Dipole model analysis of highest precision HERA data, including very low $Q^2$'s

Agnieszka Łuszczak

Krakow University of Technology and DESY Hamburg

in collaboration with Henri Kowalski (DESY)

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Motivation: We analyse, within a dipole model, the final, inclusive HERA DIS cross section data in the low $x$ region, using fully correlated errors.

We show, that these highest precision data are very well described within the dipole model framework starting from $Q^2$ values of $3.5$ GeV$^2$ to the highest values of $Q^2 = 250$ GeV$^2$.

We discuss the saturation question and the properties of the gluon density obtained in this way.

The analysis was done in the xFitter framework.
Dipole model of DIS

- Dipole picture of DIS at small $x$ in the proton rest frame

- Factorization: dipole formation + dipole interaction

$$
\sigma_{\gamma p} = \frac{4\pi^2\alpha_{em}}{Q^2} F_2 = \sum_f \int d^2r \int_0^1 dz \left| \Psi^{\gamma}(r, z, Q^2, m_f) \right|^2 \hat{\sigma}(r, x)
$$

- Dipole-proton interaction

$$
\hat{\sigma}(r, x) = \sigma_0 \left(1 - \exp\{-\hat{r}^2\}\right) \quad \hat{r} = r/R_s(x)
$$
Dipole cross section

BGK (Bartels-Golec-Kowalski) parametrization

\[ \hat{\sigma}(r, x) = \sigma_0 \left\{ 1 - \exp \left[ -\pi^2 r^2 \alpha_s(\mu^2) x g(x, \mu^2) / (3\sigma_0) \right] \right\} \]

\[ \mu^2 = C / r^2 + \mu_0^2 \] is the scale of the gluon density

\[ \mu_0^2 \] is a starting scale of the QCD evolution. \[ \mu_0^2 = Q_0^2 \]

Gluon density is evolved according to the LO or NLO DGLAP eq.

Soft gluon:

\[ x g(x, \mu_0^2) = A_g x^{\lambda_g} (1 - x)^{C_g} \]

Soft + hard gluon:

\[ x g(x, \mu_0^2) = A_g x^{\lambda_g} (1 - x)^{C_g} (1 + D_g x + E_g x^2) \]
Results of the Fits

Dipole model BGK fit with fix valence quarks and without

\begin{tabular}{|c|c|c|c|c|c|c|c|}
\hline
$Q^2_{\text{min}}$ [GeV$^2$] & $\sigma_0$ [mb] & $A_g$ & $\lambda_g$ & $C_g$ & $N_{df}$ & $\chi^2$ & $\chi^2/N_{df}$ \\
\hline
3.5 & 87.0±8.9 & 2.32±0.009 & -0.056±0.11 & 8.21±0.80 & 534 & 551.05 & 1.03 \\
8.5 & 72.36±7.4 & 2.766±0.009 & -0.042±0.123 & 6.543±0.632 & 448 & 452.48 & 1.01 \\
\hline
\end{tabular}

Table 1: BGK fit with fixed valence quarks for $\sigma_r$ for H1ZEUS-NC data in the range $Q^2 \geq 3.5$ or $8.5$ GeV$^2$ and $x \leq 0.01$. NLO fit. Soft gluon. $m_{uds} = 0.14, m_c = 1.3$ GeV. $Q^2_0 = 1.9$ GeV$^2$.

1.2 BGK NLO fit without valence quarks for $\sigma_r$ for HERA1+2-NCep-460, HERA1+2-NCep-575, HERA1+2-NCep-820, HERA1+2-NCep-920 and HERA1+2-NCem in the range $Q^2 \geq 3.5$ GeV$^2$ and $Q^2 \geq 8.5$ and $x \leq 0.01$. Soft gluon.

\begin{tabular}{|c|c|c|c|c|c|c|c|c|}
\hline
No & $Q^2$ & HF Scheme & $\sigma_0$ & $A_g$ & $\lambda_g$ & $C_g$ & $cBGK$ & $N_p$ & $\chi^2$ & $\chi^2/N_p$ \\
\hline
1 & $Q^2 \geq 3.5$ & RT OPT & 85.111 & 2.075 & -0.093 & 4.989 & 4.0 & 568 & 592.46 & 1.04 \\
2 & $Q^2 \geq 8.5$ & RT OPT & 123.31 & 1.997 & -0.0975 & 4.655 & 4.0 & 482 & 479.37 & 0.99 \\
\hline
\end{tabular}
Results of the Fits

**Dipole model BGK fit with fitted valence quarks**

1.3 BGK NLO fit with fitted valence quarks for $\sigma_r$ for HERA1+2-NCep-460, HERA1+2-NCep-575, HERA1+2-NCep-820, HERA1+2-NCep-920 and HERA1+2-NCem in the range $Q^2 \geq 3.5$ GeV$^2$ and $Q^2 \geq 8.5$ and $x \leq 0.01$. *Soft gluon.*

<table>
<thead>
<tr>
<th>No</th>
<th>$Q^2$</th>
<th>HF Scheme</th>
<th>$\sigma_0$</th>
<th>$A_g$</th>
<th>$\lambda_g$</th>
<th>$C_g$</th>
<th>$cBGK$</th>
<th>$Np$</th>
<th>$\chi^2$</th>
<th>$\chi^2/Np$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$Q^2 \geq 3.5$</td>
<td>RT OPT</td>
<td>85.111</td>
<td>1.921</td>
<td>-0.103</td>
<td>4.674</td>
<td>4.0</td>
<td>557</td>
<td>575.30</td>
<td>1.03</td>
</tr>
<tr>
<td>2</td>
<td>$Q^2 \geq 8.5$</td>
<td>RT OPT</td>
<td>93.581</td>
<td>1.665</td>
<td>-0.124</td>
<td>6.066</td>
<td>4.0</td>
<td>473</td>
<td>476.71</td>
<td>1.01</td>
</tr>
</tbody>
</table>

**HERAPDF fit with fitted valence quarks**

1.4 HERAPDF NLO fit with fitted valence quarks for $\sigma_r$ for HERA1+2-NCep-460, HERA1+2-NCep-575 HERA1+2-NCep-820, HERA1+2-NCep-920, HERA1+2-NCem, HERA1+2-CCep and HERA1+2-CCem data in the range $Q^2 \geq 3.5$ and $Q^2 \geq 8.5$ and $x \leq 1.0$.

<table>
<thead>
<tr>
<th>No</th>
<th>$Q^2$</th>
<th>HF Scheme</th>
<th>$Np$</th>
<th>$\chi^2$</th>
<th>$\chi^2/Np$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$Q^2 \geq 3.5$</td>
<td>RT</td>
<td>1131</td>
<td>1356.70</td>
<td>1.20</td>
</tr>
<tr>
<td>2</td>
<td>$Q^2 \geq 8.5$</td>
<td>RT</td>
<td>456</td>
<td>470.88</td>
<td>1.15</td>
</tr>
</tbody>
</table>
HERAPDF fit with fix valence quarks, soft + hard gluon

HERAPDF NLO fit with fix valence quarks for $\sigma_r$ for HERA1+2-NCep-460, HERA1+2-NCep-575 HERA1+2-NCep-820, HERA1+2-NCep-920, HERA1+2-NCem data in the range $Q^2 \geq 3.5$ and $x \leq 0.01$.

<table>
<thead>
<tr>
<th>No</th>
<th>$Q^2$</th>
<th>HF Scheme</th>
<th>$N_p$</th>
<th>$\chi^2$</th>
<th>$\chi^2/N_p$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$Q^2 \geq 3.5$</td>
<td>RT</td>
<td>532</td>
<td>564.80</td>
<td>1.06</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>No</th>
<th>$Q^2_{\text{min}}$ [GeV$^2$]</th>
<th>HF Scheme</th>
<th>$N_p$</th>
<th>$\chi^2$</th>
<th>$\chi^2/N_p$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3.5</td>
<td>FONLL-B</td>
<td>534</td>
<td>539.3</td>
<td>1.01</td>
</tr>
<tr>
<td>2</td>
<td>3.5</td>
<td>FONLL-B</td>
<td>532</td>
<td>537.3</td>
<td>1.01</td>
</tr>
</tbody>
</table>

Soft+Hard
Results of the Fits

\[ m_{u,d,s} = 140 \text{ MeV}, \quad m_c = 1.3 \text{ GeV} \]

\[ \hat{\sigma}(r, x) = \sigma_0 \left\{ 1 - \exp \left[ -\pi^2 r^2 \alpha_s(\mu^2) x g(x, \mu^2)/(3\sigma_0) \right] \right\} \text{ with saturation} \]

<table>
<thead>
<tr>
<th>( Q_{\text{min}}^2 ) [GeV(^2)]</th>
<th>( \sigma_0 ) [mb]</th>
<th>( A_g )</th>
<th>( \lambda_g )</th>
<th>( C_g )</th>
<th>( D_g )</th>
<th>( E_g )</th>
<th>( N_{df} )</th>
<th>( \chi^2 )</th>
<th>( \chi^2/N_{df} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.5</td>
<td>77.6±18.6</td>
<td>2.6166±0.158</td>
<td>-0.0636±0.0087</td>
<td>37.114±5.057</td>
<td>3.0597±6.510</td>
<td>1406.4±552.65</td>
<td>532</td>
<td>534.17</td>
<td>1.00</td>
</tr>
<tr>
<td>8.5</td>
<td>63.5±18.5</td>
<td>2.112±0.101</td>
<td>-0.0541±0.0065</td>
<td>21.341±4.062</td>
<td>1.098±5.764</td>
<td>867.23±423.67</td>
<td>448</td>
<td>439.04</td>
<td>0.98</td>
</tr>
</tbody>
</table>

\[ \hat{\sigma}(r, x) = \sigma_0 \left[ \pi^2 r^2 \alpha_s(\mu^2) x g(x, \mu^2)/(3\sigma_0) \right] \text{ without saturation} \]

<table>
<thead>
<tr>
<th>( Q_{\text{min}}^2 ) [GeV(^2)]</th>
<th>( Q_0^2 ) [GeV(^2)]</th>
<th>( A_g )</th>
<th>( \lambda_g )</th>
<th>( C_g )</th>
<th>( D_g )</th>
<th>( E_g )</th>
<th>( N_{df} )</th>
<th>( \chi^2 )</th>
<th>( \chi^2/N_{df} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.5</td>
<td>1.9</td>
<td>2.33±0.10</td>
<td>-0.094±0.006</td>
<td>14.8±11.5</td>
<td>9.80±14.7</td>
<td>-99.5±74.830</td>
<td>533</td>
<td>556.17</td>
<td>1.04</td>
</tr>
<tr>
<td>3.5</td>
<td>1.1</td>
<td>3.80±0.22</td>
<td>0.10±0.01</td>
<td>32.5±1.6</td>
<td>-25.2±3.49</td>
<td>1868±252</td>
<td>533</td>
<td>539.2</td>
<td>1.01</td>
</tr>
</tbody>
</table>
Comparision with HERAI+II data
Comparision with HERAI+II data

Extrapolated fits
Fits to data including the lower $Q^2 > 0.35$ GeV$^2$ region

**with saturation**

<table>
<thead>
<tr>
<th>$Q^2_0$ [GeV$^2$]</th>
<th>$\sigma_0$ [mb]</th>
<th>$A_g$</th>
<th>$\lambda_g$</th>
<th>$C_g$</th>
<th>$D_g$</th>
<th>$E_g$</th>
<th>$N_{df}$</th>
<th>$\chi^2$</th>
<th>$\chi^2/N_{df}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.9</td>
<td>38.2 ± 4.1</td>
<td>2.80 ± 0.14</td>
<td>-0.063 ± 0.006</td>
<td>46.3 ± 4.58</td>
<td>12.1 ± 6.00</td>
<td>1970.4 ± 566.0</td>
<td>653</td>
<td>790.4</td>
<td>1.21</td>
</tr>
<tr>
<td>1.1</td>
<td>196.1 ± 105</td>
<td>6.24 ± 0.53</td>
<td>0.098 ± 0.012</td>
<td>52.3 ± 6.5</td>
<td>-22.0 ± 10.64</td>
<td>2145.0 ± 835.7</td>
<td>653</td>
<td>894.1</td>
<td>1.37</td>
</tr>
</tbody>
</table>

**without saturation**

<table>
<thead>
<tr>
<th>$Q^2_0$ [GeV$^2$]</th>
<th>$A_g$</th>
<th>$\lambda_g$</th>
<th>$C_g$</th>
<th>$D_g$</th>
<th>$E_g$</th>
<th>$N_{df}$</th>
<th>$\chi^2$</th>
<th>$\chi^2/N_{df}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.9</td>
<td>3.05 ± 0.092</td>
<td>-0.022 ± 0.004</td>
<td>40.3 ± 1.067</td>
<td>-32.3 ± 3.02</td>
<td>3158.3 ± 219.3</td>
<td>654</td>
<td>1024.3</td>
<td>1.56</td>
</tr>
<tr>
<td>1.1</td>
<td>5.62 ± 0.13</td>
<td>0.158 ± 0.001</td>
<td>43.320 ± 0.15</td>
<td>-55.011 ± 8.62</td>
<td>3791.6 ± 187.7</td>
<td>654</td>
<td>999.98</td>
<td>1.53</td>
</tr>
</tbody>
</table>
The differences are disappearing at larger $Q^2$. 

saturated gluon density is higher than the non-saturated
Summary

The dipole BGK fits with the DGLAP QCD evolution, describe the highest precision HERA data, in the low x region and for $3.5 < Q^2 < 250$ GeV$^2$ very well.

For $Q^2 > 3.5$ GeV$^2$ no significant differences between the saturated and non-saturated fits were observed.

The fits to data including the lower $Q^2 > 0.35$ GeV$^2$ region show that the saturated gluon density is preferred.

In the old saturation investigation were the first set of HERA data was used, the fits with and without saturation had the same quality, even when the low $Q^2$ region was included in the fit. The present result showing that the fit with saturation has a sizably better quality than that without saturation is new and is due to the substantially improved quality of HERA data.
Back up slides