3-parton production in DIS at small x

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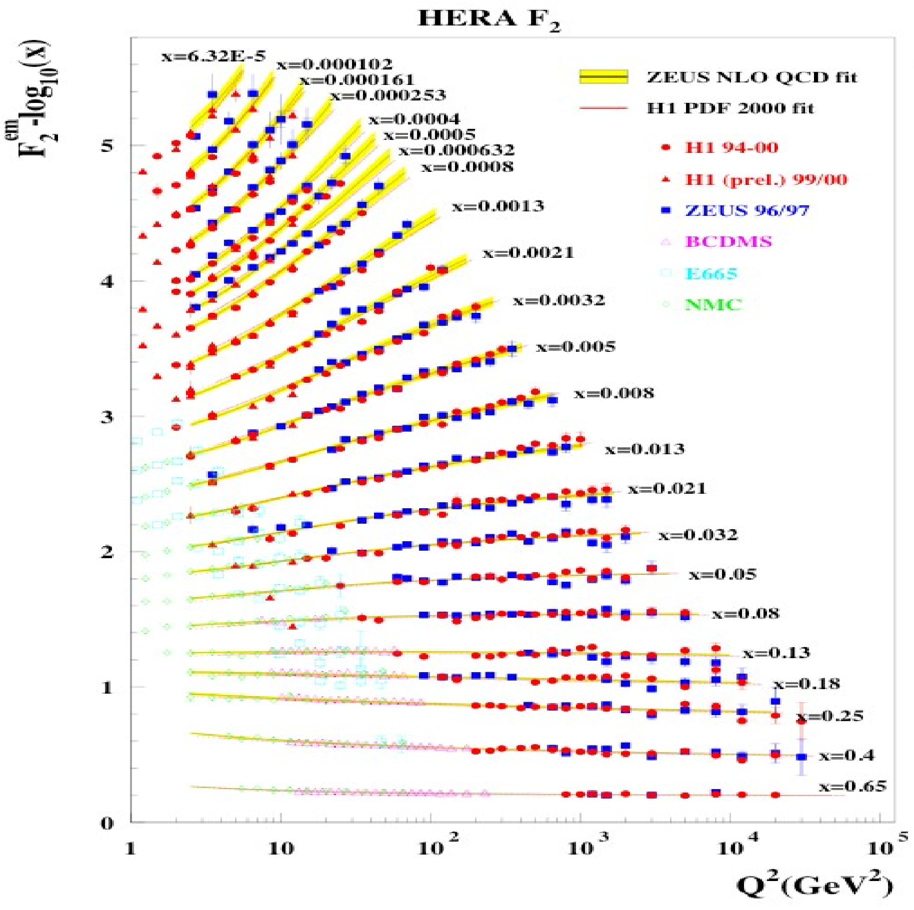
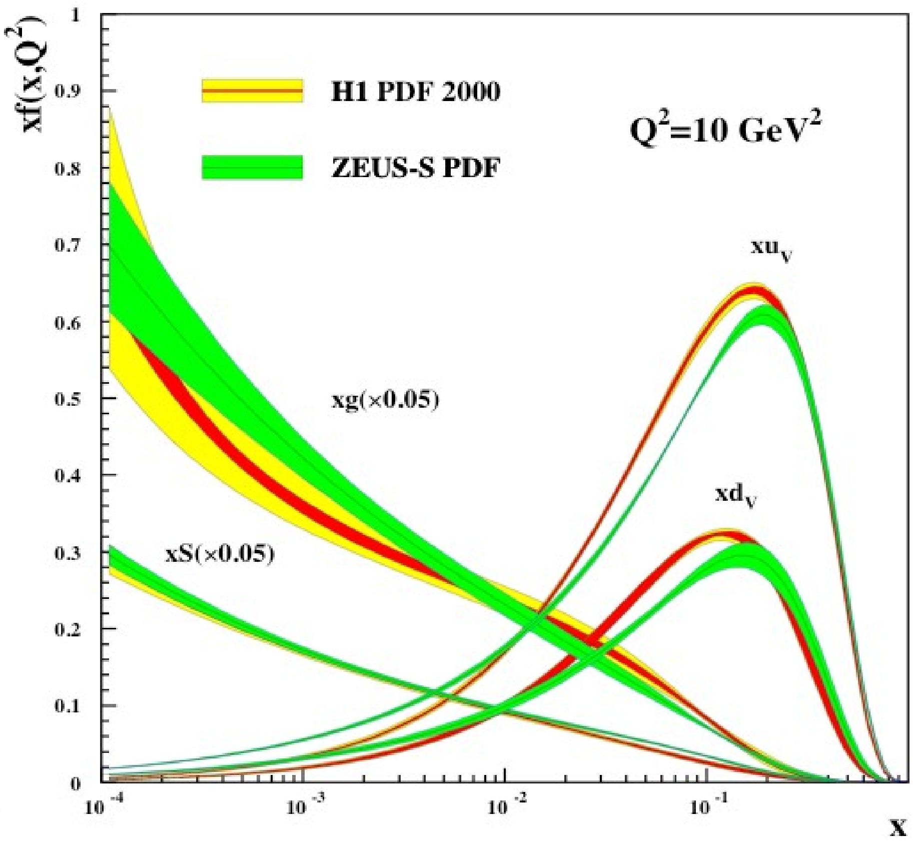
QCD structure functions

parton distributions

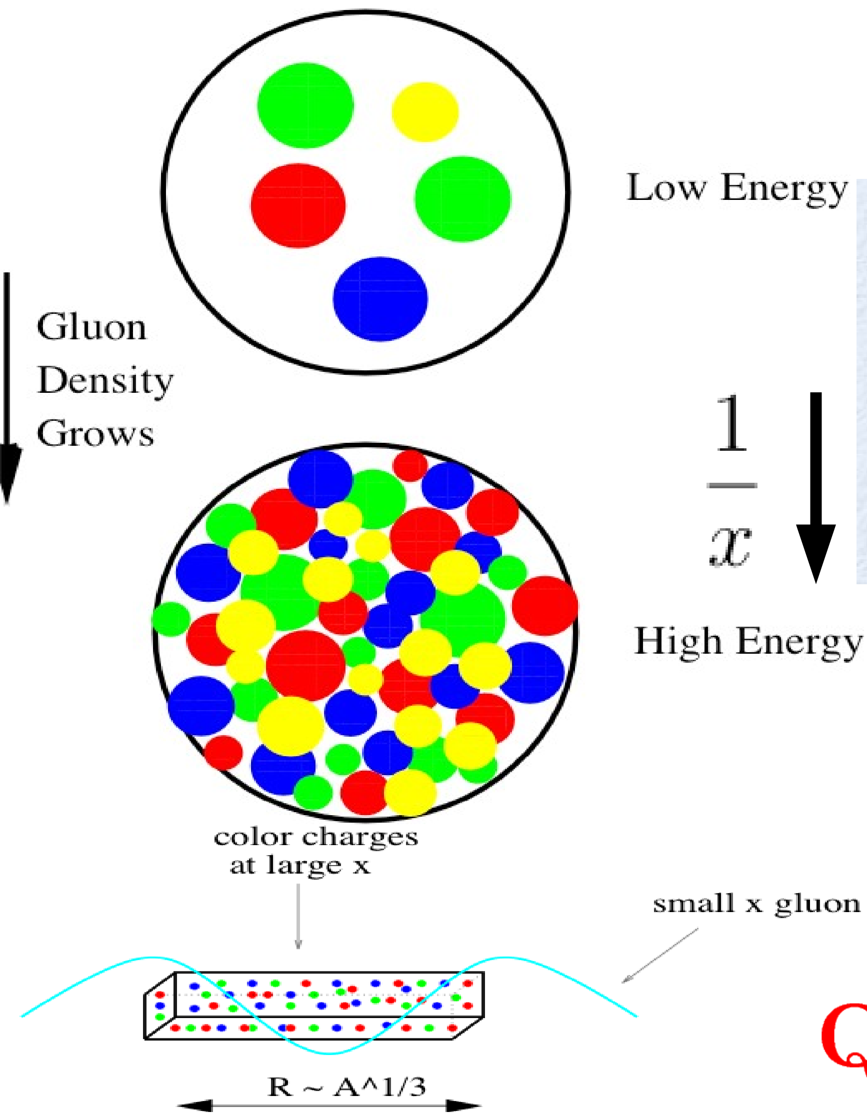




<u>*growth due to small x gluon radiation:*</u> $\mathbf{P(x) \sim 1/x}$

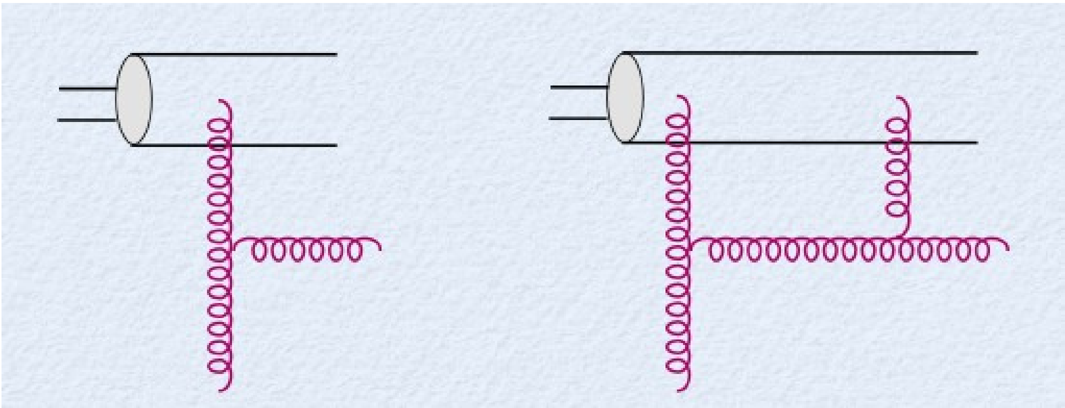
Gluon saturation

Gribov-Levin-Ryskin
Mueller-Qiu



$\frac{1}{x}$

“attractive” bremsstrahlung vs. “repulsive” recombination



$$\frac{\alpha_s}{Q^2} \frac{xG(x, b_t, Q^2)}{S_\perp} \sim 1$$

$$Q_s^2(x, b_t, A) \sim A^{1/3} \left(\frac{1}{x}\right)^{0.3}$$

Signatures:

<u>two main effects</u>

multiple scatterings

evolution with x (rapidity)

Probes:

dense-dense (AA, pA, pp) collisions

dilute-dense (pA, forward pp) collisions

spin asymmetries

DIS

structure functions ***Golec-Biernat-Wusthoff***, *PRD59 (1998) 014017*

NLO di-hadron correlations

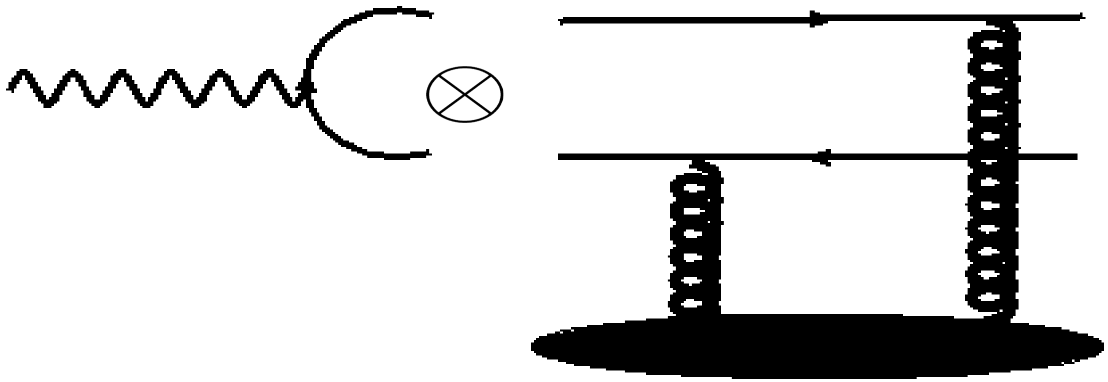
<u>3-hadron/jet azimuthal angular correlations</u>

DIS total cross section (F_2, F_L)

$$\sigma^{\rm total}_{\rm DIS} = 2\int_0^1 dz \int d^2x_t\, d^2y_t \, \left|\Psi(k^{\pm}, k_t | z, x_t, y_t)\right|^2 T(x_t, y_t)$$

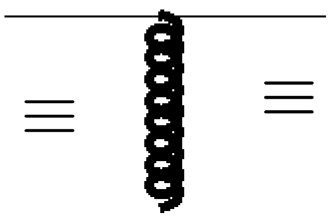
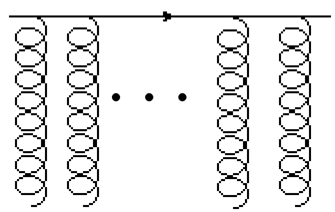
***dipole** cross section*

$$T(x_t, y_t) \equiv \frac{1}{N_c} {\rm Tr} \left\langle 1 - V(x_t) V^{\dagger}(y_t) \right\rangle$$



Golec-Biernat-Wusthoff

$$T \sim \left[1 - e^{-r_t^2/4R_0^2(x)}\right]$$

$V(x_t) \equiv$  $\equiv$  $\sim 1 + O(g\,A) + O(g^2\,A^2)$

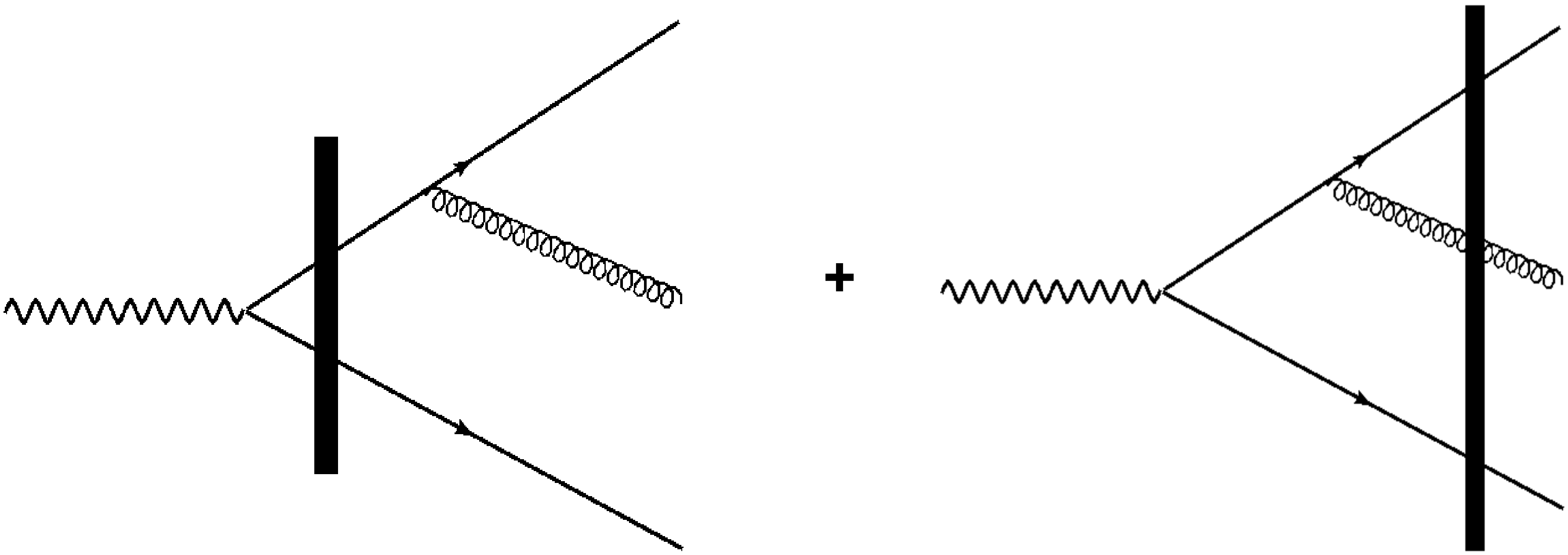
multiple scatterings *encoded in Wilson line V*

***energy (rapidity or x) dependence** via JIMWLK evolution of correlators of V's*

something with more discriminating power

angular correlations in 3-parton production in DIS

$$\gamma^\star \mathbf{T} \to \mathbf{q}\,\bar{\mathbf{q}}\,\mathbf{g}\,\mathbf{X}$$

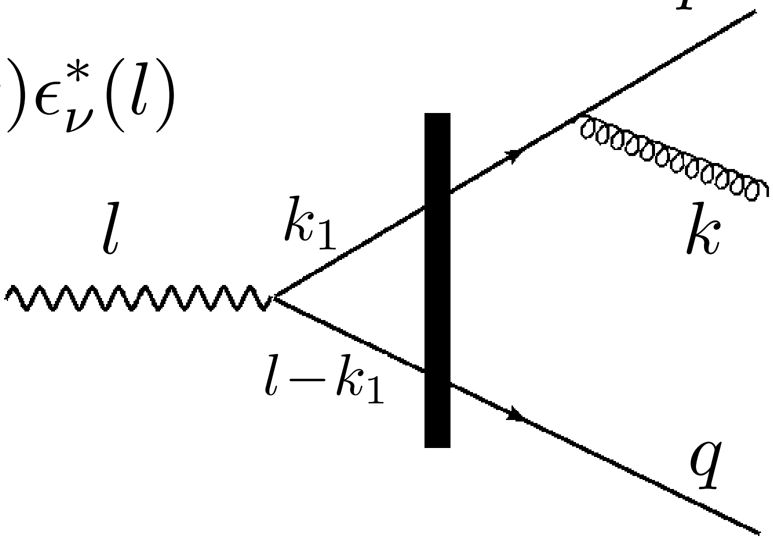


+ radiation from anti-quark

Ayala, Hentschinski , Jalilian-Marian, Tejeda-Yeomans; PLB761 (2016) 229

1^{st} diagram

$$\mathcal{A} \equiv -eg\,\bar{u}(p)\,[A]^{\mu\nu}\,v(q)\,\epsilon_\mu\,(k)\epsilon^*_\nu(l)$$



$$A_1^{\mu\nu} = \gamma^\mu\,t^a\,S_F^0(p+k)\,\tau_F(p+k,k_1)\,S_F^0(k_1)\,\gamma^\nu\,S_F^0(l-k_1)\,\tau_F(l-k_1,q)\,\frac{d^4k_1}{(2\pi)^4}$$

$$= \frac{i}{2\,l^-}\frac{\delta(l^- - p^- - q^- - k^-)}{(p+k)^2}\int d^2x_t\,d^2y_t\,e^{-i(p_t+k_t)\cdot x_t}e^{-iq_t\cdot y_t}$$

$$\gamma^\mu\,t^a\,i(\not{p}+\not{k})\gamma^-\,i\not{k}_1\,\gamma^\nu\,i(\not{l}-\not{k}_1)\,\gamma^-\,K_0\,[L\,(x_t-y_t)]$$

$$V(x_t)\,V^\dagger(y_t)$$

with

$$L^2 = \frac{q^-(p^-+k^-)}{l^-\,l^-}Q^2 \qquad k_1^- = p^- - k^- \qquad k_1^+ = \frac{k_{1t}^2 - i\epsilon}{2(p^-+k^-)} \qquad k_{1t} = -i\,\partial_{x_t-y_t}$$

spinor helicity methods

Review:
L. Dixon, hep-ph/9601359

massless quarks: helicity eigenstates

$$u_\pm(k) \equiv \frac{1}{2}(1 \pm \gamma_5)\, u(k) \qquad \overline{u_\pm(k)} \equiv \overline{u(k)}\, \frac{1}{2}(1 \mp \gamma_5)$$

$$v_\mp(k) \equiv \frac{1}{2}(1 \pm \gamma_5)\, v(k) \qquad \overline{v_\mp(k)} \equiv \overline{v(k)}\, \frac{1}{2}(1 \mp \gamma_5)$$

helicity operator $h \equiv \vec{\Sigma} \cdot \hat{p}$

$$\begin{aligned} \vec{\Sigma} \cdot \hat{p}\, u_\pm(p) &= \pm u_\pm(p) \\ -\vec{\Sigma} \cdot \hat{p}\, v_\pm(p) &= \pm v_\pm(p) \end{aligned}$$

$$u_+(k) = v_-(k) = \frac{1}{\sqrt{2}} \begin{bmatrix} \sqrt{k^+} \\ \sqrt{k^-} e^{i\phi_k} \\ \sqrt{k^+} \\ \sqrt{k^-} e^{i\phi_k} \end{bmatrix} \qquad u_-(k) = v_+(k) = \frac{1}{\sqrt{2}} \begin{bmatrix} \sqrt{k^-} e^{-i\phi_k} \\ -\sqrt{k^+} \\ -\sqrt{k^-} e^{-i\phi_k} \\ \sqrt{k^+} \end{bmatrix}$$

with $e^{\pm i\phi_k} \equiv \dfrac{k_x \pm i k_y}{\sqrt{k^+ k^-}}$ and $k^\pm = \dfrac{E \pm k_z}{\sqrt{2}}$

spinor helicity methods

notation:

$$|i^{\pm}>\equiv |k_i^{\pm}>\equiv u_{\pm}(k_i) = v_{\mp}(k_i) \qquad <i^{\pm}|\equiv <k_i^{\pm}|\equiv \overline{u}_{\pm}(k_i) = \overline{v}_{\mp}(k_i)$$

basic spinor products:

$$<i\,j> \;\equiv\; <i^-|j^+> = \overline{u}_-(k_i)\,u_+(k_j) \;=\; \sqrt{|s_{ij}|}e^{i\phi_{ij}} \qquad \cos\phi_{ij} = \frac{k_i^x k_j^+ - k_j^x k_i^+}{\sqrt{|s_{ij}|k_i^+ k_j^+}}$$

$$[i\,j] \;\equiv\; <i^+|j^-> = \overline{u}_+(k_i)\,u_-(k_j) \;=\; -\sqrt{|s_{ij}|}e^{-i\phi_{ij}} \qquad \sin\phi_{ij} = \frac{k_i^y k_j^+ - k_j^y k_i^+}{\sqrt{|s_{ij}|k_i^+ k_j^+}}$$

with

$$\begin{aligned} s_{ij} &= (k_i + k_j)^2 = 2k_i\cdot k_j \\ &= -<ij>[ij] \end{aligned}$$

and

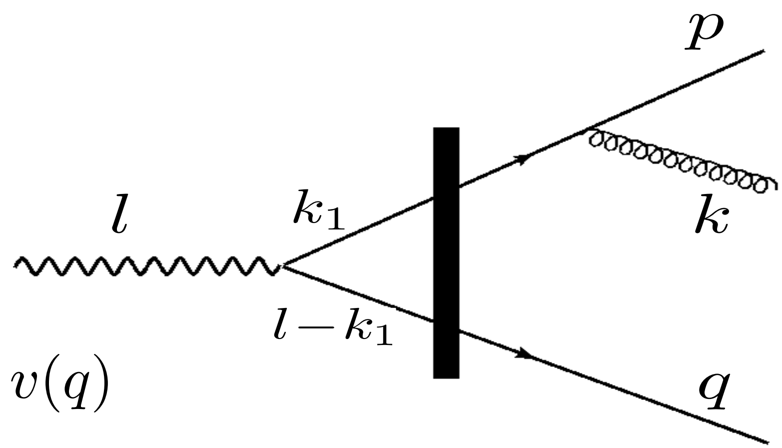
$$\begin{aligned} <ii> &= [ii] = 0 \\ <ij] &= [ij> = 0 \end{aligned}$$

any off-shell momentum $k^\mu \equiv \bar{k}^\mu + \frac{k^2}{2k^+}n^\mu$ where $\bar{k}^\mu$ is on-shell $\bar{k}^2 = 0$

any on-shell momentum $\not{p} = |p^+><p^+| + |p^-><p^-|$

work with a given helicity state

Diagram A1



Numerator: Dirac Algebra

$$a_1 \equiv \overline{u}(p)\, \not{\epsilon}^\star(k)\, (\not{p}+\not{k})\, \not{n}\, \not{k}_1\, \not{\epsilon}(l)\, (\not{k}_1 - \not{l})\, \not{n}\, v(q)$$

longitudinal photons

quark anti-quark gluon helicity: + - +

$$\not{l} = l^+ \not{\overline{n}} - \frac{Q^2}{2l^+}\not{n}$$

$$a_1^{L;+-+} \;=\; -\frac{\sqrt{2}}{[n\,k]}\frac{Q}{l^+}\,[n\,p] < k\,p > [n\,p] < n\,\overline{k}_1 > \left[n\,\overline{k}_1\right] < n\,q >$$

$$\left(< n\,\overline{k}_1 > \left[n\,\overline{k}_1\right] - l^+ < n\,\overline{n} > [n\,\overline{n}]\right)$$

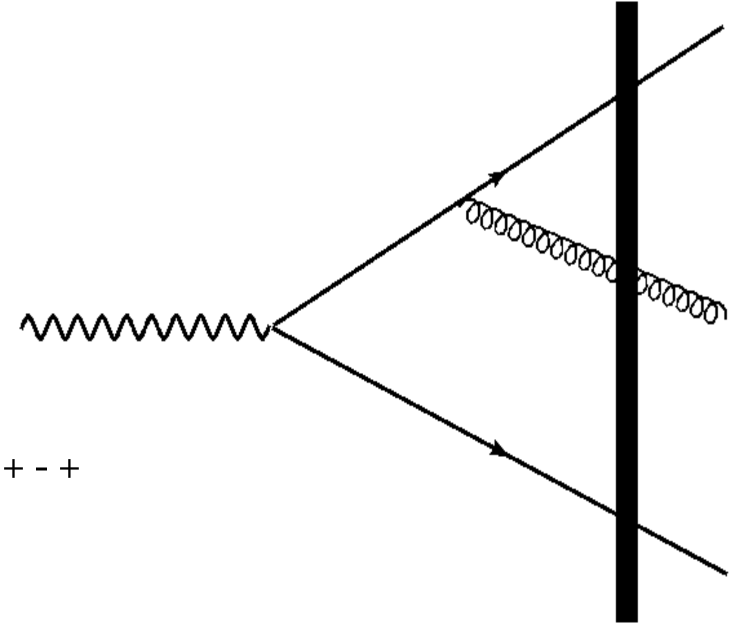
with

$$< np >= -[np] = \sqrt{2p^+}$$

transverse photons: +

$$a_1^{\perp=+;+-+} = -\frac{\sqrt{2}}{[nk]}[pn] < kp > [pn] < nk_1 > [k_1 n] < \bar{n}k_1 > [k_1 n] < nq >$$

Diagram A3



Numerator: Dirac Algebra

longitudinal photons

quark anti-quark gluon helicity: + - +

$$a_3^{L;+-+} = \frac{\sqrt{2}Q}{l^+\,[n\bar{k}_2]}[pn]\left(< n\bar{k}_1 > [\bar{k}_1 n] - < n\bar{k}_2 > [\bar{k}_2 n]\right) < \bar{k}_2\bar{k}_1 > [\bar{k}_1 n]$$

$$\left(< n\bar{k}_1 > [\bar{k}_1 n] - l^+ < n\bar{n} > [\bar{n}n]\right) < nq >$$

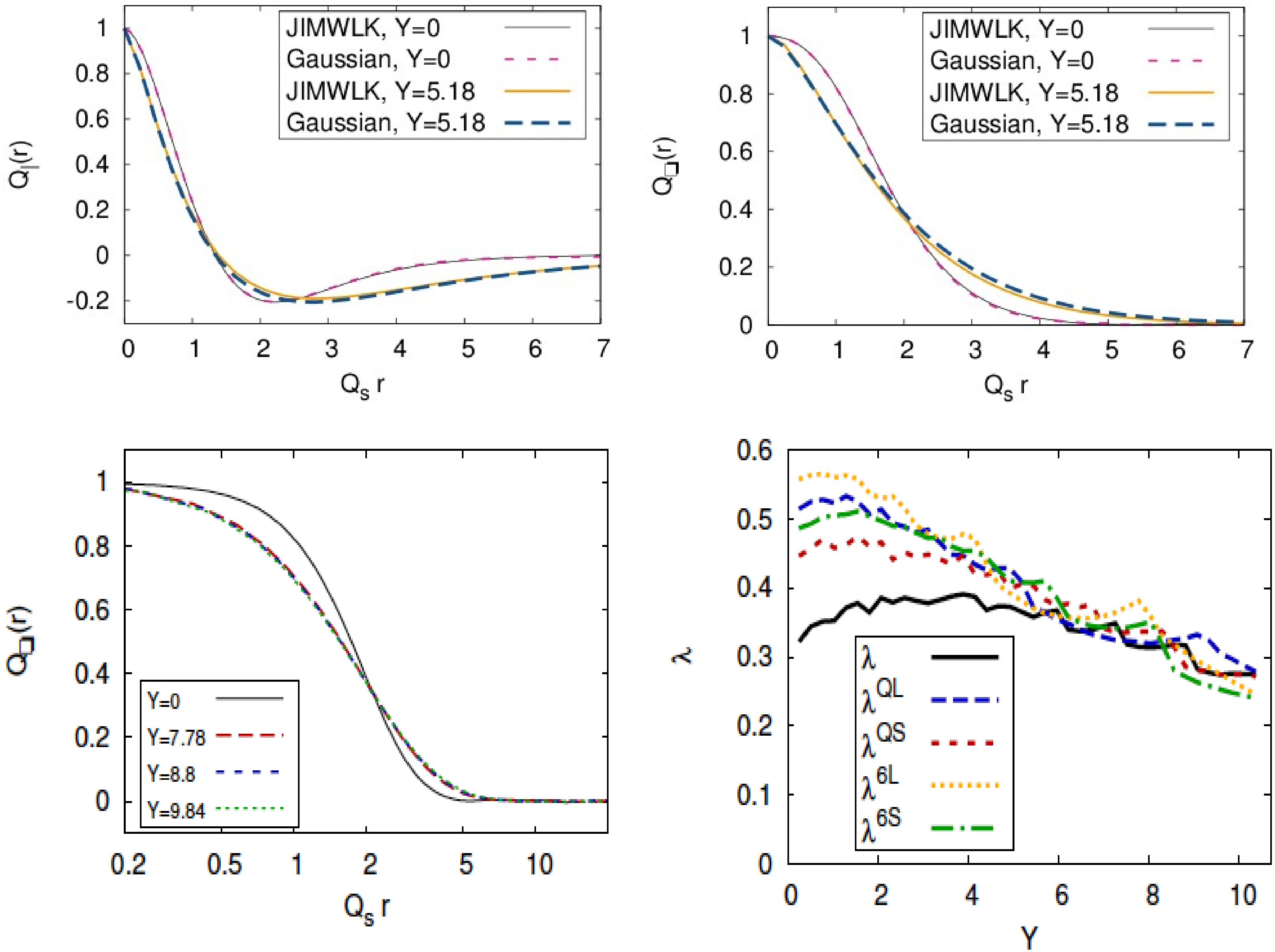
$$= -2^4 Q(l^+)^2 \frac{(z_1 z_2)^{3/2}}{z_3}\left[z_3\, k_{1\,t}\cdot\epsilon - (z_1+z_3)k_{2\,t}\cdot\epsilon\right]$$

the rest is some standard integrals

add up the amplitudes, square.., still need to deal with products of Wilson lines: Quadrupoles

Quadrupole: $< Q(r,\bar{r},\bar{s},s) > \equiv \frac{1}{N_c} < Tr\, V(r)\, V^{\dagger}(\bar{r})\, V(\bar{s})\, V^{\dagger}(s) >$

Dumitru-Jalilian-Marian-Lappi-Schenke-Venugopalan:PLB706 (2011) 219



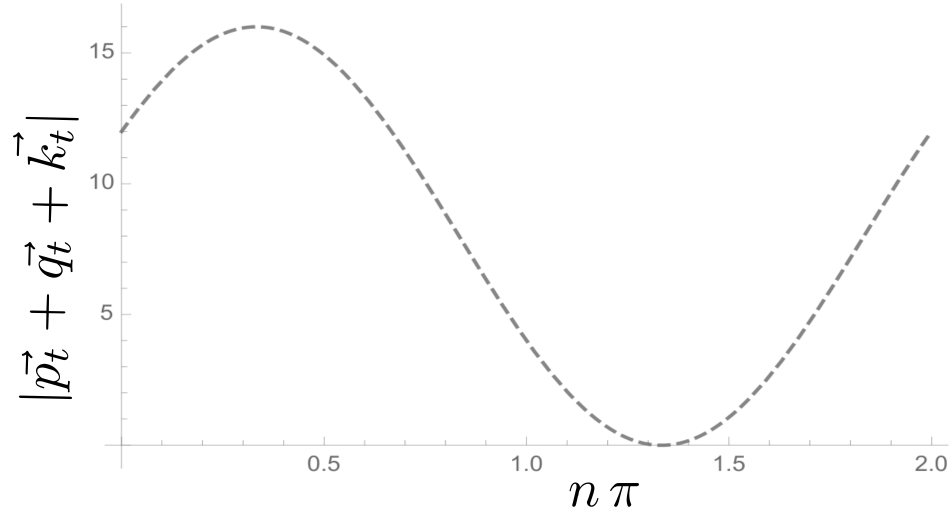
3-parton production: kinematics

<u>linear regime</u>: <u>*use ugd's*</u>

$z_1 = z_2 = 0.2, z_3 = 0.6$

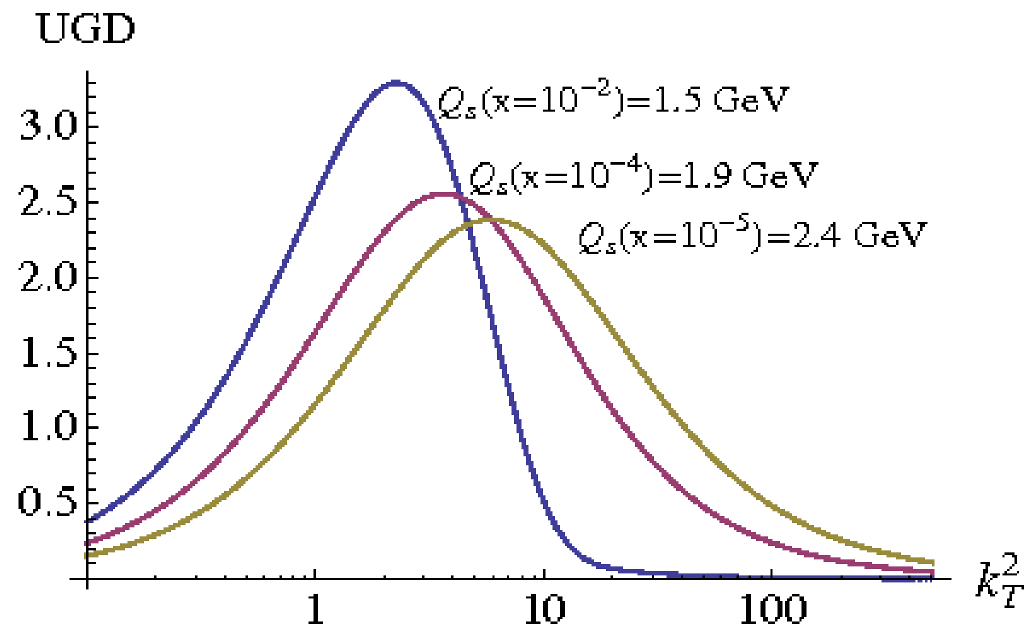
$p_t = q_t = k_t = 4\,GeV$

$Q^2 = 16\,GeV^2$



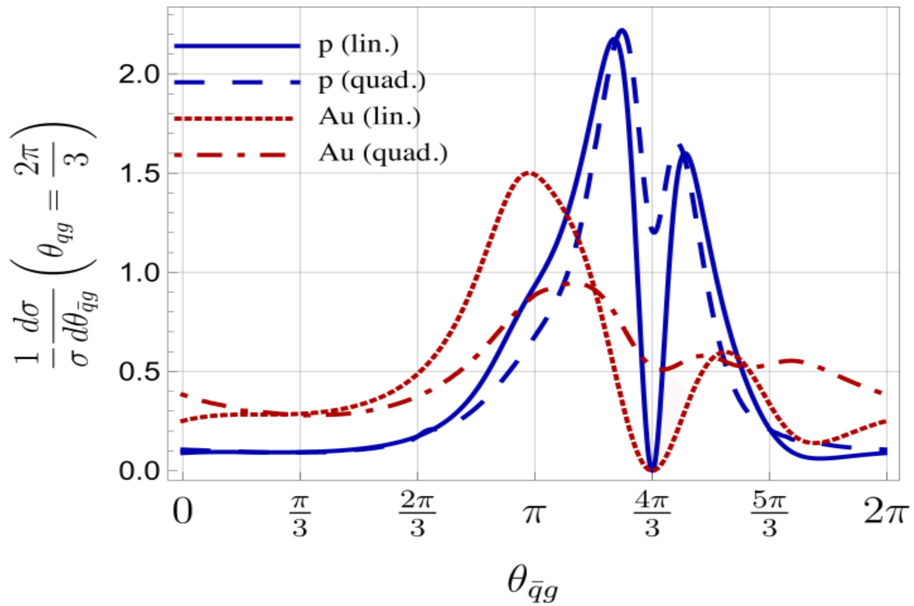
$\Delta\phi_{12} = \frac{2\pi}{3}$ vary $\Delta\phi_{13}$

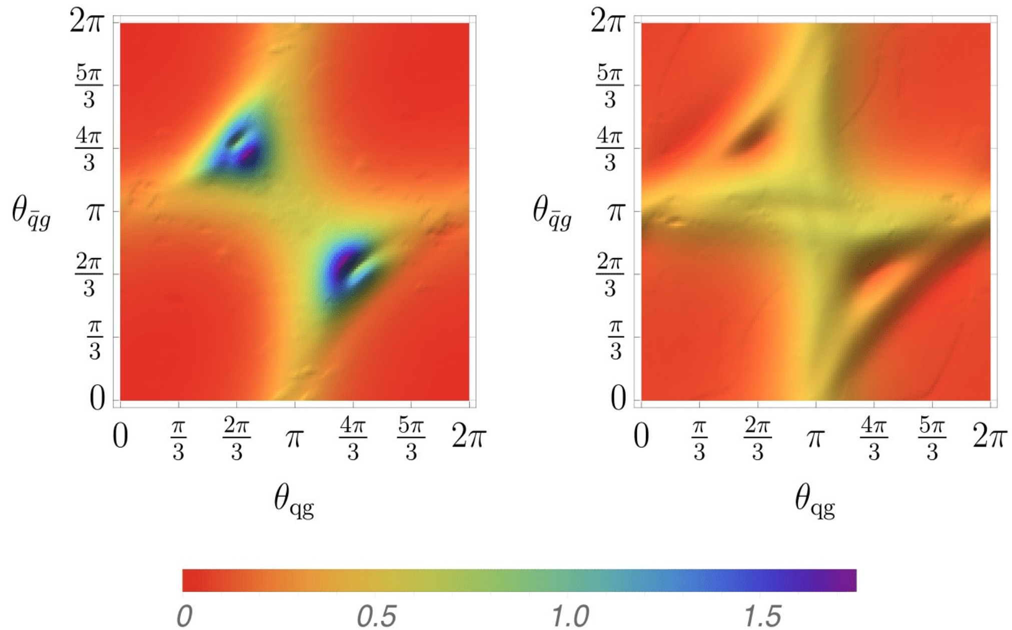
$k_t^2\,\tilde{T}(k_t)$



broadening and disappearance of the peak

3-parton azimuthal angular correlations





Possible extensions to other processes?

real photons: $Q^2 \to 0$

ultra-peripheral nucleus-nucleus collisions

crossing symmetry:

$$\gamma^{(\star)}\, T \longrightarrow q\,\bar{q}\,g\,X \quad \Longleftrightarrow \quad \left\{ \begin{array}{l} q\,T \longrightarrow q\,g\,\gamma^{(\star)}\,X \\ \bar{q}\,T \longrightarrow \bar{q}\,g\,\gamma^{(\star)}\,X \\ g\,T \longrightarrow q\,\bar{q}\,\gamma^{(\star)}\,X \end{array} \right\}$$

proton-nucleus collisions (collinear factorization in proton?)

$$pA \longrightarrow h_1\,h_2\,\gamma^{(\star)}\,X$$

MPI (double/triple parton scattering)

$$\gamma^{(\star)}\, T \longrightarrow q\,\bar{q}\,g\,X \quad \Longleftrightarrow \quad \left\{ \begin{array}{l} q\,\bar{q}\,T \longrightarrow g\,\gamma^{(\star)}\,X \\ g\,\bar{q}\,T \longrightarrow \bar{q}\,\gamma^{(\star)}\,X \\ g\,q\,T \longrightarrow q\,\gamma^{(\star)}\,X \end{array} \right\}$$

SUMMARY

CGC is a systematic approach to high energy collisions

novel aspects of QCD

Leading Log CGC works (too) well

the first phenomenological application by Golec-Biernat, Wusthoff

it has been used to fit a wealth of data; ep, eA, pp, pA, AA

Precision (NLO) studies of less-inclusive observables are needed

Azimuthal angular correlations offer a unique probe of CGC

3-hadron/jet correlations should be even more powerful

DIS is the ideal ground for precision CGC studies