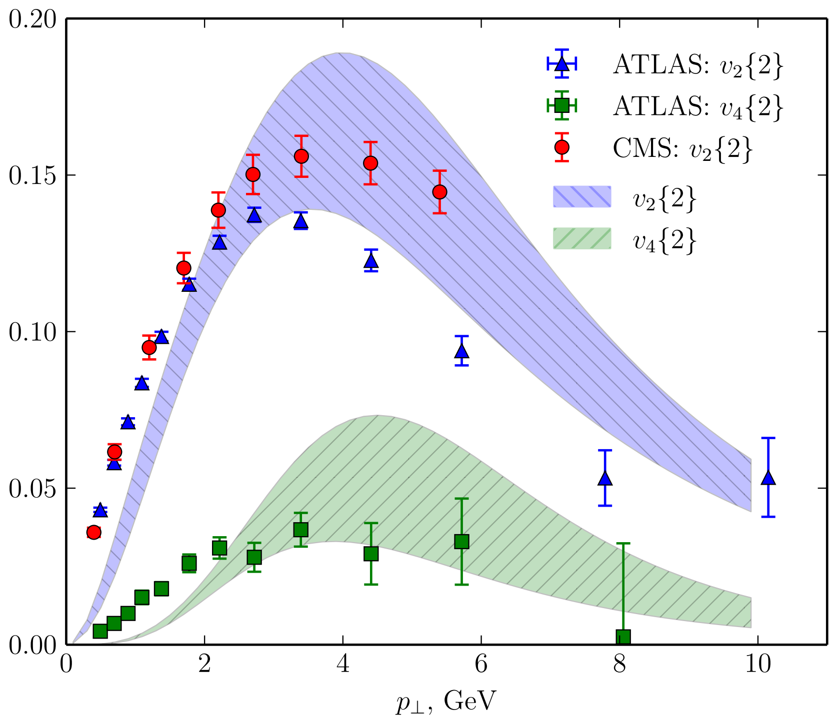
Even and Odd Moments of Flow from Initial Conditions

Flow: Collective motions of produced particles as observed in multi-particle momentum space correlations
Flow moments can and do arise from initial state anisotropies in distributions of sources. Observed in high multiplicity pp and pA collisions and AA collisions

The dominant source in heavy ion collisions arises from the final state evolution of coordinate space anisotropies into momentum space anisotropies, largely through hydrodynamic evolution. This is also probably the case for pp and pA collisions at low to intermediate transverse momentum. But flow anisotropies in pA go out to very high momentum:

(Blue and green envelopes will be described in a few pages)



Hydrodynamic description : Bozek and Broniowski

Jargonology

Color Glass Condensate:

Theory of initial distribution of gluons in hadron at small x
Due to high occupancy gluons (saturation) may be treated as classical fields
Time dilation effects (Gribov-Bjorken): Gluons may be generated by an incoherent ensemble of sources

Glasma

After a collision, highly coherent gluon field begin to evolve in time
Because of high occupancy for some time they may be described by classical equations
As phase space density lowers, evolves to a transport description
At later times, forms a thermalized Quark Gluon Plasma

Can two particle correlation reflect intrinsic correlations in the hadron wavefunction?

The Glasma interference in two gluon emission is an example

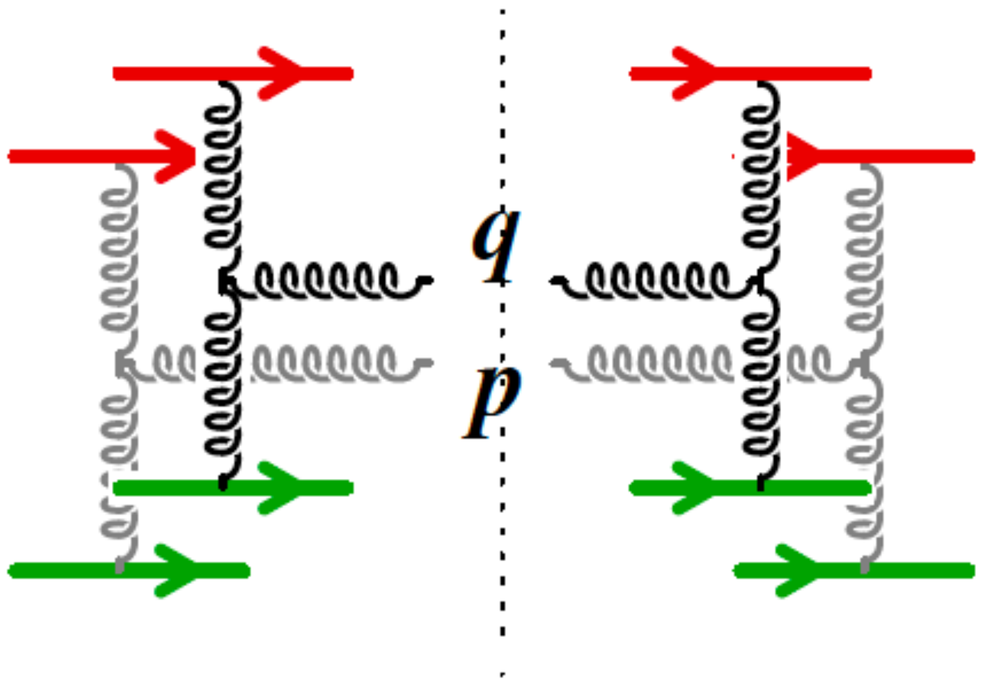
(Dumitru, Dusling, Gelis, Lappi,Venugopalan, ...)

Solid lines are sources in the colliding hadrons and squiggly lines are gluons

Contribution that does not vanish as the number of sources goes to infinity

but vanishes as 1/N_c as number of colors -> infinity

(The Glasma graph also has wrong sign for c_2(4) and needs something more)



Different from fluctuating source position diagrams that disappear as 1/N where N is number of sources

McLerran and Skokov

In the ordinary MV model of color fluctuations, there are no correlations between separated sources:

$$\langle \rho^a(\vec{x})\rho^b(\vec{y})\rangle = \delta^{ab}\delta(\vec{x}-\vec{y})\mu^2$$

Such a correlation does not insure charge neutrality

$$\langle \rho^a(\vec{k})\rho^b(\vec{q})\rangle = (2\pi)^2\delta^{ab}\delta(\vec{k}+\vec{q})\Delta(\vec{k})$$

$$\Delta(\vec{k}) = \frac{\mu^2 k^2}{k^2+m^2}$$

Because propagator vanishes at zero momentum, charge is neutralized, but this implies there are correlations on the size scale of the saturation momentum, whose size scale is of order 1/m

$$m \sim Q_{sat}$$

(Iancu. Itakura, McLerran)

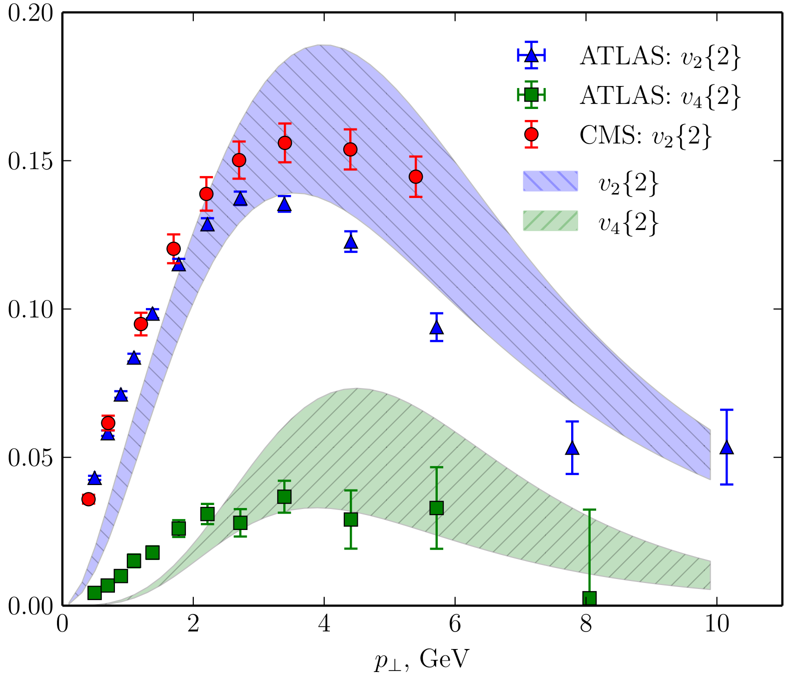
Mclerran Skokov

If one considers emission from a finite number of sources correlated on a scale 1/m, then one finds even moments of correlations in momentum space. Collectivity is generated by the wave interference of the emission combined with correlations between the sources. These correlations extend in momentum space out to a scale of order m ~ Q_{sat}. One obtains the natural scale.

These correlations are generated by fluctuations so are of order 1/N, where N is the number of sources, and are non-vanishing in the large N_c limit

They vanish for odd angular moments of the flow distribution, and this requires some final state interactions, which we will discuss later in this talk

(Blue and green envelopes are from a high p_T computation of the wave interference with a correlated source distribution)



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Can this be done in a systematic way that allows for non-perturbative Monte-Carlo computation:

Modify MV source function

$$\rho(k_T)\rho(-k_T) \rightarrow \rho(k_T)\Delta^{-1}(k_T)\rho(-k_T)$$

$$\Delta^{-1}(k_T) = \frac{1}{\mu^2}(1 + \frac{m^2}{k_T^2})$$

The first term is the local MV color fluctuation term. The second is a Coulomb interaction.

The source function describes a theory with a pointlike interaction plus an extended Coulomb interaction.

One can write this in coordinate space. Consider a theory with a finite number of sources dispersed at various points.

One can do a Monte-Carlo computation where the positions of the points are random, as well as the color orientations of the sources. One can thus compute correlation functions of the fields, for example dipole operators for the pA case. This will generate correlation functions for flow which will have even moments.

There are numerous difficult technical issues in discretizing and spreading out the sources to avoid infinities associated with the Coulomb energy and maintaining positivity of the functional. These problems are solved.

What about the odd moments? They presumably need final state interactions to be non-zero.

Simple example: pA interactions in the Glasma and CGC. Here the effect of the large nucleus is trivai: The classical solution induced by the nucleus simply gauge rotates the propagation of the proton in the forward light cone.

In first order in the proton source, the S matrix is analytically computed. There are no odd moments of the flow distribution. One needs to go to second order in the proton source, which is the first saturation correction for the proton.

Consider the reduction formula for a field in the forward light cone

$$a^+(\vec{k},\infty) = \frac{1}{i}\int_{t=0} d^3x \exp(-ik\cdot x)\overleftrightarrow{\partial_0}\phi(x) + \frac{1}{i}\int_0^\infty dt \int d^3x \exp(-ik\cdot x)\left(\Box + m^2\right)\phi(x)\,.$$

Due to the boundary condition in the forward light cone, there is a boundary term and a bulk. For free fields in the forward light cone, the second term vanishes and particle production is entirely determined by the surface contribution. This surface contribution yields no odd flow moment, and is CP even

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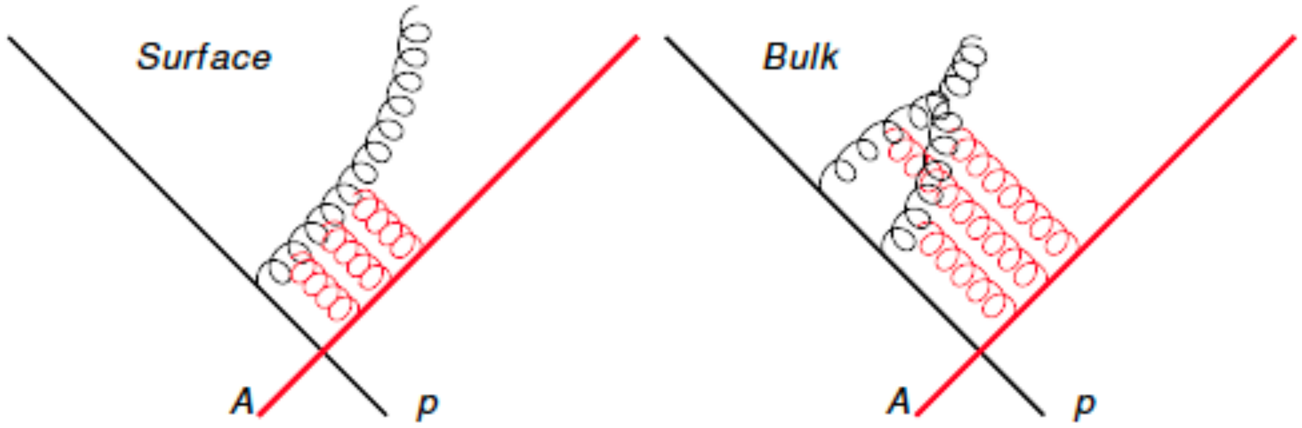
The first non-zero contribution to the odd flow moments comes from the interference of the first order solution to the classical equations with the second order contribution. It involves an overlap of a bulk term with a surface term



We can compute the odd moment explicitly in terms of sources. (We were unable to find a relationship between the odd moment and the eccentricity of the source distribution.) Our results will allow for a direct lowest order contributions of the odd moments without directly solving the classical equations for the Glasma.
Explicit computation is quite pretty

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Summary and Conclusions

One can generate angular correlations of particle momentum distributions, or flow, from initial state fluctuations.

May be from final state interactions at least for smallish transverse momentum

At high transverse momentum, even moments may be produced without final state interaction

Odd moments involve at least one final state interaction.

Explicit computations illustrate these effects.