High Energy QCQ @ NLO

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Alex Kovner, ML, and Yair Mulian
arXiv:1310.0378 (PRD), arXiv:1401.0374 (JHEP), arXiv:1405.0418 (JHEP)

ML and Yair Mulian, arXiv:1610.03453

High Energy Scattering

Target (ρ^t) Projectile (ρ^p)

$$\langle \mathbf{T}| \quad \rightarrow \qquad\qquad \leftarrow \quad |\mathbf{P}\rangle$$

S-matrix:

$$\mathbf{S}(\mathbf{Y}) \;=\; \langle \mathbf{T}\, \langle\, \mathbf{P}| \;\; \hat{\mathbf{S}}(\rho^{\mathbf{t}},\, \rho^{\mathbf{p}}) \;\; |\mathbf{P}\rangle\, \mathbf{T}\rangle$$

or, more generally, any observable $\hat{\mathcal{O}}(\rho^{\mathrm{t}},\, \rho^{\mathrm{p}})$

$$\langle \hat{\mathcal{O}} \rangle_{\mathbf{Y}} \;=\; \langle \mathbf{T}\, \langle\, \mathbf{P}| \;\; \hat{\mathcal{O}}(\rho^{\mathbf{t}},\, \rho^{\mathbf{p}}) \;\; |\mathbf{P}\rangle\, \mathbf{T}\rangle$$

The question we pose is how these averages change with increase in energy of the process

Projectile averaged S-matrix:

$$\mathbf{\Sigma(Y)} \;\equiv\; \langle\, \mathbf{P}|\, \hat{\mathbf{S}}(\rho^{\mathbf{t}},\; \rho^{\mathbf{p}})\; |\mathbf{P}\rangle \;\;=\;\; \int \mathbf{D}\rho^{\mathbf{p}}\;\; \mathbf{S}(\rho^{\mathbf{t}},\; \rho^{\mathbf{p}})\;\; \mathbf{W}^{\mathbf{p}}_{\mathbf{Y}}[\rho^{\mathbf{p}}]$$

evolve with rapidity as $\mathbf{H}$ $\rightarrow$ the HE effective Hamiltonian

$$\mathbf{\Sigma(Y + \delta Y)} \;\;=\;\; \int \mathbf{D}\rho^{\mathbf{p}}\;\; \mathbf{e}^{-\delta\mathbf{Y}\,\mathbf{H}}\;\; \mathbf{S}(\rho^{\mathbf{t}},\; \rho^{\mathbf{p}})\;\; \mathbf{W}^{\mathbf{p}}_{\mathbf{Y}}[\rho^{\mathbf{p}}]$$

Spectrum of $\mathbf{H}$ defines energy dependence of the observables.

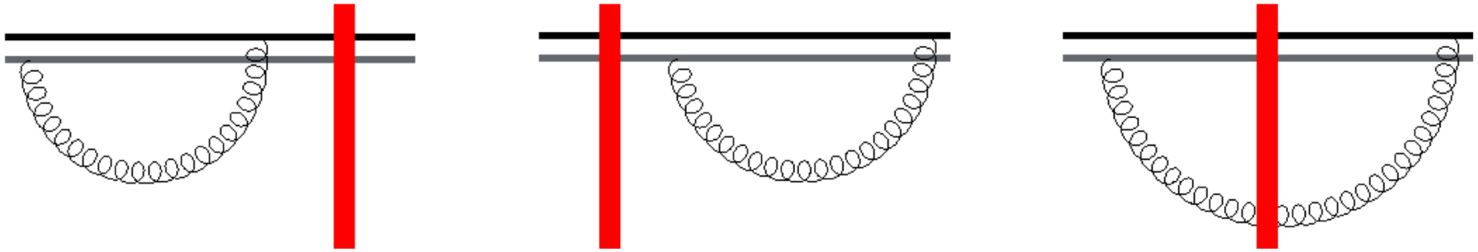
$$\mathbf{e}^{-\delta\mathbf{Y}\,\mathbf{H}} \;\simeq\; \mathbf{1} \;-\; \delta\mathbf{Y}\,\mathbf{H} \;+\; \frac{\mathbf{1}}{\mathbf{2}}\,\delta\mathbf{Y}^{\mathbf{2}}\,\mathbf{H}^{\mathbf{2}}\;\ldots$$

$$\mathbf{H} \;=\; \mathbf{H}^{\mathbf{LO}}(\alpha_{\mathbf{s}}) \;+\; \mathbf{H}^{\mathbf{NLO}}(\alpha_{\mathbf{s}}^{\mathbf{2}}) \;+\; \ldots$$

LO JIMWLK Hamiltonian

The JIMWLK Hamiltonian is a limit of H for dilute partonic system ($\rho^p \to 0$) which scatters on a dense target. It accounts for linear gluon emission + multiple rescatterings.

$$H_{LO}^{JIMWLK} = \frac{\alpha_s}{2\pi^2}\int_{x,y,z}\frac{(x-z)_i(y-z)_i}{(x-z)^2(y-z)^2}\left\{J_L^a(x)J_L^a(y) + J_R^a(x)J_R^a(y) - 2J_L^a(x)S_A^{ab}(z)J_R^b(y)\right\}$$



$$\mathbf{S_A^{cd}(z) = \mathcal{P}\,\exp\left\{i\int dx^+\,T^a\,\alpha_t^a(z,x^+)\right\}^{cd}}.$$

$$\text{"}\boldsymbol{\Delta}\text{"}\,\alpha_t = \rho_t \qquad \mathbf{(YM)}$$

The left and right SU(N) generators:

$$\mathbf{J_L^a(x)S_A^{ij}(z) = (T^aS_A(z))^{ij}\,\delta^2(x-z)} \qquad \mathbf{J_R^a(x)S_A^{ij}(z) = (S_A(z)T^a)^{ij}\,\delta^2(x-z)}$$

- $\mathbf{H^{JIMWLK}}$ **contains all the LO BFKL / BKP / TPV physics**

JIMWLK Hamiltonian @ NLO

Alex Kovner, ML and Yair Mulian (2013)

$$H^{NLO\ JIMWLK} = \int_{x,y,z} K_{JSJ}(x,y;z) \Big[J_L^a(x)J_L^a(y) + J_R^a(x)J_R^a(y) - 2J_L^a(x)S_A^{ab}(z)J_R^b(y)\Big]$$

$$+ \int_{x\,y\,z\,z'} K_{JSSJ}(x,y;z,z') \Big[f^{abc}f^{def}J_L^a(x)S_A^{be}(z)S_A^{cf}(z')J_R^d(y) - N_c J_L^a(x)S_A^{ab}(z)J_R^b(y)\Big]$$

$$+ \int_{x,y,z,z'} K_{q\bar{q}}(x,y;z,z') \Big[2\,J_L^a(x)\,tr[S_F^\dagger(z)\,t^a\,S_F(z')t^b]\,J_R^b(y) \;-\; J_L^a(x)\,S_A^{ab}(z)\,J_R^b(y)\Big]$$

$$+ \int_{w,x,y,z,z'} K_{JJSSJ}(w;x,y;z,z')f^{acb}\Big[J_L^d(x)\,J_L^e(y)\,S_A^{dc}(z)\,S_A^{eb}(z')\,J_R^a(w)$$

$$- J_L^a(w)\,S_A^{cd}(z)\,S_A^{be}(z')\,J_R^d(x)\,J_R^e(y)\Big]$$

$$+ \int_{w,x,y,z} K_{JJSJ}(w;x,y;z)\,f^{bde}\Big[J_L^d(x)\,J_L^e(y)\,S_A^{ba}(z)\,J_R^a(w) - J_L^a(w)\,S_A^{ab}(z)\,J_R^d(x)\,J_R^e(y)\Big]$$

$$+ \int_{w,x,y} K_{JJJ}(w;x,y)f^{deb}\,[J_L^d(x)\,J_L^e(y)\,J_L^b(w) \;-\; J_R^d(x)\,J_R^e(y)\,J_R^b(w)]\,.$$

Symmetries: $\mathbf{SU_L(N)\times SU_R(N)}$ **CPT, Unitarity**

Shortcuts to the Kernels

Step 1: Compute evolution of 3-quark Wilson loop operator in SU(3) (baryon)

$$\mathbf{B(u,v,w)} = \epsilon^{\mathbf{ijk}}\epsilon^{\mathbf{lmn}}\mathbf{S_F^{il}(u)S_F^{jm}(v)S_F^{kn}(w)}$$

$$\partial_{\mathbf{Y}}\,\mathbf{B(u,v,w)} = -\mathbf{H}^{\mathbf{NLO\,JIMWLK}}\,\mathbf{B(u,v,w)}$$

and compare with Grabovsky (hep-ph/1307.5414) $\quad\rightarrow\quad \mathbf{K_{JJSSJ}},\ \mathbf{K_{JJSJ}}$

Step 2: Compute evolution of quark dipole operator

$$\mathbf{s(u,v)} = \mathbf{tr[S_F(u)S_F^\dagger(v)]/N_c}$$

$$\partial_{\mathbf{Y}}\,\mathbf{s(u,v)} = -\mathbf{H}^{\mathbf{NLO\,JIMWLK}}\,\mathbf{s(u,v)}$$

and compare with Balitsky and Chirilli (hep-ph/0710.4330) $\quad\rightarrow\quad \mathbf{K_{JSSJ}},\ \mathbf{K_{JSJ}},\ \mathbf{K_{qq}}$

NLO Kernels (for gauge invariant operators)

$$K_{JJSSJ}(w;x,y;z,z') = -i\frac{\alpha_s^2}{2\,\pi^4}\left(\frac{X_iY'_j}{X^2Y'^2}\right)$$
$$\times\Big(\frac{\delta_{ij}}{2(z-z')^2}+\frac{(z'-z)_iW'_j}{(z'-z)^2W'^2}+\frac{(z-z')_jW_i}{(z-z')^2W^2}-\frac{W_iW'_j}{W^2W'^2}\Big)\ln\frac{W^2}{W'^2}$$

$$K_{JJSJ}(w;x,y;z) = -\,i\,\frac{\alpha_s^2}{4\,\pi^3}\left[\frac{X\cdot W}{X^2\,W^2}-\frac{Y\cdot W}{Y^2\,W^2}\right]\ln\frac{Y^2}{(x-y)^2}\,\ln\frac{X^2}{(x-y)^2},$$

$$K_{q\bar{q}}(x,y;z,z') = -\frac{\alpha_s^2\,n_f}{8\,\pi^4}\Big\{\frac{X'^2Y^2+Y'^2X^2-(x-y)^2(z-z')^2}{(z-z')^4(X^2Y'^2-X'^2Y^2)}\ln\frac{X^2Y'^2}{X'^2Y^2}-\frac{2}{(z-z')^4}\Big\}$$

$$X=x-z,\quad X'=x-z',\quad Y=y-z,\quad Y'=y-z',\quad W=w-z$$

$$K_{JSSJ}(x,y;z,z') = \frac{\alpha_s^2}{16\,\pi^4}\Big[-\frac{4}{(z-z')^4} + \Big\{2\frac{X^2Y'^2 + X'^2Y^2 - 4(x-y)^2(z-z')^2}{(z-z')^4[X^2Y'^2 - X'^2Y^2]}$$

$$+\frac{(x-y)^4}{X^2Y'^2 - X'^2Y^2}\Big[\frac{1}{X^2Y'^2}+\frac{1}{Y^2X'^2}\Big]+\frac{(x-y)^2}{(z-z')^2}\Big[\frac{1}{X^2Y'^2}-\frac{1}{X'^2Y^2}\Big]\Big\}\ln\frac{X^2Y'^2}{X'^2Y^2}\Big]+\tilde{K}(x,y,z,z').$$

$$K_{JSJ}(x,y;z) = -\frac{\alpha_s^2}{16\pi^3}\frac{(x-y)^2}{X^2Y^2}\Big[b\ln(x-y)^2\mu^2 - b\frac{X^2-Y^2}{(x-y)^2}\ln\frac{X^2}{Y^2}+\Big(\frac{67}{9}-\frac{\pi^2}{3}\Big)N_c-\frac{10}{9}n_f\Big]$$

$$-\frac{N_c}{2}\int_{z'}\tilde{K}(x,y,z,z').$$

Here μ is the normalization point, $b = \frac{11}{3}N_c - \frac{2}{3}n_f$

$$\tilde{K}(x,y,z,z') = \frac{i}{2}\Big[K_{JJSSJ}(x;x,y;z,z') - K_{JJSSJ}(y;x,y;z,z') - K_{JJSSJ}(x;y,x;z,z')$$

$$+K_{JJSSJ}(y;y,x;z,z')\Big]$$

The kernels are not unique though...

NLO Kernels for color non-singlets

"By inspection" of Balitsky and Chirilli (arXiv:1309.7644 [hep-ph])

$$K_{JSJ}(x,y,z) \to \bar{K}_{JSJ}(x,y,z) \equiv K_{JSJ}(x,y,z) + \\ + \frac{\alpha_s^2}{16\pi^3}\left\{\left[\frac{1}{X^2}+\frac{1}{Y^2}\right]\left[\left(\frac{67}{9}-\frac{\pi^2}{3}\right)N_c - \frac{10}{9}n_f\right] + \frac{b}{X^2}\ln X^2\mu^2 + \frac{b}{Y^2}\ln Y^2\mu^2\right\};$$

$$K_{JSSJ}(x,y;z,z') \to \bar{K}_{JSSJ}(x,y;z,z') + \\ \equiv K_{JSSJ}(x,y;z,z') + \frac{\alpha_s^2}{8\pi^4}\left[\frac{4}{(z-z')^4} - \frac{I(x,z,z')}{(z-z')^2} - \frac{I(y,z,z')}{(z-z')^2}\right];$$

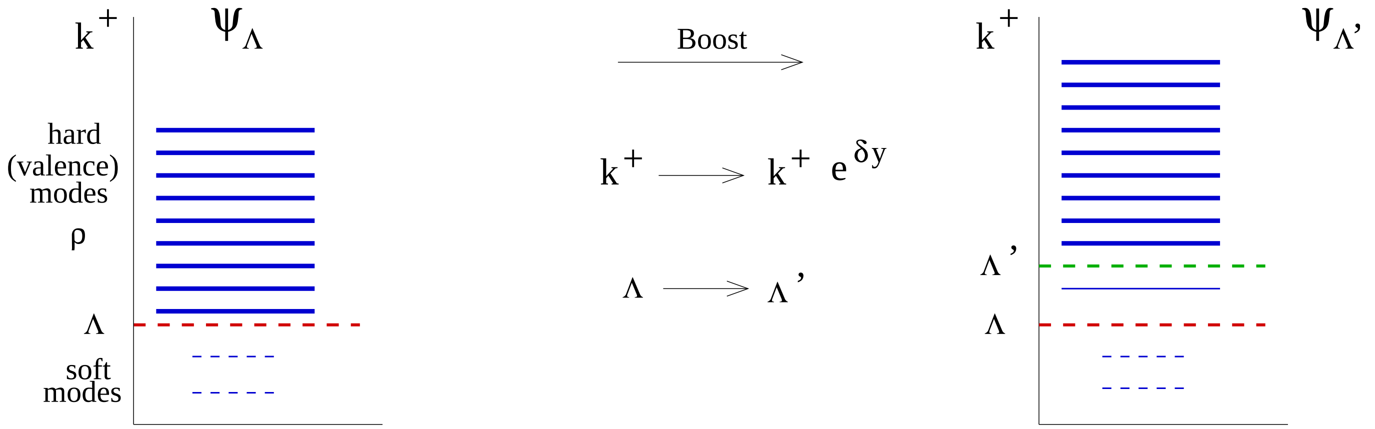
$$K_{q\bar{q}}(x,y;z,z') \to \bar{K}_{q\bar{q}}(x,y;z,z') \equiv K_{q\bar{q}}(x,y;z,z') - \frac{\alpha_s^2 n_f}{8\pi^4}\left[\frac{I_f(x,z,z')}{(z-z')^2} + \frac{I_f(y,z,z')}{(z-z')^2}\right],$$

$$I(x,z,z') = \frac{1}{X^2-(X')^2}\ln\frac{X^2}{(X')^2}\left[\frac{X^2+(X')^2}{(z-z')^2} - \frac{X\cdot X'}{X^2} - \frac{X\cdot X'}{(X')^2} - 2\right];$$

$$I_f(x,z,z') = \frac{2}{(z-z')^2} - \frac{2X\cdot X'}{(z-z')^2(X^2-(X')^2)}\ln\frac{X^2}{(X')^2}.$$

Light Cone Wave Function

$$\mathbf{H}_{\mathrm{LC\,QCD}}\,|\mathbf{\Psi}\rangle \;=\; \mathbf{E}\,|\mathbf{\Psi}\rangle$$



Hard particles with $k^+ \; > \; \Lambda$ scatter of the target. Hard (valence) modes are described by the valence density $\rho(x_\perp)$ (shock wave).

The boost opens a window above Λ with the width $\sim \; \delta y$. The window is populated by soft modes, which became hard after the boost. These newly created hard modes do scatter off the target.

In the dilute limit $\rho \; \sim \; 1$; gluon emission $\sim \alpha_s \, \rho$, LO = one gluon, NLO = 2 gluons/quarks

Denote soft glue (quark) creation and annihilation operators as $\mathbf{a}$ and $\mathbf{a}^\dagger$.

$$\mathbf{H}_{\mathrm{LC\,QCD}} = \mathbf{H}[\rho,\, \mathbf{a},\, \mathbf{a}^\dagger] = \mathbf{H}_{\mathrm{V}}[\rho] + \mathbf{H}_{\mathrm{free}}[\mathbf{a},\, \mathbf{a}^\dagger] + \mathbf{H}_{\mathrm{int}}[\rho,\, \mathbf{a},\, \mathbf{a}^\dagger]$$

LCWF with no soft gluons

$$\mathbf{H}_{\mathrm{V}}\, |\mathbf{v},\, \mathbf{0}_{\mathrm{a}}\rangle = \mathbf{E}_0\, |\mathbf{v},\, \mathbf{0}_{\mathrm{a}}\rangle\,; \qquad \mathbf{a}\, |\mathbf{v},\, \mathbf{0}_{\mathrm{a}}\rangle = \mathbf{0}\,; \qquad \mathbf{E}_0 = \mathbf{0}$$

LCWF with soft gluon/quark dressing

$$|\boldsymbol{\Psi}\rangle = \boldsymbol{\Omega}(\rho,\, \mathbf{a},\, \mathbf{a}^\dagger)\, |\mathbf{v},\, \mathbf{0}_{\mathrm{a}}\rangle\,;$$

Find Ω in perturbation theory

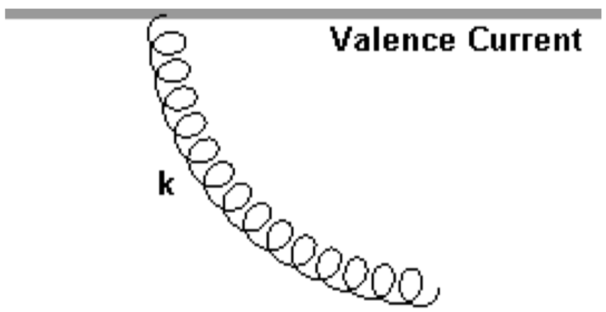
LCWF at LO

First order (g) perturbation theory

$$|\Psi_{LO}\rangle = \mathcal{N}\,|0_a\rangle - \sum_i |i\rangle \frac{\langle i|\, H_{int}\, |0_a\rangle}{E_i} \qquad \langle \Psi_{LO}|\Psi_{LO}\rangle = 1 \to \mathcal{N}$$

Eikonal coupling between valence and soft gluons due to separation of scales

$$H_{int} = -\int \frac{dk^+}{2\pi}\frac{d^2k_\perp}{(2\pi)^2}\frac{g\,k_i}{\sqrt{2}\,|k^+|^{3/2}}\left[a_i^{\dagger a}(k^+,\,k_\perp)\;\rho^a(-k_\perp) + a_i^a(k^+,\,-k_\perp)\;\rho^a(k_\perp)\right]$$



A cloud of WW gluons dressing the valence ones

$$\Sigma^{LO} = \langle \Psi_{LO}|\,\hat{S}\,|\Psi_{LO}\rangle \to \text{LO JIMWLK}$$

LCWF at NLO

ML and Yair Mulian, arXiv:1610.03453

- g^3 + normalisation at g^4

$$|\Psi_{NLO}\rangle = \mathcal{N}\,|0\rangle + \sum_i |i\rangle \Big[-\frac{\langle i|\,H_{int}\,|0\rangle}{E_i} + \frac{\langle i|\,H_{int}\,|j\rangle\,\langle j|\,H_{int}\,|0\rangle}{E_i\,E_j} +$$

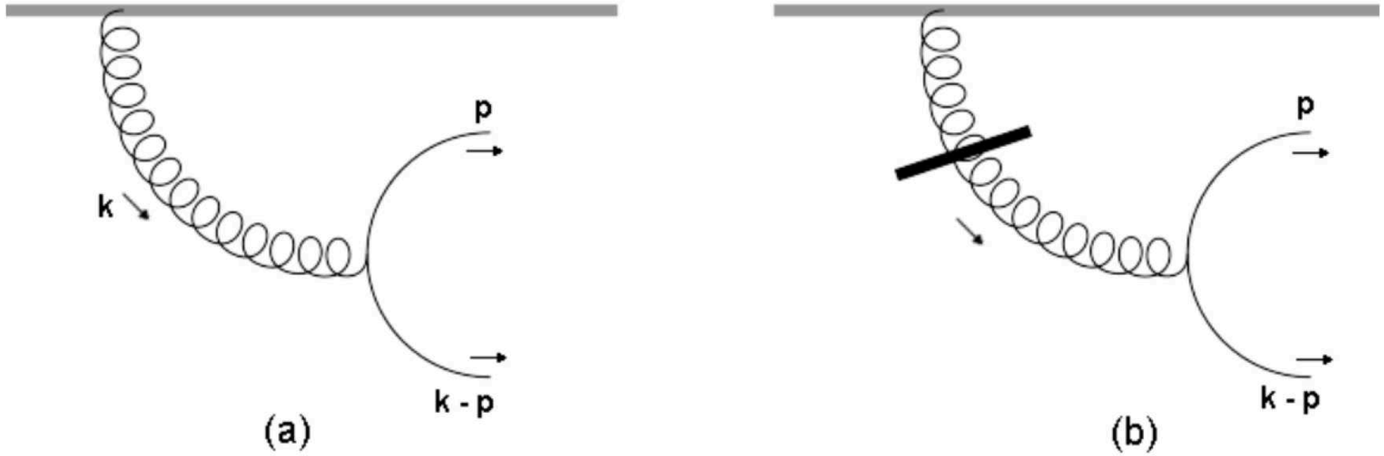
$$+ \frac{\langle i|\,H_{int}\,|0\rangle\,\langle j|\,H_{int}\,|0\rangle^2\,(2\,E_j - E_i)}{2\,E_i^2\,E_j^2} - \frac{\langle i|\,H_{int}\,|j\rangle\,\langle j|\,H_{int}\,|k\rangle\,\langle k|\,H_{int}\,|0\rangle}{E_i\,E_j\,E_k} \Big]$$

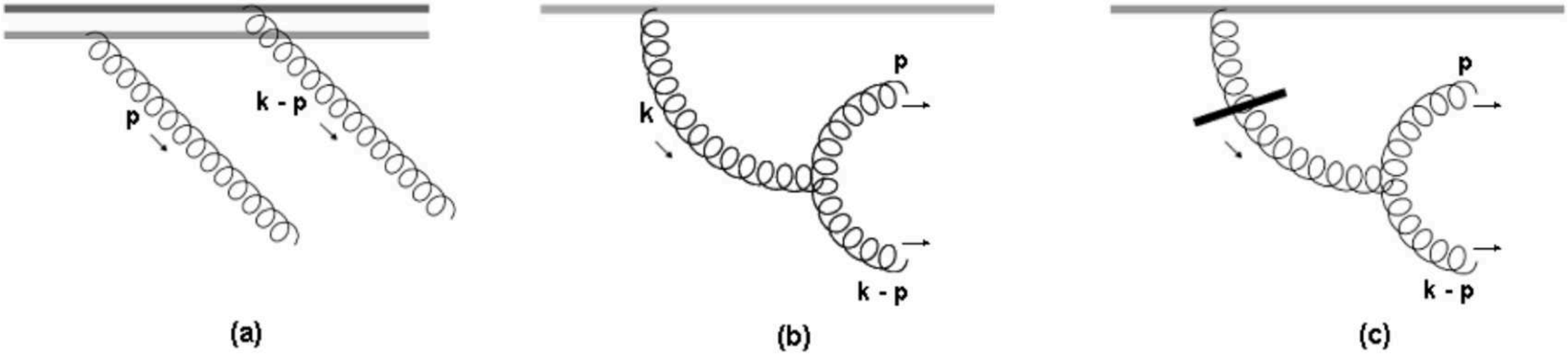
i runs over one gluon, two gluons, and two quarks

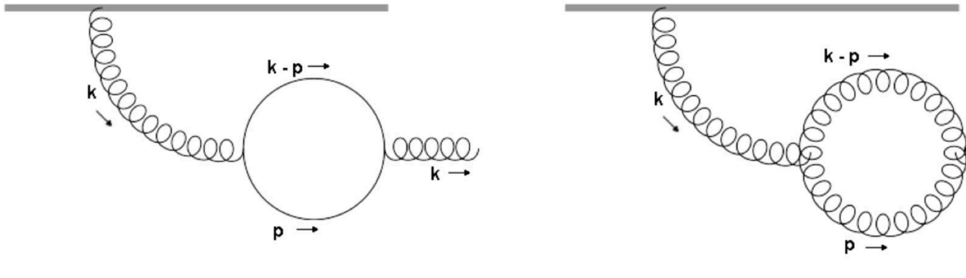
Operator valued matrix elements

Accounts for first non-linear/saturation effects in the projectile

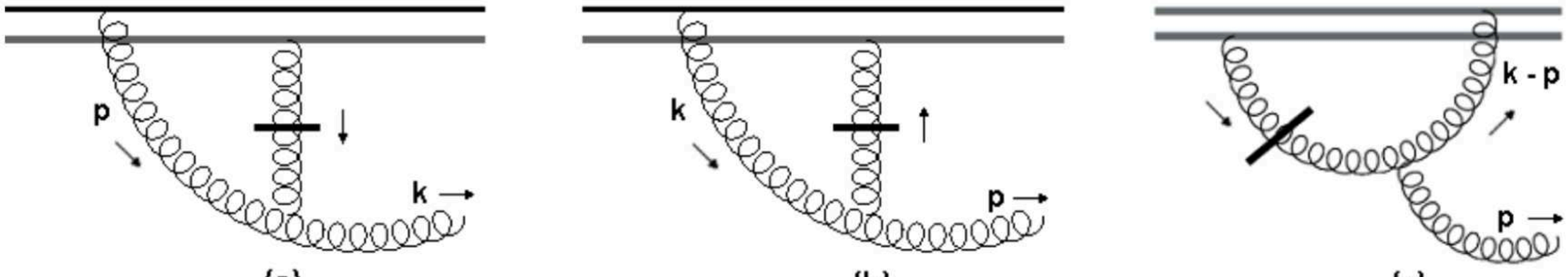
$$\langle\Psi_{NLO}|\Psi_{NLO}\rangle = 1 \to |\mathcal{N}| \; ; \qquad \mathcal{N} = |\mathcal{N}|\,e^{i\phi}$$


p
k
k - p
(a)
p
k - p
(b)

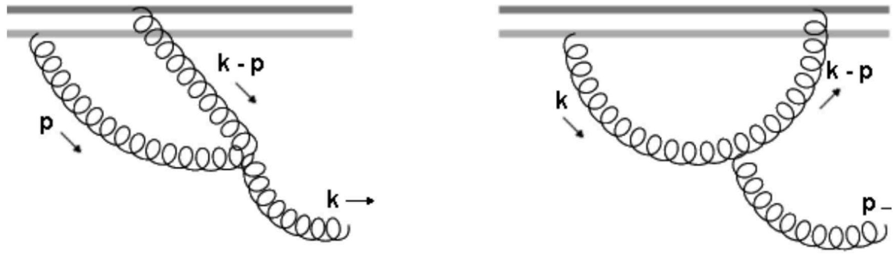

k - p
p
(a)
p
k
k - p
(b)
p
k - p
(c)



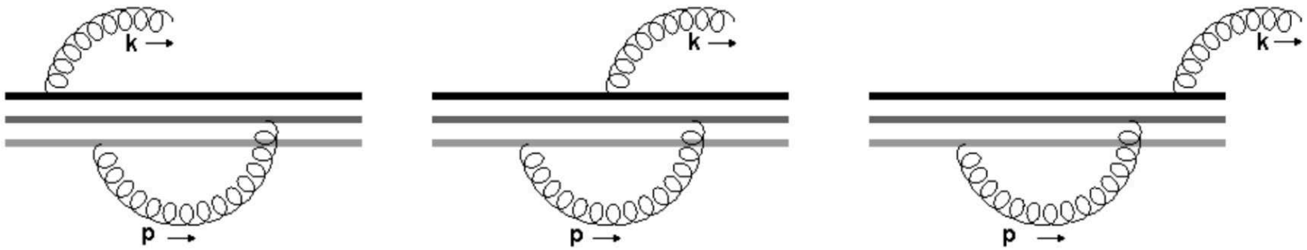
(a) (b)



(a) (b) (c)



(a) (b)



(a) (b) (c)

Phase of the LCWF @ NLO

$$\mathcal{N} \;=\; |\mathcal{N}|\, \mathbf{e}^{\mathbf{i}\phi}$$

Beyond perturbation theory: Born-Oppenheimer adiabatic approximation

$$\langle \mathbf{v}| \otimes \langle\psi|\, \mathbf{H_V}\, |\psi\rangle \otimes |\mathbf{v}\rangle \;\simeq\; \langle \mathbf{v}|\, \mathbf{H_V}\, |\mathbf{v}\rangle \qquad \mathbf{or} \qquad \langle\psi|\; \mathbf{H_V}\; |\psi\rangle_{\mathbf{soft}} \;\simeq\; \mathbf{0}$$

Berry connection

$$\langle\psi|\, \frac{\delta}{\delta\rho^{\mathbf{d}}(\mathbf{w})}\, |\psi\rangle \;=\; \mathbf{0} \qquad \to\; \phi$$

JIMWLK Hamiltonian @ NLO (again)

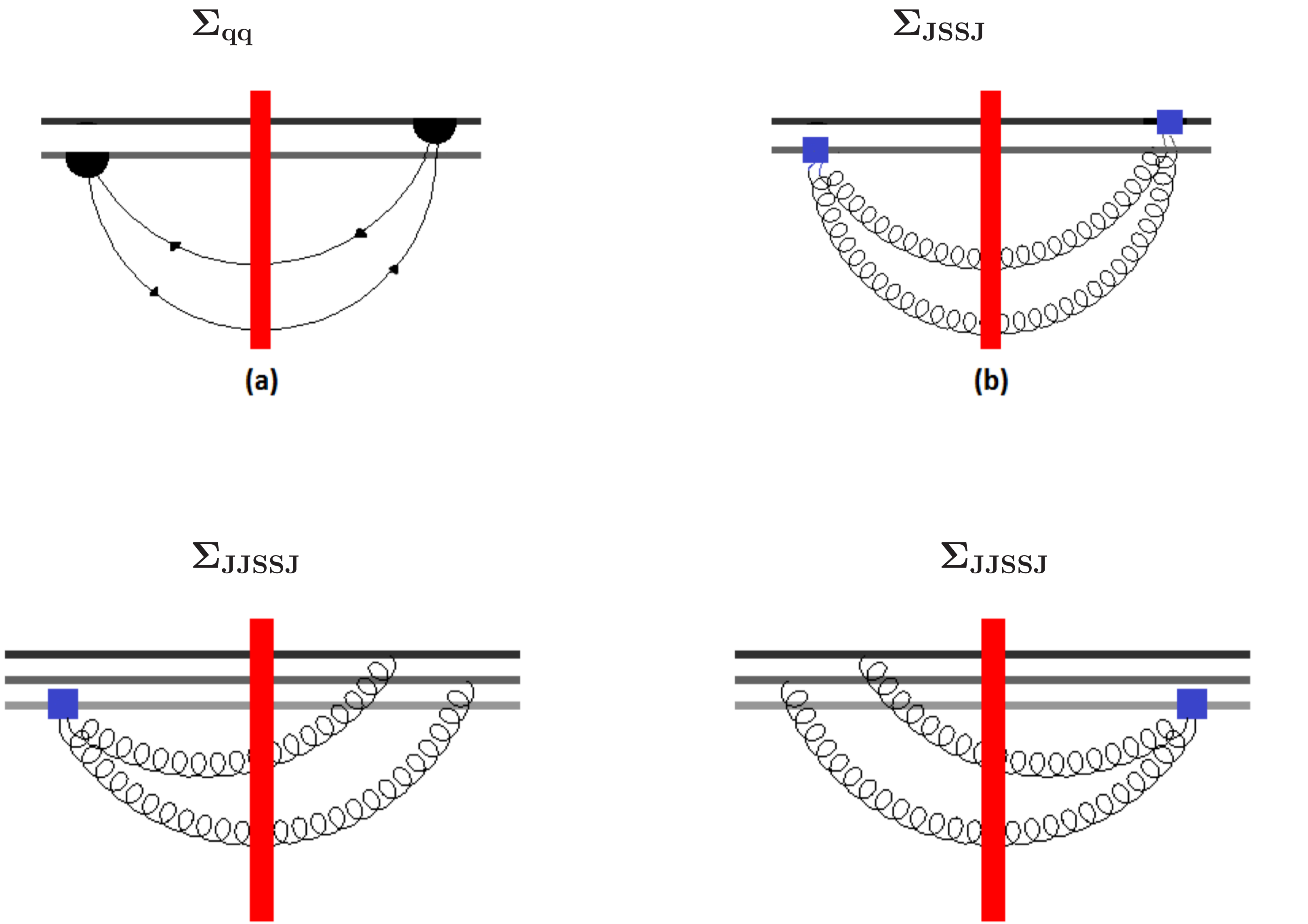
$$\mathbf{\Sigma} = \left\langle \psi^{\mathbf{NLO}} \right| \hat{\mathbf{S}} \left| \psi^{\mathbf{NLO}} \right\rangle$$

$$= \Sigma^{LO} + \Sigma_{q\bar{q}} + \Sigma_{JJSSJ} + \Sigma_{JSSJ} + \Sigma_{JJSJ} + \Sigma_{JSJ} + \Sigma_{JJSSJJ} + \Sigma_{JJJSJ} + \Sigma_{virtual}.$$

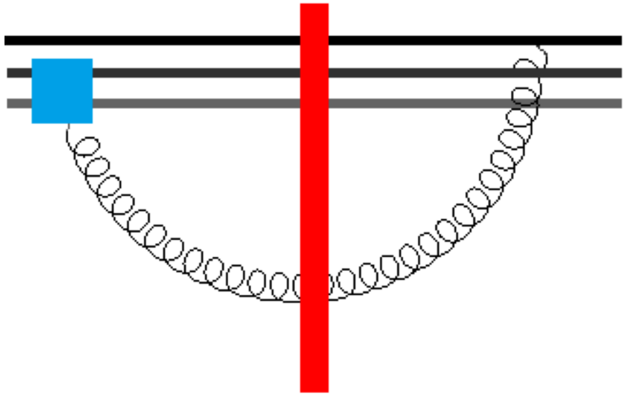
$$\mathbf{\Sigma}_{\dots} = \mathbf{\Sigma}^{\mathbf{NLO}}_{\dots}(\delta \mathsf{Y}) + \mathbf{\Sigma}^{(\delta \mathsf{Y})^{\mathbf{2}}}_{\dots}$$

$$\mathbf{\Sigma}^{(\delta \mathsf{Y})^{\mathbf{2}}} = \frac{\mathbf{1}}{\mathbf{2}} \left(\delta \mathbf{Y}\, \mathbf{H}^{\mathbf{LO}}_{\mathbf{JIMWLK}}\right)^{\mathbf{2}}$$

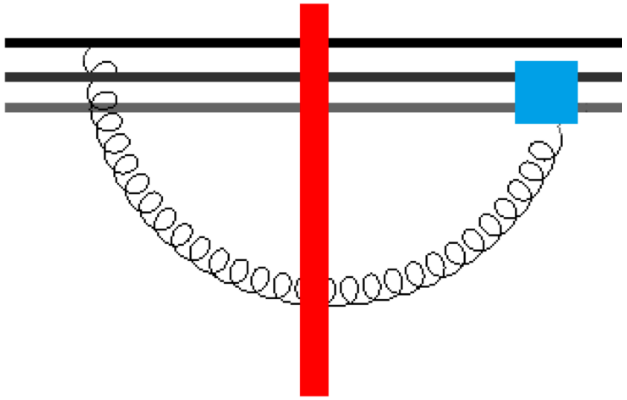
$$\mathbf{\Sigma}^{\mathbf{NLO}}(\delta \mathsf{Y}) \to \mathbf{H}^{\mathbf{NLO\,JIMWLK}}$$


Σqq
ΣJSSJ
(a)
(b)
ΣJJSSJ
ΣJJSSJ

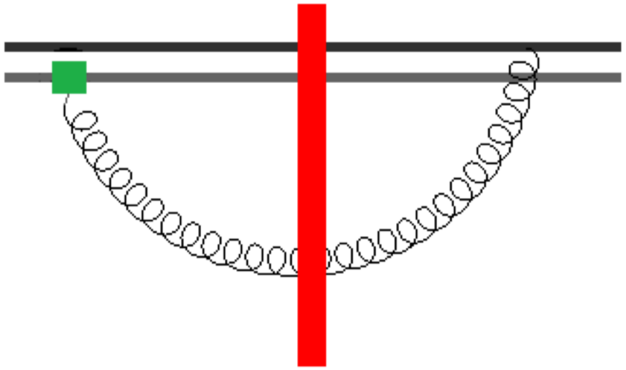
$\Sigma_{\mathbf{JJSJ}}$



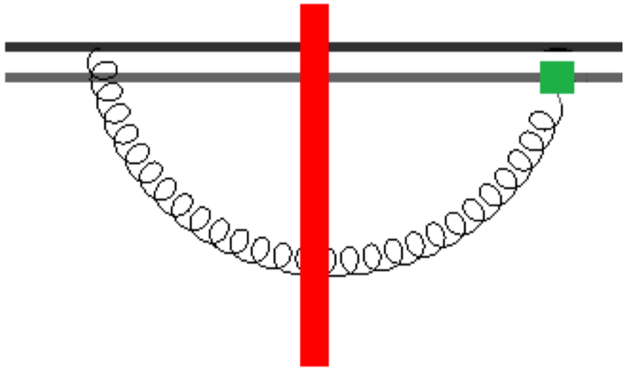
$\Sigma_{\mathbf{JJSJ}}$



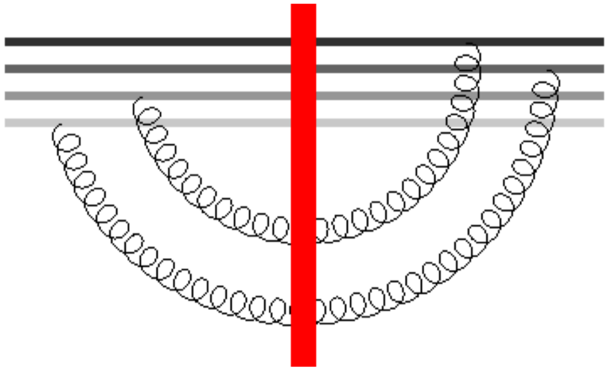
$\Sigma_{\mathbf{JSJ}}$



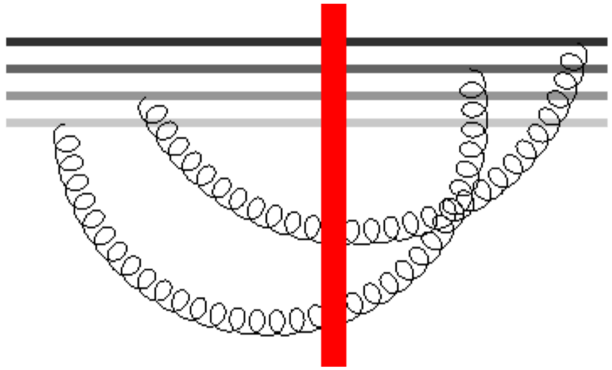
$\Sigma_{\mathbf{JSJ}}$



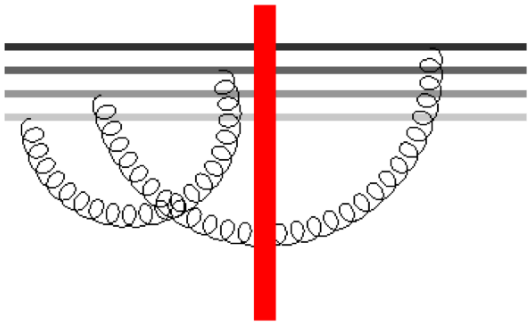
$\boldsymbol{\Sigma}_{\mathbf{JJSSJJ}}$

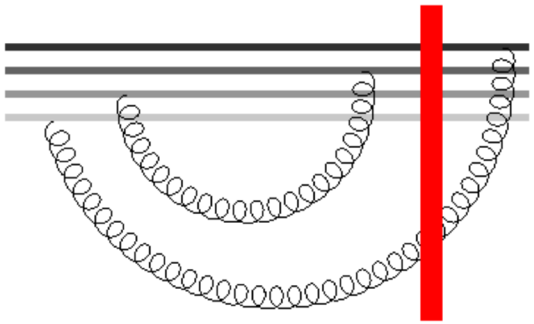


$\boldsymbol{\Sigma}_{\mathbf{JJSSJJ}}$

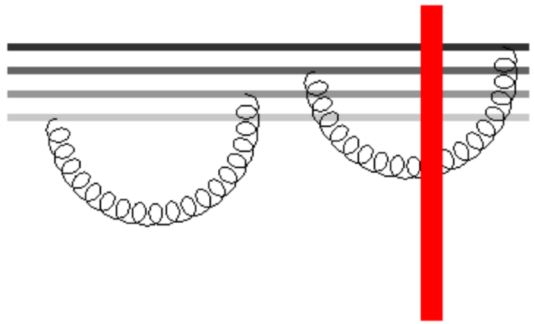


$\boldsymbol{\Sigma}_{\mathbf{JJJSJ}}$





$\boldsymbol{\Sigma}_{\mathbf{JJJSJ}}$



And also $\boldsymbol{\Sigma}_{\mathbf{virtual}}$ **;** $\quad \phi \to JJJ$

Is the JIMWLK Hamiltonian Conformally invariant?

Alex Kovner, ML and Yair Mulian (2014)

Scale invariance is trivial. Lets focus on inversion. Introduce $\mathbf{x_\pm = x_1 \pm i\, x_2}$

$$\text{Inversion transformation}: \quad x_+ \to 1/x_- \, ; \qquad\qquad x_- \to 1/x_+$$

A "naive" representation $\mathcal{I}_0$ of the inversion transformation is

$$\mathcal{I}_0 : \quad \mathbf{S(x_+, x_-) \to S(1/x_-, 1/x_+)} \; , \qquad \mathbf{J_{L,R}(x_+, x_-) \to \frac{1}{x_+ x_-} J_{L,R}(1/x_-, 1/x_+)} \; .$$

Conformal invariance (in the gauge invariant sector) @LO:

$$\mathcal{I}_0 \; \mathbf{H^{LO\,JIMWLK}} \; \mathcal{I}_0 \; = \; \mathbf{H^{LO\,JIMWLK}}$$

No (naive) Conformal invariance @NLO:

$$\mathcal{I}_0 \; \mathbf{H^{NLO\,JIMWLK}} \; \mathcal{I}_0 \; = \; \mathbf{H^{NLO\,JIMWLK}} \; + \; \mathcal{A}$$

QCD is not conformally invariant beyond tree level, but $\mathcal{N} = 4$ SUSY is.

JIMWLK Hamiltonian IS conformally invariant! (in $\mathcal{N} = 4$)

S **forms a non-trivial representation of the conformal group:**

$$\mathcal{I} : S(x) \to S(1/x) + \delta S(x)\,, \qquad \mathcal{I} : H^{LO} \to H^{LO} - \mathcal{A}$$

Here δS is of order α_s. The condition is that the net anomaly cancels:

$$\mathcal{I}\,(\mathbf{H}^{\mathbf{LO}} + \mathbf{H}^{\mathbf{NLO}})\,\mathcal{I} = \mathbf{H}^{\mathbf{LO}} + \mathbf{H}^{\mathbf{NLO}}$$

We have constructed $\mathcal{I}$ perturbatively: $\qquad \mathcal{I} = (\mathbf{1} + \mathcal{C})\,\mathcal{I}_{\mathbf{0}}\,.$

$$\mathcal{C} = -\frac{\mathbf{1}}{\mathbf{2}}\frac{\alpha_{\mathrm{s}}}{\mathbf{2}\pi^{\mathbf{2}}}\int_{\mathrm{x,y,z}} \ln\left[\frac{(\mathbf{x}-\mathbf{y})^{\mathbf{2}}\mathbf{a}^{\mathbf{2}}}{(\mathbf{x}-\mathbf{z})^{\mathbf{2}}(\mathbf{y}-\mathbf{z})^{\mathbf{2}}}\right]\times$$

$$\times\frac{(\mathbf{x}-\mathbf{y})^{\mathbf{2}}}{(\mathbf{x}-\mathbf{z})^{\mathbf{2}}(\mathbf{y}-\mathbf{z})^{\mathbf{2}}}\left\{\mathbf{J}^{\mathrm{a}}_{\mathrm{L}}(\mathbf{x})\mathbf{J}^{\mathrm{a}}_{\mathrm{L}}(\mathbf{y}) + \mathbf{J}^{\mathrm{a}}_{\mathrm{R}}(\mathbf{x})\mathbf{J}^{\mathrm{a}}_{\mathrm{R}}(\mathbf{y}) - \mathbf{2}\mathbf{J}^{\mathrm{a}}_{\mathrm{L}}(\mathbf{x})\mathbf{S}^{\mathrm{ab}}_{\mathrm{A}}(\mathbf{z})\mathbf{J}^{\mathrm{b}}_{\mathrm{R}}(\mathbf{y})\right\}$$

For an arbitrary operator $\mathcal{O}$ (s, B, H^{JIMWLK},...) we define its conformal extension:

$$\mathcal{O}^{conf} = \mathcal{O} + \frac{1}{2}[\mathcal{C},\,\mathcal{O}]\,; \qquad [s^{conf} \text{ by Balitsky and Chirilli (arXiv : 0903.5326)}]$$

CONCLUSIONS

- We have computed the LCWF@NLO. Apart of leading to JIMWLK equation, it can be used to compute semi-inclusive observables @NLO.

- We have first constructed and then independently derived the JIMWLK Hamiltonian at NLO. It fully reproduces and generalises (all?) previously known low x evolution equations at NLO, including Balitsky's hierarchy at NLO

- We have proven the conformal invariance of the NLO JIMWLK Hamiltonian (in $\mathcal{N} = 4$). For any operator, we can construct its perturbative extension, such that the resulting operator evolves with conformal kernels.

- Once expanded in the dilute limit, the NLO JIMWLK makes it possible to study evolution of any multi-gluon BKP state and transition vertices at NLO.

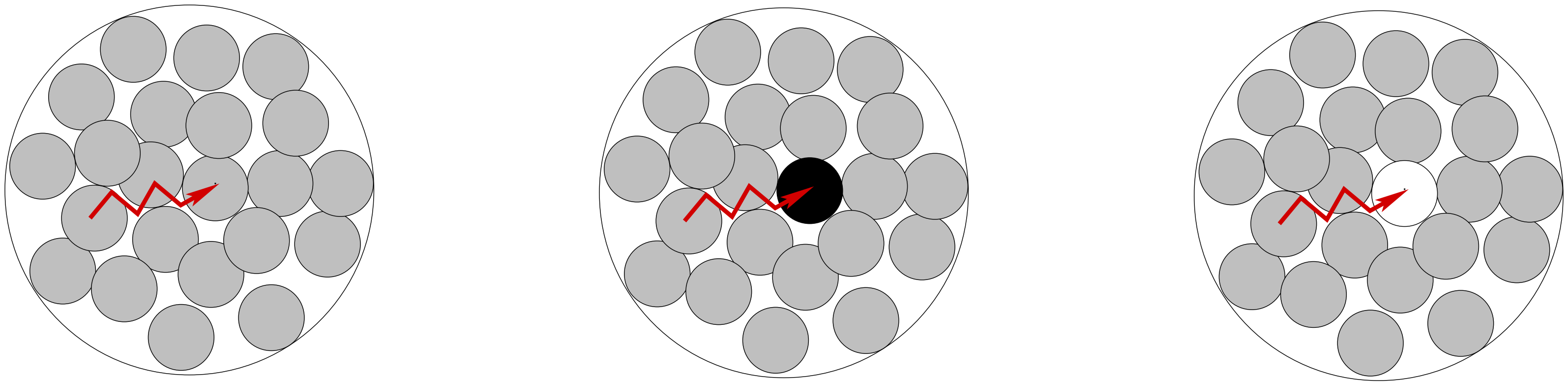
Dense limit: Inside Color Glass Condensate (CGC)

Dense regime:

(1) Hadron is almost black

(2) Emission probability is independent of density

(3) "Bleaching of color"



Random walk $\rho \sim \sqrt{\mathbf{Y}}$

LCWF – dense limit (CGC)

In the dense regime: $\boldsymbol{\Omega}(\rho \sim \mathbf{1}/\alpha_s) = \mathbf{C}\,\mathbf{B}$ $\mathbf{B}$ is a Bogolyubov operator

$$\mathbf{B} = \exp[\,\boldsymbol{\Lambda}(\rho)\,(\mathbf{a}^{\mathbf{2}} + \mathbf{a}^{\dagger\,\mathbf{2}}) + \cdots]$$

Altinoluk, Kovner, ML, Peressutti, Wiedemann (2007-2009)

Includes non-linear gluon emission effects or saturation effects in the projectile (projectile facing Pomeron fan diagrams)

This is a LCWF of a state known as CGC, and $\mathbf{1}/\boldsymbol{\Lambda}$ is the value of the Condensate