On Hermitian separability of NLO adjoint BFKL eigenvalue in super Yang-Mills

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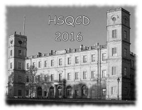
Based on work with A.Prygarin: arXiv:1510.00589, arXiv:1603.01093.



Hermitian separability

- LO: Holomorphic separability

$$H = h + h^* \, , \; E = \epsilon + \tilde{\epsilon} \, ,$$

exists for singlet and adjoint eigenvalues.

- NLO: Hermitian separability

$$\omega_{nlo}(\nu, n) = \omega_{nlo} \, (\, f_i(\nu, n) \,) \; , \quad f_i = f_i^+ + f_i^-$$

guarantees of reality of eigenvalue at real ν, not true for adjoint eigenvalue;

see in A.V.Kotikov and L.N.Lipatov, "DGLAP and BFKL equations in the N=4 supersymmetric gauge theory", Nucl. Phys. B **661**, 19 (2003), (Nucl. Phys. B **685**, 405 (2004)).

BFKL Green´s function

- Consider variable change in the Green´s function

$$G_\omega \propto \sum_{n=-\infty}^{\infty} \int_{-\infty}^{\infty} d\,\mu\, \frac{f^*_{\mu\, n}(\vec{k}^{\,\prime}, \vec{q}^{\,\prime})\, f_{\mu\, n}(\vec{k}, \vec{q})}{\omega - \omega(\mu, n)} = \sum_{n=-\infty}^{\infty} \int_{-\infty}^{\infty} d\,\nu\, \frac{f^*_{\nu\, n}(\vec{k}^{\,\prime}, \vec{q}^{\,\prime})\, f_{\nu\, n}(\vec{k}, \vec{q})}{\omega - \omega^0(\nu, n)}$$

- Variable´s change

$$\nu = f(\mu, n)$$

- In the perturbative framework

$$\nu = \mu + a\, f_1(\mu, n)\,,$$

similarly to B.Basso, S.Caron-Huot and A.Sever, "Adjoint BFKL at finite coupling: a short-cut from the collinear limit", JHEP **1501**, 027 (2015) .

Adjoint BFKL kernel eigenfunctions

- Regular eigenfunctions for the NLO adjoint BFKL kernel

$$f^{0}_{\nu\, n} \;=\; (p)^{\imath\,\nu\,+\,n/2}\,(p^{*})^{\imath\,\nu\,-\,n/2}$$

- Eigenfunctions after the anomalous dimension variable change:

$$f_{\mu\, n} \;=\; (p)^{\imath\,(\mu\,+\,a\,f_1(\mu\,,n))\,+\,n/2}\,(p^{*})^{\imath\,(\mu\,+\,a\,f_1(\mu\,,n))\,-\,n/2}\left(1\,+\,\frac{1}{2}\,a\,\imath\,(\,D_{\mu} f_1\,)\right)$$

here $f_1(\mu\,,n)$ is undefined (yet) function which we can determine on the base of some additional requirements – residual freedom. At LO these functions does not change eigenvalue, at NLO some new eigenvalues are produced. The procedure can be continued for the higher perturbative orders as well.

Eigenfunctions orthogonality and completeness conditions

- The orthogonality condition

$$\int \frac{d^2 k}{\pi k^2 (q-k)^2} f^*_{\mu' \, n'}(\vec{k}, \vec{q}) \, f_{\mu \, n}(\vec{k}, \vec{q}) \; = \; 2\pi \, (1 + \imath \, a \, (D_\mu \, f_1)) \, \delta(f_1^{'} \; - \; f_1) \delta_{n' \, n} \; =$$

$$= \; 2\pi \delta(\mu^{'} \; - \; \mu) \delta_{n' \, n} \, ,$$

the completeness condition

$$\sum_{n \, = \, - \, \infty}^{\infty} \int_{-\infty}^{\infty} d \, \mu \, f^*_{\mu \, n}(\vec{k}^{'}, \vec{q}^{'}) \, f_{\mu \, n}(\vec{k}, \vec{q}) \; =$$

$$= \sum_{n=-\infty}^{\infty} \int_{-\infty}^{\infty} d \, \mu \, f^{*0}_{f(\mu,n) \, n}(\vec{k}^{'}, \vec{q}^{'}) \, f^{0}_{f(\mu,n) \, n}(\vec{k}, \vec{q}) \; (1 + \imath \, a \, (D_\mu \, f_1)) \; =$$

$$= \sum_{n \, = \, - \, \infty}^{\infty} \int_{-\infty}^{\infty} d \, \nu \, f^{*0}_{\nu \, n}(\vec{k}^{'}, \vec{q}^{'}) \, f^{0}_{\nu \, n}(\vec{k}, \vec{q}) \; = \; 2\pi^2 \, \delta^2(k \; - \; k^{'}) \frac{k^2 \, (q-k)^2}{q^2}$$

NLO eigenvalue

- Eigenvalue with regular eigenfunctions:

$$K^8 \otimes f^0_{\nu\, n} = \omega^0(\nu,\, n)\, f^0_{\nu\, n} = -a(E_{\nu\, n} + a\, E^1_{\nu\, n})\, f^0_{\nu\, n}\,,$$

with

$$E_{\nu\, n} = \frac{1}{2}\,(\,\psi(\imath\,\nu + n/2) + \psi(-\imath\,\nu - n/2) + \psi(\imath\,\nu - n/2) + \psi(-\imath\,\nu + n/2)) - 2\,\psi(1)\,,$$

$$E^1_{\nu\, n} = -\frac{1}{4}\, D^2_\nu\, E_{\nu\, n} + \frac{1}{2}\, V\,\left(\, D_\nu\, E_{un}\,\right)\,,$$

where

$$V = \frac{\imath\, u}{u^2 + n^2\,/\,4}\,.$$

Correction to NLO eigenvalue

- After the redefinition of the anomalous dimension variable :

$$K^8 \otimes f_{\mu\, n} = \omega(\mu,\, n)\, f_{\mu\, n} = -a(E_{\mu + a\, f_1(\mu,n)\, n} + a\, E^1_{\mu\, n})\, f_{\mu\, n}\,,$$

where

$$E_{\mu + a\, f_1\, n} = \frac{1}{2}\,(\psi(\imath\, \mu + \imath\, a\, f_1 + n/2) + \psi(-\imath\, \mu - \imath\, a\, f_1 - n/2) +$$

$$+\ \psi(\imath\, \mu + \imath\, a\, f_1 - n/2) + \psi(-\imath\, \mu - \imath\, a\, f_1 + n/2)) - 2\,\psi(1) = E_{\mu\, n} + a\,(\Delta\, E_{\mu\, n})$$

with

$$\Delta\, E_{\mu\, n} = \imath\, \frac{f_1(\mu, n)}{2}\, \frac{\partial}{\imath\, \partial\, \nu}\, (\psi(\imath\, \mu + n/2) + \psi(-\imath\, \mu - n/2) + \psi(\imath\, \mu - n/2) + \psi(-\imath\, \mu + n/2)) =$$

$$= \imath\, f_1(\mu, n)\,(\, D_\mu\, E_{\mu\, n})\,.$$

Redefined NLO eigenvalue

- New eigenvalue with NLO precision:

$$\omega(\mu, n) = -a(E_{\mu\, n} + \imath\, a\, f_1(\mu\, , n)\, (D_\mu\, E_{\mu\, n}) + a\, E^1_{\mu\, n})\,,$$

it is similar to scale change in the running coupling in BFKL equation, see in :

1. V.S.Fadin and L.N.Lipatov, "BFKL pomeron in the next-to-leading approximation" Phys. Lett. B **429**, 127 (1998);
2. V.S.Fadin and L.N.Lipatov, "BFKL equation for the adjoint representation of the gauge group in the next-to-leading approximation at N=4 SUSY", Phys. Lett. B **706**, 470 (2012);
3. A.V.Kotikov and L.N.Lipatov, "NLO corrections to the BFKL equation in QCD and in supersymmetric gauge theories", Nucl. Phys. B **582**, 19 (2000); "DGLAP and BFKL equations in the N=4 supersymmetric gauge theory", Nucl. Phys. B **661**, 19 (2003), (Nucl. Phys. B **685**, 405 (2004)).

- Residual freedom: we can use some f_1 function in order to redefine/adjust eigenvalue $\omega(\mu, n)$ for the same NLO kernel, but we have no precise prescription how to choose f_1 function.

Fixing residual freedom: Hermitian separability

- Let's introduce the following $f_1(\mu, n)$ function

$$\imath\, f_1(\mu,\, n) \;=\; \frac{1}{2}\,\left(\,\psi(1\,+\,\imath\,\mu\,+\,|n|/2)\,-\,\psi(1\,-\,\imath\,\mu\,+\,|n|/2)\,\right)\,,$$

for the NLO part of the eigenvalue we will obtain:

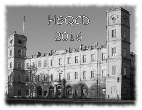
$$\omega_{NLO}(\mu,n) = -a^2\left(E^1_{\mu\,n} + (\,\Delta E_{\mu\,n})\right) = a^2\left(\frac{1}{4}D^2_\mu\,E_{\mu\,n} - \frac{1}{2}\tilde{V}\,D_\mu E_{\mu\,n} + \zeta_2\,E_{\mu\,n} + 3\zeta_3\right)$$

with

$$\tilde{V} \;=\; \frac{\imath\,\mu}{\mu^2\,+\,n^2\,/\,2} + 2\,\imath\,f_1 \;=\; \frac{\imath\,\mu}{\mu^2\,+\,n^2\,/\,2} + \psi(1\,+\,\imath\,\mu\,+\,n/2) - \psi(1\,-\,\imath\,\mu\,+\,n/2)\,,$$

$$\lim_{\mu\to 0}\,\omega_{NLO}(\mu,0)\;=\;0\,.$$

Correspondingly to the definition of (3) the eigenvalue acquires the Hermitian separability property (similarly to NLO BFKL Pomeron eigenvalue), see in S.Bondarenko, A.Prygarin: arXiv:1510.00589.



Speculation: holomorphic separability

- Let's introduce another $f_1(\mu, n)$ function:

$$\imath\, f_1(\mu,\, n) \;=\; -\frac{1}{2}\,\frac{\imath\,\mu}{\mu^2 \,+\, n^2\,/\,4} \;=\; -\,\frac{1}{2}\,V\,.$$

In this case the NLO part of adjoint BFKL eigenvalue becomes

$$\omega_{NLO}(\mu, n) \;=\; -a^2\;\left(E^1_{\mu\,n} \,+\, (\,\Delta\,E_{\mu\,n}\,)\right) \;=\; -\,a^2\,\left(-\frac{1}{4}\,D^2_\mu\,E_{\mu\,n} \,-\, \zeta_2\,E_{\mu\,n} \,-\, 3\,\zeta_3\right)\,,$$

but in this case

$$\lim_{\mu\to 0}\,\omega_{NLO}(\mu, 0) \;=\; 2\,a^2\,\zeta_3\,.$$

We can choose (simplest but not only possibility):

$$\imath\,f_1 \;=\; -\,\frac{1}{2}\,V \,+\, \frac{2\,\zeta_3}{(\,D_\mu\,E_{\mu n})}\,,$$

where correspondingly

$$\omega_{NLO}(\mu, n) \;=\; -\,a^2\,\left(-\frac{1}{4}\,D^2_\mu\,E_{\mu\,n} \,-\, \zeta_2\,E_{\mu\,n} \,-\, \zeta_3\right)\,.$$

Summary

- Exists possibility to redefine BFKL eigenfunctions in each order of BFKL kernel – possibility to redefine BFKL eigenvalue;
- The amplitude is not changing, there is a redistribution of NLO corrections between impact factors and eigenvalue;
- The correction to the adjoint NLO BFKL eigenvalue will depend on some arbitrary function $f_1(\mu, n)$, which can not be fixed from inside the approach (there are some ideas how to fix it in general). The request of Hermitian separability of the NLO eigenvalue fix this function, the redefined eigenvalue is the same as in the string based calculations of of B.Basso, S.Caron-Huot and A.Sever;
- Can we request a holomorphic separability of the adjoint NLO eigenvalue or even redefine the singlet BFKL NLO eigenvalue – these are interesting question for the further investigation.