

# Manifestations of BFKL-evolution effects at collider energies

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## Outline:

- **Motivation: high energy asymptotics**
- **BFKLP: NLA BFKL within generalized BLM**
- **$\gamma^*\gamma^*$ - collisions**
- **Dijets from QCD dynamics: GLAPD vs. BFKL**
- **Forward dijets at LHC: dijet “K-factor” vs  $|\eta|$**
- **Forward dijets at LHC: azimuthal decorrelations vs  $|\eta|$**
- **Summary**

# High energy asymptotics

## - Large-angle scattering:

### QCD in Bjorken limit

- **GLAPD: V. Gribov & L. Lipatov (71-72); L. Lipatov (74);  
G. Altarelli & G. Parisi (77); Yu. Dokshitzer (77)**

## - Small-angle scattering:

### QED in Gribov-Regge limit

- **V. Gribov, V. Gorshkov, L. Lipatov & G. Frolov (67-70)  
H. Cheng & T. Wu (66-70)**

### QCD in Gribov-Regge limit

- **BFKL: V. Fadin, E. Kuraev & L. Lipatov (75-78)  
I. Balitsky & L. Lipatov (78)**

# High-energy QCD asymptotics: GLAPD and BFKL

$$s=(p_1+p_2)^2$$
$$t=(p_1-p_3)^2 \quad Q^2=-t$$

Scattering in the Standard Model (QCD) at high energies:

Large logarithms: as  $\log(s)$ , as  $\log(Q^2)$

**Bjorken limit (large-angle scattering):**

$$s \sim Q^2 \gg m^2$$

$$Q^2/s = x \sim 1$$

**Gribov-Lipatov-Altarelli-Parisi-Dokshitzer (GLAPD):**

(as  $\log(Q^2)$ )<sup>n</sup> resummation

Inclusive cross section  $\sim 1/Q^4$

**Gribov-Regge limit (small-angle scattering):**

$$s \gg Q^2 \gg m^2$$

$$Q^2/s = x \Rightarrow 0$$

**Balitsky-Fadin-Kuraev-Lipatov (BFKL):**

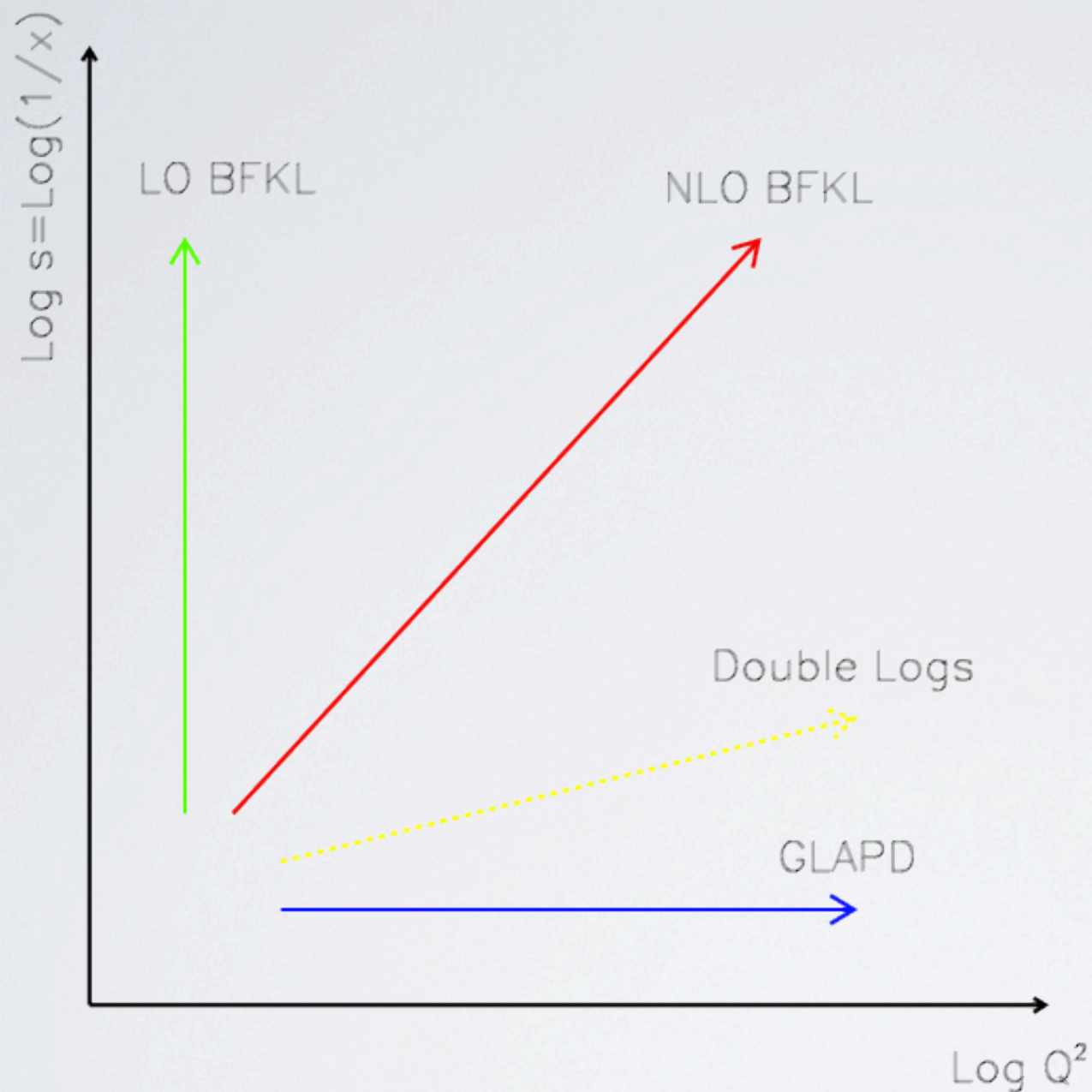
(as  $\log(s)$ )<sup>n</sup> resummation

Total cross section  $\sim s^{(a_P-1)}$

$a_P$  – Pomeron intercept

soft scattering data:  $a_P = 1.1$

# x-section asymptotics



**Bjorken limit (GLAPD):**

$$s \sim Q^2 \gg m^2$$

$$Q^2/s = x \sim 1$$

**Large-angle (large-x) scattering**

**Gribov-Regge limit (BFKL):**

$$s \gg Q^2 \gg m^2$$

$$Q^2/s = x \rightarrow 0$$

**Small-angle (small-x) scattering**

# Leading Log (LL) BFKL: problems

**LL BFKL: designed for infinite collision energies**

**LL BFKL problems (at finite energies):**

- **fixed (non-running) coupling  $\alpha_s$**
- **energy-momentum conservation**
- **transverse momentum conservation**

**Cross section in LL BFKL:**

$$\sigma = \sigma_0 (S/S_0)^{(\alpha_P - 1)}$$

$$\alpha_P = 1 + C \alpha_s \approx 1.5 - 1.6$$

**Data:  $\alpha_P \approx 1.2 - 1.3$**

# **BFKL: next-to-leading logs (NLL)**

**V.S. Fadin & L.N. Lipatov (89-98)**

**C. Camici & M. Ciafaloni (96-98)**

**next-to-leading log approximation (NLL) BFKL**  
**MSbar-renormalization scheme: large corrections**

**S.J. Brodsky, V.S. Fadin, VK, L.N. Lipatov, G.B. Pivovarov (98-99) BFKLP**

**BFKLP: NLL BFKL + resummation of running coupling  $\alpha_s$**

**(Brodsky, Lepage & Mackenzie - 83) BLM approach**

# NLL BFKL: BLM?

**S. Brodsky, P. Lepage & P. Mackenzie (83) BLM approach for NLO**

- **QCD – asymptotically conformal**
- **non-conformal corrections (running coupling corrections) are resummed into optimal scale**

**BLM in high orders: S. Mikhailov & A. Kataev (2015),  
PMC - S. Brodsky et al. (2012-15)**

**Naïve BLM application at NLO does not work (!):**

- **NLL BFKL in  $\overline{\text{MS}}$  scheme**
- **Upsilon  $\rightarrow$  ggg decay in NLO in  $\overline{\text{MS}}$  scheme**

**$\overline{\text{MS}}$ -renormalization scheme: nonphysical RG scheme (!)**

**S. Brodsky, Rathmann et al (1997)**



# **BFKLP: NLL BFKL within generalized BLM**

**Naïve BLM application does not work (!):**

- **NLL BFKL in  $\overline{MS}$  scheme**
- **Upsilon  $\rightarrow$  ggg decay in NLO in  $\overline{MS}$  scheme**

**$\overline{MS}$ -scheme: nonphysical RG scheme (!)**

**numerically close to V-scheme (heavy quark potential) – Abelian in LO**

**physical RG scheme: MOM scheme (gauge dependent)**

- **NLL BFKL in non-Abelian in LO**
- **Upsilon  $\rightarrow$  ggg decay in non-Abelian in LO**

**one can use MOM-scheme based on ggg-vertex non-Abelian in LO**

**BLM generalized on non-Abelian case:**

**S.J. Brodsky, V.S. Fadin, VK, L.N. Lipatov, G.B. Pivovarov(98-99) BFKLP**

**BFKLP: NLL BFKL + resummation of running coupling as**

**BLM resummation depends on non-Abelian structure in LO**

# BFKLP: NLL BFKL within generalized BLM

$$\omega_{\overline{MS}}(Q_1^2, \nu) = \int d^2 Q_2 K_{\overline{MS}}(\mathbf{Q}_1, \mathbf{Q}_2) \left( \frac{Q_2^2}{Q_1^2} \right)^{-\frac{1}{2} + i\nu} \quad \sigma \sim S^{\alpha_{IP} - 1} = S^{\omega^{\max}}$$

$$= N_C \chi_L(\nu) \frac{\alpha_{\overline{MS}}(Q_1^2)}{\pi} \left[ 1 + r_{\overline{MS}}(\nu) \frac{\alpha_{\overline{MS}}(Q_1^2)}{\pi} \right],$$

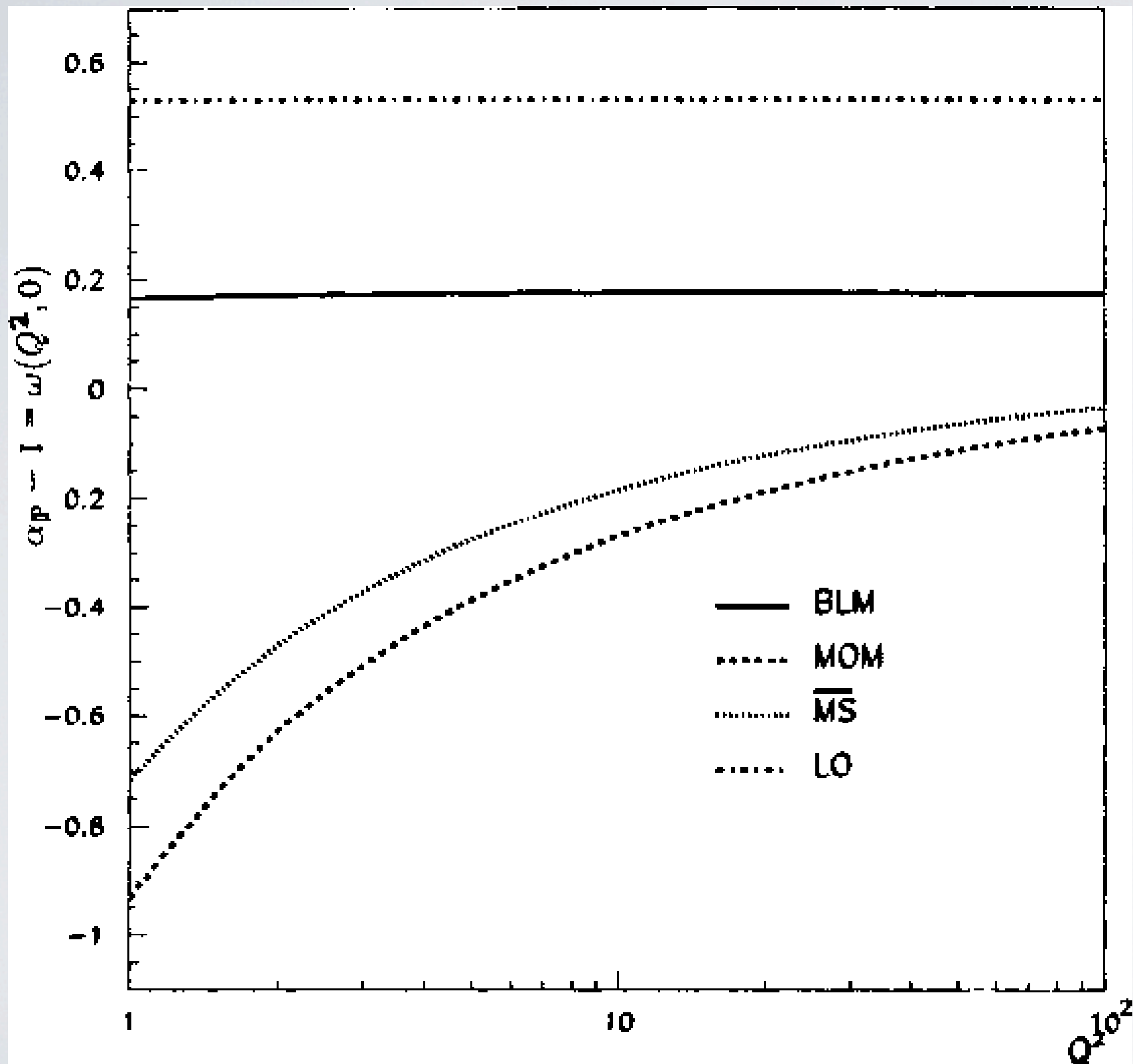
$$\chi_L(\nu) = 2\psi(1) - \psi(1/2 + i\nu) - \psi(1/2 - i\nu)$$

$$r_{\overline{MS}}(\nu) = r_{\overline{MS}}^{\beta}(\nu) + r_{\overline{MS}}^{\text{conf}}(\nu)$$

$$r_{\overline{MS}}^{\beta}(\nu) = -\frac{\beta_0}{4} \left[ \frac{1}{2} \chi_L(\nu) - \frac{5}{3} \right]$$

$$r_{\overline{MS}}^{\text{conf}}(\nu) = -\frac{N_C}{4\chi_L(\nu)} \left[ \frac{\pi^2 \sinh(\pi\nu)}{2\nu \cosh^2(\pi\nu)} \left( 3 + \left( 1 + \frac{N_F}{N_C^3} \right) \frac{11 + 12\nu^2}{16(1 + \nu^2)} \right) - \chi_L''(\nu) + \frac{\pi^2 - 4}{3} \chi_L(\nu) - \frac{\pi^3}{\cosh(\pi\nu)} - 6\zeta(3) + 4\varphi(\nu) \right]$$

# BFKLP: NLL BFKL within generalized BLM



$$\sigma \sim S^{\alpha_{IP} - 1} = S^{\omega^{\max}}$$

# **BFKLP: NLL BFKL within generalized BLM**

**V.S. Fadin & L.N. Lipatov (89-98)**

**C. Camici & M. Ciafaloni (96-98)**

**next-to-leading log approximation (NLL) BFKL  
MSbar-renormalization scheme: large corrections**

**S.J. Brodsky, V.S. Fadin, VK, L.N. Lipatov, G.B. Pivovarov (98-99) BFKLP**

**D. Colferai, M. Ciafaloni & G. Salam (99) ...**

**BFKLP: NLL BFKL + resummation of running coupling  $\alpha_s$   
in physical renormalization scheme**

**BFKLP: Conformal BFKL kernel in NLL  $\rightarrow$  SUSY N=4**

**Pomeron intercept:  $a_P = 1.2 - 1.3$**

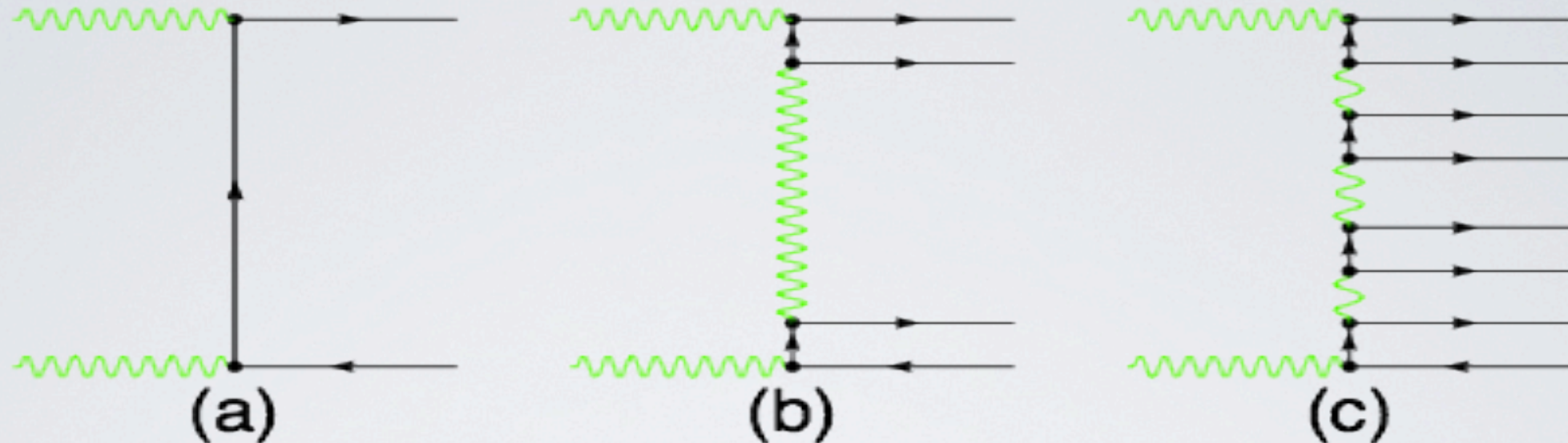
**Cross section:  $\sigma_0 (S/S_0)^{(a_P-1)}$   $a_P = 1 + C \alpha_s$**

**L.N. Lipatov, A.V. Kotikov et al. (2000-06)**

**SUSY N=4 BFKL-Pomeron**

**Anomalous dimensions: test of AdS/CFT**

# Asymptotics of QED cross sections



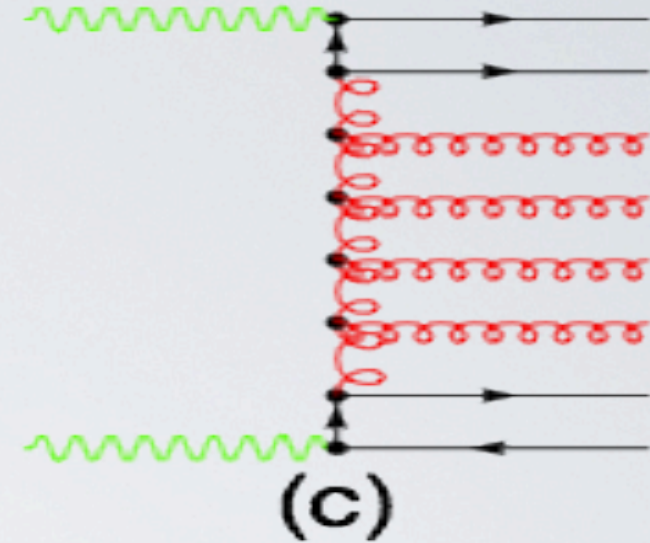
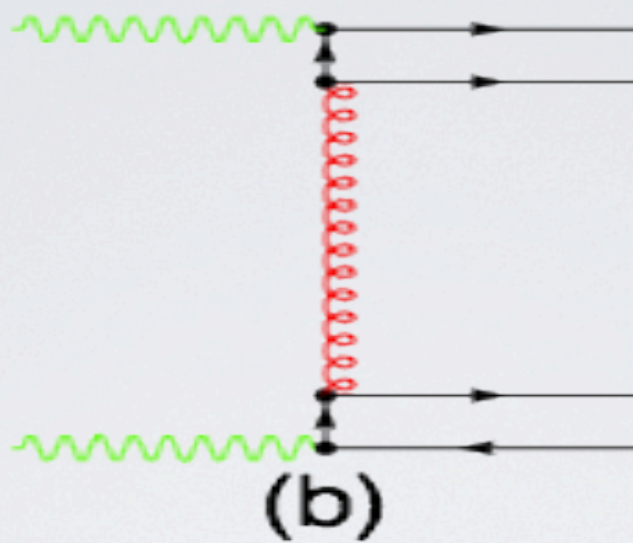
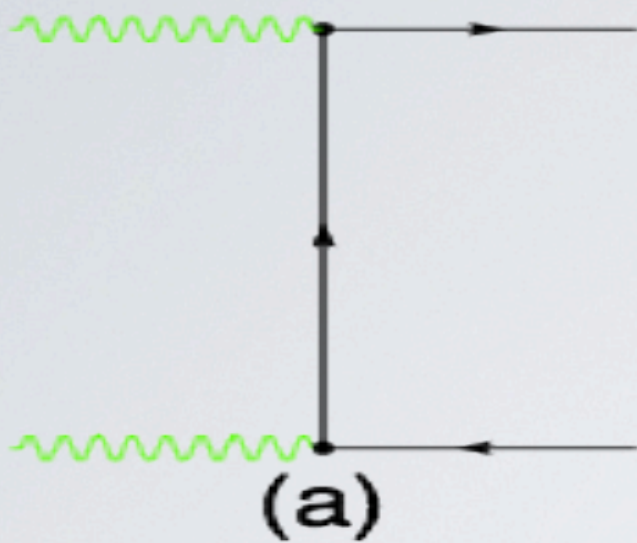
$$\sigma \sim (a_{\text{QED}})^2 \log(s)/s \quad \sigma \sim (a_{\text{QED}})^4 \text{const}(s)$$

**All orders:** V.N. Gribov, L.N. Lipatov, G.V. Frolov & V.G. Gorshkov (69-71)  
H. Cheng & T.T. Wu (69-70)

**Cross section at  $s \rightarrow \infty$ :**  $\sim (a_{\text{QED}})^4 (S/S_0)^{(a_P-1)}$

$$a_P = 1 + C (a_{\text{QED}})^2 \approx 1.002$$

# Asymptotics of QCD cross sections: $\gamma\gamma$



$$\sigma \sim (a_{\text{QED}})^2 \log(s)/s$$

$$\sigma \sim (a_{\text{QED}})^2 (a_s)^2 \text{const}(s)$$

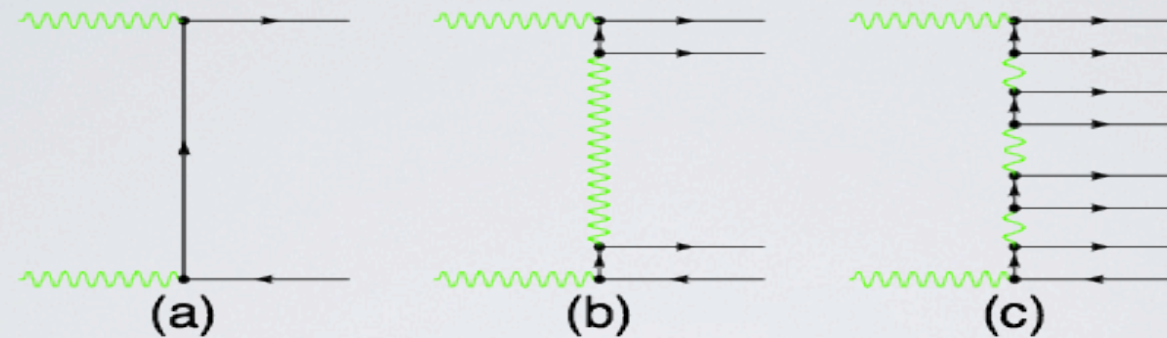
**All orders: LL BFKL**

**Cross section at  $s \rightarrow \infty$ :  $\sim (a_{\text{QED}})^2 (a_s)^2 (S/S_0)^{a_P-1}$**

**$a_P = 1 + C(a_s) \approx 1.5$  LL BFKL S. Brodsky & F. Hautmann (96)**

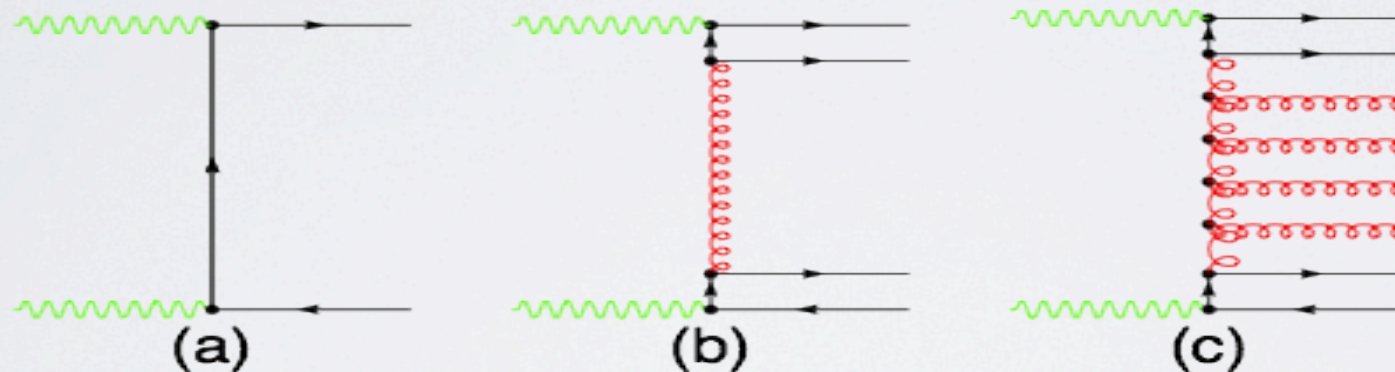
**$a_P = 1 + C(a_s) \approx 1.2$  NLL BFKL S. Brodsky, V Fadin, VK,  
L. Lipatov, G. Pivovarov (2001-02)**

## Asymptotics of QED cross sections



**V.N. Gribov, L.N. Lipatov, G.V. Frolov & V.G. Gorshkov (69-71)**  
**Cheng & T.T. Wu (69-71)**

## Asymptotics of QCD cross sections



**LL BFKL**

**J. Bartels et al (96), S.J. Brodsky & Hautmann (97)**

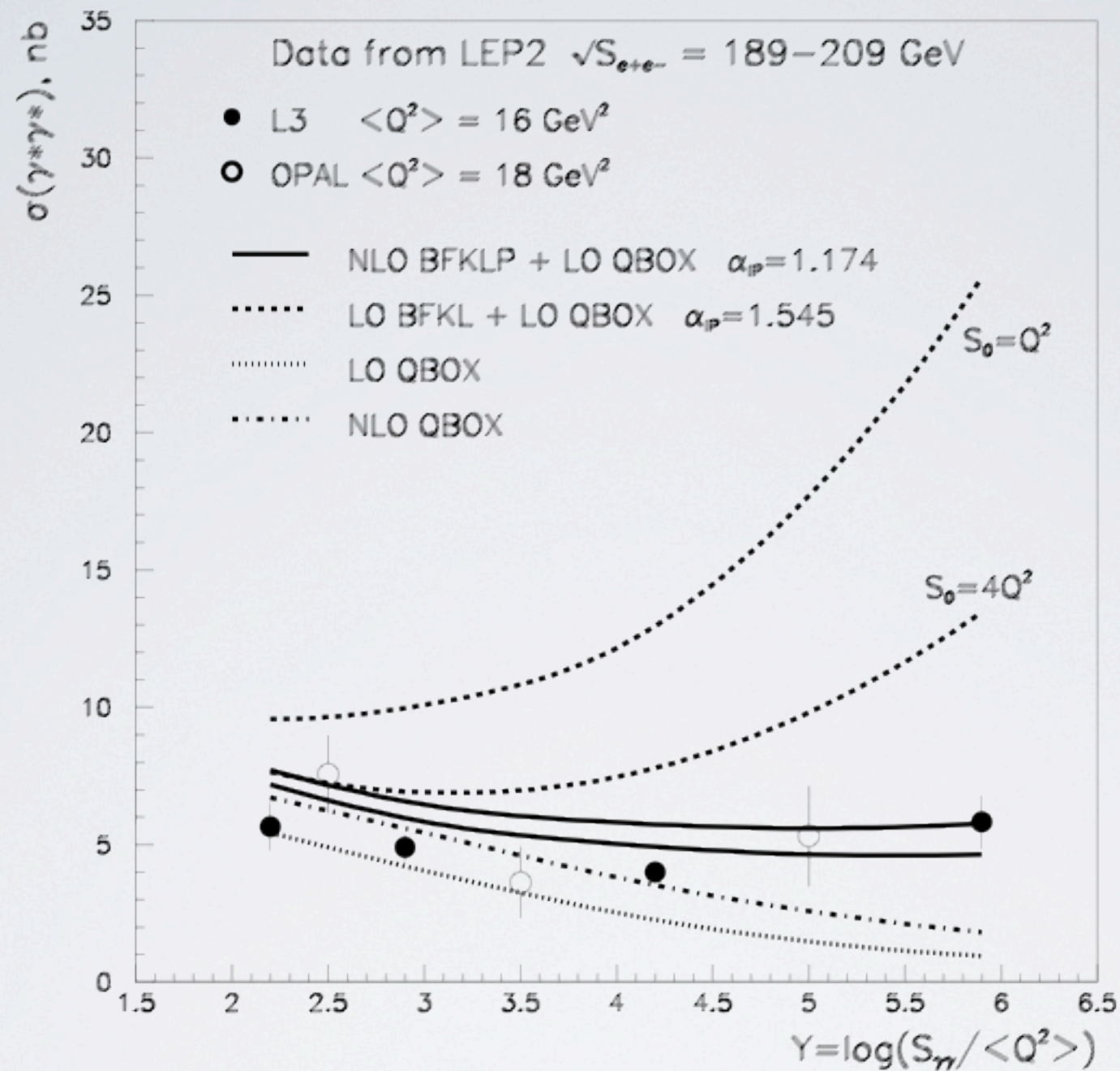
**NLL BFKL (with LO impact factors)**

**S.J. Brodsky, V.S. Fadin, VK, L.N. Lipatov & G.B. Pivovarov (2001-02)**

**NLO impact factors and full NLL BFKL:**

**I. Balitsky, J.Chirilli, J. Bartels et al.**

# Highly virtual photon scattering at LEP-2

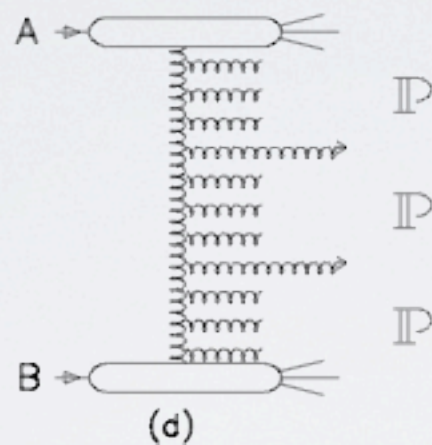
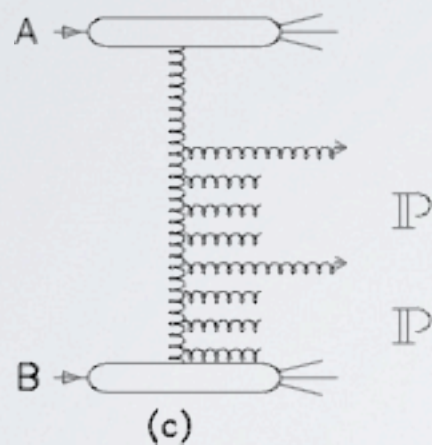
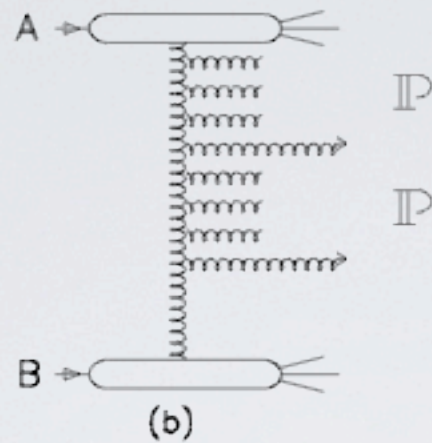
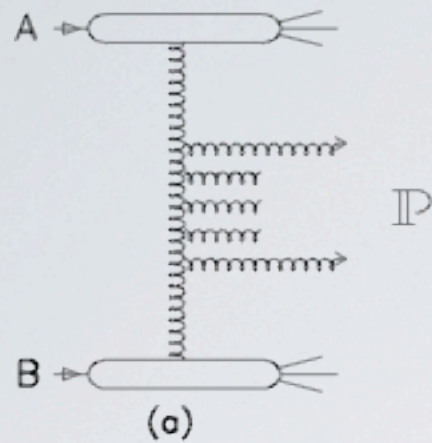


**S.J Brodsky, VK, L.N. Lipatov, V.S. Fadin & G.B. Pivovarov (2002)**  
**BFKLP: NLL BFKL + generalized BLM**

**LL BFKL: ruled out**



# BFKL: dijet processes



**Jet production**

**GLAPD: ordering on  $\kappa T$   
 $y$  – no ordering**

**BFKL: ordering on  $y$   
 $\kappa T$  – no ordering**

**A. Mueller & H. Navelet, Nucl. Phys. (87)**

**Most forward/backward (Mueller-Navelet) dijets:  $x$ -section  $\sim \exp(|\Delta|y)$**

**V.T. Kim & G.B. Pivovarov, Phys. Rev. (96)**

**Inclusive dijets**

**J.C. Collins, R.K. Ellis (91), S. Catani et al (91)**

**E.M.Levin, M.G.Ryskin, Yu.M.Shabelsky, A.G.Shuvaev (91)**

**$kT$ -factorization**

# Dijet K-factor

**K-factor = x-section / Born x-section**

**GLAPD: x-section  $\rightarrow C_1 \alpha_s^2 + C_2 \alpha_s^3 + \dots$**   
**Born x-section  $\rightarrow C_1 \alpha_s^2$**

**K-factor =  $(1 + C_2 / C_1 \alpha_s + C_3 / C_1 \alpha_s^2 + \dots)$**

**Mueller-Navelet (87):**

**BFKL  $\rightarrow$  enhanced  $(\alpha_s \Delta y)$ -terms**  
**x-section  $\rightarrow B_1 \alpha_s^2 \Delta y + B_2 \alpha_s^3 \Delta y^2 + \dots$**   
**Born x-section  $\rightarrow B_1 \alpha_s^2 \Delta y$**

**K-factor\_MN  $\rightarrow \exp(\alpha_s \Delta y)$**

**$\Delta y = |y_1 - y_2|$**

# **Dijet K-factor: not measurable**

**K-factor = x-section / Born x-section**

**Born x-section: no real and no virtual corrections**

**only a theoretical quantity - > not measurable (!)**

**Experiment: one cannot forbid virtual corrections  
by kinematical conditions**

**Exclusive dijet x-section: always contains virtual  
corrections**

**VK & G. Pivovarov:**

**Using dijets with extra jet veto  
instead of Born dijets**

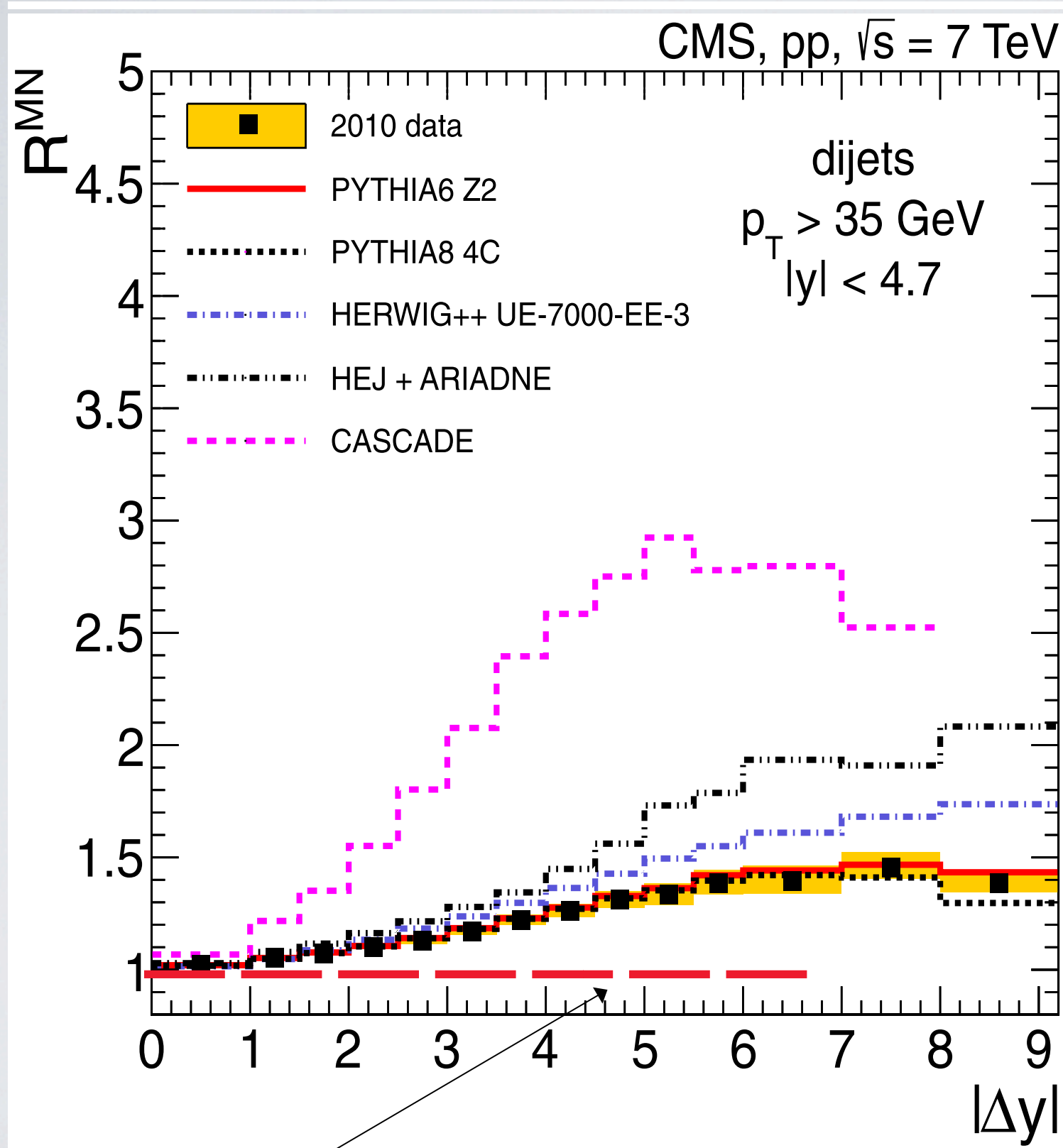
# Forward dijets at Tevatron and LHC

**Tevatron : D0 ->  $|\Delta y| < 6$   $p_{Tmin} = 20$  GeV**  
- azimuthal decorr. (1997)  
- 1800/630 GeV x-section ratio (2001)

**LHC: ATLAS ->  $|\Delta y| < 6$   $70$  GeV  $< p_T < 90$  GeV**  
- (inverse) “K-factor” (2011)

**LHC: CMS ->  $|\Delta y| < 9.4$   $p_{Tmin} = 35$  GeV**  
- “K-factor” (2012)  
- azimuthal angle decorr. (prel. 2013, LHCP2015)

# CMS: dijet “K-factor”



**EPJ C 72 (2012) 2216**  
**7 TeV,  $p_{T\_min} = 35$  GeV**  
 **$\Delta y = |\eta| < 9.4$**

**GLAPD**

# Forward dijets at LHC:

## Color coherence and AO effects

**GLAPD: strong  $k_T$ -ordering & no rapidity ordering**

**BFKL: strong rapidity ordering & no  $k_T$ -ordering**

**Color coherence effects  $\Rightarrow$  rapidity ordering**

**Polar angle ordering (AO):**

**jet cone veto for larger cone angles  $\Rightarrow$  rapidity ordering**

**Pythia 6 and 8: GLAPD + AO (AO cannot be fully switched off!)**

**Herwig++: GLAPD + color coherence (CC cannot be switched off)**

**No pure GLAPD MC generators (!) available  
at present: Pythia and Herwig generators contain  $|\Delta y|$ -effects**

**small CC and AO  $|\Delta y|$ -effects in GLAPD-regime  
can be large in BFKL-regime at large  $|\Delta y|$**

# Forward dijets at LHC

**GLAPD generators Pythia 6 and 8 (with AO) are consistent with CMS dijet “K-factor” data rather well:**

- 1) no sizeable BFKL effects?**
- 2) or BFKL effects cancels out in dijet ratio**

**in the latter case the “K-factor” with extra jet veto  
can be more sensitive BFKL effects**

**2-jet “exclusive” events: impose an extra jet veto  $p_{T\text{veto}} < p_{T\text{min}}$**

# **Forward dijets: azimuthal angle decorrelations**

## **Cosines**

**V. Del Duca & C. Schmidt (94)**

**J. Stirling (94)**

**Cosine ratios → GLAPD cancellation  
→ more sensitive to BFKL (!)**

**A. Sabio Vera et al (2011)**



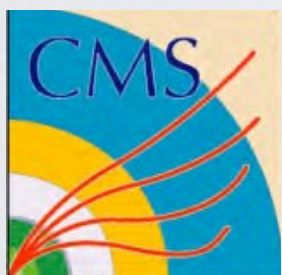
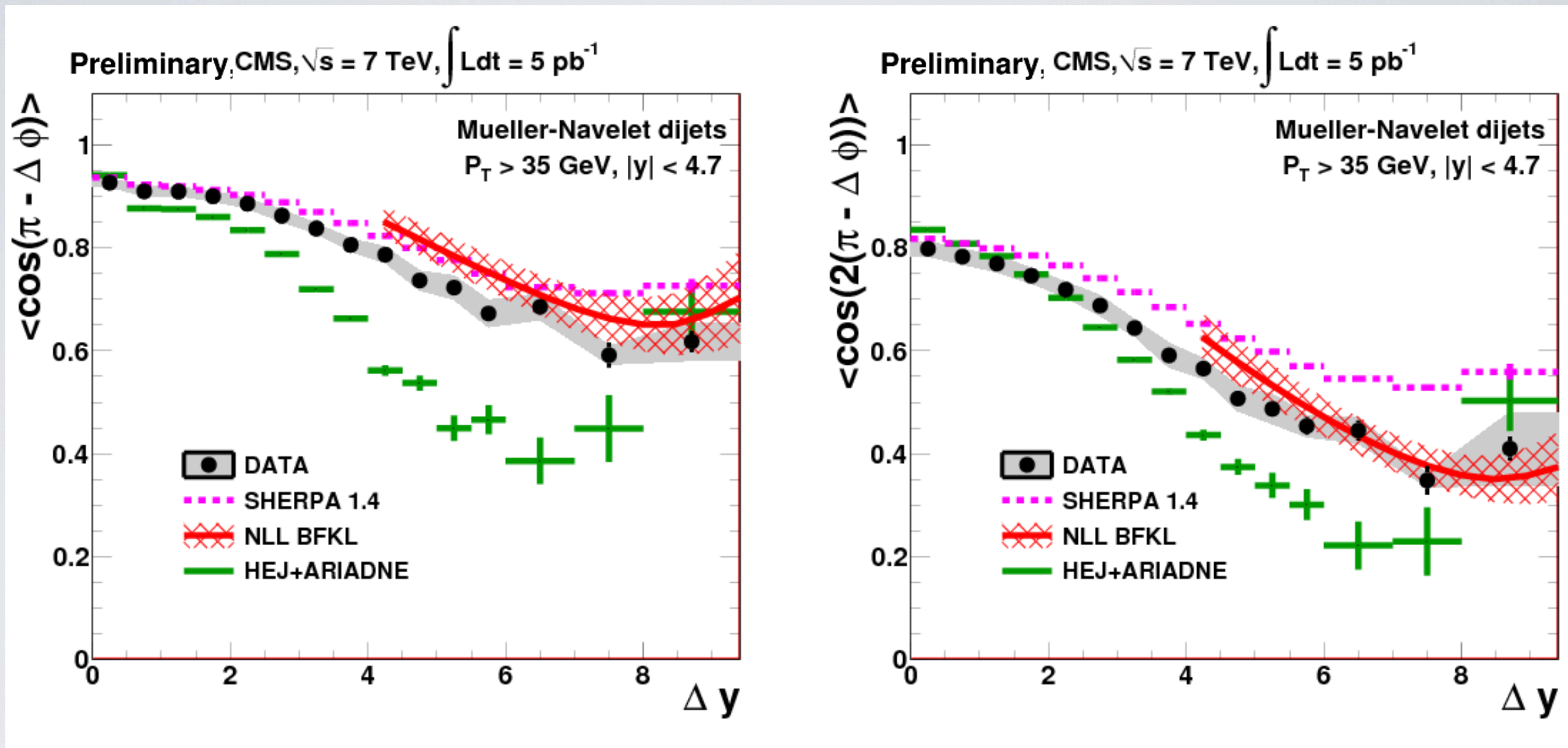
# Forward dijets: azimuthal decorrelations

$$\frac{1}{\sigma} \frac{d\sigma}{d(\Delta\phi)}(\Delta y, p_{T\min}) = \frac{1}{2\pi} \left[ 1 + 2 \sum_{n=1}^{\infty} C_n(\Delta y, p_{T\min}) \cdot \cos(n(\pi - \Delta\phi)) \right]$$

$$C_n(\Delta y, p_{T\min}) = \langle \cos(n(\pi - \Delta\phi)) \rangle, \text{ where } \Delta\phi = \phi_1 - \phi_2$$

**V. del Duca & C. Schmidt (94-95) Strling (94)**  
**V. Kim & G. Pivovarov (96-98)**  
**A. Sabio Vera et al (2007-11)**

# Dijets: $\langle \cos \rangle$ vs NLL BFKL+BFKLP



**LHCP2015, St. Petersburg, Aug. 31- Sep. 5, 2015**

**CMS arXiv:1601.06713**

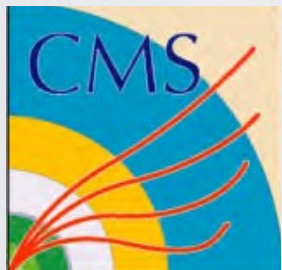
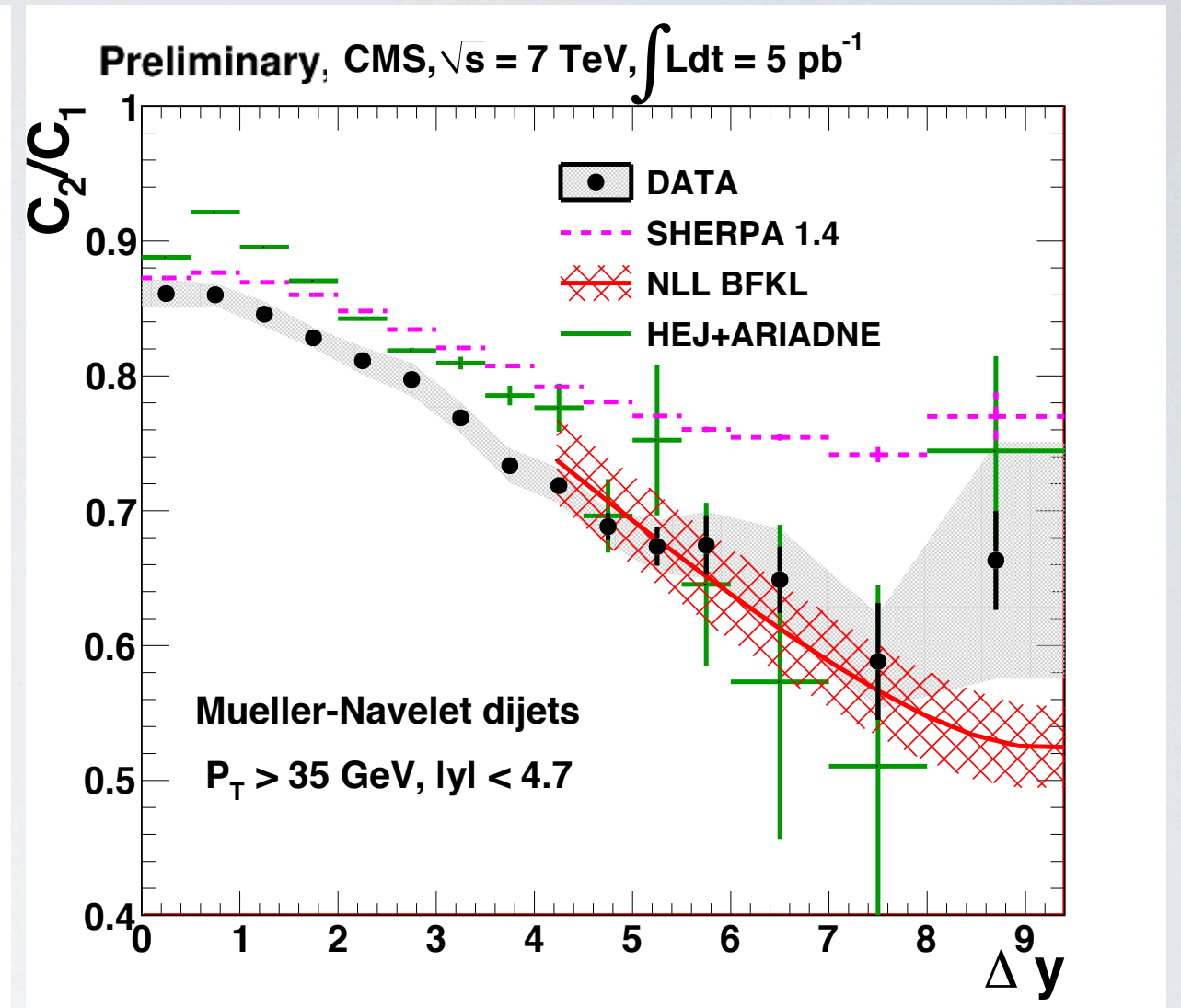
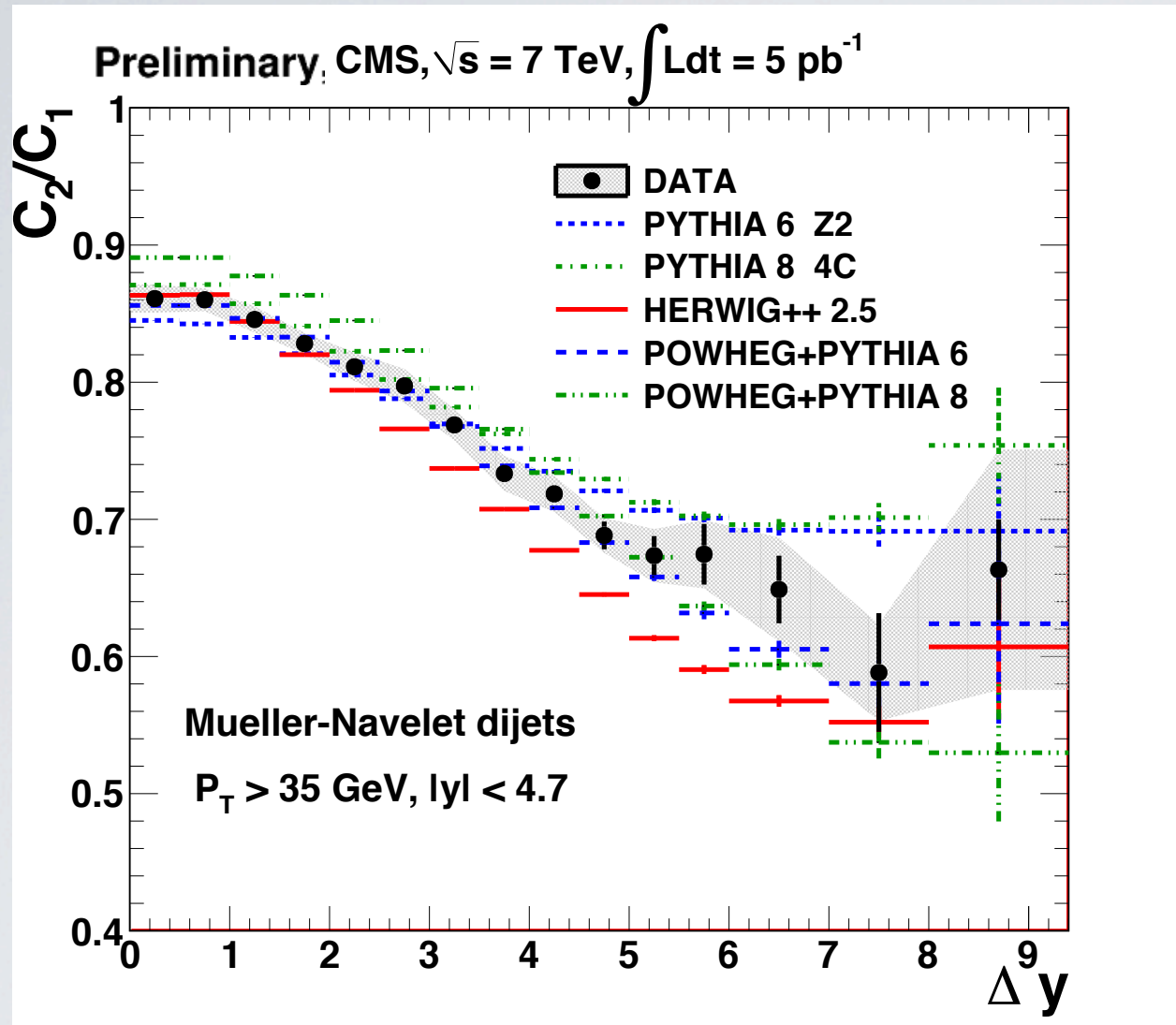
**7 TeV,  $p_{T\_min} = 35 \text{ GeV}$**

**$\Delta y = | | < 9.4$**

**NLL BFKL + BFKLP (2014)**

**B. Ducloue, L. Szymanowski & S. Wallon**

# Dijets: $\langle \cos^2 \rangle / \langle \cos \rangle$ vs NLL BFKL + BFKLP



**LHCP2015, St. Petersburg, Aug. 31- Sep. 5, 2015**

**CMS arXiv:1601.06713**

**7 TeV,  $p_{T\_min} = 35 \text{ GeV}$**

**$\Delta y < 9.4$**

**NLL BFKL + BFKLP (2014)**

**B. Ducloue, L. Szymanowski & S. Wallon**

# Summary – 1

## $\gamma^*\gamma^*$ - collisions at LEP2

**NLL BFKL improved by BFKLP (generalized BLM) (2001-02):**

**Indication on BFKL evolution**

**Outlooks: Future linear colliders**

# Summary - 2:

- **Forward dijet “K-factor” by CMS at 7 TeV :**  
**moderate rise with increasing  $|\Delta y|$** 
  - **Pythia describes the rise, Herwig overshoots the rise**
  - **however: pure GLAPD  $\rightarrow$  const ? Indication on BFKL evolution**
- **Azimuthal angle decorrelations (AAD) of CMS dijets:**
  - **agreement with NLL BFKL improved by BFKLP (generalized BLM)**
  - **GLAPD generators (Pythia, Herwig) describes AAD differently because different color coherence (CC) implementations**

**Indication on BFKL evolution**
- > **The first indication on BFKL at LHC ?**  
**Issues: No pure LL GLAPD predictions**  
**(now only LL GLAPD with color coherence, angle ordering, ...)**
- Other observables:**
  - **K-factor with extra jet veto, number of extra jets, ... ?**

**LHC Run 2 at 13 TeV ?!**