On a Four-Dimensional Formulation of Dimensionally Regulated Amplitudes

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This talk is based on the paper

A.R. Fazio, P. Mastrolia, E. Mirabella, W.J. Torres,
"On a four-dimensional formulation of dimensionally regulated amplitudes"

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and references therein.

Outline

The NLO computations of hard processes

Four Dimensional Feynman Rules

Generalized Internal legs

Four point one-loop massless color ordered amplitudes

The all helicity-plus four gluons planar amplitude with a gluonic loop

The effective coupling of Higgs to gluons in NLO amplitudes

Conclusions and perspectives

The NLO computations of hard processes

A hard parton-level cross section $2 \to m$ at NLO is made by

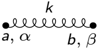
$$\sigma^{NLO} = \int_m d\sigma^B + \int_m \left(d\sigma^V + \int_1 d\sigma^A \right) + \int_{m+1} \left(d\sigma^R - d\sigma^A \right)$$

- $d\sigma^B$ is the Born exclusive regularization scheme independent differential cross section $(\bar{\Sigma} A^B A^{B*})$.
- $d\sigma^V$ is the virtual correction $(\bar{\Sigma} \Re[A^B A^{V*}])$. It involves loop diagrams whose UV (ultraviolet divergent) part is made finite in a given **renormalization scheme** and therefore the UV divergences are **regularization scheme** independent.
- $d\sigma^R$ is the real corrections, affected (together with $d\sigma^V$) by soft and collinear divergences.
- $d\sigma^A$ and $\int_1 d\sigma^A$ are the unintegrated and integrated counterterms (allowing to compute real emission of massless particles in 4 dimensions).

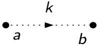
Four Dimensional Feynman Rules for gauge theories bare one-loop dimensionally regularized diagrams

The external legs are treated as usual four dimensional states.

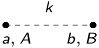
- The pure Yang-Mills (YM) loop propagators in Feynman-'t Hooft gauge



$$= -i\,\delta^{ab}\,\frac{g^{\alpha\beta}}{k^2-\mu^2+i\varepsilon} \quad \text{(gluon)},$$



$$= i\,\delta^{ab}\,\frac{1}{k^2-\mu^2+i\varepsilon} \quad \text{(ghost)},$$



$$= -i\,\delta^{ab}\,\frac{G^{AB}}{k^2-\mu^2+i\varepsilon} \quad \text{(scalar)},$$

The scalars come from a dimensional reduction of $D = 4 - 2\epsilon$ dimensional gluons vector fields.

In $D = 4 - 2\epsilon$ dimensions we perform the decomposition of the loop momentum $\bar{k}^\alpha$ in a 4-dimensional part k^α and in its orthogonal complement the -2ϵ-dimensional **fixed** vector μ^α

$$\bar{k}^\alpha = k^\alpha + \mu^\alpha \quad \mu^\alpha \mu_\alpha = -\mu^2$$
$$\bar{g}^{\alpha\beta} = g^{\alpha\beta} + \tilde{g}^{\alpha\beta} \quad \tilde{g}^{\alpha\beta} \to G^{AB} \quad \mu^\alpha \to i\mu Q^A$$

where the A and B label the components of the complementary space of dimension $D - 4$.
The metric G^{AB} and the vector Q^A needed to reformulate the Feynman rules satisfy

$$G^{AB}G^{BC} = G^{AC}, \qquad G^{AA} = 0, \qquad G^{AB} = G^{BA}$$
$$Q^A G^{AB} = Q^B, \qquad Q^A Q^A = 1.$$

- Fermion propagator in a loop
  Dirac matrices have the following splitting

$$\bar{\gamma}^{\alpha} = \gamma^{\alpha} + \tilde{\gamma}^{\alpha}$$

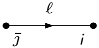
and satisfy in D dimensions the Clifford algebra

$$\{\bar{\gamma}^{\alpha}, \bar{\gamma}^{\beta}\} = 2\bar{g}^{\alpha\beta}.$$

A possible 4-dimensional representation of $\tilde{\gamma}$ matrices is in terms of γ^5 by the replacement

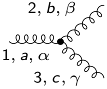
$$\tilde{\gamma}^{\alpha} \to \gamma^5 \Gamma^A.$$

By imposing the rule $Q^A \Gamma^A = 1$ needed to recover $\not{\mu}\not{\mu} = -\mu^2$,

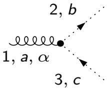


$$= i\delta^{i}_{\jmath} \frac{\not{k} + m - i\mu\gamma^5}{k^2 - m^2 - \mu^2 + i\varepsilon}\,.$$

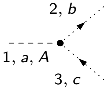
- Four Dimensional Interaction Vertices



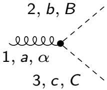
$$= -g\, f^{abc} \left[(k_1 - k_2)^\gamma g^{\alpha\beta} + (k_2 - k_3)^\alpha g^{\beta\gamma} + (k_3 - k_1)^\beta g^{\gamma\alpha} \right],$$



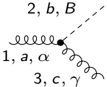
$$= -g\, f^{abc}\, k_2^\alpha\,,$$



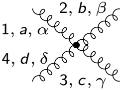
$$= -ig\, f^{abc}\, \mu Q^A\,,$$



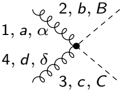
$$= -g\, f^{abc}\, (k_2 - k_3)^\alpha\, G^{BC}\,,$$



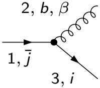
$$= \mp g\, f^{abc}\, (i\mu)\, g^{\gamma\alpha}\, Q^B \quad (\tilde{k}_1 = 0, \quad \tilde{k}_3^\gamma = \pm\mu^\gamma)$$



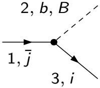
$$= -ig^2 \Big[+ f^{xad} f^{xbc} \left(g^{\alpha\beta} g^{\delta\gamma} - g^{\alpha\gamma} g^{\beta\delta} \right) + f^{xac} f^{xbd} \left(g^{\alpha\beta} g^{\delta\gamma} - g^{\alpha\delta} g^{\beta\gamma} \right) + f^{xab} f^{xdc} \left(g^{\alpha\delta} g^{\beta\gamma} - g^{\alpha\gamma} g^{\beta\delta} \right) \Big] ,$$



$$= 2ig^2 \, g^{\alpha\delta} \left(f^{xab} f^{xcd} + f^{xac} f^{xbd} \right) G^{BC} ,$$



$$= -ig \left(t^b \right)^i_{\bar{j}} \gamma^\beta ,$$



$$= -ig \left(t^b \right)^i_{\bar{j}} \gamma^5 \Gamma^B$$

- **Selection rules (-2ϵ SRs) or about the algebra in the $D-4$ dimensional complementary space.**
  In the -2ϵ-dimensional vector space the following rules

$$
\begin{aligned}
G^{AB}G^{BC} &= G^{AC}, & G^{AA} &= 0, & G^{AB} &= G^{BA}, \\
\Gamma^{A}G^{AB} &= \Gamma^{B}, & \Gamma^{A}\Gamma^{A} &= 0, & Q^{A}\Gamma^{A} &= 1, \\
Q^{A}G^{AB} &= Q^{B}, & Q^{A}Q^{A} &= 1
\end{aligned}
$$

  completely define our four dimensional formulation in agreement with the **Four Dimensional Helicity scheme** up to spurious terms as explicitly checked in reproducing the integrand numerator of QCD amplitudes of the following processes

$$
\begin{aligned}
q\,\bar{q} &\to t\,\bar{t}\,, & g\,g &\to t\,\bar{t}\,, & t\,\bar{t} &\to t\,\bar{t}\,, \\
g\,g &\to g\,g\,, & q\,\bar{q} &\to t\,\bar{t}\,g\,, & g\,g &\to t\,\bar{t}\,g\,, \\
q\,\bar{q} &\to t\,\bar{t}\,q'\,\bar{q}'\,.
\end{aligned}
$$

Generalized Internal legs

- Generalized subluminal Dirac equation.
  Given the ℓ four dimensional vector

$$\left(\not\ell - i\mu\gamma^5 - m\right)\ u_\lambda\left(\ell\right) = 0\,,$$
$$\left(\not\ell - i\mu\gamma^5 + m\right)\ v_\lambda\left(\ell\right) = 0\,,$$
$$\ell^\mu = \ell^{\flat\mu} + \frac{m^2+\mu^2}{2l\cdot q_\ell} q^\mu{}_\ell; \quad \left(\ell^\flat\right)^2 = 0 = q_\ell^2\,.$$

- Solutions of the generalized Dirac equation

$$u_+\left(\ell\right) = \left|\ell^\flat\right\rangle - \frac{(m-i\mu)}{\left[\ell^\flat\, q_\ell\right]}\left|q_\ell\right]\,,$$
$$u_-\left(\ell\right) = \left|\ell^\flat\right] - \frac{(m+i\mu)}{\left\langle\ell^\flat\, q_\ell\right\rangle}\left|q_\ell\right\rangle\,,$$
$$v_-\left(\ell\right) = \left|\ell^\flat\right\rangle + \frac{(m-i\mu)}{\left[\ell^\flat\, q_\ell\right]}\left|q_\ell\right]\,, \quad v_+\left(\ell\right) \ = \left|\ell^\flat\right] + \frac{(m+i\mu)}{\left\langle\ell^\flat\, q_\ell\right\rangle}\left|q_\ell\right\rangle\,.$$

- Polarization sum of the solutions of the generalized Dirac equation

$$\sum_{\lambda=\pm} u_\lambda(\ell)\,\bar{u}_\lambda(\ell) = \not{\ell} - i\mu\gamma^5 + m\,,$$
$$\sum_{\lambda=\pm} v_\lambda(\ell)\,\bar{v}_\lambda(\ell) = \not{\ell} - i\mu\gamma^5 - m\,.$$

- **BCFW** (Britto, Cachazo, Feng, Witten) recursive relations

$$A_n = \sum_{\text{partitions}} \sum_{\lambda} A_L(\hat{p}_i, \hat{P}^*) \frac{u_\lambda(\hat{P})\bar{u}_\lambda(\hat{P})}{P^2 - m^2 - \mu^2} A_R(-\hat{P}^*, \hat{p}_j)$$

$$= \sum_{\text{partitions}} A_L(\hat{p}_i, \hat{P}^*) \frac{\hat{\not{P}} - m - i\mu\gamma_5}{P^2 - m^2 - \mu^2} A_R(-\hat{P}^*, \hat{p}_j)$$

The hatted are the shifted complex momenta and P^* shows that the amplitude has been stripped of his external spinor wave function.

PROOF OF THE COMPLETNESS RELATIONS

Chirality projectors

$$\omega_\pm = \frac{\mathbb{I} \pm \gamma^5}{2} \ ,$$

and we show that:

$$\begin{aligned}
&\frac{|q_\ell][\ell^\flat| - |\ell^\flat][q_\ell|}{[\ell^\flat q_\ell]} = \\
&= \frac{|q_\ell]\langle q_\ell\, \ell^\flat\rangle[\ell^\flat| + |\ell^\flat]\langle \ell^\flat\ q_\ell\rangle[q_\ell|}{2\ell^\flat \cdot q_\ell} \\
&= \frac{(|q_\ell]\langle q_\ell|)(|\ell^\flat\rangle[\ell^\flat|) + (|\ell^\flat]\langle \ell^\flat|)(|q_\ell\rangle[q_\ell|)}{2\ell^\flat \cdot q_\ell} \\
&= \frac{\omega_- \not{q}_\ell \omega_+ \not{\ell}^\flat + \omega_- \not{\ell}^\flat \omega_+ \not{q}_\ell}{2\ell^\flat \cdot q_\ell} \\
&= \frac{\omega_-^2 \{\not{q}_\ell\,, \not{\ell}^\flat\}}{2\ell^\flat \cdot q_\ell} = \omega_- \ ,
\end{aligned} \tag{7a}$$

and similarly

$$\frac{|\ell^\flat\rangle\langle q_\ell| - |q_\ell\rangle\langle\ell^\flat|}{\langle q_\ell\, \ell^\flat\rangle} = \omega_+ \,.$$

Therefore

$$\sum_{\lambda=\pm} u_\lambda(\ell)\bar{u}_\lambda(\ell) =$$

$$\left(|\ell^\flat\rangle + \frac{(m - i\mu)}{[\ell^\flat\, q_\ell]}|q_\ell]\right)\left([\ell^\flat| + \frac{(m + i\mu)}{\langle q_\ell\, \ell^\flat\rangle}\langle q_\ell|\right) +$$
$$\left(|\ell^\flat] + \frac{(m + i\mu)}{\langle \ell^\flat\, q_\ell\rangle}|q_\ell\rangle\right)\left(\langle\ell^\flat| + \frac{(m - i\mu)}{[q_\ell\, \ell^\flat]}[q_\ell|\right) =$$
$$= \not{\ell}^\flat + \frac{m^2 + \mu^2}{2\ell^\flat \cdot q_\ell}\not{q}_\ell + (m - i\mu)\frac{|q_\ell][\ell^\flat| - |\ell^\flat][q_\ell|}{[\ell^\flat\, q_\ell]}$$
$$+(m + i\mu)\frac{|\ell^\flat\rangle\langle q_\ell| - |q_\ell\rangle\langle\ell^\flat|}{\langle q_\ell\, \ell^\flat\rangle} =$$
$$= \not{\ell}^\flat + \frac{m^2 + \mu^2}{2\ell^\flat \cdot q_\ell}\not{q}_\ell + (m - i\mu)\omega_- + (m + i\mu)\omega_+ =$$
$$= \not{\ell} + i\mu\gamma^5 + m \,. \tag{8}$$

- D dimensional Polarization Vectors

  In Arnowitt-Fickler gauge the helicity sum of the transverse D-dimensional polarization vectors is

$$\sum_{i=1}^{D-2} \varepsilon^{\alpha}_{i\,(D)}\left(\bar{\ell},\bar{\eta}\right) \varepsilon^{*\beta}_{i\,(D)}\left(\bar{\ell},\bar{\eta}\right) = -\bar{g}^{\alpha\beta} + \frac{\bar{\ell}^{\alpha}\,\bar{\eta}^{\beta} + \bar{\ell}^{\beta}\,\bar{\eta}^{\alpha}}{\bar{\ell}\cdot\bar{\eta}} - \frac{\bar{\eta}^{2}\,\bar{\ell}^{\alpha}\,\bar{\ell}^{\beta}}{(\bar{\eta}\cdot\bar{\ell})^{2}}$$

$$\bar{\ell}\cdot\bar{\eta} \neq 0$$

  From the gauge invariance in D dimensions the choice of the fixed D-dimensional gauge vector

$$\bar{\eta}^{\alpha} = \mu^{\alpha}$$

  allows for the disentanglement

$$\begin{aligned} \sum_{i=1}^{D-2} \varepsilon^{\alpha}_{i\,(D)}\left(\bar{\ell},\bar{\eta}\right) \varepsilon^{*\beta}_{i\,(D)}\left(\bar{\ell},\bar{\eta}\right) &= \left(-g^{\alpha\beta} + \frac{\ell^{\alpha}\ell^{\beta}}{\mu^{2}}\right) \\ &- \left(\tilde{g}^{\alpha\beta} + \frac{\mu^{\alpha}\mu^{\beta}}{\mu^{2}}\right) . \end{aligned}$$

- Generalized Polarization Vectors

  Once again let us decompose the massive four-dimensional vector ($\ell^2 = \mu^2$)

$$\ell^\alpha = \ell^{\flat\,\alpha} + \hat{q}_\ell^\alpha$$

the μ-massive polarizations vectors are

$$\varepsilon_+^\alpha(\ell) = -\frac{\left[\ell^\flat\left|\gamma^\alpha\right|\hat{q}_\ell\right\rangle}{\sqrt{2}\mu}\,, \qquad \varepsilon_-^\alpha(\ell) = -\frac{\left\langle\ell^\flat\left|\gamma^\alpha\right|\hat{q}_\ell\right]}{\sqrt{2}\mu}\,,$$

$$\varepsilon_0^\alpha(\ell) = \frac{\ell^{\flat\alpha} - \hat{q}_\ell^\alpha}{\mu}$$

with the usual Proca's completness relation

$$\sum_{\lambda=\pm,0} \varepsilon_\lambda^\alpha(\ell)\,\varepsilon_\lambda^{*\beta}(\ell) = -g^{\alpha\beta} + \frac{\ell^\alpha\ell^\beta}{\mu^2}$$

$$\varepsilon_\pm^2(\ell) = 0\,, \qquad \varepsilon_\pm(\ell)\cdot\varepsilon_\mp(\ell) = -1\,,$$

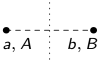
$$\varepsilon_0^2(\ell) = -1\,, \qquad \varepsilon_\pm(\ell)\cdot\varepsilon_0(\ell) = 0\,,$$

$$\varepsilon_\lambda(\ell)\cdot\ell = 0 \quad \lambda = \pm, 0.$$

The numerator of cut propagator of the scalar can be expressed in terms of the (-2ϵ)-SRs:

$$\tilde{g}^{\alpha\beta} + \frac{\mu^\alpha \mu^\beta}{\mu^2} \quad \rightarrow \quad \hat{G}^{AB} \equiv G^{AB} - Q^A Q^B \,.$$

The factor $\hat{G}^{AB}$ can be easily accounted for by defining the cut propagator as



$$= \hat{G}^{AB}\,\delta^{ab}\,.$$

The previous Feynman rules and cuts prescriptions fully reconstruct the μ^2 dependence of the $\epsilon-$ dimensional numerator of scattering amplitudes of renormalized gauge theories.

Four point massless one-loop color ordered amplitudes A_4

From the reduction theorem a dimensionally regularized A_4 is decomposed in a cut-constructible part and in a rational part ($\mathcal{R}$) expressed in terms of scalar integrals in $D = 4 - 2\epsilon$ dimensions. The coefficients c_i are rational functions of the external momenta and polarizations.

$$A_4 = \frac{1}{(4\pi)^{2-\epsilon}} \Big[c_{1|2|3|4;\,0}\, I_{1|2|3|4} + \big(c_{12|3|4;\,0}\, I_{12|3|4} + c_{1|2|34;\,0}\, I_{1|2|34} + c_{1|23|4;\,0}\, I_{1|23|4} + c_{2|3|41;\,0}\, I_{2|3|41} \big) + \big(c_{12|34;\,0}\, I_{12|34} + c_{23|41;\,0}\, I_{23|41} \big) \Big] + \mathcal{R} + O(\epsilon)\,,$$

$$\mathcal{R} = \frac{1}{(4\pi)^{2-\epsilon}} \Big[c_{1|2|3|4;\,4}\, I_{1|2|3|4}[\mu^4] + \big(c_{12|3|4;\,2}\, I_{12|34}[\mu^2] + c_{1|2|34;\,2}\, I_{1|2|34}[\mu^2] + c_{1|23|4;\,2}\, I_{1|23|4}[\mu^2] + c_{2|3|41;\,2}\, I_{2|3|41}[\mu^2] \big) + \big(c_{12|34;\,2}\, I_{12|34}[\mu^2] + c_{23|41;\,2}\, I_{23|41}[\mu^2] \big) \Big] .$$

By the separation

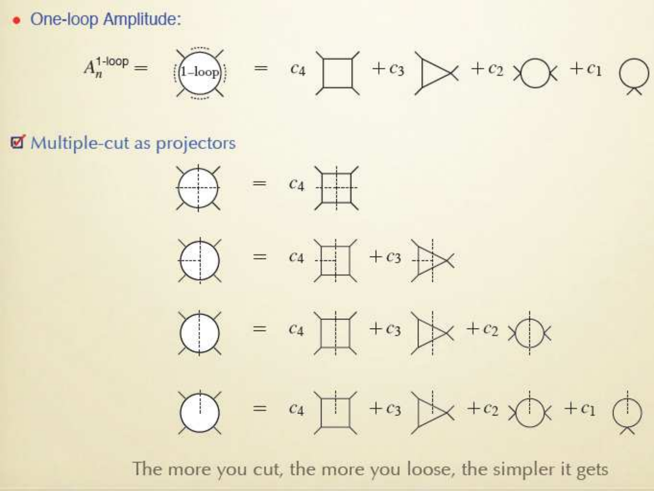
$$\int \frac{d^D \bar{\ell}}{(2\pi)^D} = \int \frac{d^{-\epsilon}(\mu^2)}{(2\pi)^{-2\epsilon}} \int \frac{d^4 \ell}{(2\pi)^4}.$$

and using polar coordinates in the -2ϵ dimensional Euclidean vector space, all the integrals in $\mathcal{R}$ can be computed. In particular

$$\lim_{\epsilon \to 0} I^{4-2\epsilon}_{1|2|3|4}[\mu^4] = \lim_{\epsilon \to 0} \left(\epsilon(\epsilon - 1) I^{8-2\epsilon}_{1|2|3|4} \right) = -\frac{1}{6}.$$

We found a way of computing the rational part of scattering amplitudes by unitarity cuts with loop momenta in $D = 4$.

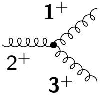
Generalized unitarity



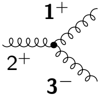
Inside our **FDF** scheme we are using the **generalized unitarity** to compute the full one loop amplitude including the **cut-constructible** and the full gauge invariant **rational part**.

The all helicity-plus four gluons planar amplitude with a gluonic loop

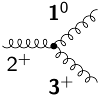
In order to reconstruct the full μ dependence and to obtain by cut construction the rational coefficients of the master integral decomposition, the following color-ordered trees amplitudes are needed. With all outgoing complex momenta



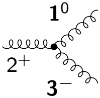
$$= 0\,,$$



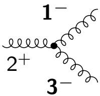
$$= ig\left(\frac{[\mathbf{1}^\flat|2][\hat{q}_\mathbf{1}|2]}{\mu} + \frac{\langle r_2|\mathbf{1}|2]}{\langle r_2|2\rangle}\right)$$



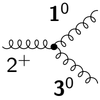
$$= 0\,,$$



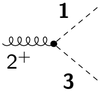
$$= \frac{\sqrt{2} i g \, [\hat{q}_{\mathbf{1}}|2]^2}{\mu} \, ,$$



$$= i g \, \frac{[\hat{q}_{\mathbf{1}}|2] \, [\hat{q}_{\mathbf{3}}|2] \, \langle \mathbf{1}^\flat | \mathbf{3}^\flat \rangle}{\mu^2} \, ,$$



$$= -\, i g \, \frac{\langle r_2 | \mathbf{1} | 2]}{\langle r_2 | 2 \rangle} \Big\{ 1 - \frac{(1+\xi)}{\xi \, \mu^2} \Big[(1+\xi) \, \mu^2 + \xi \, \langle \hat{q}_{\mathbf{1}} | 2 | \hat{q}_{\mathbf{1}}] \Big] \Big\} \, ,$$



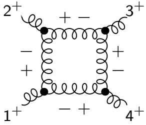
$$= \frac{i g}{\sqrt{2}} \, (\mathbf{3} - \mathbf{1})^\mu \, \varepsilon^+_\mu \, (2, r_2) \, G^{AB}$$

$$= - i g \, \frac{\langle r_2 | \mathbf{1} | 2]}{\langle r_2 | 2 \rangle} G^{AB}$$

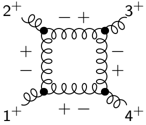
where $\hat{q}_3 = \xi \hat{q}_1$.

The box coefficients are obtained by the following attaching procedure, with the external legs of the trees on the generalized mass-shell

$C^{[0]}_{1|2|3|4} =$



$+$



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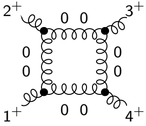


,

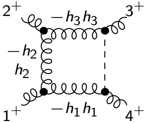
$$c^{[0]}_{1|2|3|4;\ 4} = 3g^4 i \frac{[12]\,[34]}{\langle 12\rangle\,\langle 34\rangle}$$

in which the relations

$$\langle \mathbf{j}^\flat | \hat{q}_\mathbf{j} \rangle = [\hat{q}_\mathbf{j} | \mathbf{j}^\flat] = \mu$$

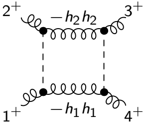
allow to obtain a polinomial numerator in μ.

$$C^{[1]}_{1|2|3|4} = \sum_{h_i=\pm,0} \mathcal{T}_1$$

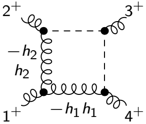


$$+\ \text{c.p.}\ ,$$

$$C^{[2]}_{1|2|3|4} = \sum_{h_i=\pm,0} \mathcal{T}_1^2$$

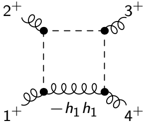


$$+\ \mathcal{T}_2$$



$$+\ \text{c.p.}\ ,$$

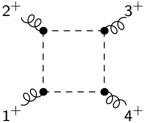
$$C^{[3]}_{1|2|3|4} = \sum_{h_1=\pm,0} \mathcal{T}_3$$



$$+\ \text{c.p.}\ ,$$

,

$$C^{[4]}_{1|2|3|4} = \mathcal{T}_4$$



$$\begin{aligned}
\mathcal{T}_1 &= Q^A \hat{G}^{AB} Q^B &&= \;\; 0\,,\\
\mathcal{T}_2 &= Q^A \hat{G}^{AB} G^{BC} \hat{G}^{CD} Q^D &&= \;\; 0\,,\\
\mathcal{T}_3 &= Q^A \hat{G}^{AB} G^{BC} \hat{G}^{CD} G^{DE} \hat{G}^{EF} Q^F &&= \;\; 0\,,\\
\mathcal{T}_4 &= \mathrm{tr}\left(G\, \hat{G}\, G\, \hat{G}\, G\, \hat{G}\, G\, \hat{G} \right) &&= -1\,.
\end{aligned}$$

$$c_{1|2|3|4;\,4} = c^{[0]}_{1|2|3|4;\,4} + c^{[4]}_{1|2|3|4;\,4} = 3g^4 i \frac{[12]\,[34]}{\langle 12\rangle\,\langle 34\rangle} - ig^4 \frac{[12]\,[34]}{\langle 12\rangle\,\langle 34\rangle}\,,$$

$$A_4^{1-\mathrm{loop}}\left(1_g^+, 2_g^+, 3_g^+, 4_g^+\right) = \frac{2ig^4}{16\pi^2} \times \left(-\frac{1}{6}\right) \times \frac{[12]\,[34]}{\langle 12\rangle \langle 34\rangle}\,.$$

The effective coupling of Higgs to gluons in NLO amplitudes

- For 2 gluons $\to$ Higgs we use an effective operator with $m_{\rm top} \to \infty$.

$$L_{\rm int} = \frac{C}{2} H \mathrm{Tr} F_{\mu\nu} F^{\mu\nu}$$

- The leading tree-level color ordered amplitude $0 \to gggH$

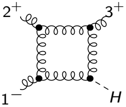
$$A_{4,H}^{\rm tree}(1^- 2^+ 3^+ H) = i \frac{[23]^4}{[12]\,[23]\,[31]}\,.$$

- As an example of application of our regularization scheme to an effective field theory consider at **NLO** in the large $m_{\rm top}$ limit the color ordered primitive amplitude

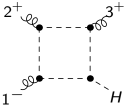
$$A_{4,H}^{\text{1-loop}}(1^- 2^+ 3^+ H)$$

To see just how the procedure works consider firstly some quadruple cuts:

$$C_{1|2|3|H} =$$



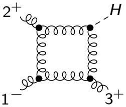
$$+$$



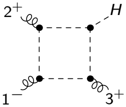
,

$$c_{1|2|3|H;\,0} = -\frac{1}{2} A_{4,H}^{\rm tree} s_{12} s_{23}\,,$$
$$c_{1|2|3|H;\,4} = 0\,;$$

$$C_{1|2|H|3} =$$



$$+$$

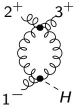


,

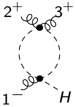
$$c_{1|2|H|3;\,0} = -\frac{1}{2} A_{4,H}^{\rm tree} s_{13} s_{12}\,,$$
$$c_{1|2|H|3;\,4} = 0\,.$$

......and by omitting for reasons of time many other contributions.....and just considering some among the double cuts

$$C_{23|H1} =$$



$+$



$$c_{23|H1;\,0} = 0$$

$$c_{23|H1;\,2} = 4A^{\text{tree}}_{4,H}\frac{s_{12}s_{13}}{s_{23}^3}\,.$$

By collecting all master integrals coefficients we recognize a full agreement with the the full Feynman diagrams calculations in the FDH scheme performed by Schmidt in 1997, the result is

$$
\begin{aligned}
A_4^{1-loop}\left(1^-,2^+,3^+,H\right) = r_\Gamma A_4^{tree} \times \\
&\left\{ \frac{1}{\epsilon^2}\left[(-s_{12})^{-\epsilon}+(-s_{13})^{-\epsilon}+(-s_{23})^{-\epsilon}\right] - \frac{\pi^2}{2} \right. \\
&+ \left[2\mathrm{Li}_2\left(1-\frac{s_{12}}{m_H^2}\right) + 2\mathrm{Li}_2\left(1-\frac{s_{13}}{m_H^2}\right) + 2\mathrm{Li}_2\left(1-\frac{s_{23}}{m_H^2}\right) \right] \\
&+ \left[\log\left(\frac{s_{12}}{m_H^2}\right)\log\left(\frac{s_{23}}{m_H^2}\right) + \log\left(\frac{s_{12}}{m_H^2}\right)\log\left(\frac{s_{13}}{m_H^2}\right) \right. \\
&\left. + \log\left(\frac{s_{13}}{m_H^2}\right)\log\left(\frac{s_{23}}{m_H^2}\right) \right] \\
&\left. - \frac{1}{3}\frac{s_{12}s_{13}+s_{12}s_{23}+s_{13}s_{23}}{m_H^4} + 1 \right\}
\end{aligned}
$$

Conclusions and perspectives

- A four-dimensional formulation (*FDF*) of dimensional regularization of one-loop scattering amplitudes has been applied to generalized unitarity techniques. At one loop the cut-constructible part and the rational part of scattering amplitudes have been computed by the same on-shell methods.
- The *FDF* Feynman rules have been extended to the recursive methods for generating the integrand of one-loop amplitudes.
- The inclusion of the fermion mass for a one loop amplitude like $0 \to gg t\bar{t}$ at one loop in *FDF* will be analysed.
- More loops and more jets in *FDF* is another goal to achieve.
- An important issue is to apply FDF for real corrections and corresponding subtraction terms of infrared divergences.