Properties of the BFKL Amplitude with Running Coupling

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Eigenfunctions of BFKL Kernel with running coupling

$$\int dt' \sqrt{\alpha_s(t)}\,\mathcal{K}(t,t')\sqrt{\alpha(t')} f_\omega(t') = \omega f_\omega(t)$$

$$\left(t = \ln(k_T^2/\Lambda^2\right)$$

Boundary Conditions:

$$f_\omega(t) \stackrel{t\to\infty}{\longrightarrow} \to 0$$

$$f_\omega(t) \stackrel{t\to-\infty}{\longrightarrow} \propto \sin\left(\nu_0 t + \phi_\omega\right)$$

ϕ_ω is determined by the IR properties of QCD

Solution in semi-classical approximation

$$f_\omega(t) \sim \sqrt{\frac{1}{\alpha_s(t)\omega}} Ai(z(t))$$

where

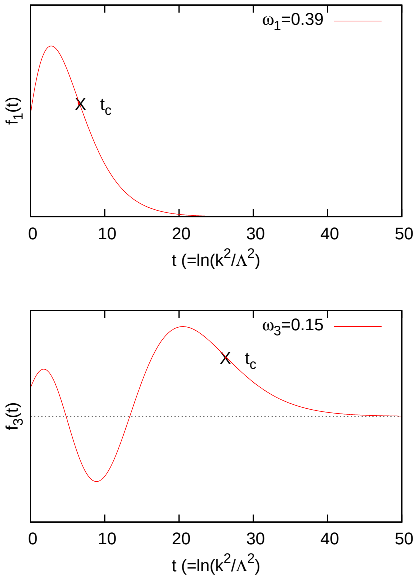
$$z(t) \; = \; \left[\frac{3}{2}\int_{t_c}^{t} \nu_\omega(t')dt'\right]^{2/3}$$

$$\omega \; = \alpha_s(t)\left[-\gamma_E - \Re e\left\{\Psi\left(\frac{1}{2} + i\nu_\omega(t)\right)\right\}\right]$$

Ai is Airy function which obeys UV boundary condition

$$Ai(z(t)) \xrightarrow{t\to\infty} 0$$

Infrared condition can only be satisfied for specific values of ω - discrete spectrum



ω1=0.39
f1(t)
X tc
ω2=0.21
f2(t)
X tc
ω3=0.15
f3(t)
X tc
ω4=0.11
f4(t)
X tc
0 10 20 30 40 50
t (=ln(k2/Λ2)

As t increases, leading poles are attenuated and sub-leading poles become more important.

This means that the rate of rise of cross-section with rapidity diminishes for large increasing transverse momentum.

Fitting Structure Functions at low-x

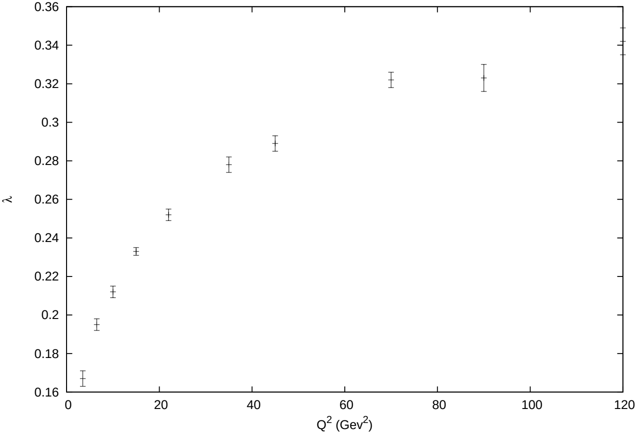
A popular parameterization of structure functions:

$$F_2(x,Q^2) = A(Q^2)x^{-\lambda(Q^2)}$$

Not motivated by either a BFKL or DGLAP analysis, but nevertheless seems to work very well (Caldwell, 2015)
To match the discrete BFKL pomeron we need to find a fit such that

$$A(Q^2)x^{-\lambda(Q^2)} \approx \sum_n C_n f_{\omega_n}(Q^2)x^{-\omega_n}$$

Since $f_\omega(t)$ decreases for sufficiently large t, we expect that for sufficiently large Q^2, $\lambda(Q^2)$ is a decreasing function of Q^2.

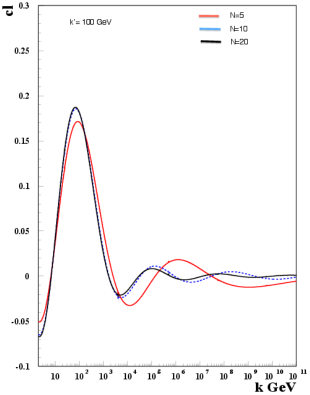


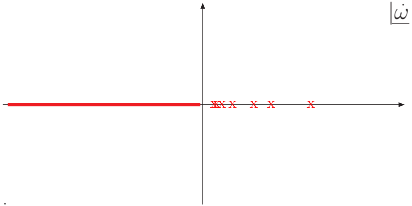
0.36
0.34
0.32
0.3
0.28
0.26
0.24
0.22
0.2
0.18
0.16
λ
0
20
40
60
80
100
120
Q2 (Gev2)

Completeness

The eigenfunctions are orthonormal, but they do NOT obey the completeness relation:

$$\lim_{N\to\infty}\sum_{n=1}^{N} f_n(t) f_n^*(t') = \delta(t-t')$$





As well as a set of discrete poles for positive ω there is a cut along negative real axis.

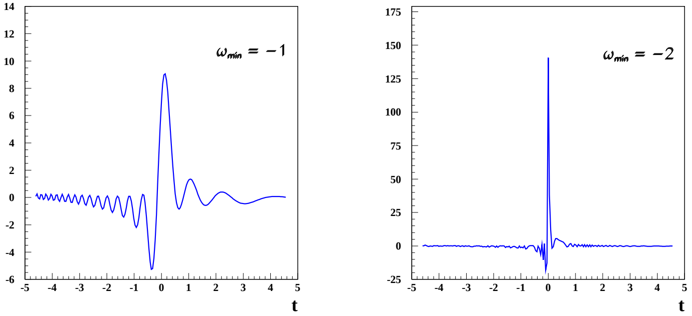
The contribution from this cut is needed in order to reproduce the required completeness relation.

This cut contribution generates a small but non-negligible contribution to unintegrated gluon density for low-x (despite $x^{-|\omega|}$ suppression).

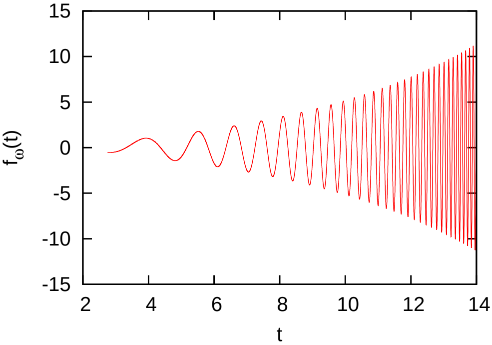
Improved Completeness relation

Include the continuum of states with $\omega < 0$

$$\lim_{N\to\infty}\sum_{n=1}^{N} f_n(t) f_n^*(t') + \lim_{\omega_{min}\to -\infty}\int_{\omega_{min}}^{0} f_{-|\omega|}(t) f_{-|\omega|}^*(t') \; = \; \delta(t-t')$$



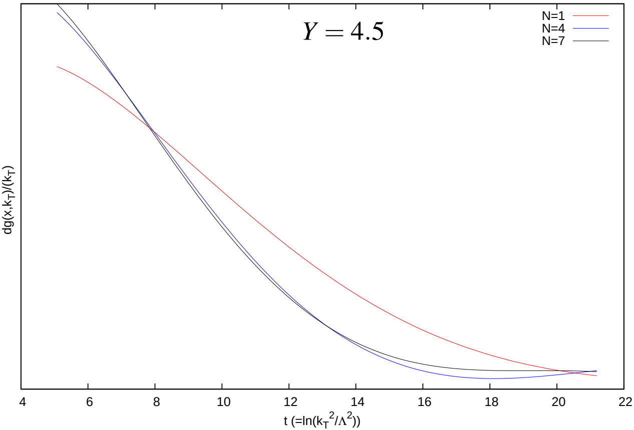
For large t (Q^2) the continuum of states with negative ω becomes more relevant.



Negative ω contributions are difficult to handle numerically as they acquire higher frequency oscillations with larger amplitudes as t increases

Convergence for Sum Over the First Few Poles

However a sum over the discrete, positive ω poles converges rapidly up to very large t for rapidity $Y >\sim 4$

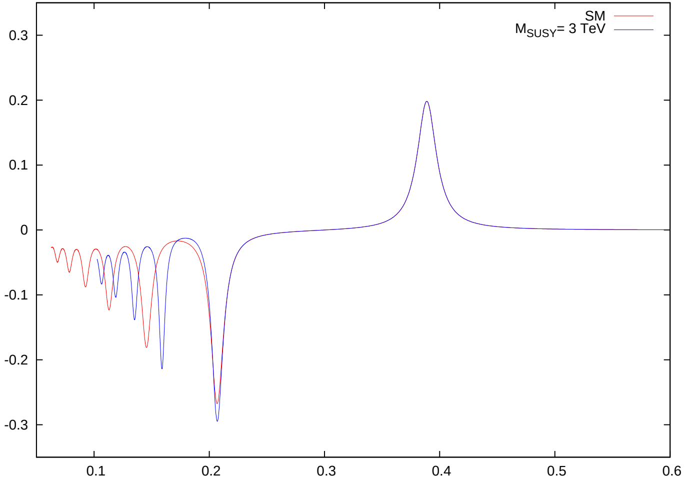


Detecting Signals For New Physics at Very High Energy

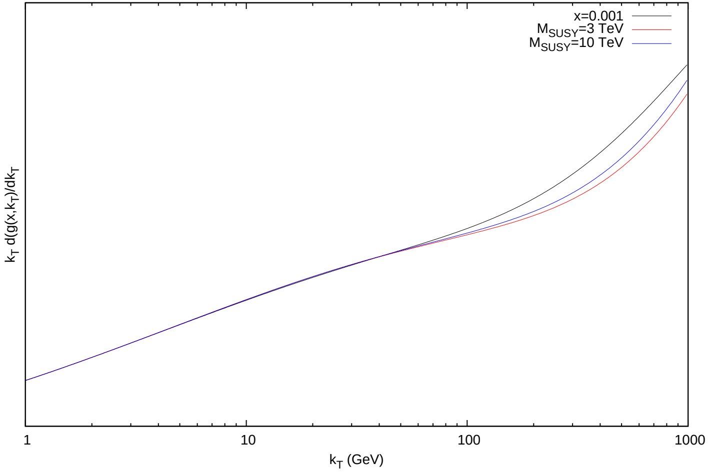
New physics at energy scale E_{thresh} alters the running of the coupling,
(and also affect the NLO characteristic function)
This, in turn, affects the value of t_c at which the eigenfunction changes behaviour from oscillation to decay.
This affects both the positions and the residues of the poles at $\omega = \omega_n$ in the case where

$$t_c(n) > \ln\left(\frac{E_{thresh}^2}{\Lambda^2}\right)$$

Effect on Position of Poles and Residues at $k_T = 100$ GeV from SUSY at 3 TeV



Effect of SUSY on Unintegrated Gluon Density at $x = 0.001$



CONCLUSIONS

- The eigenfunctions of the BFKL operator with running coupling, (eigenvalue ω) oscillate with t ($\equiv \ln(k_T^2/\Lambda^2)$) for $t < t_c(\omega)$ and decay exponentially for $t > t_c(\omega)$
- The Green function for the BFKL equation with running coupling is not uniquely defined in the infrared region ($t \ll t_c(\omega)$.
- For poles n for which $t > t_c(\omega_n)$, the residues decrease with increasing t so that the sub-leading poles (larger n) dominate.
- For poles n for which $t < t_c(\omega_n)$ the residues oscillate with t.
- This implies that $\lambda(Q^2)$ in the low-x parameterization $x^{-\lambda(Q^2)}$ should decrease with large increasing Q^2. (data shows a hint that this is the case)
- For sufficiently large t all the discrete solution (positive ω) decay away and the continuum of states with negative ω will start to dominate over the (Regge) poles in diffraction amplitudes. Such values of g correspond to gluon transverse momenta, which are much larger than current experimental reach, but even at current energies the contribution from the continuum cannot be neglected.

CONCLUSIONS

- The effect on the running coupling from possible new physics at very high energies affects both the positions and the residues of the subleading poles - since for these poles t_c is greater than the threshold for new physics.
- The modification of the positions and residues of the poles feeds down to a modification of the BFKL amplitude for the unintegrated gluon density in such a way that the quality of a fit to data could be affected by the presence of such new physics at very high energies.