

# Exclusive diffractive processes with NLO accuracy in the shockwave framework

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To appear soon

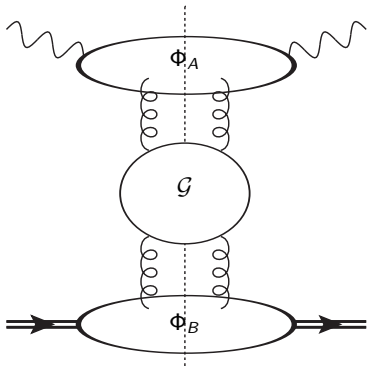
# Probing QCD in the Regge limit and towards saturation

## Observables to probe small- $x$ QCD and saturation physics

- Perturbation theory should apply : a **hard scale**  $Q^2$  is required
- One needs **semihard kinematics** :  $s \gg p_T^2 \gg \Lambda_{QCD}^2$  where all the typical transverse scales  $p_T$  are of the same order
- Saturation is reached when  $Q^2 \sim Q_s^2 \propto \left(\frac{A}{x}\right)^{\frac{1}{3}}$  : **the smaller  $x \sim \frac{Q^2}{s}$  is and the heavier the target ion, the easier saturation is reached.**
- Typical processes : DIS, Mueller-Navelet double jets, ultraperipheral events at the LHC...

# Precision tests of BFKL dynamics

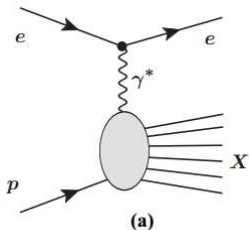
- BFKL and B-JIMWLK kernels are known at NLL accuracy, resumming  $\alpha_s(\alpha_s \log s)^n$  corrections (Lipatov, Fadin ; Camici, Ciafaloni ; Balitsky, Chirilli ; Kovner, Lublinsky, Mulian)
- Very few impact factors are known at NLO accuracy
  - $\gamma^* \rightarrow \gamma^*$  (Bartels, Colferai, Gieseke, Kyrielis, Qiao; Balitsky, Chirilli)
  - Forward jet production (Bartels, Colferai, Vacca ; Caporale, Ivanov, Murdaca, Papa, Perri ; Chachamis, Hentschinski, Madrigal, Sabio Vera)
  - Inclusive production of a pair of hadrons separated by a large interval of rapidity (Ivanov, Papa)
  - $\gamma_L^* \rightarrow V_L$  in the forward limit (Ivanov, Kostsky, Papa)



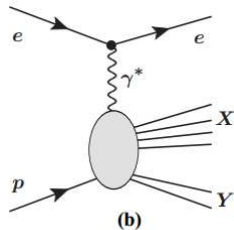
# Diffraction DIS

## Rapidity gap events at HERA

Experiments at HERA : about 10% of scattering events reveal a **rapidity gap**



DIS events



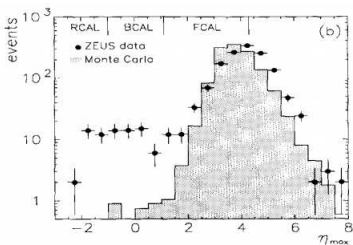
DDIS events

DIS : Deep Inelastic Scattering, DDIS : Diffractive DIS

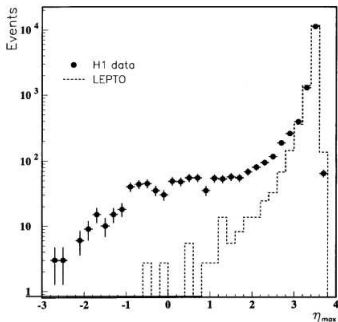
## Diffractive DIS

Rapidity gap events at HERA

Experiments at HERA : about 10% of events reveal a **rapidity gap**



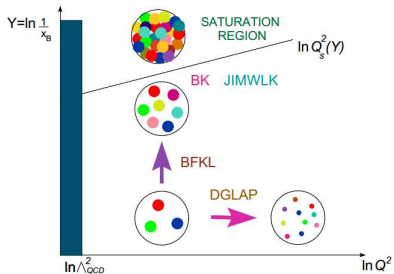
ZEUS, 1993



H1, 1994

# Diffraction DIS

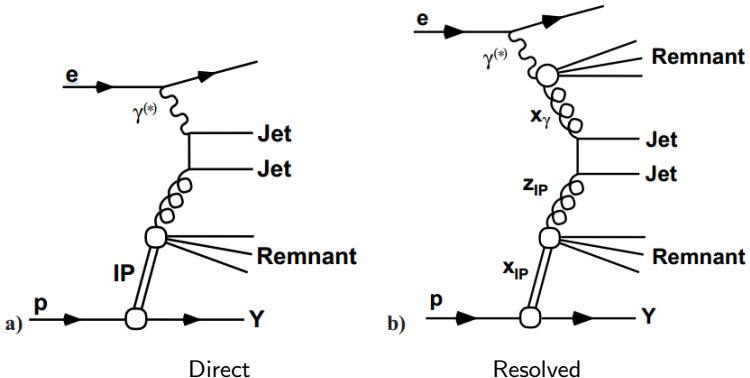
## Theoretical approaches for DDIS using pQCD



- **Collinear factorization approach**
  - Relies on a QCD factorization theorem, using a hard scale such as the **virtuality  $Q^2$**  of the incoming photon
  - One needs to introduce a **diffraction distribution function** for partons *within a pomeron*
- **$k_T$  factorization approach** for two exchanged gluons
  - low- $x$  QCD approach :  $s \gg Q^2 \gg \Lambda_{QCD}$
  - The pomeron is described as a **two-gluon color-singlet state**

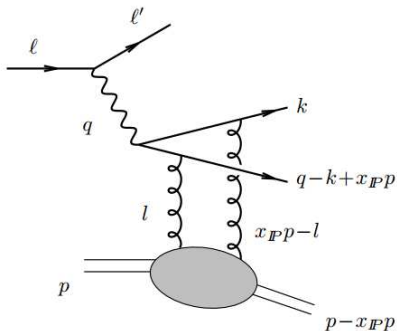
# Theoretical approaches for DDIS using pQCD

## Collinear factorization approach



# Theoretical approaches for DDIS using pQCD

$k_T$ -factorization approach : two gluon exchange



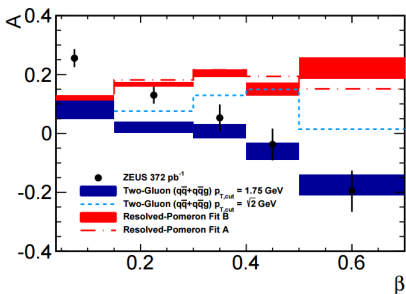
Bartels, Ivanov, Jung, Lotter, Wüsthoff

Braun and Ivanov developed a similar model in [collinear factorization](#)



# Theoretical approaches for DDIS using pQCD

## Confrontation of the two approaches with HERA data



ZEUS collaboration, 2015

# NLO computations in the **shockwave** framework

# The shockwave approach

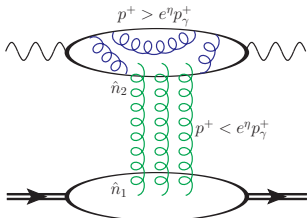
One decomposes the gluon field  $\mathcal{A}$  into an **internal field** and an **external field** :

$$\mathcal{A}^\mu = A^\mu + b^\mu$$

The internal one contains the gluons with  $p_g^+ > e^\eta p_\gamma^+$  and the external one contains the gluons with  $p_g^+ < e^\eta p_\gamma^+$ . One writes :

$$b_\eta^\mu(z) = \delta(z^+) B_\eta(\vec{z}) n_2^\mu$$

Intuitively, large boost  $\Lambda$  along the + direction :



$$b^+(x^+, x^-, \vec{x}) \rightarrow \frac{1}{\Lambda} b^+ \left( \Lambda x^+, \frac{1}{\Lambda} x^-, \vec{x} \right)$$

$$b^-(x^+, x^-, \vec{x}) \rightarrow \Lambda b^- \left( \Lambda x^+, \frac{1}{\Lambda} x^-, \vec{x} \right)$$

$$b^i(x^+, x^-, \vec{x}) \rightarrow b^i \left( \Lambda x^+, \frac{1}{\Lambda} x^-, \vec{x} \right)$$

Lightcone variables  $x^+ \equiv \frac{x^0 + x^3}{\sqrt{2}}$ ,  $x^- \equiv \frac{x^0 - x^3}{\sqrt{2}}$

# Propagator through a shockwave

$$G(z_2, z_0) = - \int d^4 z_1 \theta(z_2^+) \delta(z_1^+) \theta(-z_0^+) G(z_2 - z_1) \gamma^+ G(z_1 - z_0) U_1$$

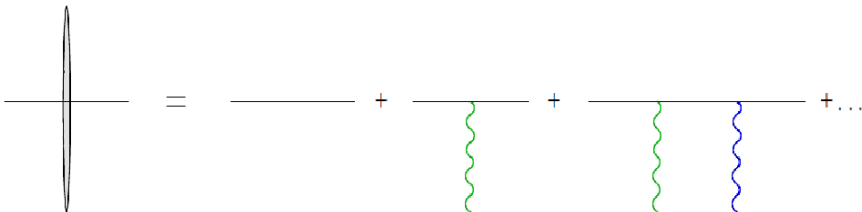
$$G(q, p) = (2\pi) \theta(p^+) \int d^D q_1 \delta(q + q_1 - p) G(q) \gamma^+ \tilde{U}_{q_1} G(p)$$

Wilson lines :

$$U_i = U_{\vec{z}_i} = U(\vec{z}_i, \eta) = P \exp \left[ ig \int_{-\infty}^{+\infty} b_{\eta}^{-}(z_i^+, \vec{z}_i) dz_i^+ \right]$$

$$U_i = 1 + ig \int_{-\infty}^{+\infty} b_{\eta}^{-}(z_i^+, \vec{z}_i) dz_i^+ + (ig)^2 \int_{-\infty}^{+\infty} b_{\eta}^{-}(z_i^+, \vec{z}_i) b_{\eta}^{-}(z_j^+, \vec{z}_j) \theta(z_{ji}^+) dz_i^+ dz_j^+$$

...



## Balitsky's hierarchy of equations

Dipole operator

$$\mathbf{U}_{12}^\eta = \frac{1}{N_c} \text{Tr} \left( U_1^\eta U_2^{\eta\dagger} \right) - 1$$

B-JIMWLK equation

[Balitsky, Jalilian-Marian, Iancu, McLerran, Weigert, Leonidov, Kovner]

$$\frac{\partial \mathbf{U}_{12}}{\partial \eta} = \frac{\alpha_s N_c}{2\pi^2} \int d\vec{z}_3 \frac{\vec{z}_{12}^2}{\vec{z}_{13}^2 \vec{z}_{23}^2} [\mathbf{U}_{13} + \mathbf{U}_{32} - \mathbf{U}_{12} - \mathbf{U}_{13} \mathbf{U}_{32}]$$

$$\frac{\partial \mathbf{U}_{13} \mathbf{U}_{32}}{\partial \eta} = \dots$$

# The BK equation

Mean field approximation, or 't Hooft limit  $N_c \rightarrow \infty$  in Balitsky's equation



⇒ **BK equation** [Balitsky, 1995] [Kovchegov, 1999]

$$\frac{\partial \mathbf{U}_{12}}{\partial \eta} = \frac{\alpha_s N_c}{2\pi^2} \int d\vec{z}_3 \frac{\vec{z}_{12}^2}{\vec{z}_{13}^2 \vec{z}_{23}^2} [\mathbf{U}_{13} + \mathbf{U}_{32} - \mathbf{U}_{12} - \mathbf{U}_{13} \mathbf{U}_{32}]$$

Evolves a **dipole** into a **double dipole**

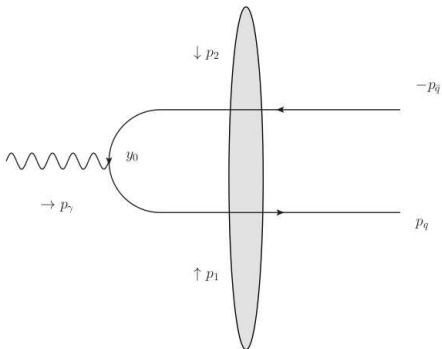
Non-linear term : **saturation**

# Assumptions

- Regge-Gribov limit :  $s \gg Q^2 \gg \Lambda_{QCD}$
- Otherwise **completely general kinematics**
- **Shockwave (CGC) Wilson line approach**
- **Transverse dimensional regularization  $d = 2 + 2\epsilon$ , longitudinal cutoff**

$$p_g^+ < \alpha p_\gamma^+$$

## Leading Order

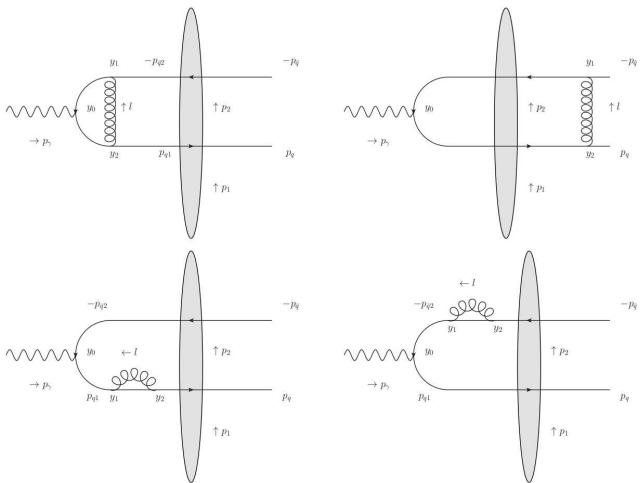


$$\mathcal{A}_0 = \varepsilon_\alpha N_c \int d^d \vec{p}_1 d^d \vec{p}_2 \Phi_0^\alpha(\vec{p}_1, \vec{p}_2) \delta(\vec{p}_{q1} + \vec{p}_{\bar{q}2}) \tilde{\mathbf{U}}_{12}$$

$$p_{ij} \equiv p_i - p_j$$

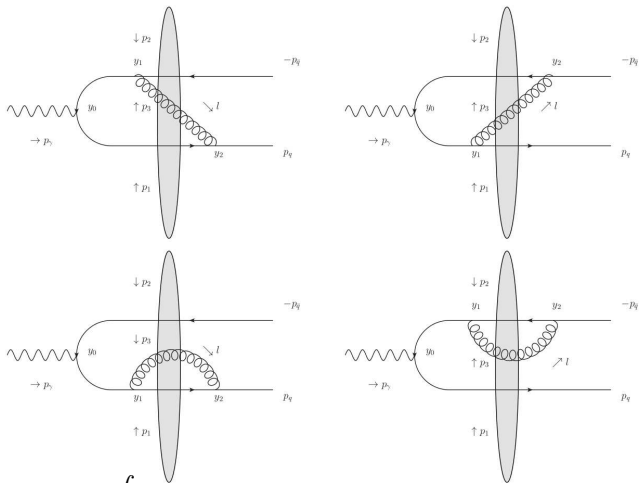


# First kind of virtual corrections



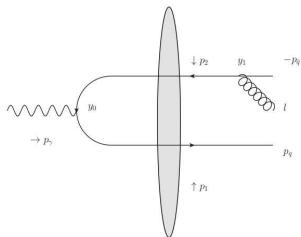
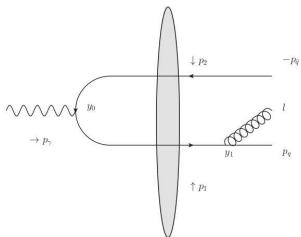
$$A_{V1} \propto \varepsilon_\alpha N_c \int d^d \vec{p}_1 d^d \vec{p}_2 \Phi_{V1}^\alpha(\vec{p}_1, \vec{p}_2) \delta(\vec{p}_{q1} + \vec{p}_{q2}) \left( \frac{N_c^2 - 1}{N_c} \right) \tilde{U}_{12}$$

## Second kind of virtual corrections



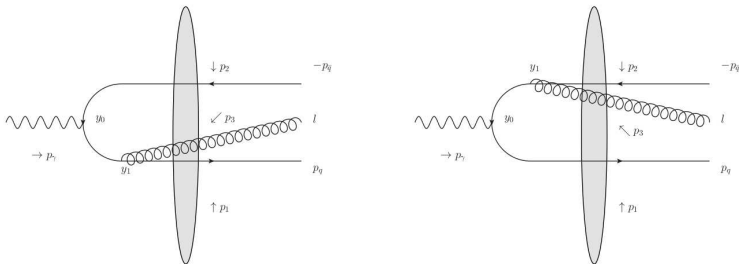
$$\mathcal{A}_{V2} \propto \varepsilon_\alpha N_c \int d^d \vec{p}_1 d^d \vec{p}_2 d^d \vec{p}_3 \Phi_{V2}^\alpha(\vec{p}_1, \vec{p}_2, \vec{p}_3) \delta(\vec{p}_{q1} + \vec{p}_{\bar{q}2} - \vec{p}_3) \left[ \delta(\vec{p}_3) \left( \frac{N_c^2 - 1}{N_c} \right) \tilde{\mathbf{U}}_{12} + N_c \left( \tilde{\mathbf{U}}_{13} \tilde{\mathbf{U}}_{32} + \tilde{\mathbf{U}}_{13} + \tilde{\mathbf{U}}_{32} - \tilde{\mathbf{U}}_{12} \right) \right]$$

## First kind of real corrections



$$\mathcal{A}_{R1} = \varepsilon_\alpha N_c \int d^d \vec{p}_1 d^d \vec{p}_2 \Phi_{R1}^\alpha(\vec{p}_1, \vec{p}_2) \delta(\vec{p}_{q1} + \vec{p}_{q2} + \vec{p}_g) \left( \frac{N_c^2 - 1}{N_c} \right) \tilde{\mathbf{U}}_{12}$$

## Second kind of real corrections

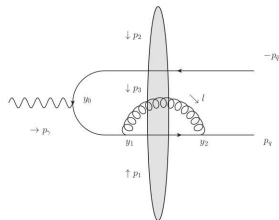
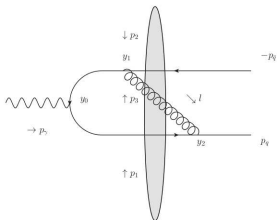


$$\begin{aligned}
 \mathcal{A}_{R2} = & \varepsilon_\alpha N_c \int d^d \vec{p}_1 d^d \vec{p}_2 d^d \vec{p}_3 \Phi_{R2}^\alpha(\vec{p}_1, \vec{p}_2, \vec{p}_3) \delta(\vec{p}_{q1} + \vec{p}_{\bar{q}2} + \vec{p}_{g3}) \\
 & \left[ \left( \frac{N_c^2 - 1}{N_c} \right) \tilde{\mathbf{U}}_{12} \delta(\vec{p}_3) + N_c \left( \tilde{\mathbf{U}}_{13} \tilde{\mathbf{U}}_{32} + \tilde{\mathbf{U}}_{13} + \tilde{\mathbf{U}}_{32} - \tilde{\mathbf{U}}_{12} \right) \right]
 \end{aligned}$$

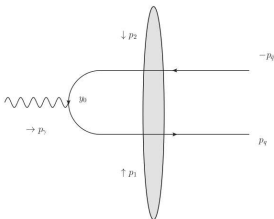
# Divergences

- Rapidity divergence  $p_g^+ \rightarrow 0$   $\Phi_{V2}\Phi_0^* + \Phi_0\Phi_{V2}^*$
- UV divergence  $\vec{p}_g^2 \rightarrow +\infty$   $\Phi_{V1}\Phi_0^* + \Phi_0\Phi_{V1}^*$
- Soft divergence  $p_g \rightarrow 0$   $\Phi_{V1}\Phi_0^* + \Phi_0\Phi_{V1}^*, \Phi_{R1}\Phi_{R1}^*$
- Collinear divergence  $p_g \propto p_q$  or  $p_{\bar{q}}$   $\Phi_{R1}\Phi_{R1}^*$
- Soft and collinear divergence  $p_g = \frac{p_g^+}{p_q^+} p_q$  or  $\frac{p_g^+}{p_{\bar{q}}^+} p_{\bar{q}}$ ,  $p_g^+ \rightarrow 0$   $\Phi_{R1}\Phi_{R1}^*$

# Rapidity divergence



Double dipole virtual correction  $\Phi_{V2}$



**B-JIMWLK evolution** of the LO term :  $\Phi_0 \otimes \mathcal{K}_{BK}$

## Rapidity divergence

## B-JIMWLK equation

$$\frac{\partial \tilde{\mathbf{U}}_{12}^\alpha}{\partial \log \alpha} = 2\alpha_s N_c \mu^{2-d} \int \frac{d^d \vec{k}_1 d^d \vec{k}_2 d^d \vec{k}_3}{(2\pi)^{2d}} \delta(\vec{k}_1 + \vec{k}_2 + \vec{k}_3 - \vec{p}_1 - \vec{p}_2) \left( \tilde{\mathbf{U}}_{13} \tilde{\mathbf{U}}_{32} + \tilde{\mathbf{U}}_{13} + \tilde{\mathbf{U}}_{32} - \tilde{\mathbf{U}}_{12} \right) \\ \times \left[ 2 \frac{(\vec{k}_1 - \vec{p}_1) \cdot (\vec{k}_2 - \vec{p}_2)}{(\vec{k}_1 - \vec{p}_1)^2 (\vec{k}_2 - \vec{p}_2)^2} + \frac{\pi^{\frac{d}{2}} \Gamma(1 - \frac{d}{2}) \Gamma^2(\frac{d}{2})}{\Gamma(d-1)} \left( \frac{\delta(\vec{k}_2 - \vec{p}_2)}{[(\vec{k}_1 - \vec{p}_1)^2]^{1-\frac{d}{2}}} + \frac{\delta(\vec{k}_1 - \vec{p}_1)}{[(\vec{k}_2 - \vec{p}_2)^2]^{1-\frac{d}{2}}} \right) \right]$$

$\eta$  **rapidity divide**, which separates the upper and the lower impact factors

$$\tilde{\mathbf{U}}_{12}^\alpha \Phi_0 \rightarrow \Phi_0 \tilde{\mathbf{U}}_{12}^\eta + \log \left( \frac{e^\eta}{\alpha} \right) \mathcal{K}_{BK} \Phi_0 \left( \tilde{\mathbf{U}}_{13} \tilde{\mathbf{U}}_{32} + \tilde{\mathbf{U}}_{13} + \tilde{\mathbf{U}}_{32} - \tilde{\mathbf{U}}_{12} \right)$$

# Rapidity divergence

Virtual contribution

$$(\Phi_{V2}^\mu)_{div} \propto \Phi_0^\mu \left\{ 4 \ln \left( \frac{x\bar{x}}{\alpha^2} \right) \left[ \frac{1}{\epsilon} + \ln \left( \frac{\vec{p}_3^2}{\mu^2} \right) \right] - \frac{6}{\epsilon} \right\}$$

BK contribution

$$(\Phi_{BK}^\mu)_{div} \propto \Phi_0^\mu \left\{ 4 \ln \left( \frac{\alpha^2}{e^{2\eta}} \right) \left[ \frac{1}{\epsilon} + \ln \left( \frac{\vec{p}_3^2}{\mu^2} \right) \right] \right\}$$

Sum : the  $\alpha$  dependence cancels

$$(\Phi'_{V2}{}^\mu)_{div} \propto \Phi_0^\mu \left\{ 4 \ln \left( \frac{x\bar{x}}{e^{2\eta}} \right) \left[ \frac{1}{\epsilon} + \ln \left( \frac{\vec{p}_3^2}{\mu^2} \right) \right] - \frac{6}{\epsilon} \right\}$$



# Rapidity divergence

Cancellation of the remaining  $1/\epsilon$  divergence

Convolution

$$\begin{aligned}
 (\Phi'_{V_2}{}^\mu \otimes \mathbf{UU}) &= \int d^d \vec{p}_1 d^d \vec{p}_2 d^d \vec{p}_3 \left\{ 4 \ln \left( \frac{x\bar{x}}{e^{2\eta}} \right) \left[ \frac{1}{\epsilon} + \ln \left( \frac{\vec{p}_3^2}{\mu^2} \right) \right] - \frac{6}{\epsilon} \right\} \\
 &\times \delta(\vec{p}_{q_1} + \vec{p}_{\bar{q}_2} - \vec{p}_3) \left[ \tilde{\mathbf{U}}_{13} + \tilde{\mathbf{U}}_{32} - \tilde{\mathbf{U}}_{12} - \tilde{\mathbf{U}}_{13} \tilde{\mathbf{U}}_{32} \right] \Phi_0^\mu(\vec{p}_1, \vec{p}_2)
 \end{aligned}$$

Rq :

- $\Phi_0(\vec{p}_1, \vec{p}_2)$  only depends on one of the  $t$ -channel momenta.
- The double-dipole operators **cancel**s when  $\vec{z}_3 = \vec{z}_1$  or  $\vec{z}_3 = \vec{z}_2$ .

This permits one to show that the convolution **cancel**s the remaining  $\frac{1}{\epsilon}$  **divergence**.

Then  $\tilde{\mathbf{U}}_{12}^\alpha \Phi_0 + \Phi_{V_2}$  is **finite**

# Divergences

- Rapidity divergence

- UV divergence  $\vec{p}_g^2 \rightarrow +\infty$

$$\Phi_{V1}\Phi_0^* + \Phi_0\Phi_{V1}^*$$

- Soft divergence  $p_g \rightarrow 0$

$$\Phi_{V1}\Phi_0^* + \Phi_0\Phi_{V1}^*, \Phi_{R1}\Phi_{R1}^*$$

- Collinear divergence  $p_g \propto p_q$  or  $p_{\bar{q}}$

$$\Phi_{R1}\Phi_{R1}^*$$

- Soft and collinear divergence  $p_g = \frac{p_g^+}{p_q^+} p_q$  or  $\frac{p_g^+}{p_{\bar{q}}^+} p_{\bar{q}}$ ,  $p_g^+ \rightarrow 0$

$$\Phi_{R1}\Phi_{R1}^*$$

## Constructing a finite cross section

Exclusive diffractive production of a forward dijet

From partons to jets

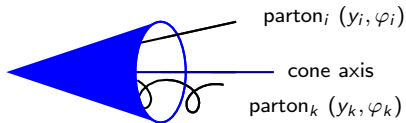
# Soft and collinear divergence

## Jet cone algorithm

We define a **cone** width for each pair of particles with momenta  $p_i$  and  $p_k$ , rapidity difference  $\Delta Y_{ik}$  and relative azimuthal angle  $\Delta\varphi_{ik}$

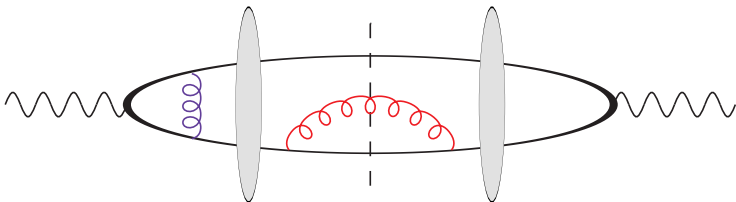
$$(\Delta Y_{ik})^2 + (\Delta\varphi_{ik})^2 = R_{ik}^2$$

If  $R_{ik}^2 < R^2$ , then the two particles together define a **single jet** of momentum  $p_i + p_k$ .



Applying this in the small  $R^2$  limit cancels our **soft and collinear** divergence.

# Remaining divergence



- UV divergence  $\vec{p}_g^2 \rightarrow +\infty$

$$\Phi_{V1}\Phi_0^* + \Phi_0\Phi_{V1}^*$$

- Soft divergence  $p_g \rightarrow 0$

$$\Phi_{V1}\Phi_0^* + \Phi_0\Phi_{V1}^*, \Phi_{R1}\Phi_{R1}^*$$

- Collinear divergence  $p_g \propto p_q$  or  $p_{\bar{q}}$

$$\Phi_{R1}\Phi_{R1}^*$$

# Remaining divergence

## Soft real emission

$$(\Phi_{R1} \Phi_{R1}^*)_{soft} \propto (\Phi_0 \Phi_0^*) \int_{\text{outside the cones}} \left| \frac{p_q^\mu}{(p_q \cdot p_g)} - \frac{p_{\bar{q}}^\mu}{(p_{\bar{q}} \cdot p_g)} \right|^2 \frac{dp_g^+}{p_g^+} \frac{d^d p_g}{(2\pi)^d}$$

## Collinear real emission

$$(\Phi_{R1} \Phi_{R1}^*)_{col} \propto (\Phi_0 \Phi_0^*) (\mathcal{N}_q + \mathcal{N}_{\bar{q}})$$

Where  $\mathcal{N}$  is the number of jets in the quark or the antiquark

$$\mathcal{N}_k = \frac{(4\pi)^{\frac{d}{2}}}{\Gamma(2 - \frac{d}{2})} \int_{\alpha p_\gamma^+}^{p_{jet}^+} \frac{dp_g^+ dp_k^+}{2p_g^+ 2p_k^+} \int_{\text{in cone } k} \frac{d^d \vec{p}_g d^d \vec{p}_k}{(2\pi)^d \mu^{d-2}} \frac{\text{Tr}(\hat{p}_k \gamma^\mu \hat{p}_{jet} \gamma^\nu) d_{\mu\nu}(p_g)}{2p_{jet}^+ (p_k^- + p_g^- - p_{jet}^-)^2}$$

Those two contributions **cancel exactly the virtual divergences** (both UV and soft)

# Cancellation of divergences

## Total divergence

$$(d\sigma_1)_{div} = \alpha_s \frac{\Gamma(1-\epsilon)}{(4\pi)^{1+\epsilon}} \left( \frac{N_c^2 - 1}{2N_c} \right) (S_V + S_V^* + S_R + \mathcal{N}_{jet1} + \mathcal{N}_{jet2}) d\sigma_0$$

## Virtual contribution

$$S_V = \left[ 2 \ln \left( \frac{x_j x_{\bar{j}}}{\alpha^2} \right) - 3 \right] \left[ \ln \left( \frac{x_j x_{\bar{j}} \mu^2}{(x_j \vec{p}_{\bar{j}} - x_{\bar{j}} \vec{p}_j)^2} \right) - \frac{1}{\epsilon} \right]$$

$$+ 2i\pi \ln \left( \frac{x_j x_{\bar{j}}}{\alpha^2} \right) + \ln^2 \left( \frac{x_j x_{\bar{j}}}{\alpha^2} \right) - \frac{\pi^2}{3} + 6$$

## Real contribution

$$S_R + \mathcal{N}_{jet1} + \mathcal{N}_{jet2} = 2 \left[ \ln \left( \frac{(x_{\bar{j}} \vec{p}_j - x_j \vec{p}_{\bar{j}})^4}{x_{\bar{j}}^2 x_j^2 R^4 \vec{p}_{\bar{j}}^2 \vec{p}_j^2} \right) \ln \left( \frac{4E^2}{x_j x_{\bar{j}} (\rho_{\gamma}^+)^2} \right) \right.$$

$$+ 2 \ln \left( \frac{x_{\bar{j}} x_j}{\alpha^2} \right) \left( \frac{1}{\epsilon} - \ln \left( \frac{x_{\bar{j}} x_j \mu^2}{(x_{\bar{j}} \vec{p}_j - x_j \vec{p}_{\bar{j}})^2} \right) \right) - \ln^2 \left( \frac{x_{\bar{j}} x_j}{\alpha^2} \right)$$

$$\left. + \frac{3}{2} \ln \left( \frac{16\mu^4}{R^4 \vec{p}_{\bar{j}}^2 \vec{p}_j^2} \right) - \ln \left( \frac{x_j}{x_{\bar{j}}} \right) \ln \left( \frac{x_j \vec{p}_{\bar{j}}^2}{x_{\bar{j}} \vec{p}_j^2} \right) - \frac{3}{\epsilon} - \frac{2\pi^2}{3} + 7 \right]$$

# Cancellation of divergences

## Total divergence

$$\begin{aligned}div &= S_V + S_V^* + S_R + \mathcal{N}_{jet1} + \mathcal{N}_{jet2} \\ &= 4 \left[ \frac{1}{2} \ln \left( \frac{(x_j \vec{p}_j - x_{\bar{j}} \vec{p}_{\bar{j}})^4}{x_j^2 x_{\bar{j}}^2 R^4 \vec{p}_j^2 \vec{p}_{\bar{j}}^2} \right) \left( \ln \left( \frac{4E^2}{x_j x_{\bar{j}} (p_\gamma^+)^2} \right) + \frac{3}{2} \right) \right. \\ &\quad \left. + \ln(8) - \frac{1}{2} \ln \left( \frac{x_j}{x_{\bar{j}}} \right) \ln \left( \frac{x_j \vec{p}_j^2}{x_{\bar{j}} \vec{p}_{\bar{j}}^2} \right) + \frac{13 - \pi^2}{2} \right]\end{aligned}$$

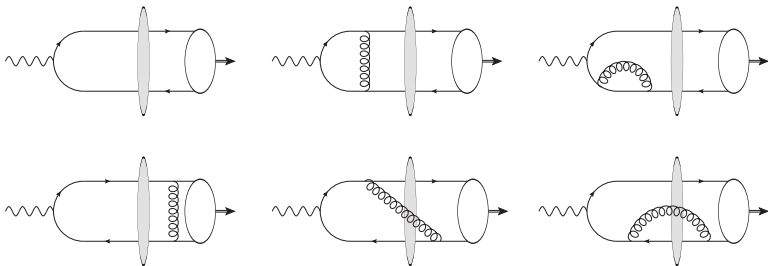
Our cross section is thus **finite**



## Constructing a **finite amplitude**

Exclusive diffractive production of a light neutral vector meson

## Exclusive diffractive production of a light neutral vector meson



$$\begin{aligned}
 \mathcal{A}_0 &= -\frac{e_V f_V \varepsilon_\beta}{N_c} \int_0^1 dx \varphi_{\parallel}(x) \int \frac{d^d \vec{p}_1}{(2\pi)^d} \frac{d^d \vec{p}_2}{(2\pi)^d} \\
 &\times (2\pi)^{d+1} \delta(p_V^+ - p_\gamma^+) \delta(\vec{p}_V - \vec{p}_\gamma - \vec{p}_1 - \vec{p}_2) \\
 &\times \Phi_0^\beta(x, \vec{p}_1, \vec{p}_2) \tilde{\mathbf{U}}_{12}.
 \end{aligned}$$

Leading twist for a longitudinally polarized meson

Otherwise **general kinematics**, including transverse virtual photon (twist 3) contributions, and the photoproduction limit (for large  $t$ -channel momentum transfer)

## ERBL evolution equation

Evolution equation for the distribution amplitude in the  $\overline{MS}$  scheme

$$\frac{\partial \varphi(x, \mu^2)}{\partial \ln \mu^2} = -\frac{\alpha_s C_F}{2\pi} \frac{\Gamma(1-\epsilon)}{(4\pi)^\epsilon} \int_0^1 dz \varphi(z, \mu^2) \mathcal{K}(x, z),$$

where we parameterize the **ERBL kernel** for consistency as

$$\begin{aligned} \mathcal{K}(x, z) &= \frac{x}{z} \left[ 1 + \frac{1}{z-x} \right] \theta(z-x-\alpha) \\ &+ \frac{1-x}{1-z} \left[ 1 + \frac{1}{x-z} \right] \theta(x-z-\alpha) \\ &+ \left[ \frac{3}{2} - \ln \left( \frac{x(1-x)}{\alpha^2} \right) \right] \delta(z-x). \end{aligned}$$

It is **equivalent to the usual ERBL kernel**

## Infrared finiteness

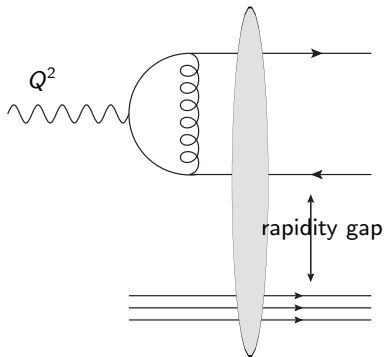
The amplitude we obtain is finite. For example the dipole  $\gamma_L^* \rightarrow V_L$  contribution reads

$$\begin{aligned}
 \Phi_1^+(x) &= \int_0^x dz \left( \frac{x-z}{x} \right) \Phi_0^+(x-z) \\
 &\times \left[ 1 + \left( 1 + \left[ \frac{1}{z} \right]_+ \right) \ln \left( \frac{\left( ((\bar{x}+z)\vec{p}_1 - (x-z)\vec{p}_2)^2 + (x-z)(\bar{x}+z)Q^2 \right)^2}{\mu_F^2(x-z)(\bar{x}+z)Q^2} \right) \right] \\
 &+ \frac{1}{2} \Phi_0^+(x) \left[ \frac{1}{2} \ln^2 \left( \frac{\bar{x}}{x} \right) + 3 - \frac{\pi^2}{6} - \frac{3}{2} \ln \left( \frac{\left( (\bar{x}\vec{p}_1 - x\vec{p}_2)^2 + x\bar{x}Q^2 \right)^2}{x\bar{x}\mu_F^2Q^2} \right) \right] \\
 &+ \frac{(p_\gamma^+)^2}{2x\bar{x}} \int_0^x dz \left[ (\phi_5)_{LL} |_{\vec{p}_3=\vec{0}} + (\phi_6)_{LL} |_{\vec{p}_3=\vec{0}} \right]_+ + (x \leftrightarrow \bar{x}, \vec{p}_1 \leftrightarrow \vec{p}_2).
 \end{aligned}$$

**No end point singularity**, even for a transverse photon and even in the photoproduction limit.

# General amplitude

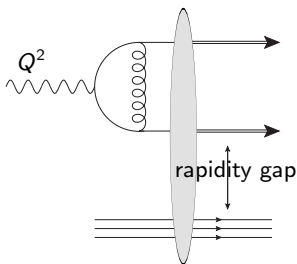
- Most general kinematics
- The hard scale can be  $Q^2$ ,  $t$ ,  $M_X^2$ ...
- The target can be either a **proton** or an **ion**, or another impact factor.
- **Finite results for  $Q^2 = 0$**
- One can study **ultraperipheral collision** by tagging the particle which emitted the photon, in the limit  $Q^2 \rightarrow 0$ .



The general amplitude

## Phenomenological applications : exclusive dijet production at NLO accuracy

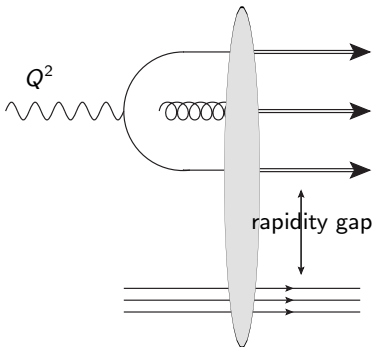
- HERA data for exclusive dijet production in diffractive DIS can be fitted with our results
- We can also give predictions for the same process in a future electron-ion or electron-proton collider (EIC, LHeC...)
- For  $Q^2 = 0$  we can give predictions for ultraperipheral  $pp$  and  $pA$  collisions at the LHC



Amplitude for diffractive dijet production

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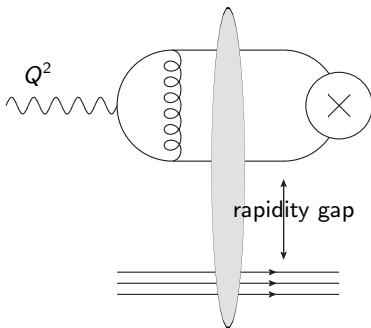


Amplitude for diffractive trijet production

[Ayala, Hentschinski, Jalilian-Marian, Tejeda-Yeomans]

# Phenomenological applications

- Most general kinematics
- The hard scale can be  $Q^2$ ,  $t$  or  $m^2$ .
- The target can be either a **proton** or an **ion**, or another impact factor.
- **Finite results for  $Q^2 = 0$**
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Amplitude for diffractive  $V$  production



## Comparison with previous results [Work in progress]

- The  $\gamma_L^* \rightarrow V_L$  contribution in the **forward limit** should coincide with previous results of **Ivanov, Kotsky, Papa**
- The comparison is non-trivial due to additional contributions from the formal BFKL/BK transition

$$\Phi_{BK} \otimes \mathcal{K}_{BK} \otimes \Phi'_{BK} = (\Phi_{BFKL} \otimes \mathcal{O})(\mathcal{O}^{-1} \otimes \mathcal{K}_{BFKL} \mathcal{O})(\mathcal{O}^{-1} \otimes \Phi'_{BFKL})$$

$\mathcal{O}$  was obtained to prove the **kernel equivalence**, but never checked on an impact factor

## Conclusion

- We provided the **full computation** of the impact factor for the exclusive diffractive production of a forward dijet and of a light neutral vector meson with **NLO accuracy** in the **shockwave approach**
- It leads to an enormous number of possible phenomenological applications to test QCD in its Regge limit and towards saturation **in past, present and future  $ep$ ,  $eA$ ,  $pp$  and  $pA$  colliders**
- Several theoretical extensions could be obtained with slight modifications to our result