Evidence of higher twist contributions in inclusive DIS

Workshop on QCD and Diffraction – Sat 1000+

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Outline:

- Higher twists in proton structure: motivation
- Higher twists at high energy: QCD picture
- Higher twists corrections to DIS from GBW
- Breakdown of DGLAP in DIS and interpretation in terms of higher twists
- Combined DGLAP + GBW inspired HT fits to HERA data
- Conclusions

Based on analysis done by

M. Sadzikowski, W. Słomiński, K. Wichmann and LM

Entanglement / complementarity / problem with linearization

- The higher twist problem at small x is complicated theoretically and experimentally: parallel and entangled efforts are necessary to find meaningful results
- Theory efforts pioneered by Furmanski and Petronzio, Petersburg Gribov School, Bartels, Kwieciński, Praszłowicz, Golec-Biernat and Wusthoff (more than 30 years of theoretical physics!)
- Only recently beautiful analysis of final HERA data provided a strong experimental input
- Fruitful dialogue of theory and experiment → 3 following talks on the topic of HERA data beyond DGLAP from different perspectives. Talk by K. Wichmann: based on published results, this analysis is completed and will be out soon

General motivation for higher twist investigation program

- Standard QCD descriptions based on leading-twist DGLAP is very successful and precise
- However, theory of twist-related issue of multiple scattering is not yet satisfactory and higher twist corrections to DGLAP are unknown
- Good understanding of higher twists →

  – broadening of QCD applicability

  – better precision, qualitative determination of DGLAP limitations

  – better determination of parton densities

  – novel observables in proton structure

Deeply Inelastic Scattering: how?

- Unpolarised structure functions
  F_1, F_2 or F_2, F_L

$$\frac{d\sigma}{d\Omega dE'} = \frac{\alpha_{\rm em}}{Q^4} L_{\mu\nu} W^{\mu\nu}(p,q)$$

$$W^{\mu\nu} = -F_1 g^{\mu\nu} + F_2 \frac{p^\mu p^\nu}{\nu}$$

- OPE: product of local operators in separated points

$$W^{\mu\nu} = \sum_\tau \left(\frac{\Lambda}{Q}\right)^{\tau-2} \sum_i C^{\mu\nu}_{\tau,i} \otimes f_{\tau,i}(Q^2/\Lambda^2)$$

- Twist = dimension – spin: gives the Q dependence
- Leading twist = 2: DGLAP evolution (high precision)

$$\frac{\partial f_i(Q^2)}{\partial \log(Q^2)} = \alpha_s(Q^2) P_{ji} \otimes f_j(Q^2)$$

- \`Easy', efficient but... limited at moderate Q^2

Twists at small x in a nutshell (1)

- Higher twists effects: power suppressed by hard scale:

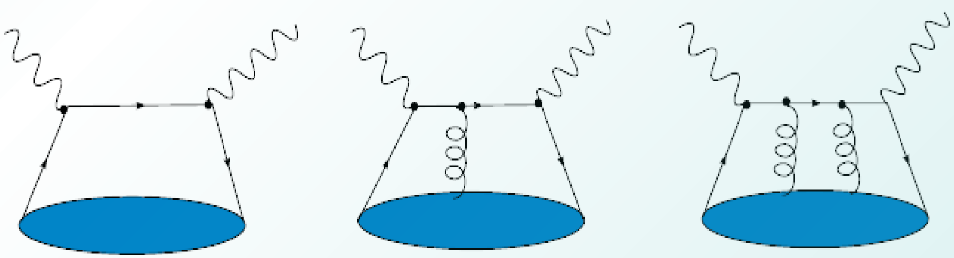
$$W^{\mu\nu} = \sum_\tau \left(\frac{\Lambda}{Q}\right)^{\tau-2} \sum_i C^{\mu\nu}_{\tau,i} \otimes f_{\tau,i}(Q^2/\Lambda^2)$$

- Typical operators:

$$\langle p|\bar{q}\gamma_{\{\mu_1}D_{\mu_2}\ldots D_{\mu_n\}}q|p\rangle = \langle x^n\rangle_q\, p_{\mu_1}\cdots p_{\mu_n}$$

- What is known on higher twists in proton?

Complete twist 4 analysis o $q\bar{q}gg$ evolution [Ellis, Furmanski and Petronzio, 1983]



- Understanding of twist-4 gluonic (gggg) operators – still on the way
- However – dominant contribution should come from **quasipartonic operators** *(for which: twist = number of free partons in t-channel)*

$$(\partial_. A^\perp_\alpha)^2(\partial_. A^\perp_\beta)^2,\ \bar\psi\psi\bar\psi\psi$$

Twists at low x in a nutshell (2)

- Evolution of quasi-partonic operators: n-tchannel partons + pairwise (non-forward) DGLAP interactions [Bukhvostov, Frolov, Lipatov, Kuraev, 1985]

- More rapid QCD evolution of higher twists with x

$$\frac{\text{Twist 4}}{\text{Twist 2}} \sim \frac{1}{Q^2 R^2} \exp\left(\sqrt{b \log(Q^2) \log(1/x)}\right)$$

- Significant corrections to precise parton determination, dependent on x and Q^2

- Quasi-partonic operators: relation of higher twists to multiple scattering, multiple parton densities and parton correlations

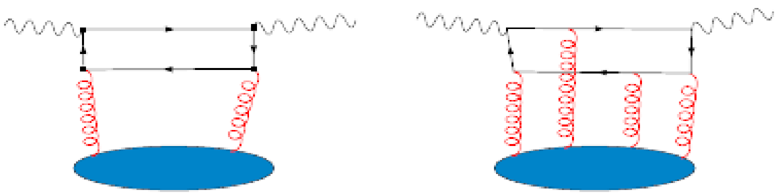
- Higher twists: expected to affect some LHC measurements that reach much lower x than HERA: important to control them

Difficulties in rigorous treatment of higher twists

- First-principle theory of higher twists: highly involved, few studies done within decades, not complete
- To provide reliable predictions: a lot of input from measurements is necessary – missing so far
- So → adopt at first a simplified picture: QCD guided model of rescattering with unitarity constraints
- Most advanced QCD studies of rescattering provided so far in the high energy limit, in kT-factorisation approach and small-x resummations (of logs(1/x))
- Efficient tool to address the problem of multiple scattering: QCD guided saturation model

QCD insight: 4-gluon evolution at twist 4

At small the dominant contribution should come from diagrams of the type:

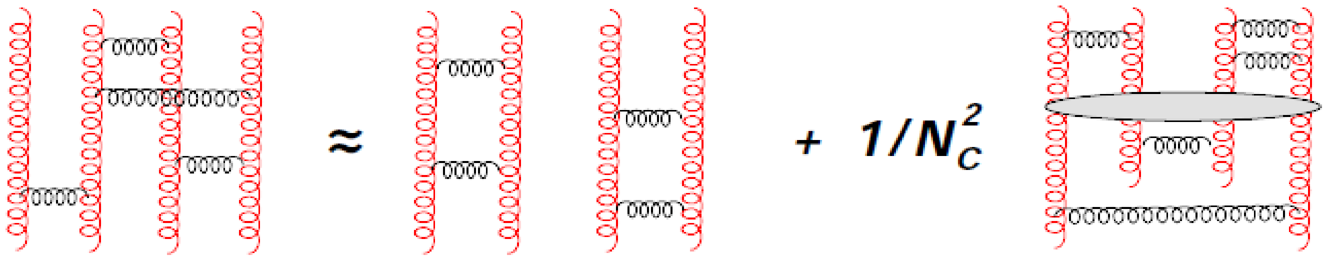


For twist-4, $N_c \to \infty$, in the leading $\alpha_s \log(Q^2) \log(1/x)$ approximation dominant singularity:

$$\gamma = \frac{4N_c\alpha}{\pi}\frac{1}{\omega}$$

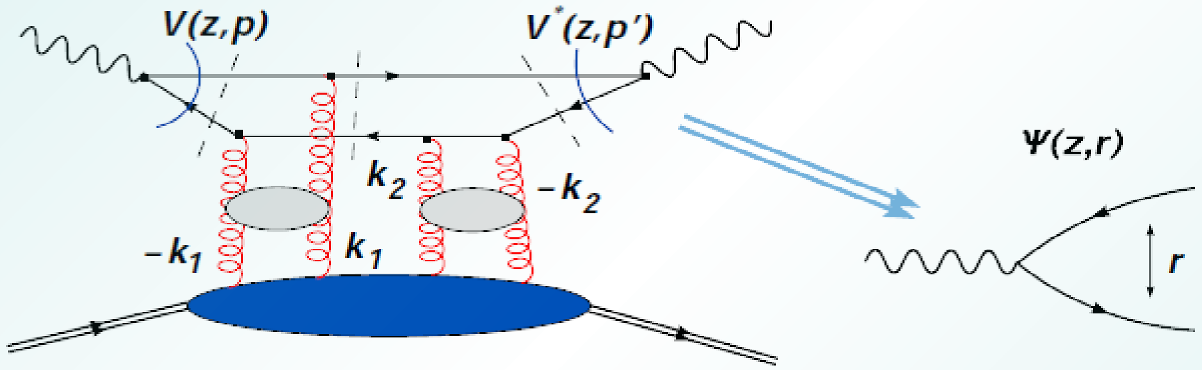
coming from two independent DGLAP evolutions

Corrections — color reconnections between ladders supressed by $\sim 1/N_c^2$ [Bartels, Ryskin, 1993]



Multiple scattering in DIS at high energies

Structure: $\Delta^{(2n)}\sigma_{\gamma^* p} \sim \int \prod_{i=1}^{2n} \frac{d^2 k_i}{k_i^4} \delta\left(\sum_i \boldsymbol{k}_i\right) \; G_{2n}^{\{a_i\}}(x, \{\boldsymbol{k}_i\}) \; \Phi_{2n}^{\{a_i\}}(\{\boldsymbol{k}_i\})$



Multi-gluon coupling in high energy limit $\longrightarrow$ photon-gluon vertex fusion governs all couplings

$$\Phi_{2n} \sim \alpha_s^n \int d^2p \int dz \sum_F \text{Color}(F) \; V^*(z, \boldsymbol{p}'(F)) \, V(z, \boldsymbol{p})$$

After projection on symmetric multiple color singlet, and the Fourier transform result is simple

$$\Phi_{2n} \sim \alpha_s^n \int d^2r \int dz \; \Psi^*(z, \boldsymbol{r}) \prod_{i=1}^{n} \left[2 - e^{i\boldsymbol{k}_i \boldsymbol{r}} - e^{-i\boldsymbol{k}_i \boldsymbol{r}}\right] \Psi(z, \boldsymbol{r})$$

Saturation model vs DGLAP

Taking factorized and symmetric form of unintegrated multi-gluon density

$$G_{2n}^{\{a_i\}}(x, \{k_i^2\}) \sim \sum_\sigma \delta^{a_{\sigma(1)}a_{\sigma(2)}} \ldots \delta^{a_{\sigma(2n-1)}a_{\sigma(2n)}} f(x, \boldsymbol{k}_{\sigma(1)}, \boldsymbol{k}_{\sigma(2)}) \ldots f(x, \boldsymbol{k}_{\sigma(2n-1)}, \boldsymbol{k}_{\sigma(2n)})$$

Invoking AGK rules one obtains the Glauber-Mueller form used by GBW

$$\Delta^{(2n)}\sigma \sim \frac{(-1)^{n+1}}{n!} R^2 \int d^2r\, dz\, |\Psi(z, \boldsymbol{r})|^2 \prod_{i=1}^{n} \underbrace{\left\{ \int \frac{d^2k_i}{k_i^4} \frac{\alpha_s f(x, \boldsymbol{k}_i^2)}{R^2} \left[2 - e^{i\boldsymbol{k}_i\boldsymbol{r}} - e^{-i\boldsymbol{k}_i\boldsymbol{r}}\right] \right\}}_{\text{single dipole scattering xs: } \sigma_1(x,r^2)/R^2}$$

In collinear limit ($k^2 \ll C/r^2$) dipole cross section coincides with DGLAP improved saturation model [Bartels, Golec-Biernat, Kowalski]

$$\sigma_1(x, r^2) \simeq \alpha_s(C/r^2) \int^{C/r^2} \frac{dk^2}{k^4} f(x, k^2)\,(k^2 r^2) \simeq r^2 \alpha_s(C/r^2)\, xg(x, C/r^2)$$

Resummed cross section:

$$\sigma_d(x, r^2) \simeq R^2 \left[1 - \exp(-\sigma_1(x, r^2)/R^2)\right]$$

Higher twist extraction from original GBW

Simple Q^2-Mellin structure of the GBW model – simple poles of γ^* impact factor * simple poles of the dipole cross-section – analytic twist decomposition of saturation [Bartels, Golec-Biernat, Peters, 2000]:

- Twist 2

$$\sigma_T^{(\tau=2)} = \frac{\alpha_{em}\sigma_0}{\pi} \langle e^2 \rangle \frac{Q_{\rm sat}^2}{Q^2} \left\{ \log(Q^2/Q_{\rm sat}^2) + \gamma_E + 1/6 \right\}$$

$$\sigma_L^{(\tau=2)} = \frac{\alpha_{em}\sigma_0}{\pi} \langle e^2 \rangle \frac{Q_{\rm sat}^2}{Q^2}$$

- Twist 4

$$\sigma_T^{(\tau=4)} = \frac{3}{5}\frac{\alpha_{em}\sigma_0}{\pi} \langle e^2 \rangle \frac{Q_{\rm sat}^4}{Q^4}$$

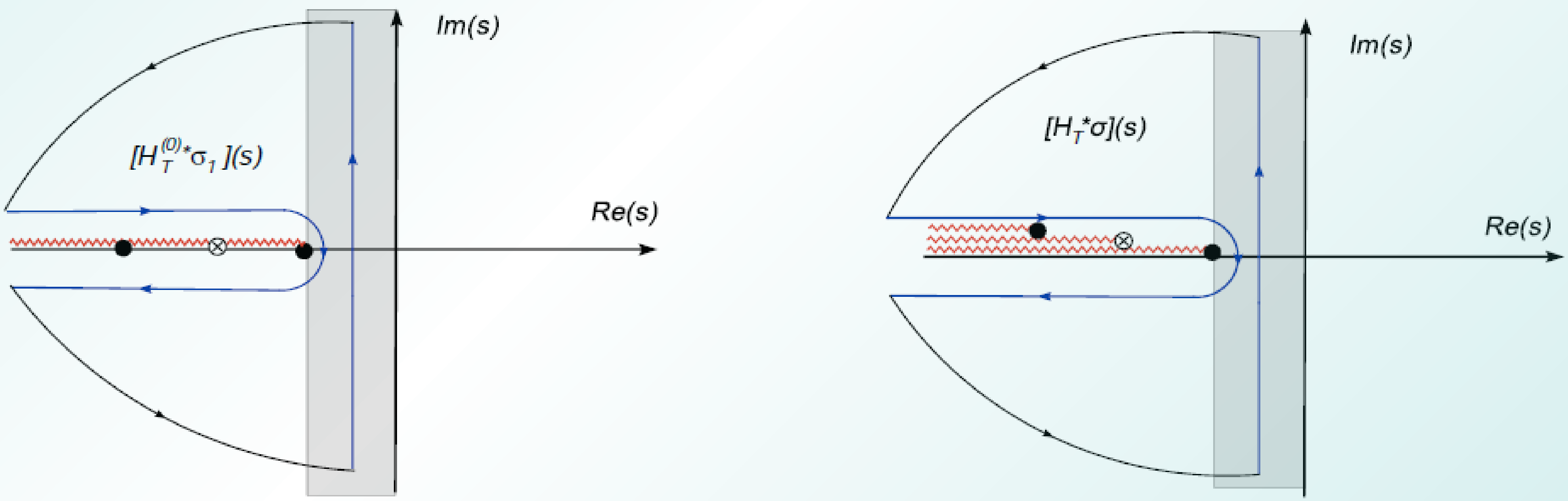
$$\sigma_L^{(\tau=4)} = -\frac{4}{5}\frac{\alpha_{em}\sigma_0}{\pi} \langle e^2 \rangle \frac{Q_{\rm sat}^4}{Q^4} \left\{ \log(Q^2/Q_{\rm sat}^2) + \gamma_E + 1/15 \right\}$$

Inclusion of QCD evolution (B-GB-K): Mellin structure of DIS [Bartels, Golec-Biernat, LM 2009]

Mellin transform of perturbative part od dipole cross section – term by term

$$\mathcal{M}[\sigma](x,s) \;=\; \mathcal{M}\left[\sum_{n=1}^{\infty}\frac{(-1)^n}{n!}\sigma_1^n\right](x,s) \;=\; \sum_{n=1}^{\infty}\mathcal{M}[\sigma_n](x,s)$$

$$\mathcal{M}_{r^2}[\sigma_1^n](x,s) \;\propto\; \mathcal{M}_{\mu^2}[(\alpha_s^n(xg)^n](x,s+n)$$



Single ladder exchanges — Multiple ladder exchange

Results in DIS: generic features [Bartels, Golec-Biernat, LM 2009]

Key information: twist 2 and twist 4 poles of the box diagram

$$\tilde{\Phi}_T(\gamma) \sim \frac{+a_T^{(2)}}{(\gamma-1)^2} \;\Rightarrow\; \sigma_T^{(2)} \sim \frac{a_T^{(2)}}{Q^2} \sum_f e_f^2 \frac{\alpha_{em}}{\pi} \int_{Q_0^2}^{Q^2} \frac{dQ'^2}{Q'^2} \alpha_s(Q'^2) x g(x, Q'^2)$$

$$\tilde{\Phi}_L(\gamma) \sim \frac{+b_L^{(2)}}{\gamma-1} \;\Rightarrow\; \sigma_L^{(2)} \sim \frac{b_L^{(2)}}{Q^2} \sum_f e_f^2 \frac{\alpha_{em}}{\pi} \alpha_s(Q^2) x g(x, Q^2)$$

$$\tilde{\Phi}_T(\gamma) \sim \frac{+b_T^{(4)}}{\gamma-2} \;\Rightarrow\; \sigma_T^{(4)} \sim \frac{+b_T^{(4)}}{Q^4} \sum_f e_f^2 \frac{\alpha_{em}}{\pi} [\alpha_s(Q^2) x g(x, Q^2)]^2$$

$$\tilde{\Phi}_L(\gamma) \sim \frac{-a_L^{(4)}}{(\gamma-2)^2} \;\Rightarrow\; \sigma_L^{(4)} \sim \frac{-a_L^{(4)}}{Q^4} \sum_f e_f^2 \frac{\alpha_{em}}{\pi} \int_{Q_0^2}^{Q^2} \frac{dQ'^2}{Q'^2} [\alpha_s(Q'^2) x g(x, Q'^2)]^2$$

F_T: twist-2: $\alpha_s\, x^{-\lambda} \log(Q^2)/Q^2$ — large, twist-4 : $\alpha_s^2\, x^{-2\lambda}/Q^4$ — suppressed correction

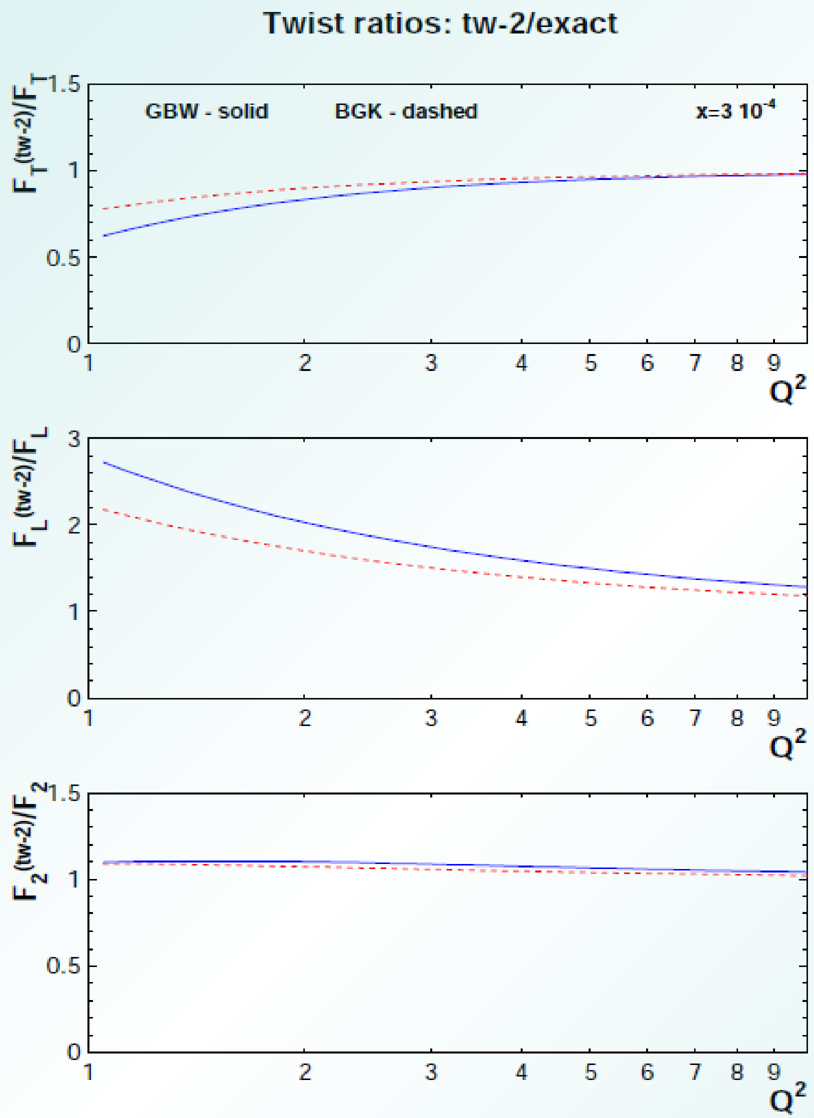
F_L: twist-2: $\alpha_s\, x^{-\lambda}/Q^2$ — small, twist-4: $-\alpha_s^2 x^{-2\lambda} \log(Q^2)/Q^4$ — enhanced correction

F_2: twist-2: $\alpha_s\, x^{-\lambda} \log(Q^2)/Q^2$

F_2: twist-4 : $[b_T^{(4)} - a_T^{(4)} \log(Q^2)]\, \alpha_s^2\, x^{-2\lambda}/Q^4$ — correction supressed by the sign structure

Pattern of HT corrections from GBW

[Bartels, Golec-Biernat, LM 2009]



Higher twist contribution at $x = 3 \cdot 10^{-4}$ and $Q^2 = 10$ GeV^2:

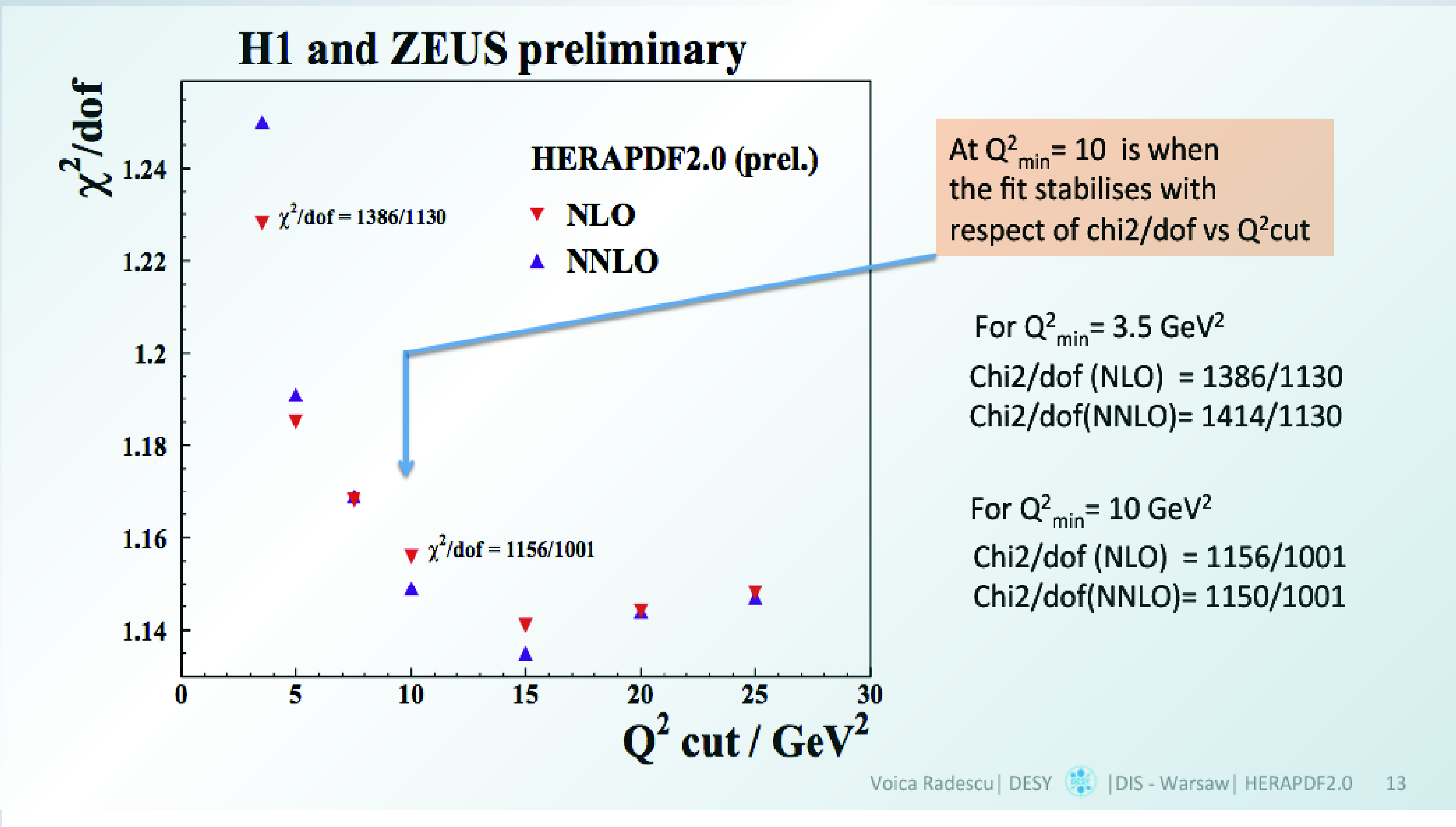
F_T: $\sim 1\%$

F_L: $\sim 20\%$

F_2: $\sim 1\%$

Fresh new twist in higher twists at small x - DIS2014

- Key source of progress: combination of all HERA DIS data
  - → talk by K.Wichmann



DGLAP fit problems at low x and low Q^2

HERAPDF2.0 (prel.)
V. Radescu (DESY)
On behalf of H1 and ZEUS

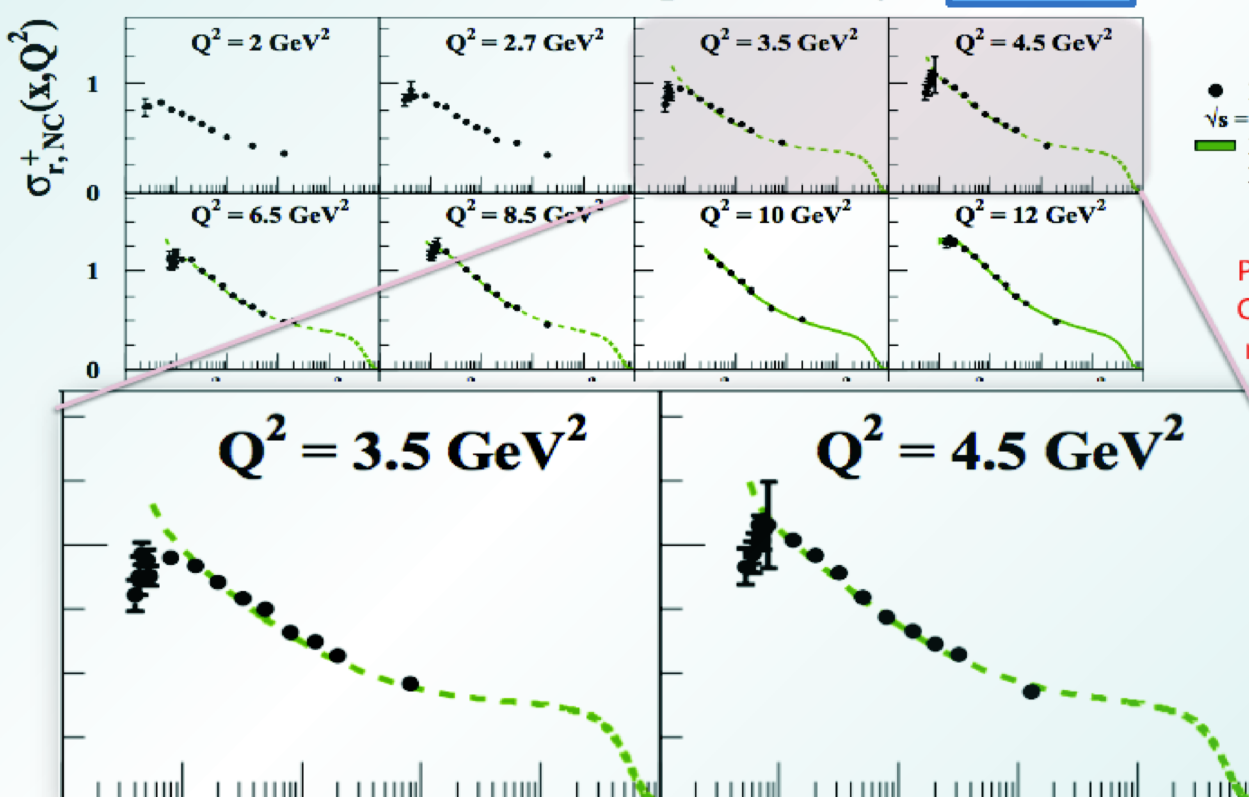
DIS 2014
Warsaw, 28 April - 2 May 2014
XXII. International Workshop on Deep-Inelastic Scattering and Related Subjects

Low Q^2 Data vs HERAPDF2.0 (Q^2_{min}=10 GeV^2)

- How does fit performed with Q2min=10 GeV2 describe the low Q2 data?

H1 and ZEUS preliminary

NNLO Fit



HERA NC e^+p (prel.) 0.5 fb^{-1}
$\sqrt{s}$ = 318 GeV
HERAPDF2.0 (prel.)
NNLO, Q^2_{min} = 10 GeV^2

Poor description of the data for $Q<10\ GeV^2$ when these data are not included in the fit:

-- predictions systematically get worse for low x, Q^2

(higher order do not help)

Voica Radescu | DESY | DIS - Warsaw | HERAPDF2.0 15

Strategy for higher twist analysis in low x HERA data

- Use full range of precision data: dominance of the DGLAP regime → need to maintain the highest quality of twist-2 DGLAP analysis
- Provide QCD inspired model of twist-4 correction
- Perform combined fit of DGLAP (input parameters) and the model of higher twists
- Analyse results in terms of χ^2 / d.o.f. for data set with $Q^2 > Q^2_{min}$
- Successful approach to higher twists in DDIS [Sadzikowski, Słomiński, LM, 2012]

Higher twist model inspired by GBW (new analysis)

- Twist 4 from GBW: [Bartels, Golec-Biernat, Peters, 2000], note the geometric scaling property

$$\sigma_T^{(\tau=4)} = \frac{3}{5}\frac{\alpha_{em}\sigma_0}{\pi}\langle e^2\rangle\frac{Q^4_{\rm sat}}{Q^4}$$

$$\sigma_L^{(\tau=4)} = -\frac{4}{5}\frac{\alpha_{em}\sigma_0}{\pi}\langle e^2\rangle\frac{Q^4_{\rm sat}}{Q^4}\left\{\log(Q^2/Q^2_{\rm sat}) + \gamma_E + 1/15\right\}$$

- More flexible parameterisation of the twist-4 effects with geometric scaling:

$$F_{T/L}^{(\tau=4)} = \frac{Q_0^2(x)}{Q^2}\,x^{-2\lambda}\left[c_{T/L}^{(0)} + c_{T/L}^{(log)}\left(\log\frac{Q_0^2}{Q^2} + \lambda\log\frac{1}{x}\right)\right]$$

- Note steep x-dependence of twist-4 corrections
- Constraint from photon impact factor: $c_T^{(\log)} = 0$

Parton saturation and DGLAP input

- In collinear framework QCD saturation effects manifest themselves in two ways: by modification of inputs at all twists (rescattering below factorisation scale) and by higher twist corrections (resc. above factorisation scale)

- We modify gluon and sea quark input

$$f_g(x, k_T^2) = \frac{3\sigma_0}{4\pi^2} \frac{k_T^4}{Q_s^2(x)} \exp(-Q^2/Q_s^2(x))$$

$$xg(x, Q^2) = \int^{Q^2} \frac{dk^2}{k^2} f_g(x, k^2) \sim \sigma_0 Q^4/Q_s^2(x) \sim x^\lambda \text{ for } Q \ll Q_s(x)$$

$$xg(x, Q^2) \sim \sigma_0 Q_s^2(x) \sim x^{-\lambda} \text{ for } Q \gg Q_s(x)$$

$$\sigma_T^{(\gamma^* p)} \sim \sigma_0 (1 + c \log(Q_s^2(x)/Q^2)) \sim \text{const}(x) \text{ modulo logs}$$

- For Q below $Q_s(x)$: $xg(x,Q^2) \sim x^\lambda$, and $xq_{sea}(x,Q^2) \rightarrow$ const(x)

  Input `Freezing' at very small x

Parton `input freezing'

- Standard DGLAP input without `freezing' :
  – gluon small x rise tamed by a negative correction
  – sea quark small x rise not tamed

- Our approach with `freezing':

$$f_g(x) \propto \frac{x^{-\mathfrak{b}_g}}{1+\left(\frac{\hat{x}}{x}\right)^{2\mathfrak{b}_g}} \underset{x\to 0}{\approxeq} x^{\mathfrak{b}_g}$$

$$f_{\rm sea}(x) \propto \frac{x^{-\mathfrak{b}_q}}{1+\left(\frac{\hat{x}}{x}\right)^{\mathfrak{b}_q}} \underset{x\to 0}{\approxeq} 1$$

Data fitting scheme

- Take all combined HERA data with Q > 1 GeV (1200+ points)

$$\sigma_{\rm red} = F_{\rm T} + \frac{2(1-y)}{1+(1-y)^2} F_{\rm L} \equiv F_2 - \frac{y^2}{1+(1-y)^2} F_{\rm L}$$

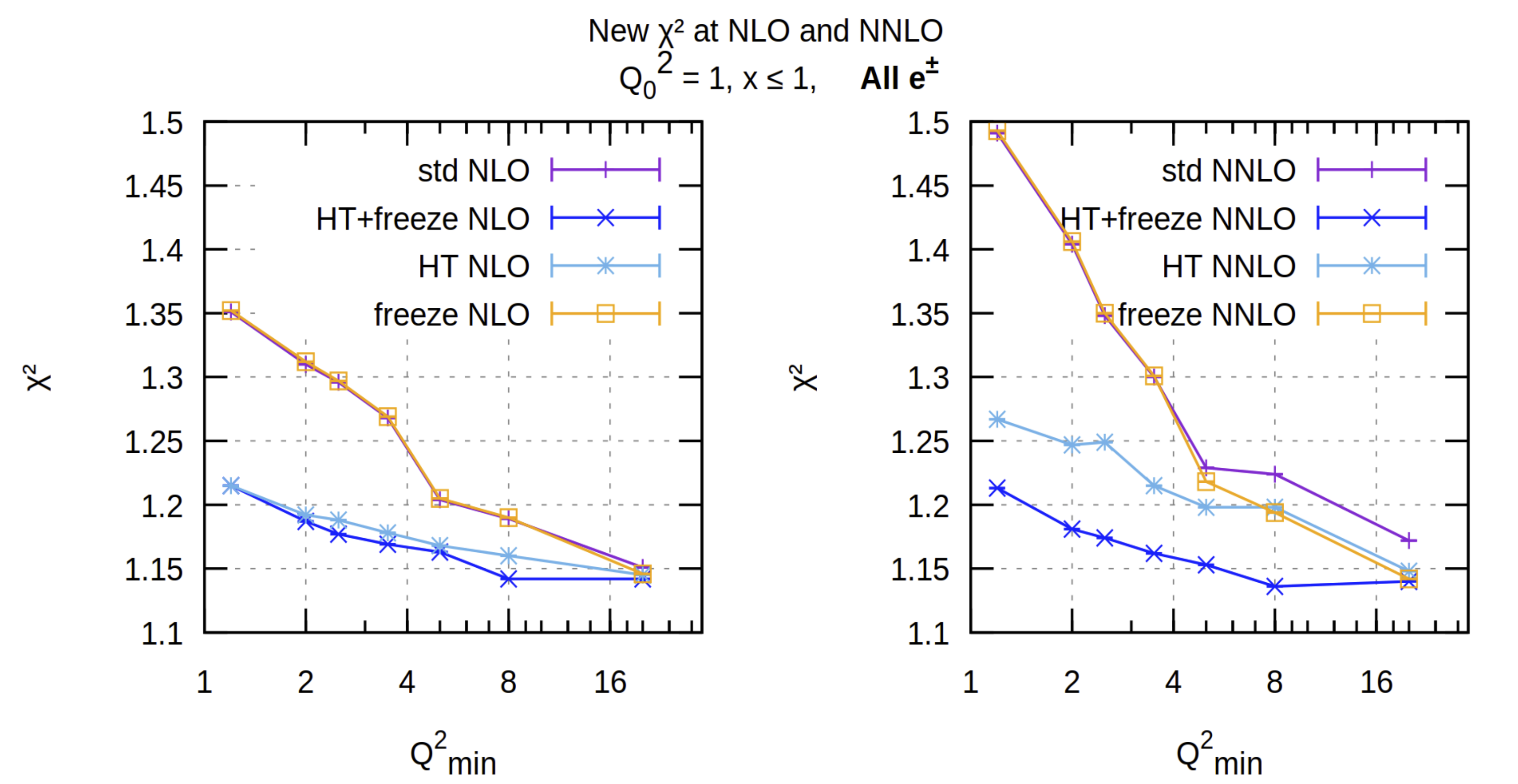
- Fit with standard DGLAP: NLO and NNLO and with
  DGLAP NLO and NNLO with twist-4 corrections and parton freezing

- RTOPT heavy flavour treatment

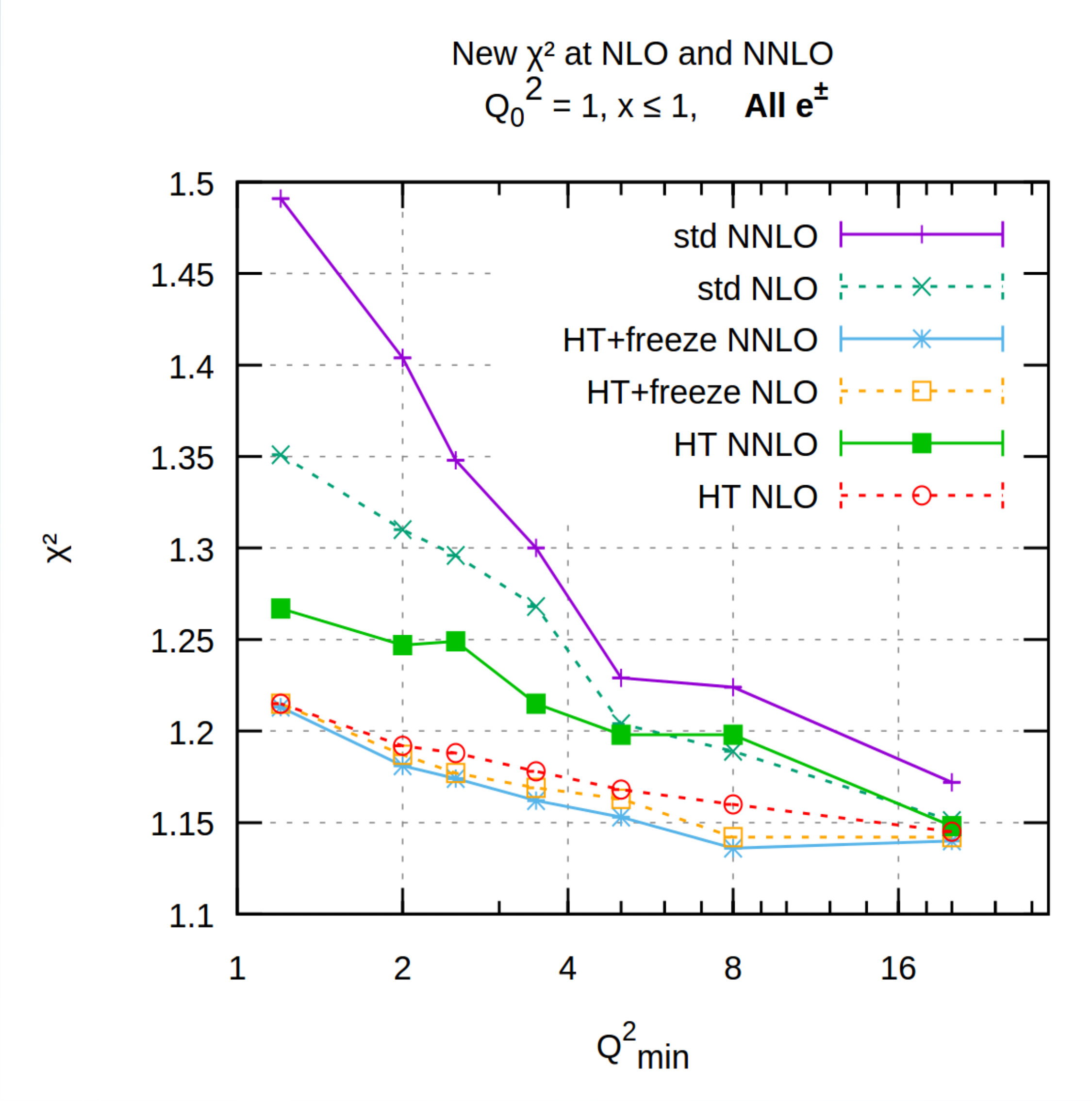
- Systematic,statistical, correlated and uncorrelated data uncertainties treated with HERA-FITTER

- Vary lower cut-off Q_{min} on Q values and check dependence of χ^2 and fit parameters

Fit results with and without higher twist corrections:



Fit results – continued

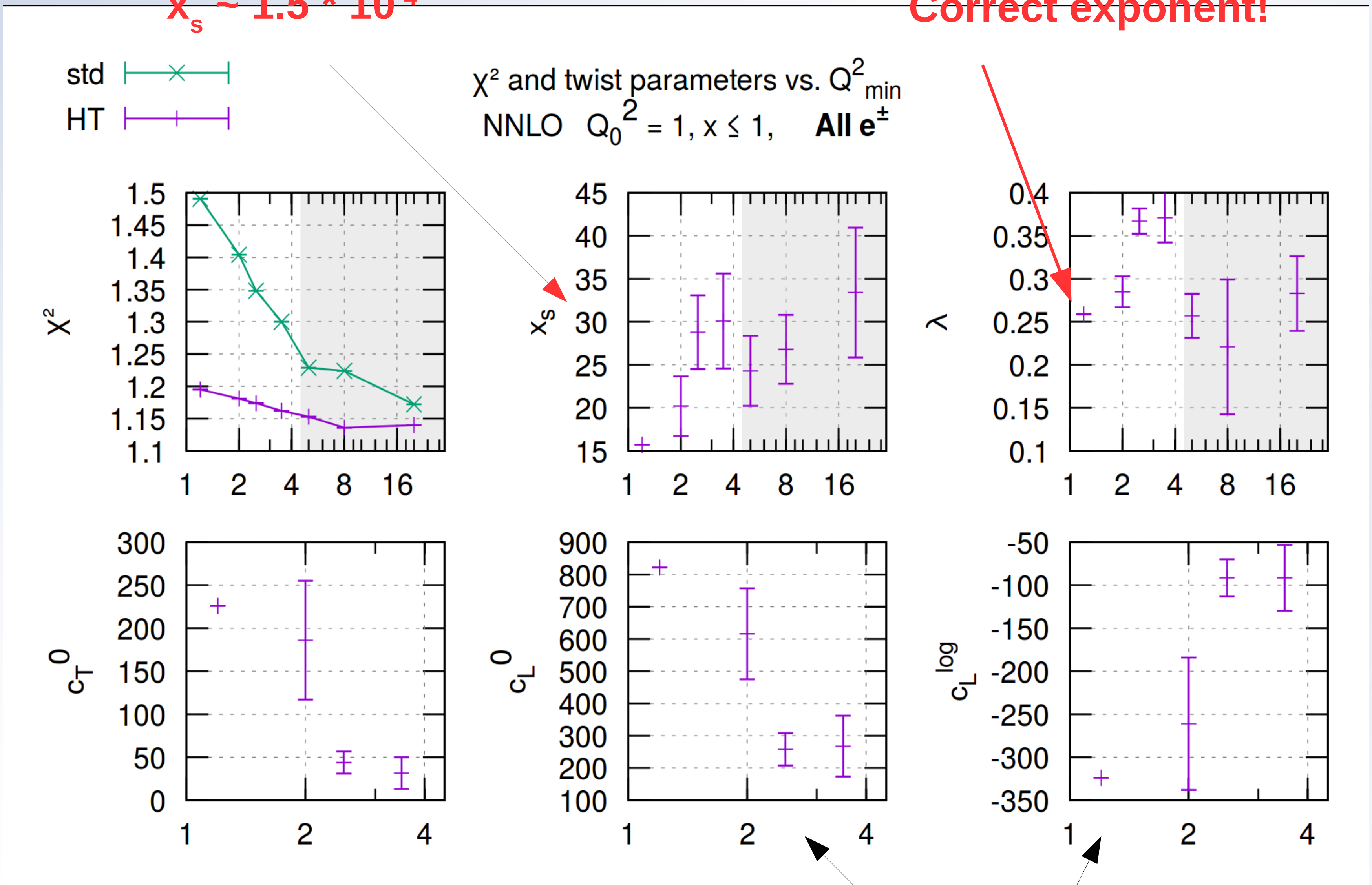


- Higher twist model improves data description
- Improvement much stronger with NNLO DGLAP
- Input freezing helps
- Nearly flat $\chi^2(Q^2_{min})$ obtained
- Effects of HT visible above 10 GeV^2

Resulting higher twist parameters

$x_s \sim 1.5 * 10^{-4}$

Correct exponent!



`Wrong' Sign!

Interpretation and conclusions:

- DGLAP + GBW inspired model of twist-4 works very well for the combined HERA data
- A good description of precision DIS data down to Q=1GeV
- The "saturation parameters": x_0 and λ are consistent with expectations, `double ladder' x-dependence found
- Theoretical puzzle twist 4 correction to F found with the opposite sign! → go back to more sophisticated QCD HT analysis by Bartels and Bontus?
- Possible need for saturation effects in the DGLAP input


Dear Krzysztof and Mark!
Warmest congratulations to you and many, many thanks!

BACKUP

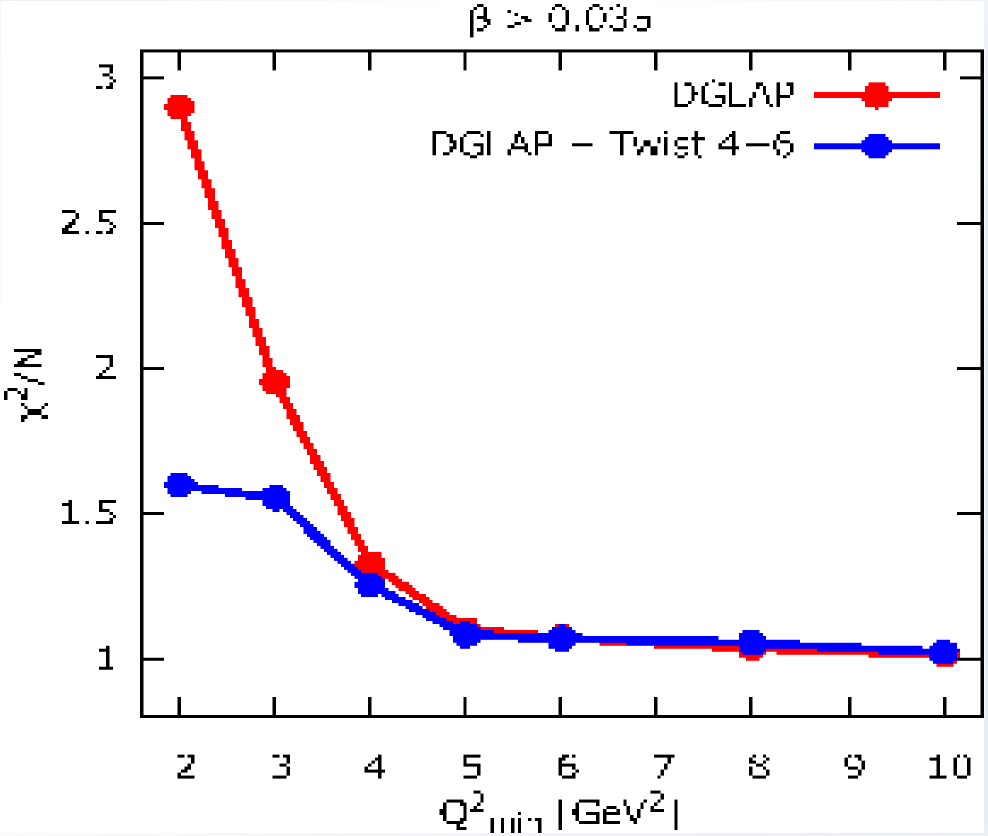
Higher twists in DDIS: impact on χ^2

Procedure:

1) Take data with $Q^2 > Q^2_{min}$

2) Fit with DGLAP, calculate χ^2

3) Fith with DGLAP and free α_s in higher twist terms and calculate χ^2

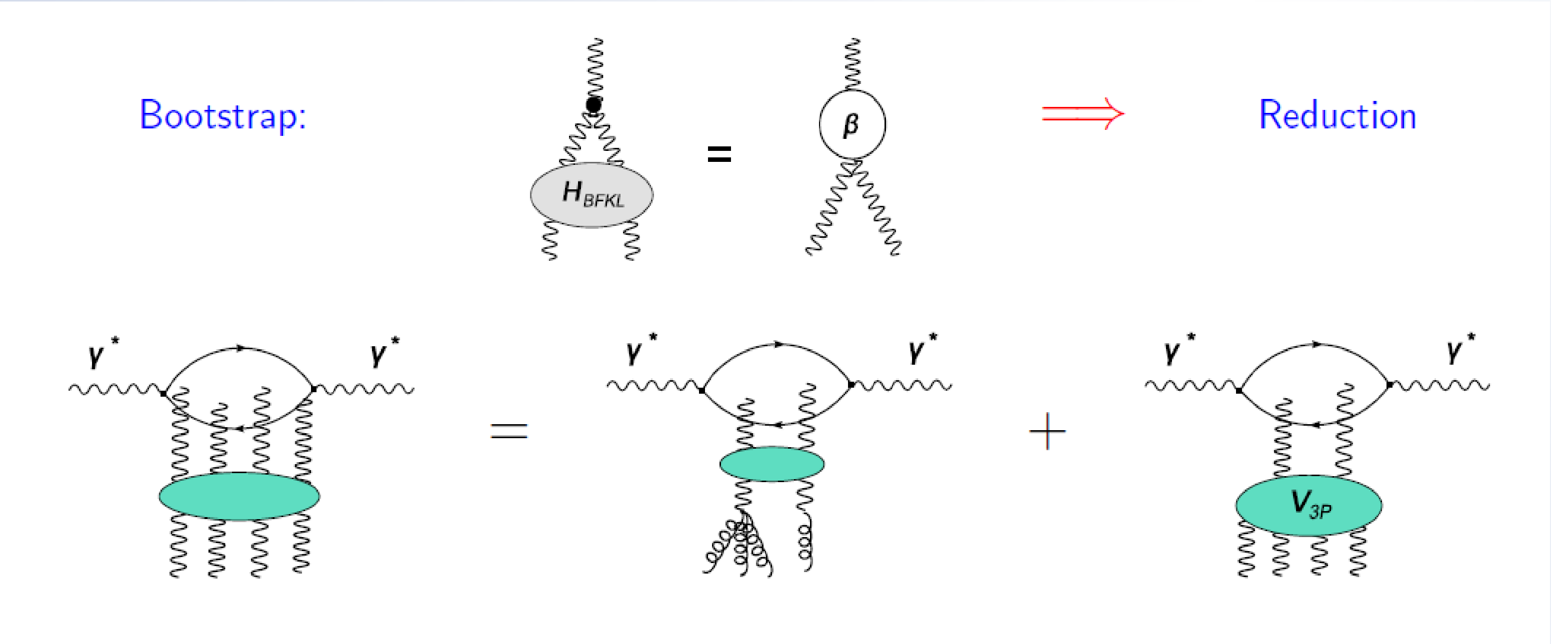


Fit with higher twists gives much better description but still not perfect

“+”: higher twist description strongly constrained

“-” arbitrary cut-off in twist series

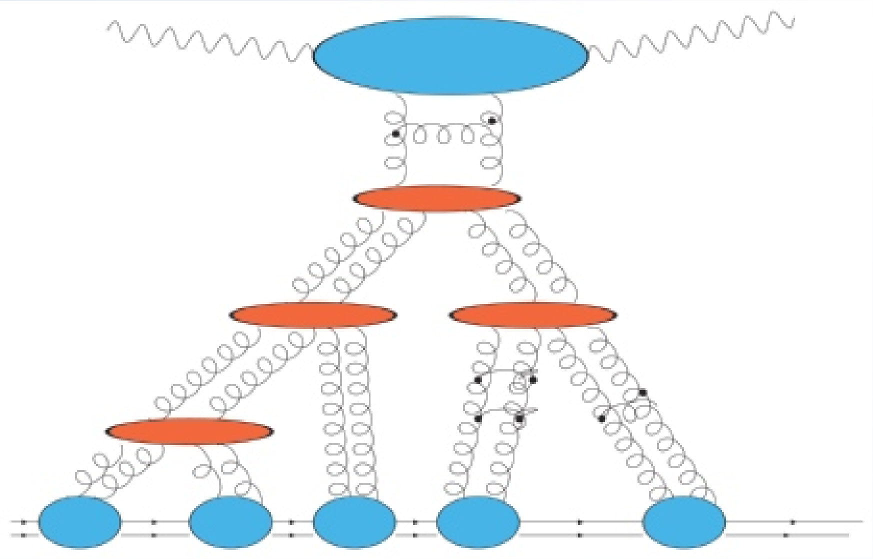
Higher twists from small-x resummations



- BFKL bootstrap (LL) → only one (reggeized, composite) gluon couples to one fundamental (quark) line
- Common double logarithmic limit of BFKL nad DGLAP evolutions → eikonal multi-gluon coupling is unrealistic → cut off of some higher twists
- Singularity structure of BFKL is well known

Multiple scattering with small x resummation

- The basis: Balitsky-Fadin-Kuraev-Lipatov equation: resummation of leading and next-to-leading log(1/x)
- Multi-gluon exchanges: Bartels-Kwieciński-Praszałowicz equation (pairwise BFKL interactions)
- Change of the number of t-channel gluons – possible: Bartels, Ewerz (EGLLA), Balitsky, Kovchegov



- Double logarithmic limit – strong ordering in BFKL / BK evolution -
  – triple pomeron vertex vanishes
  [J. Bartels, K. Kutak]
- BK evolution reduces to DLA BFKL/DGLAP evolution
- In this limit saturation effects appear in inputs for twist evolution
- Consistent with collinear calculations of quasi-multipartonic operator evolution by Bukhvostov, Frolov, Lipatov, Kuraev, 1985

In BFKL/BK pattern of higher twists modified substantially, different from GBW

- Twist 2 – similar, but no multiple ladder growth in twist 4 and higher twists [M. Sadzikowski; LM, 2014]

$$\sigma_T^{(2)} = R_p^2 A_0 \sqrt{\pi} \left(\frac{Q_0}{Q}\right)^2 \frac{t^{1/4} e^{2\sqrt{\bar{\alpha}_s Y t}}}{3(\bar{\alpha}_s Y)^{3/4}}$$

$$\sigma_L^{(2)} = R_p^2 A_0 \sqrt{\pi} \left(\frac{Q_0}{Q}\right)^2 \frac{e^{2\sqrt{\bar{\alpha}_s Y t}}}{3(\bar{\alpha}_s Y t)^{1/4}}$$

$$\sigma_T^{(4)} = R_p^2 A_0 \sqrt{\pi} \left(\frac{Q_0}{Q}\right)^4 \frac{e^{2\sqrt{\bar{\alpha}_s Y t} - 2\bar{\alpha}_s Y}}{5(\bar{\alpha}_s Y t)^{1/4}}$$

$$\sigma_L^{(4)} = -R_p^2 A_0 \sqrt{\pi} \left(\frac{Q_0}{Q}\right)^4 \frac{4 t^{1/4} e^{2\sqrt{\bar{\alpha}_s Y t} - 2\bar{\alpha}_s Y}}{15(\bar{\alpha}_s Y)^{3/4}}$$

- Exponential suppression with rapidity: at very small x and moderate Q much smaller contribution of higher twists
- Curiously, twist-2 effective cross-section of BFKL found to be close to twist 2+4 effective cross-section of GBW