low *x* and low scales

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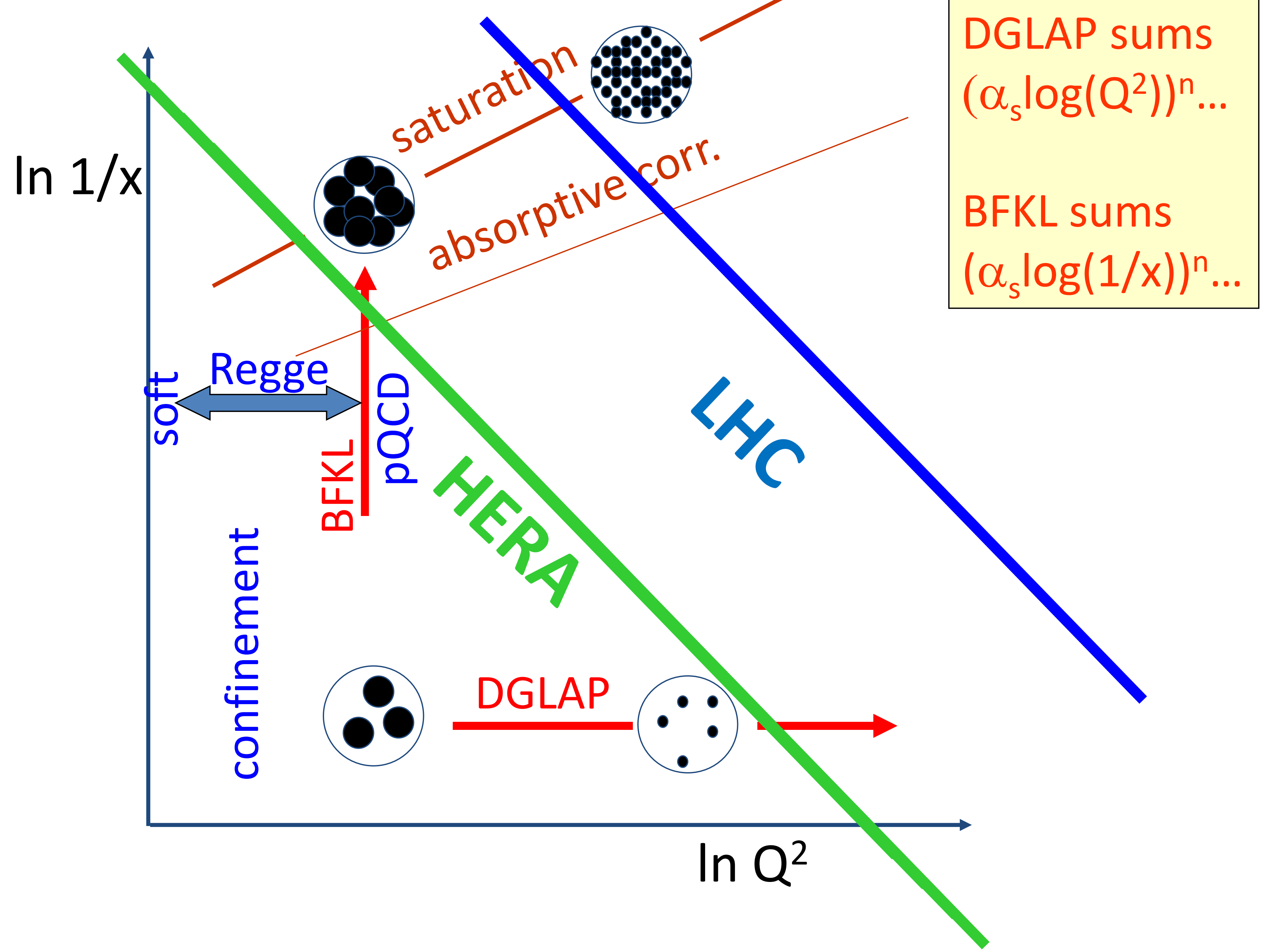
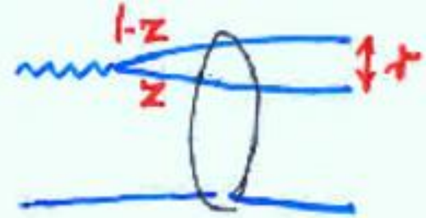
Workshop on QCD & Diffraction
--- plus **SATURATION 1000+**
5-8th Dec. 2016 Krakow, Poland



DGLAP sums
$(\alpha_s \log(Q^2))^n$...
BFKL sums
$(\alpha_s \log(1/x))^n$...
ln 1/x
saturation
absorptive corr.
soft
Regge
BFKL
pQCD
HERA
LHC
confinement
DGLAP
ln Q^2

Dipole formulation

Nikolaev & Zakharov
Mueller

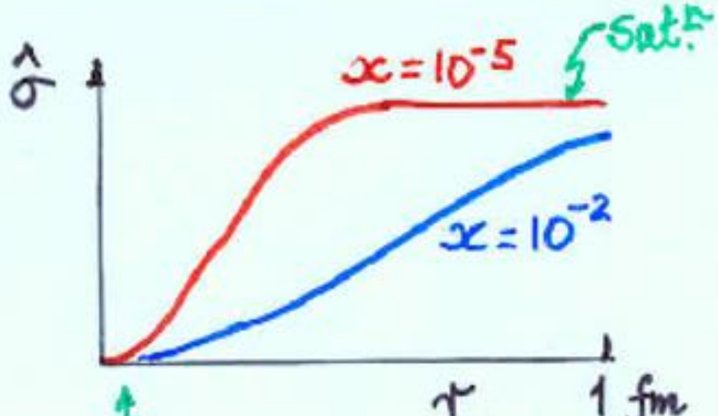


$\gamma^* p \to X(q\bar{q})p$ $\qquad$ $\gamma^* \to q\bar{q}$ $\qquad$ $\sigma(q\bar{q}\text{-}p)$

DIS : $\sigma_{T,L}(x,Q^2) = \int d^2r \int_0^1 dz \, \left|\psi_{T,L}(\vec{r},z,Q^2)\right|^2 \, \hat{\sigma}(x,\vec{r})$

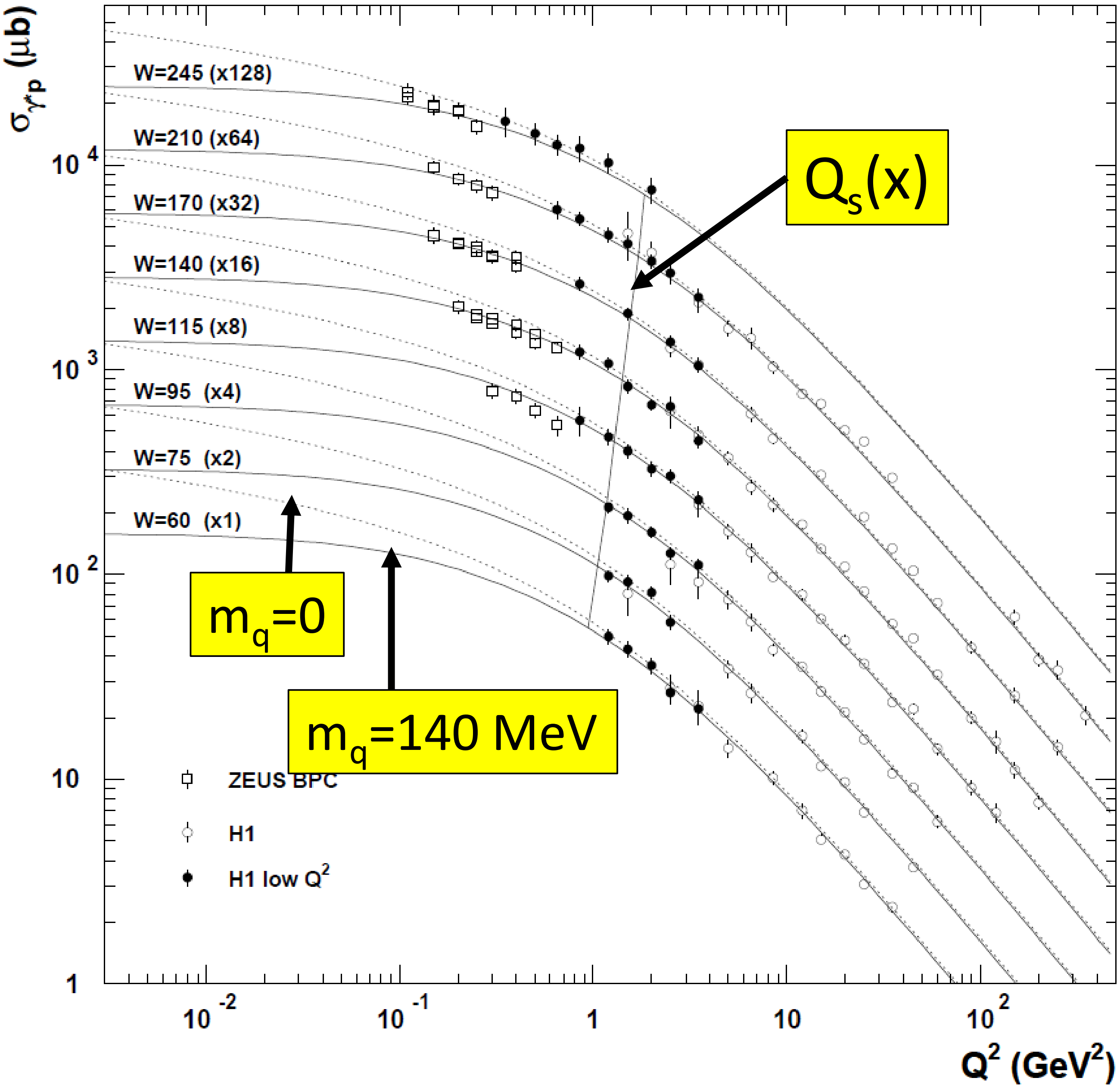
DDIS $\sigma_{T,L}^{\text{diffractive}} = \frac{1}{16\pi B} \int d^2r \int_0^1 dz \, \left|\psi_{T,L}(\vec{r},z,Q^2)\right|^2 \, \hat{\sigma}^2(x,\vec{r})$

Golec-Biernat & Wüsthoff saturation model (1998)



$\hat{\sigma} \sim r^2$ (pQCD)

Original Golec-Biernat, Wusthoff fit (1998)

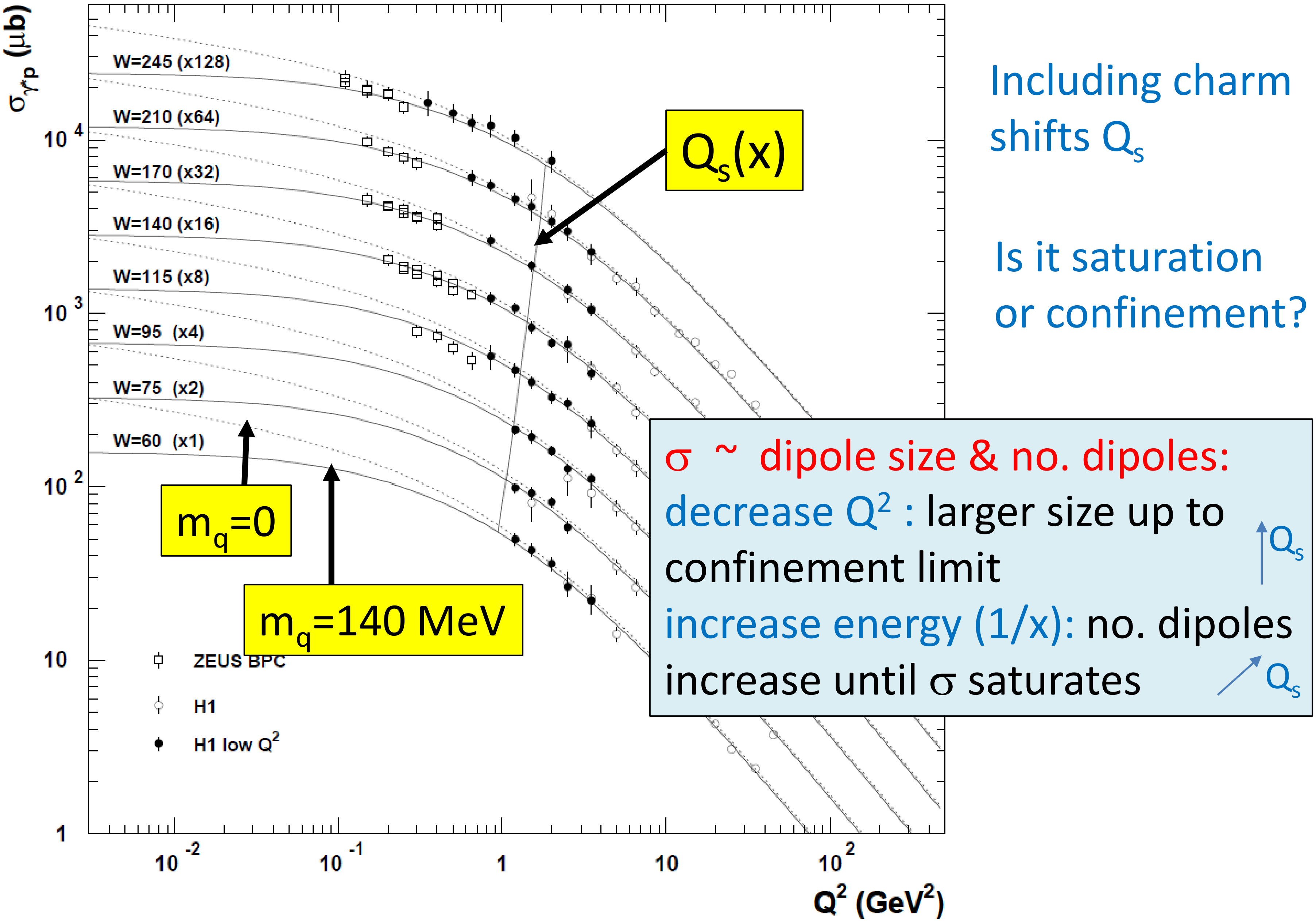


Including charm shifts Q_s

Is it saturation or confinement?

There are other dipole fits without saturation

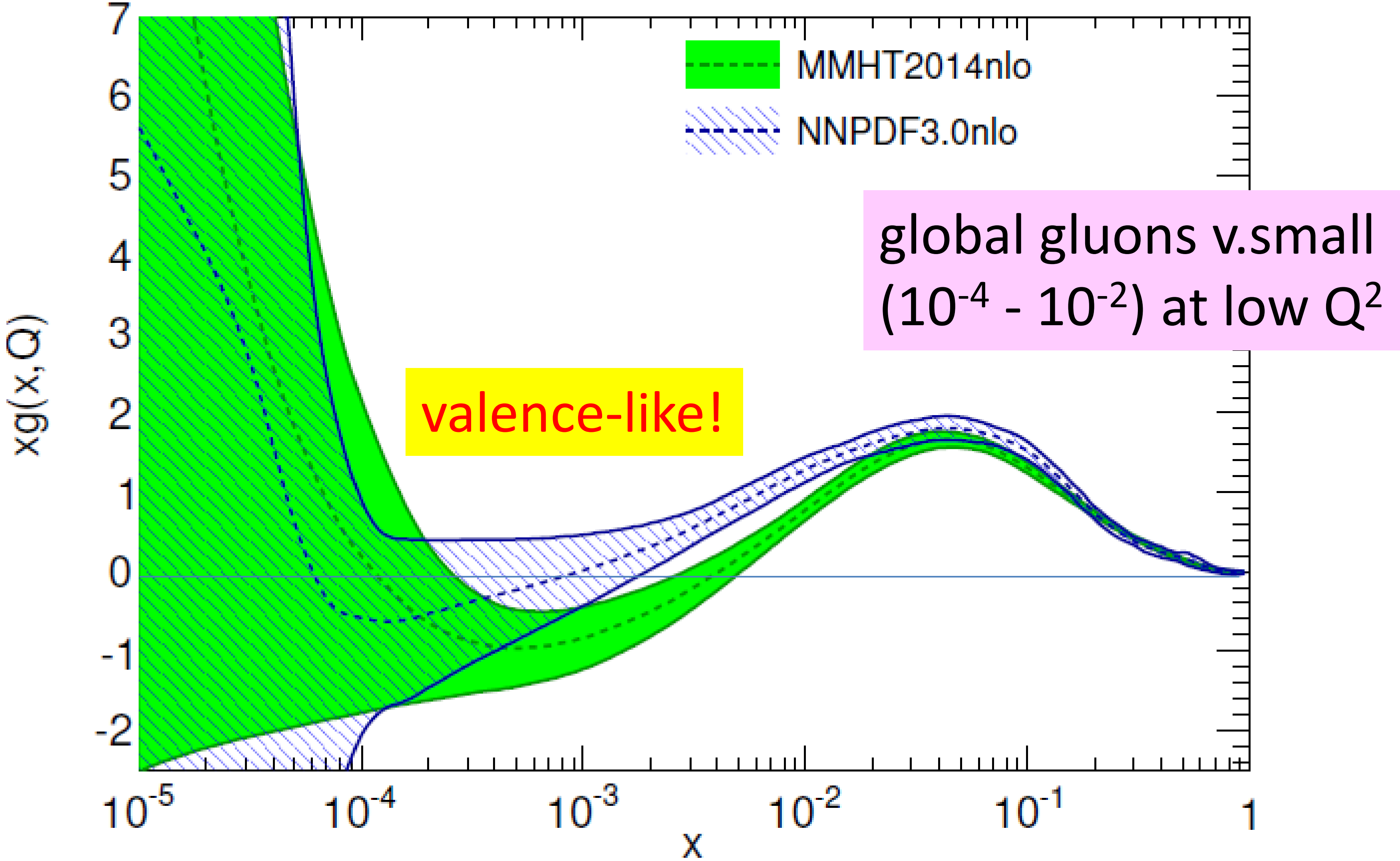
Original Golec-Biernat, Wusthoff fit (1998)

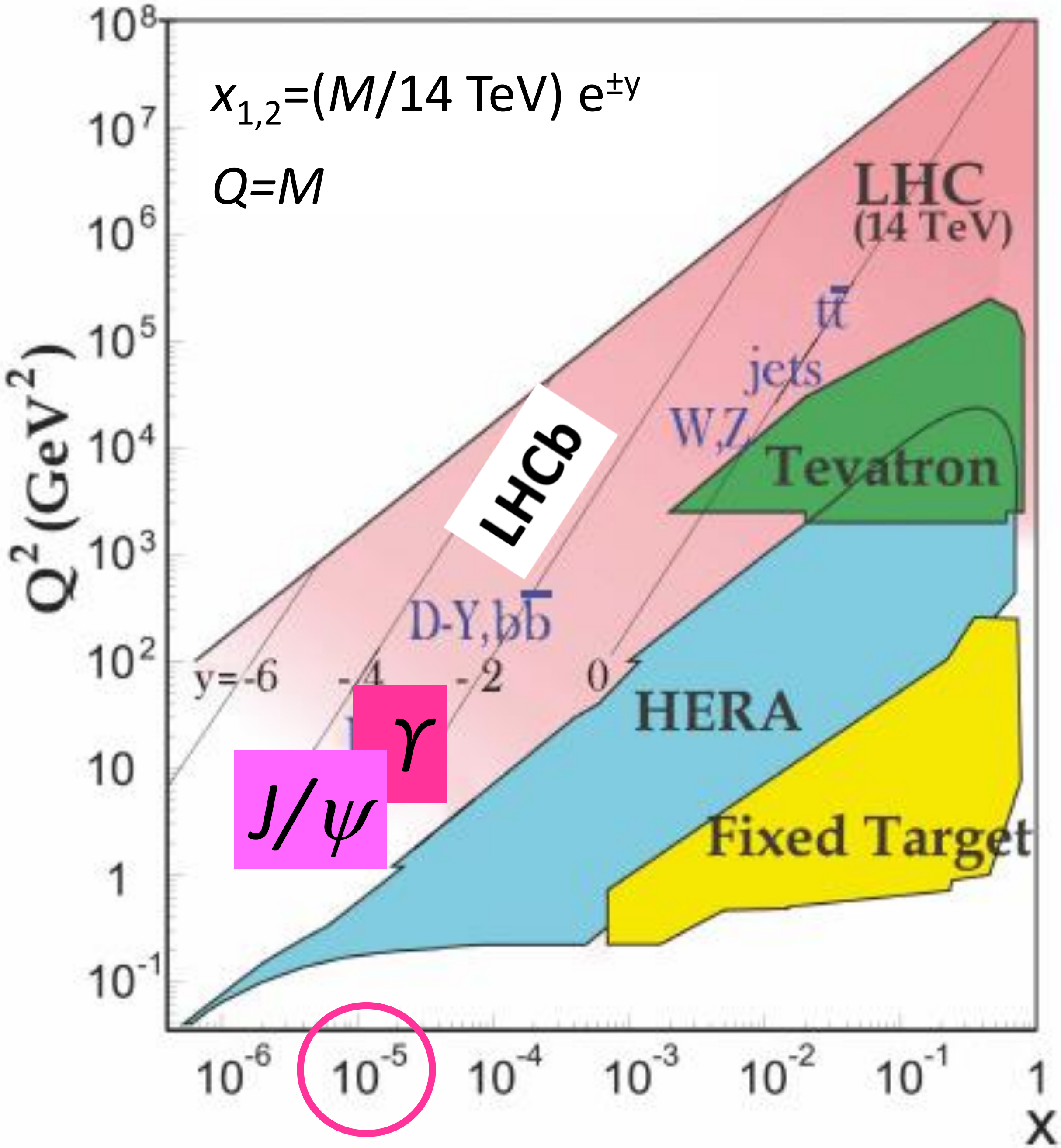


Including charm shifts Q_s

Is it saturation or confinement?

The gluon at low x and Q^2=1.21 GeV^2





LHCb with $2 < y < 4.5$
can probe gluon
down to x ~ 10^{-5}

exclusive J/ψ, Υ prod;
inclusive $c\bar{c}$, $b\bar{b}$ prod.

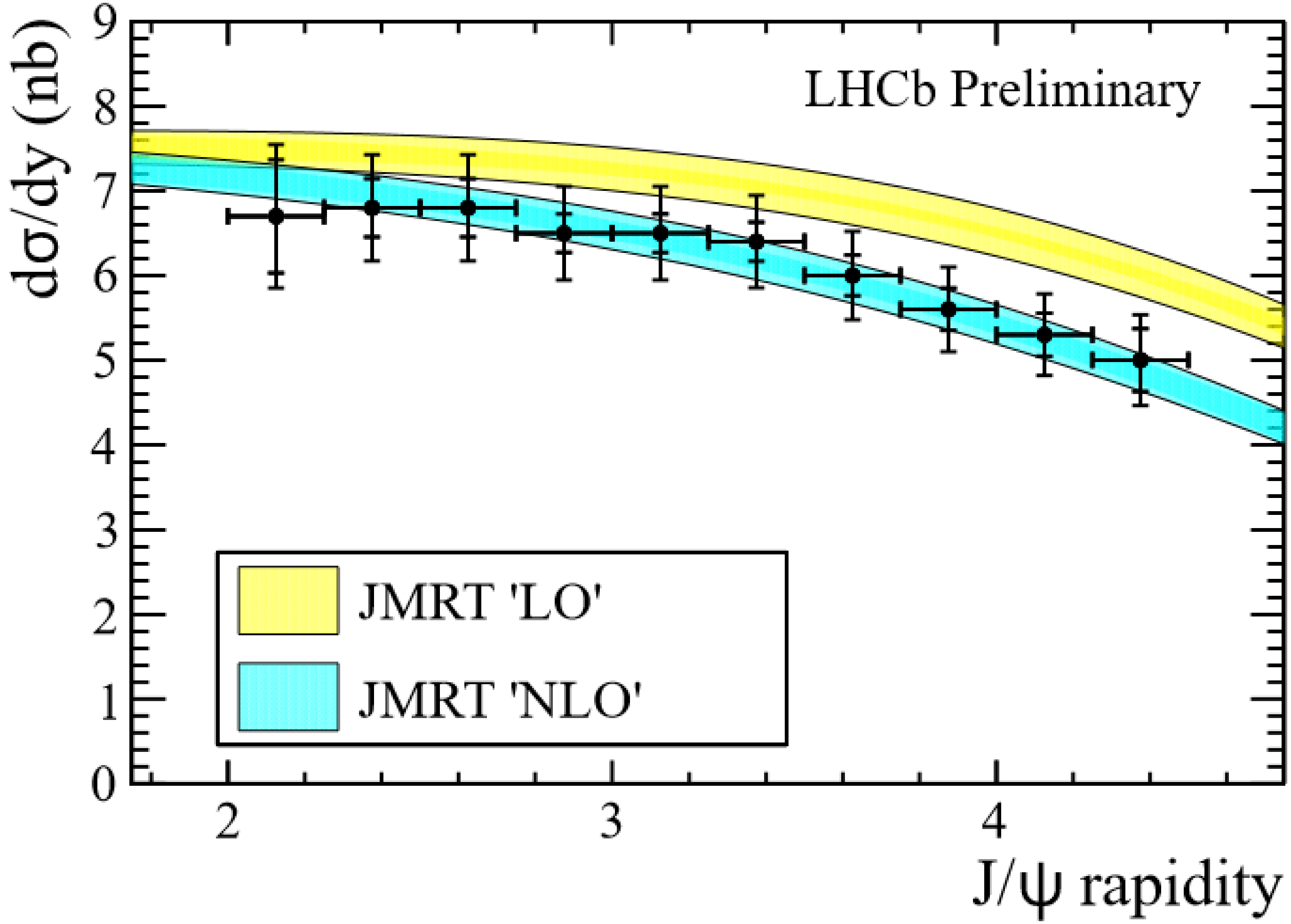
Why are these
LHC data not used
in global PDF fits ??

Start with J/ψ

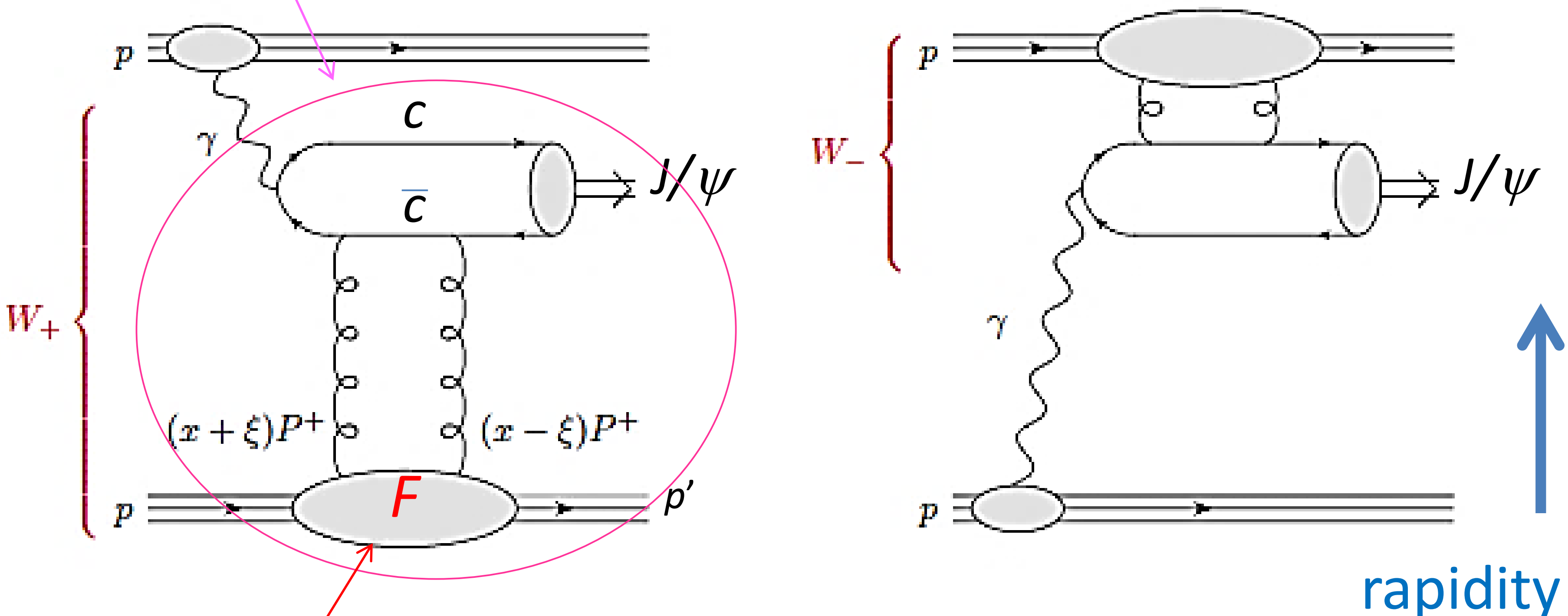
LHCb data for pp → p + J/ψ + p at 13 TeV

with new HERSCHEL forward shower counters to improve the exclusivity of the events

LHCb-CONF-2016-007



$\gamma^* p \rightarrow J/\psi + p$ is the quasi-elastic process which drives the LHC data for $pp \rightarrow p + J/\psi + p$



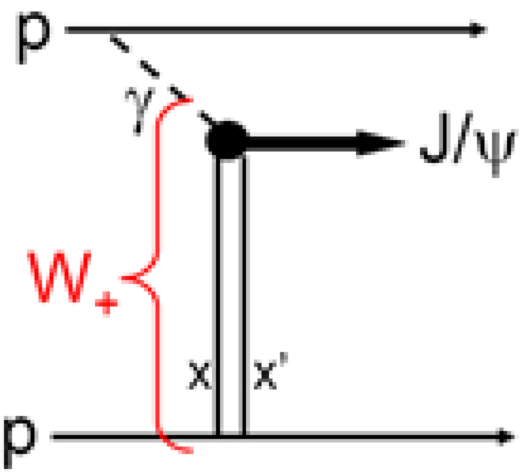
GPD: $F(x,\xi,\mu_F^2)$

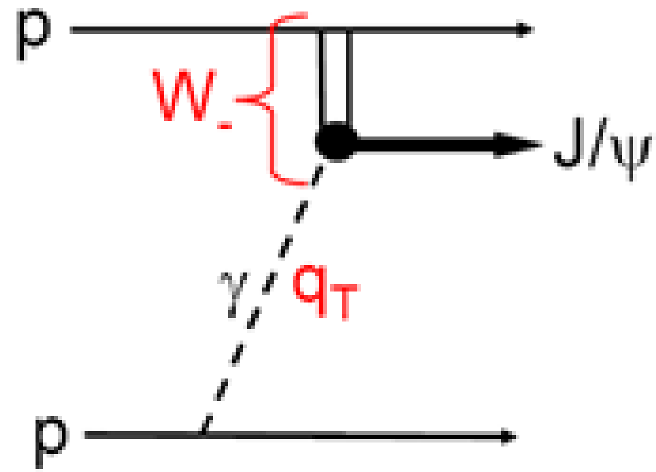
$(p'-p = \xi(p+p'))$

pp → p + J/ψ + p at the LHC

$$W_\pm^2 = M_{J/\psi}\sqrt{s}\; e^{\pm|y|}$$

γp, pγ ambiguity





|y|=4

$x \sim M_{J/\psi}\exp(-|y|)\;/\sqrt{s} \sim 10^{-5}$

$x \sim M_{J/\psi}\exp(|y|)\;/\sqrt{s} \sim 0.02$

fit LHCb

show

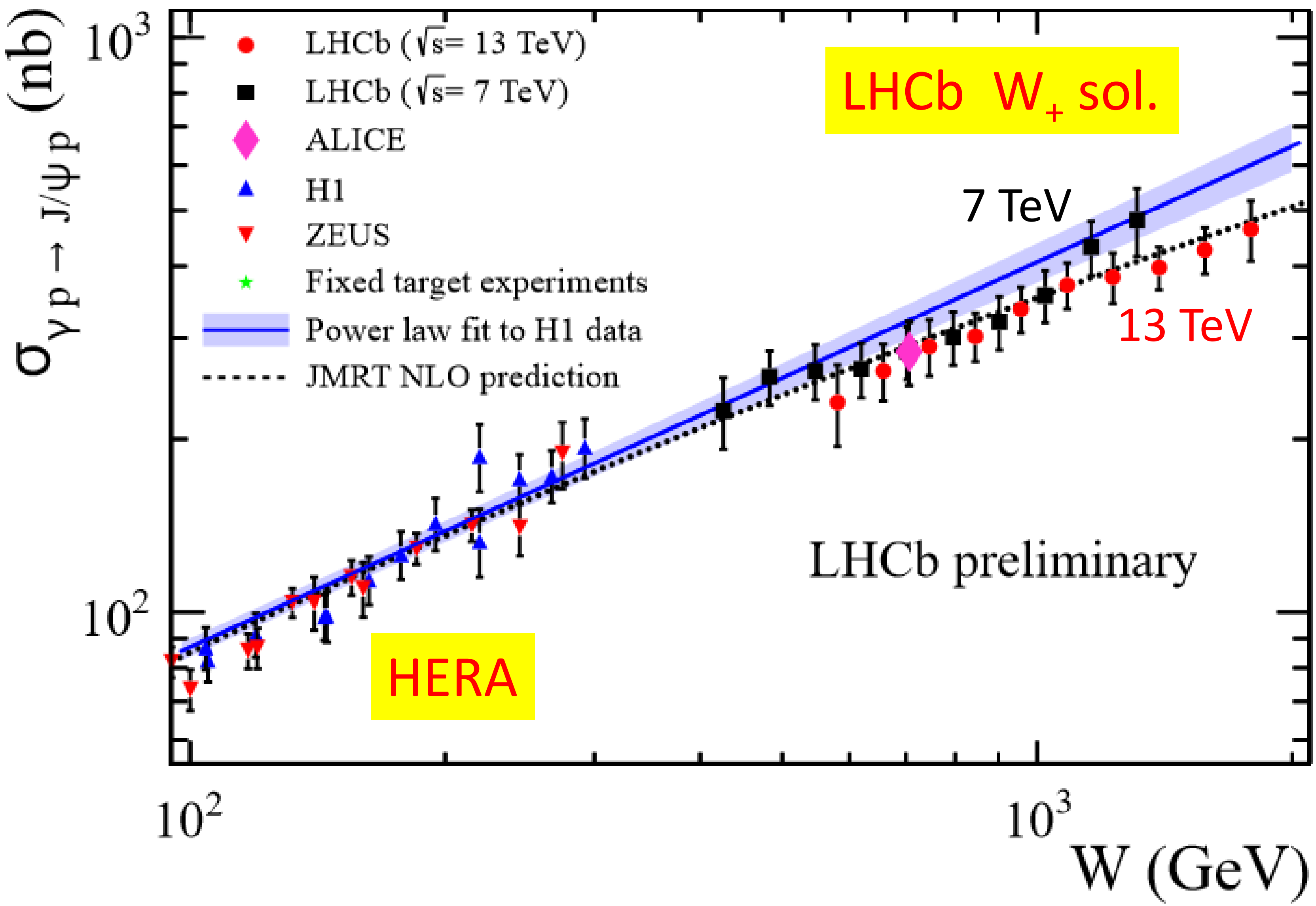
$$\frac{d\sigma^{\rm th}(pp)}{dy} = S^2(W_+)\left(k_+\frac{dn}{dk_+}\right)\sigma_+^{\rm th}(\gamma p) + S^2(W_-)\left(k_-\frac{dn}{dk_-}\right)\sigma_-^{\rm th}(\gamma p)$$

where (…) is photon flux for photon energy $k_\pm$

and S^2 are survival probabilities of LRG

HERA and LHCb extracted data for $\sigma(\gamma p \to J/\psi\ p)$

from LHCb-CONF-2016-007



Problems of using exclusive J/ψ data in global PDF fits?

1. Process described by GPD's

→ however not a problem for $\xi < |x| << 1$

GPD(ξ,x) = PDF(x') ⊗ Shuvaev(ξ,x,x')

hep-ph/9902410
Krzysztof also author

2. Bad convergence of LO, NLO,… pert. series using collinear factorization at low ξ and low scales

additional gluons = $\langle n \rangle \simeq \frac{\alpha_s N_C}{\pi} \ln(1/\xi)\,\Delta \ln \mu_F^2$ ~ 5

whereas **NLO** allows the addition of only 1 gluon !

So why is the JMRT "NLO" prediction so reasonable?

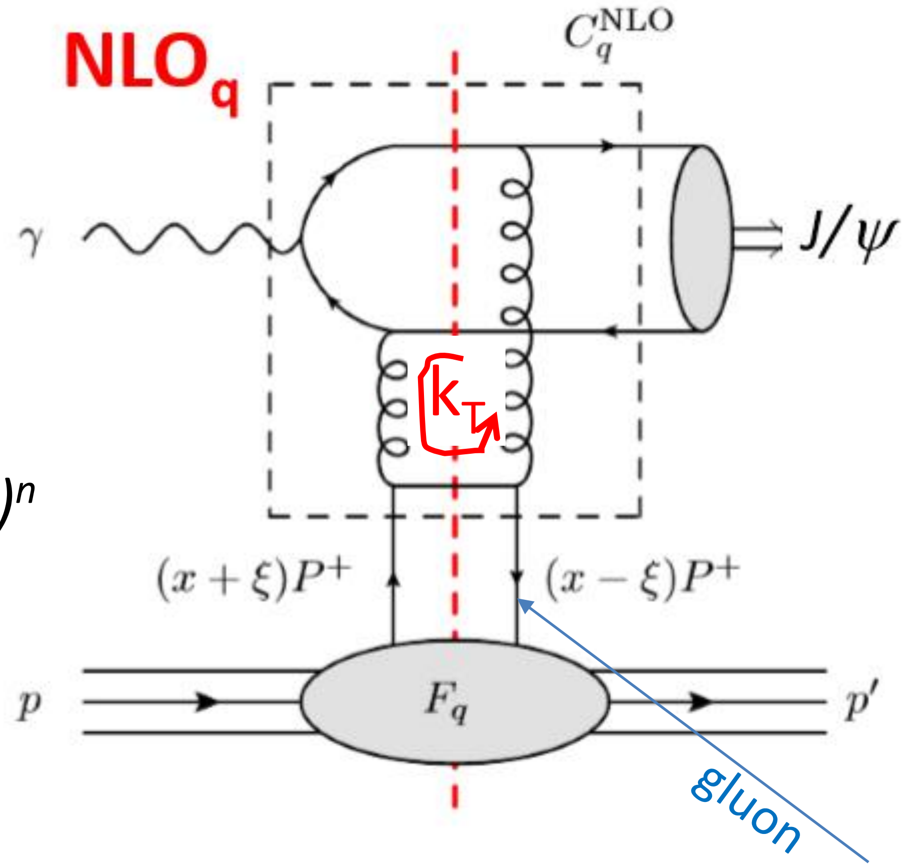
1307.7099

It uses **k_T fact^n** scheme and resums the ln(1/ξ) diagrams

k_T fact^n procedure

Obtain approx NLO corr^{ns} to coeff. fns by performing explicit k_T integration in the last step of evolution, and using an input PDF with resummed $(\ln(1/\xi)\ln\mu_F^2)^n$ terms arising from ladder diag^s.

Not the complete NLO, but includes most important diagrams at low x and low μ_F^2



also NLO$_g$ coeff. fn.

Need gluon PDF unintegrated over k_T

$$f(x,k_T^2) = \partial\left[xg(x,k_T^2)T(k_T^2,\mu^2)\right]/\partial\ln k_T^2$$

known Sudakov factor T so no additional gluons > k_T emitted

"NLO" formula for γ^*p → J/ψ+p

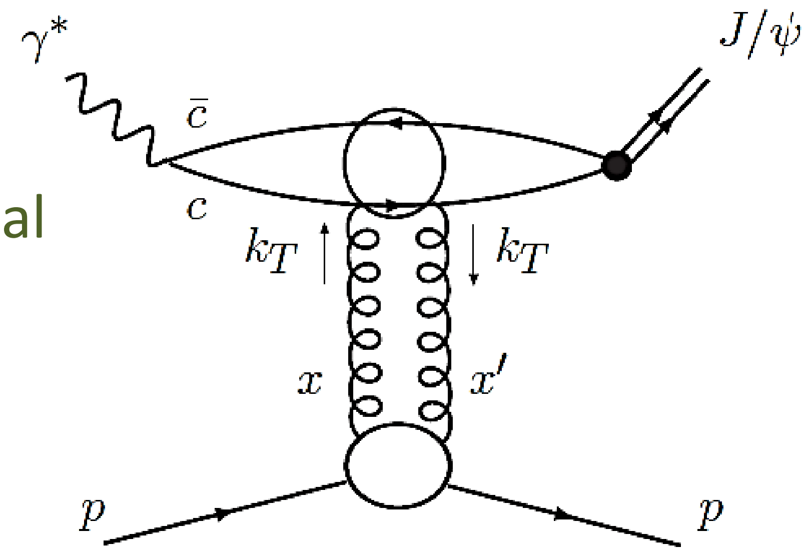
Start with LO formula

Ryskin 1993

$$\frac{\mathrm{d}\sigma}{\mathrm{d}t}(\gamma^* p \to J/\psi\ p)\Big|_{t=0} = \frac{\Gamma_{ee} M^3_{J/\psi}\pi^3}{48\alpha}\left[\frac{\alpha_s(\bar{Q}^2)}{\bar{Q}^4}\underline{xg(x,\bar{Q}^2)}\right]^2\left(1+\frac{Q^2}{M^2_{J/\psi}}\right)$$

$$x = (Q^2 + M^2_{J/\psi})/(W^2+Q^2)$$

$$\bar{Q}^2 = (Q^2 + M^2_{J/\psi})/4$$



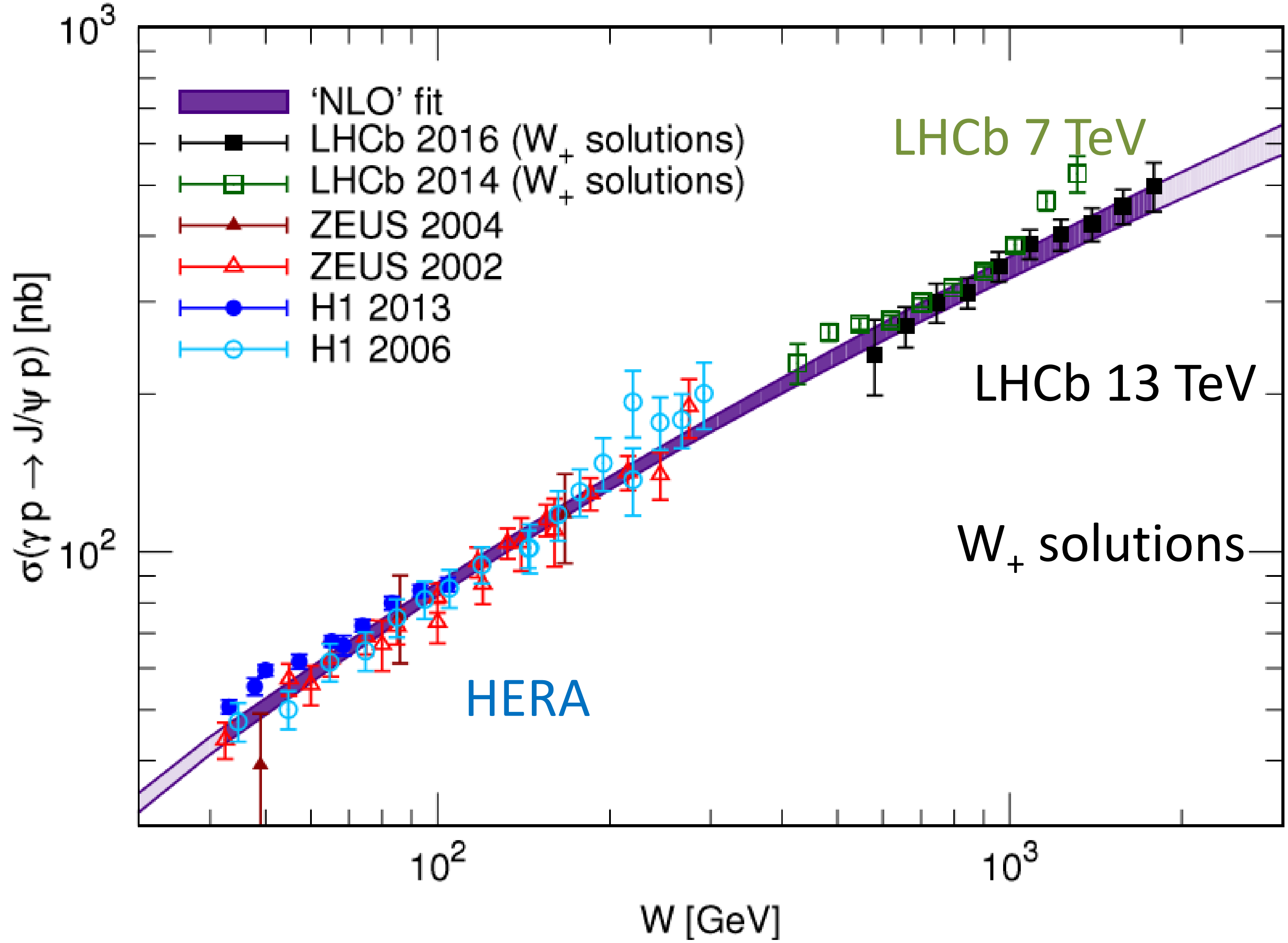
Allow for skewing (x≠x') a la Shuvaev et al

Allow for real part

Mimic NLO by including $k_T{}^2$ integration in last step of evol^n (a la Kimber at al)

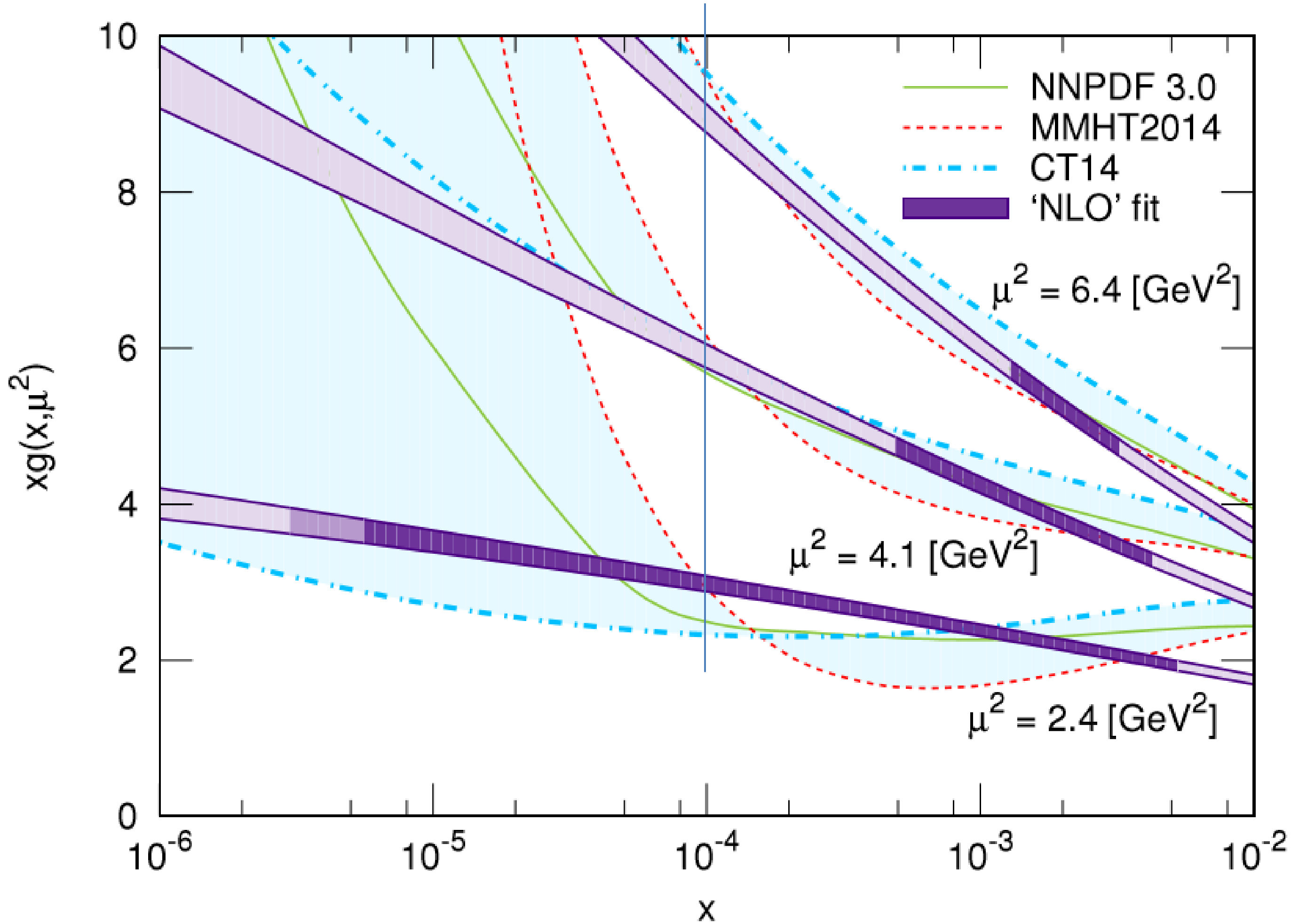
$$\left[\frac{\alpha_s(\bar{Q}^2)}{\bar{Q}^4}xg(x,\bar{Q}^2)\right] \longrightarrow \int_{Q_0^2}^{(W^2-M^2_{J/\psi})/4}\frac{\mathrm{d}k_T^2\,\alpha_s(\mu^2)}{\bar{Q}^2(\bar{Q}^2+k_T^2)}\frac{\partial\left[xg(x,k_T^2)\sqrt{T(k_T^2,\mu^2)}\right]}{\partial k_T^2}$$

+ Q_0 contribution



JMRT refit (1611.03711) but parameters N,a,b unchanged from 2013 fit

$$xg(x,\mu^2) \;=\; N x^{-a}\left(\frac{\mu^2}{Q_0^2}\right)^b \exp\left[\sqrt{16(N_c/\beta_0)\ln(1/x)\ln(G)}\right] \qquad \text{with} \quad G = \frac{\ln(\mu^2/\Lambda^2_{\rm QCD})}{\ln(Q_0^2/\Lambda^2_{\rm QCD})}$$



J/ψ "NLO" gluon PDF compared with central values of global PDF sets

but.... physical k_T fact^n scheme

collinear MS(bar) scheme

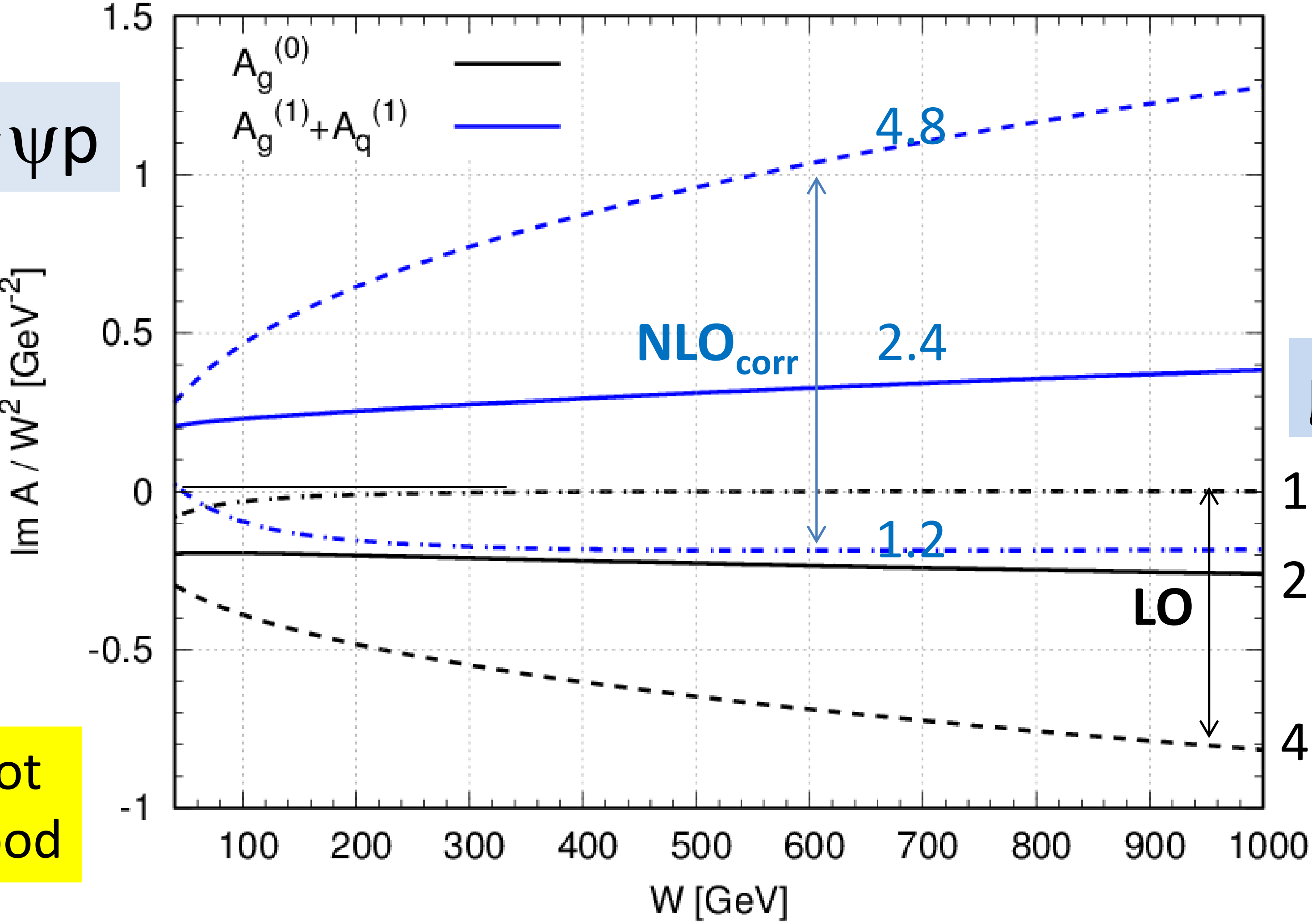
NLO known in MS(bar) scheme, but problems:

D. Ivanov, B.Pire, L.Szymanowski, J.Wagner, 1411.3750
S.P.Jones, A.D.Martin, M.Ryskin, T.Teubner, 1507.06942

A. Bad perturbative convergence $|NLO_{correctn.}| > |LO|$ **and opp. sign**

B. Strong dependence on scale μ_F

$\gamma p \rightarrow \psi p$



μ_F^2

1.2 GeV^2

2.4

4.8

Does not look good

We saw why it is a problem at low ξ

gluons emitted = $\langle n \rangle \simeq \frac{\alpha_s N_C}{\pi} \ln(1/\xi)\, \Delta \ln \mu_F^2$ ~ 5

for ξ << 1 and reasonable variation of μ_F

whereas NLO only allows emission of one gluon !

however can resum $(\alpha_s \ln(1/\xi) \ln\mu_F^2)^n$ terms and move into LO contrib. by choosing $\mu_F = m_c$ (see JMRT, 1507.06942)

$$\boxed{A(\mu_f) = C^{LO} \otimes GPD(\mu_F) + C^{NLO}_{rem}(\mu_F) \otimes GPD(\mu_f)}$$

Use explicit NLO to calculate small remainder C_{rem}.

Residual dependence on scale μ_f is small

Aside: choice of renormalization scale

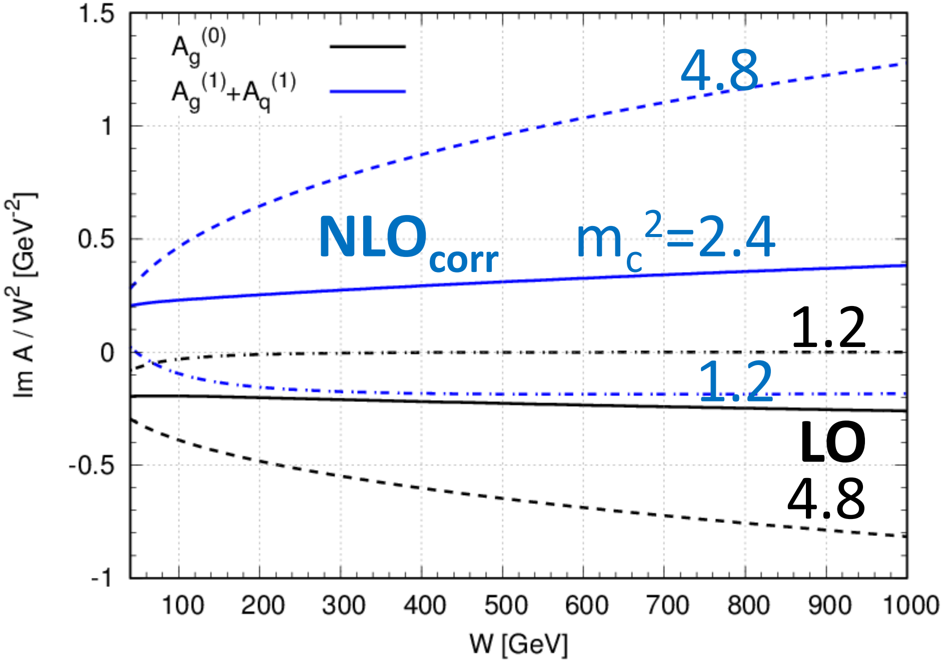
Choose $\mu_R = \mu_F$. Two reasons:

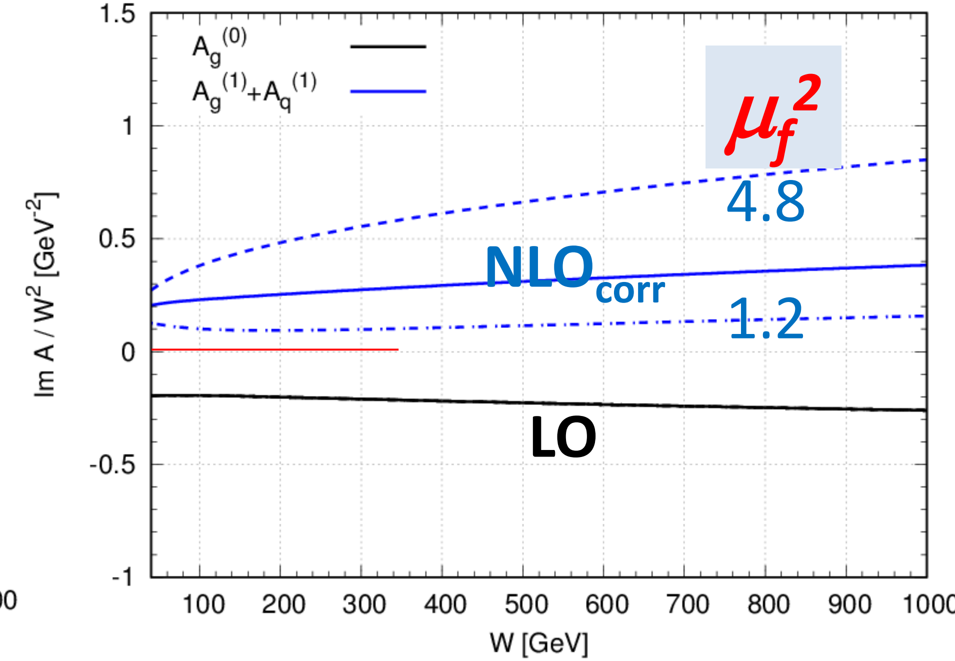
1. Corresponds to BLM prescription --- eliminates NLO $\beta_0 \ln(\mu_R/\mu_F)$ term
2. New q loop in g propagator appears twice:
   (a) part for scales $\mu<\mu_F$ by virtual comp^t of LO splitting in DGLAP evolution.
   (b) part for scales $\mu>\mu_R$ from running α_S behaviour after regular^n of UV divergence.

Not to miss part and/or to avoid double counting take

$$\mu_R = \mu_F$$

$$A(\mu_f) \;=\; C^{LO} \otimes GPD(\mu_F) \\ +\; C^{NLO}_{rem}(\mu_F) \otimes GPD(\mu_f)$$





scale dependence now weaker

A. But still have very bad perturbative convergence

NLO$_{correction}$ ~ LO and opposite sign

Have we missed something? **YES.** Effect of important Q_0 cut

Q_0^2/μ_F^2 power corr^{ns}.

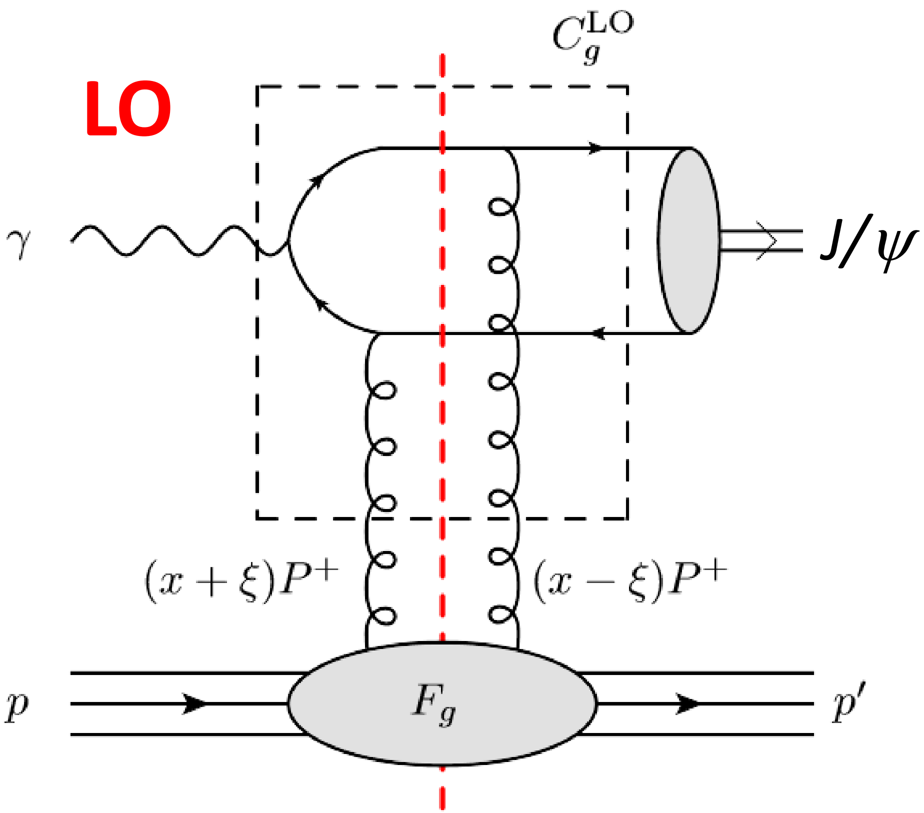
Start DGLAP evol. at Q_0

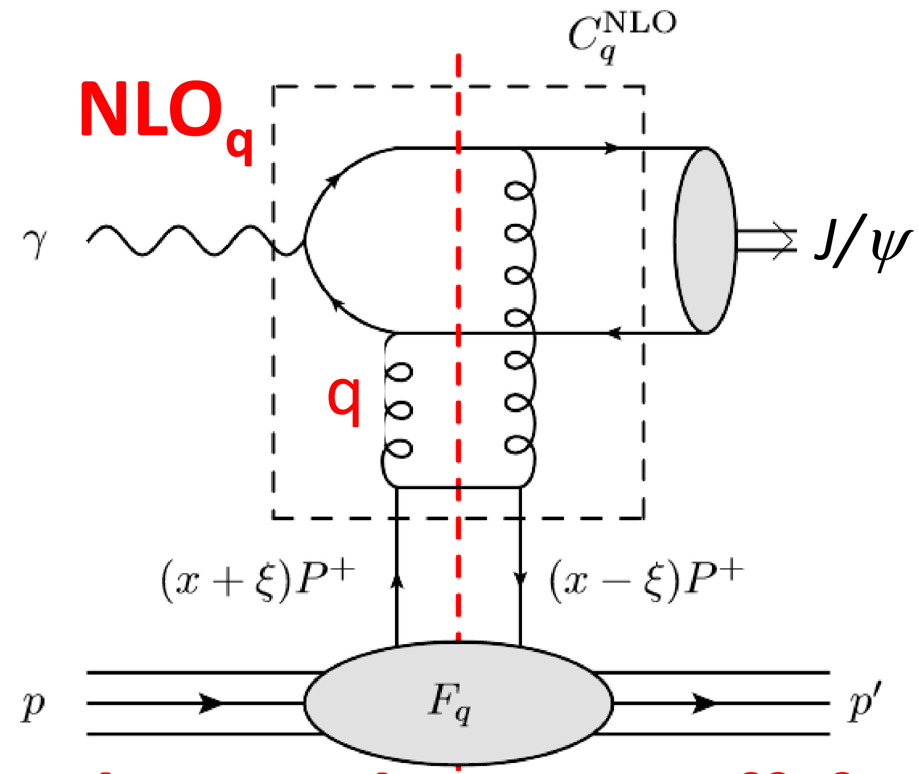
At LO everything below Q_0 is included in input PDF(Q_0)

At NLO the contrib^n from $|q^2| < Q_0^2$ is double counting

Need to subtract NLO($|q^2|<Q_0^2$) contrib^n for both *q* & *g*

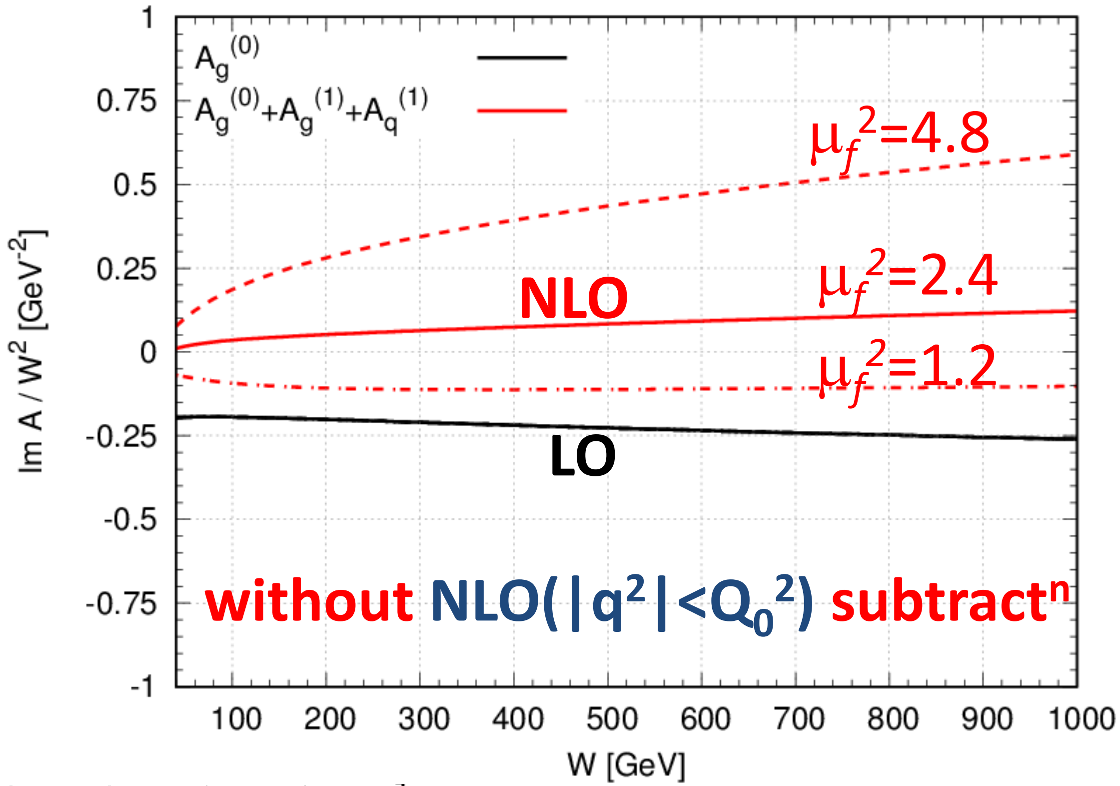
See appendix of 1610.02272

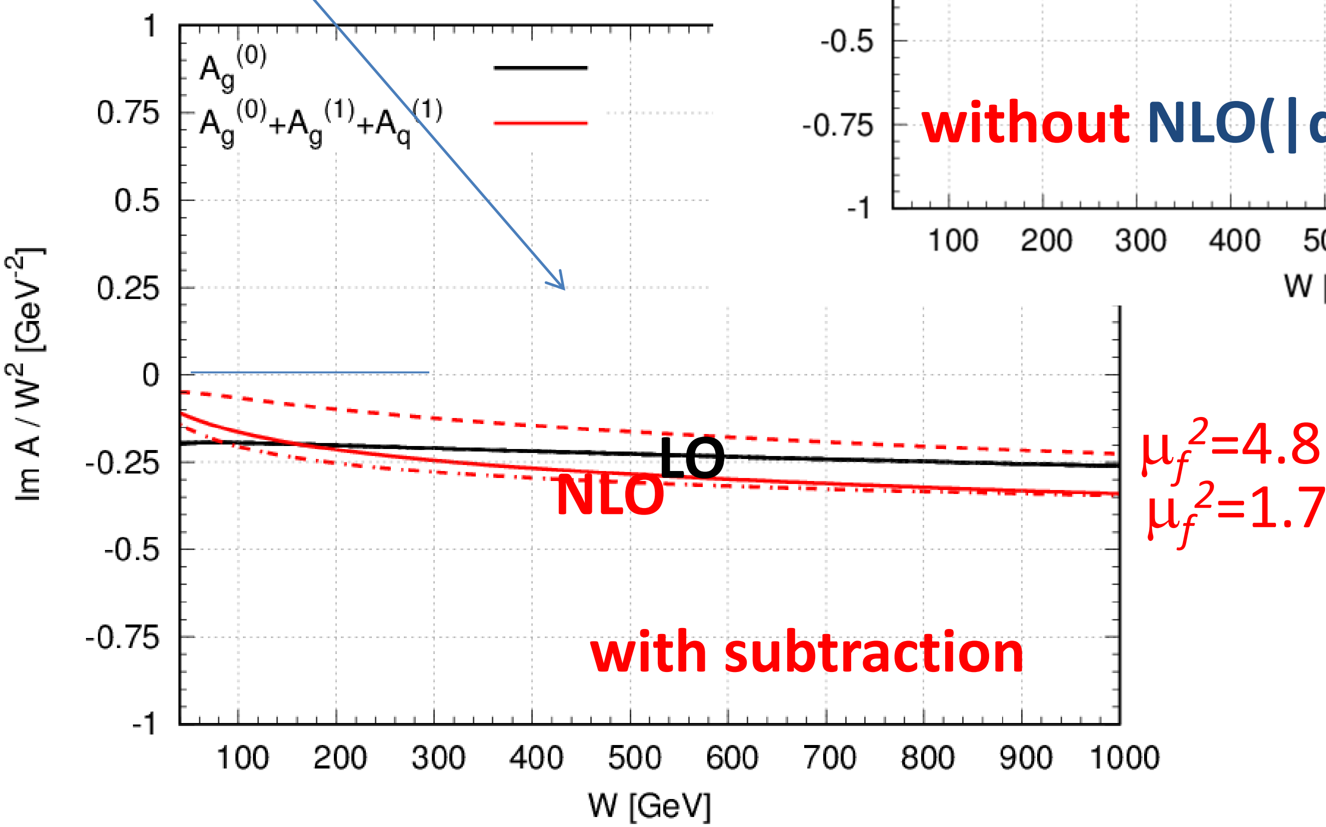




also need NLO_g coeff. fn

$\textbf{NLO}_{\textbf{corr}}$ is (i) now **small** and (ii) much less dependent on choice of (residual) fact^n scale μ_f





Conclusion for exclusive J/ψ

Subtract^{n} of NLO($|q^2|<Q_0^2$)   plus   choice $\mu_F = M_\psi/2$

(no double counting)   (resum of double logs)

leads to small NLO correction & small residual scale (μ_f) dependence --- so provides reasonable accuracy for the NLO pp → p+J/ψ+p amplitude in the collinear (MS(bar)) fact^{n} scheme

Υ even more reliable theoretically, but less events

Single inclusive open $c\bar{c}$ (and $b\bar{b}$) prod^n

Oliveira, M, Ryskin
1610.06034

We find optimum scale to resum $(\alpha_s \ln \mu_F^2 \ln(1/x))^n$ terms is

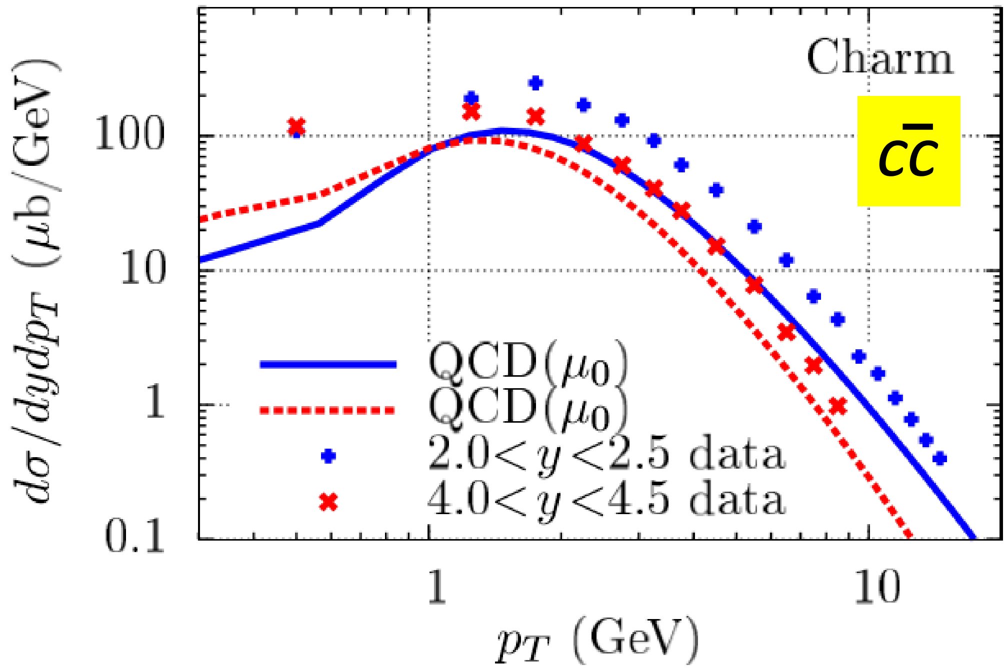
$\mu_F^2 = 0.72\,(p_T^2 + m_c^2)$

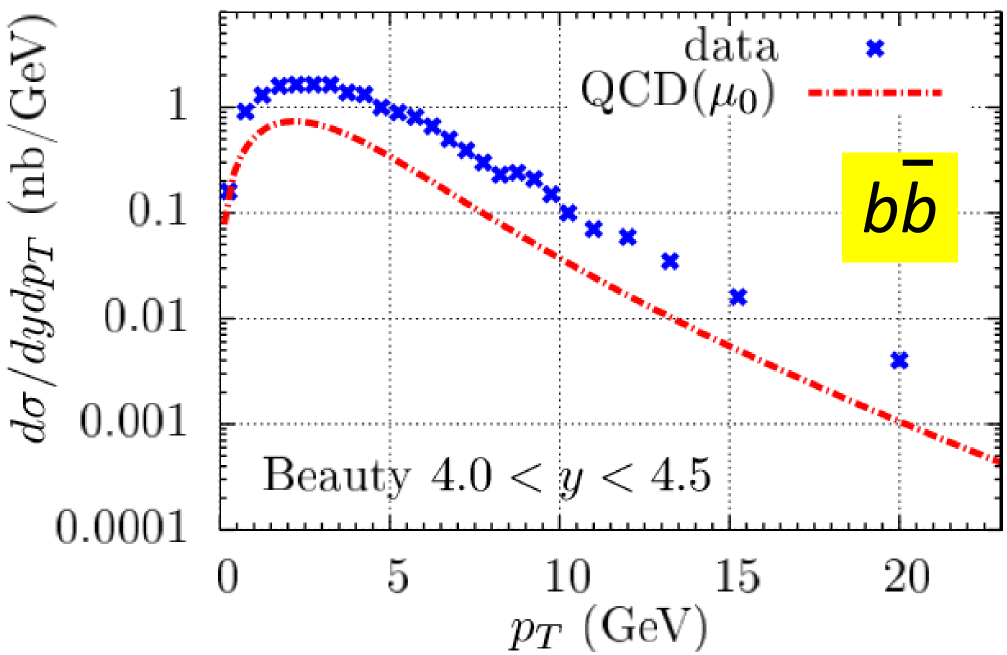
Though reduced, we find still rather strong scale dependence coming from numerically large 2→2 ($gg \rightarrow c\bar{c}$) terms at NLO and higher orders

We discuss the optimum scale to resum 2→2 these effects

We then compare with LHCb data using MCFM and FONLL
taking Prob ($c \rightarrow D^+$) = 0.25 and $p_D \sim 0.75\, p_c$
Prob ($b \rightarrow B^+$) = 0.25 and $p_B \sim 0.9\, p_b$

Conclusion for $c\bar{c}$, $b\bar{b}$





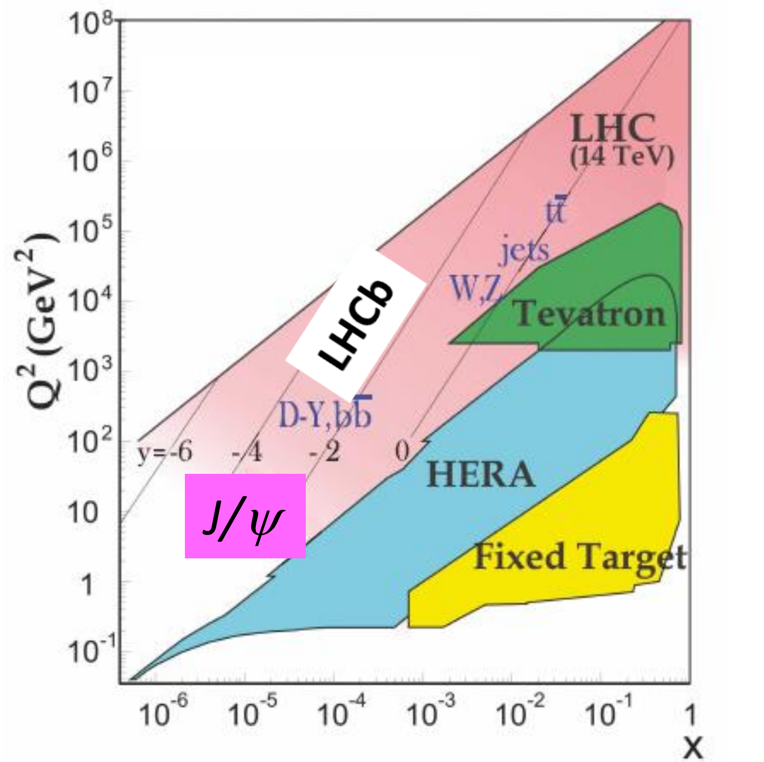
QCD predictions undershoot the LHCb $c\bar{c}$ and $b\bar{b}$ data indicating input low x gluon of CT14 NLO PDF should be larger.

Similar for other global PDFs.

Has implications for the flux of prompt atmospheric ν's, recall Anna Stasto's talk

General conclusion

Have the possibility of high precision data for exclusive J/ψ (Υ), and single inclusive $c\bar{c}$, $b\bar{b}$, production at high |y| at the LHC, being included in global parton analyses to determine the low x gluon PDF at low scales



LO approximation uses non-relativistic J/ψ wave fn.

Hoodbhoy (hep-ph/9611207) shows that the relativistic corrections, written in terms of the experimentally measured J/ψ width Γ_{ee}, are small, ~O(4%).

Suppose input g(x)=0, then only have NLO q term

But gluons cannot be smaller than density of gluons emitted before evol^n starts, so have opposite sign LO term

Input gluons cannot be freely parametrised in global fits, g(x) needs to be sufficiently positive

Pure global fits should take account of absorptive corrections which occur at low x, low Q^2.
Small/negative global input g(x) mimics this effect.

So much for conventional MS(bar) global PDFs at low x and low Q^2