)^2 - q^2 \right]^3 \left\{ k^2 \left[(k-q)^2 - q^2 \right] \left[ k^2 \left[(k-q)^2 + q^2 \right] - (q^2)^2 - (k-q)^2 \right] \right.$$
$$+ q^2 (k-q)^2 \left[ k^2 \left[(k-q)^2 + q^2 \right] - 2 (k^2)^2 - 2 [(k-q)^2 - q^2]^2 \right] \ln \left( \frac{(k-q)^2}{q^2} \right) \right\} - \frac{1}{4 q^2 (k-q)^2} \frac{1}{[(k-q)^2 - q^2]^6} \left\{ k^2 \left[ (k-q)^2 - q^2 \right]^3 \right.$$  
$$- [k^2 \left[(k-q)^2 - q^2 \right] + q^2 (k-q)^2 \left[ k^2 \left[(k-q)^2 + q^2 \right] - (q^2)^2 - (k-q)^2 \right] \right.$$
$$+ q^2 (k-q)^2 \left[ k^2 \left[(k-q)^2 + q^2 \right] - 2 (k^2)^2 - 2 [(k-q)^2 - q^2]^2 \right] \ln \left( \frac{(k-q)^2}{q^2} \right) \right\}$$

Now it is fully determined from a formal calculation!

But this result is valid only for $y = 0$
Conjecture for the $\alpha_s$ corrected $k_T$-factorization formula

$$\frac{d^3\sigma}{d^2k_T dy} = \frac{2C_F}{\pi^2} \frac{1}{k^2} \int d^2q \ \overline{\phi_{h_1}(q, y)} \overline{\phi_{h_2}(k-q, Y-y)} \frac{\alpha_s \left( \Lambda_{\text{coll}}^2 e^{-5/3} \right)}{\alpha_s \left( Q^2 e^{-5/3} \right)} \frac{\alpha_s \left( Q^* e^{-5/3} \right)}{\alpha_s \left( Q^2 e^{-5/3} \right)}$$

$$\overline{\phi_{h_i}(k, y)} = \alpha_s \phi_{h_i}(k, y)$$

$$\ln \frac{Q^2}{\mu_{\text{MS}}^2} = \frac{1}{2} \ln \frac{q^2 (k-q)^2}{\mu_{\text{MS}}^4} - \frac{1}{4 q^2 (k-q)^2 \left[(k-q)^2-q^2\right]^2} \left\{ k^2 \left[(k-q)^2-q^2\right]^3 \\
\times \left[ \left[(k-q)^2-q^2\right] \left[(k-q)^2+(k-q)^2-q^2\right] + 2k^2 \left[(q^2)^3-[(k-q)^2]^3\right] \\
- q^2 (k-q)^2 \left[2 (k^2)^2+3 [(k-q)^2-q^2]^2 - 3k^2 [(k-q)^2+q^2] \right] \ln \left( \frac{(k-q)^2}{q^2} \right) \right] \right\}$$

$$+ i \left[(k-q)^2-q^2\right]^3 \left\{ k^2 \left[(k-q)^2-q^2\right] \left[ k^2 \left[(k-q)^2+q^2\right] - (q^2)^2 - [(k-q)^2]^2 \right] \\
+ q^2 (k-q)^2 \left[ k^2 \left[(k-q)^2+q^2\right] - 2 (k^2)^2 - 2 [(k-q)^2-q^2]^2 \right] \ln \left( \frac{(k-q)^2}{q^2} \right) \right\}$$

$$\times \sqrt{2 q^2 (k-q)^2 + 2 k^2 (k-q)^2 + 2 q^2 k^2 - (k^2)^2 - (q^2)^2 - [(k-q)^2]^2} \right\},$$

Horowitz and Kovchegov, NPA 849, 72 (2011)
What is the impact of the $\alpha_s$ corrections in the conjectured $k_T$-factorization formula?

Re: assume a simple setup and calculate some observables considering each “version” of the $k_T$-factorization formula previously mentioned and compare the results.

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In light of the above “answer” this is a qualitative work!
Ingredients of our calculation

KLN ugd: $\phi_{KLN}(k, y) = \frac{2C_F}{3\pi^2}, \quad k \leq Q_s$

$= \frac{2C_F}{3\pi^2} \frac{Q^2_s}{k^2}, \quad k > Q_s$

Local Parton Hadron Duality: meaning rapidity distribution of partons and hadrons only change by a numerical factor.

Introduces an effective mass that incorporates nonperturbative effects:

$m_h = 0.350 \text{ GeV} \quad \text{for all cases}$

Simplified nuclear geometry: no Monte Carlo fluctuation on the nucleon's position for nuclei.

Bottom line: simplest setup possible! Motivation: isolate the impact of the $\alpha_s$ corrections in the $k_T$-factorization formula.
Ingredients of our calculation

Fixed coupling (FC): \( \alpha_s = 0.25 \)

("Naive") running coupling: two different prescription for \( Q^2 \)

**RC1:** scales fixed as in Albacete, Dumitru, Fujii and Nara, NPA 897, 1 (2013)

\[
Q^2 = \max\{|q + k|^2/4, |q - k|^2/4\} ; \quad \phi_{h_1} \propto \frac{1}{\alpha_s(q)} ; \quad \phi_{h_2} \propto \frac{1}{\alpha_s(k - q)}
\]

**RC2:** scales fixed as in Levin and Rezaeian, PRD 82, 014022 (2010)

\[
Q^2 = k^2 \quad \phi_{h_i} \propto \frac{1}{\alpha_s(Q_s(x_i))}
\]

\( \alpha_s \) corrected formula (CF): \( \alpha_s(\Lambda_{\text{coll}}^2 e^{-5/3}) = 0.25 \)
Results for $dN/dy$ vs $y$: $p+p$ collisions

Change in the normalization from 0.9 TeV to 7 TeV:

- FC ~ 20.0%
- RC1 ~ 26.0%
- RC2 ~ 35.0%
- CF ~ 0.94%
**Results for** $dN/dy$ vs $y$: $p(d)+A$ collisions

Change in the normalization from 0.2 TeV to 5.02 TeV:

- FC $\sim 41.0\%$
- RC1 $\sim 48.42\%$
- RC2 $\sim 55.52\%$
- CF $\sim 17.42\%$
Results for $dN/dy$ vs $y$: A+A collisions

Change in the normalization from 0.13 TeV to 2.76 TeV:

- FC $\sim 10.0\%$
- RC1 $\sim 22.0\%$
- RC2 $\sim 43.0\%$
- CF $\sim 25.0\%$
Results for $dN/d\eta|_{\eta=0}$ vs $\sqrt{s}$: all coll. systems

$\langle N_{\text{part}} \rangle$ from minimum bias pPb@LHC
ALICE Collab. PRL 110, 032301

$\langle N_{\text{part}} \rangle$ from 3% most central AuAu collisions @ 130 GeV
Phobos Collab. PRC 65, 061901(R) (2002)
Conclusions and remarks

Using a simple setup, KLN ugd + Local Parton Hadron Duality + simplified model for nuclear geometry we showed:

- the impact of $\alpha_s$ corrections on the observables is small $\rightarrow$ predictions of the distinct approaches for the pseudorapidity distributions and charged hadron multiplicities being similar;

- the biggest difference was found in the energy dependence of the observables, with the corrected formula predicting a weaker energy dependence;

- Our results motivate a more robust calculation, employing better ugd models and a realistic model for nuclear geometry;
Conclusions and remarks

- From rapidity distributions: the corrected $k_T$ - factorization formula seems to encode important energy dependent corrections improving the energy behavior of the observables.

need further studies for confirmation!
Backup slides