HSQCD, June 28 2016

B. I. Ermolaev

Model for non-perturbative inputs for parton distributions in K_T - Factorization

talk based on results obtained in collaboration with M. Greco and S.I. Troyan

Need for QCD factorization:

Description of hadronic reactions involves QCD calculations at both high and low energies. However, QCD is poorly known at low energies; the confinement problem has not been solved, so approximation methods are needed to mimic the straightforward QCD calculations at low energies. QCD factorization is the most popular approximation method.

Essence of QCD factorization:

First, non-perturbative inputs are introduced through either models or fits.
Second, the inputs are evolved with perturbative means (evolution equations).

Non-perturbative inputs for parton distributions in hadrons are introduced through the models and fits. Alternatively, there are lattice calculations

Models:

Dmitri Diakonov, V. Petrov, P. Pobylitsa, Maxim V. Polyakov; H. Avakian, A.V. Efremov, P. Schweitzer, F. Yuan;Ivan Vitev, Leonard Gamberg, Zhongbo Kang, Hongxi Xing; Asmita Mukherjee, Sreeraj Nair, Vikash Kumar Ojha;

K. Golec-Biernat, M. Wustoff; H. Jung; A.V. Lipatov, G.I. Lykasov, A.A. Grinyuk, N.P. Zotov; Jon Pumplin;

Fits:

G. Altarelli, R. Ball, S. Forte, G. Ridolfi; E. Leader, A.V. Sidorov, D.B. Stamenov; J. Blumlen, H. Botcher; M. Hirai

Most actively used in the context of factorization

Recent Lattice Calculations:

Yan-Quing Ma, Jian-Wei Qui; Marta Constantinou

I apologize if I have overlooked some name(s) and willingly accept corrections

Non-perturbative inputs for parton distributions in hadrons are introduced through the models and fits. Alternatively, there are lattice calculations

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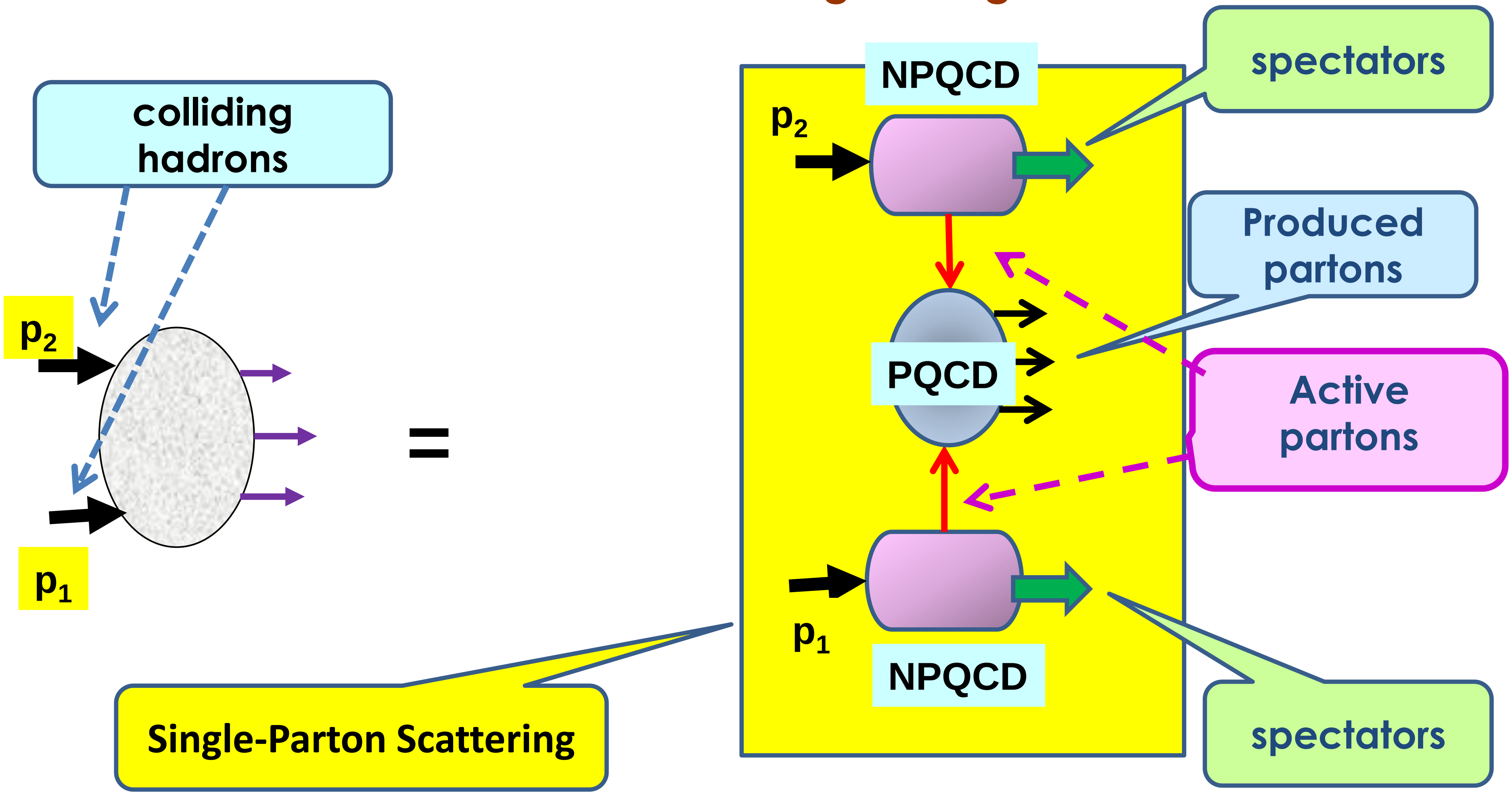
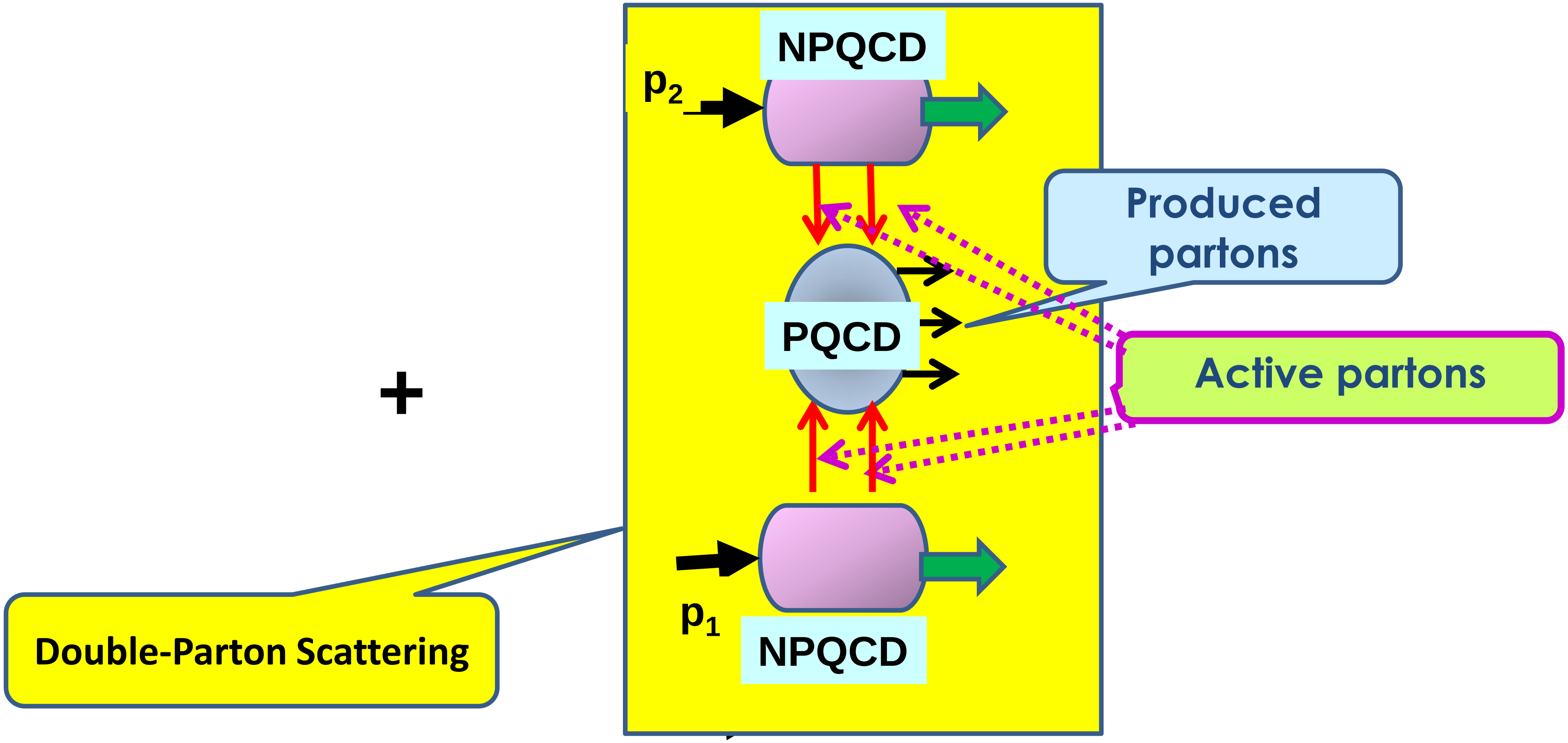
Scenarious of hadronic collisions at high energies



+

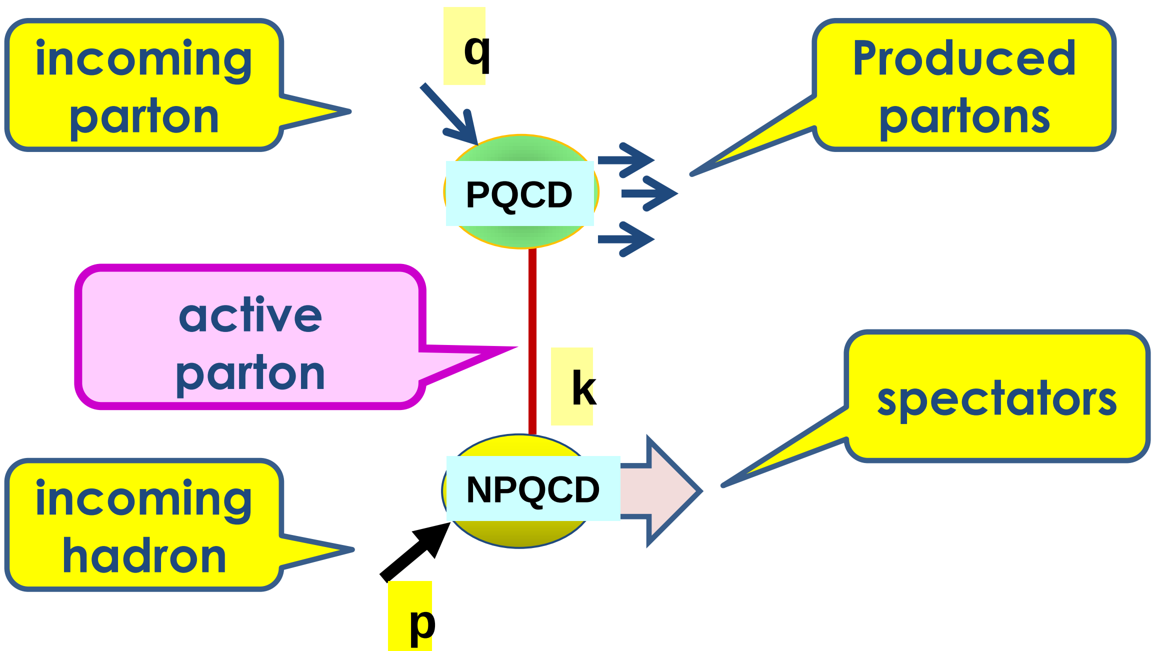
$+$



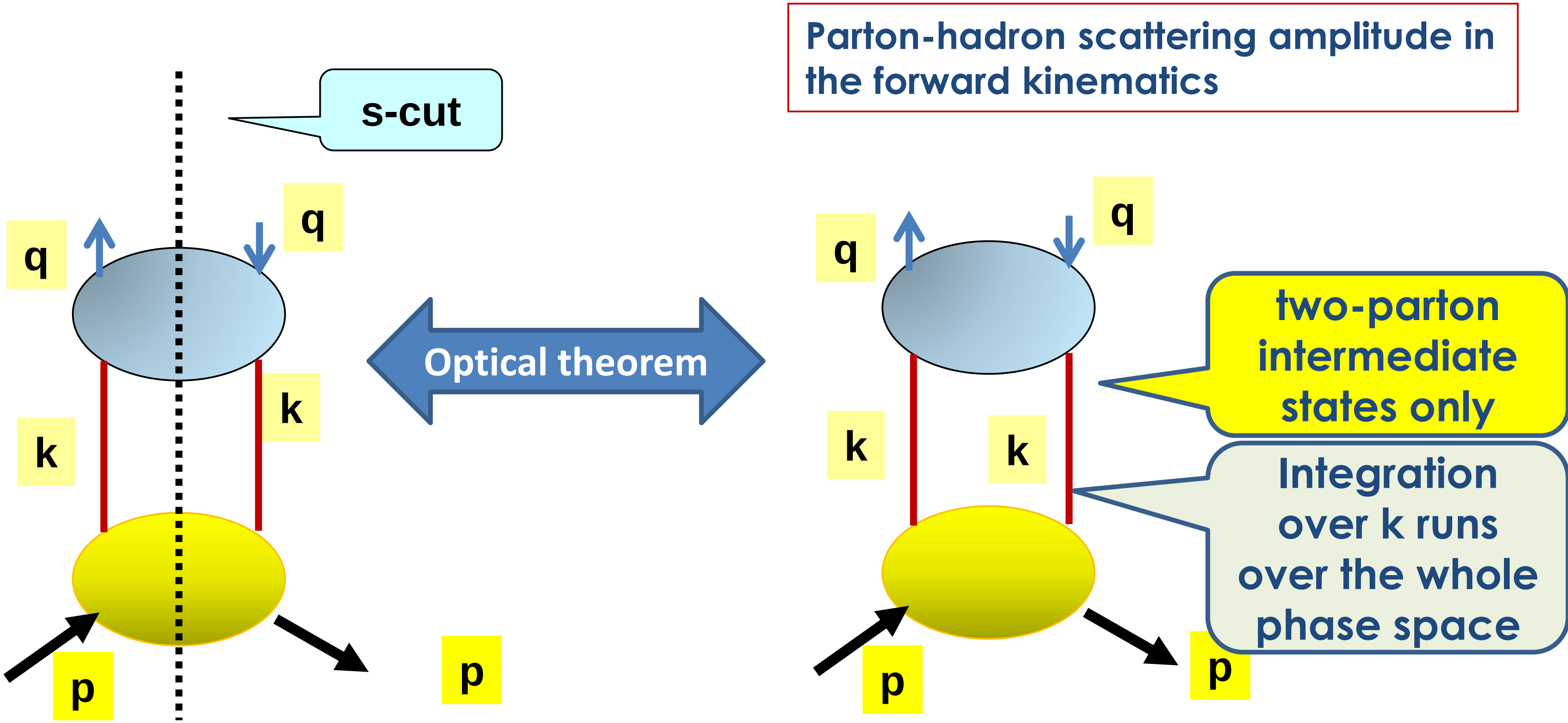
$+$ contributions of more complicated Multi-Parton states

Single-Parton Scenario is much more popular than Multi-Parton one, so in the present talk I will focus on **SINGLE-PARTON COLLISIONS** though a generalization to Multi-Parton Scattering is easy to obtain

Single-Parton Scenario for the parton-hadron scattering



Getting it squared, we arrive at the parton distribution



The kinds of QCD factorization available in the literature:

Collinear Factorization

Amati-Petronzio-Veneziano, Efremov-Ginzburg-Radyushkin, Libby-Sterman, Brodsky-Lepage, Collins-Soper-Sterman

K_T- Factorization/High-Energy Factorization

S. Catani - M. Ciafaloni – F. Hautmann

J.C. Collins, R.K. Ellis

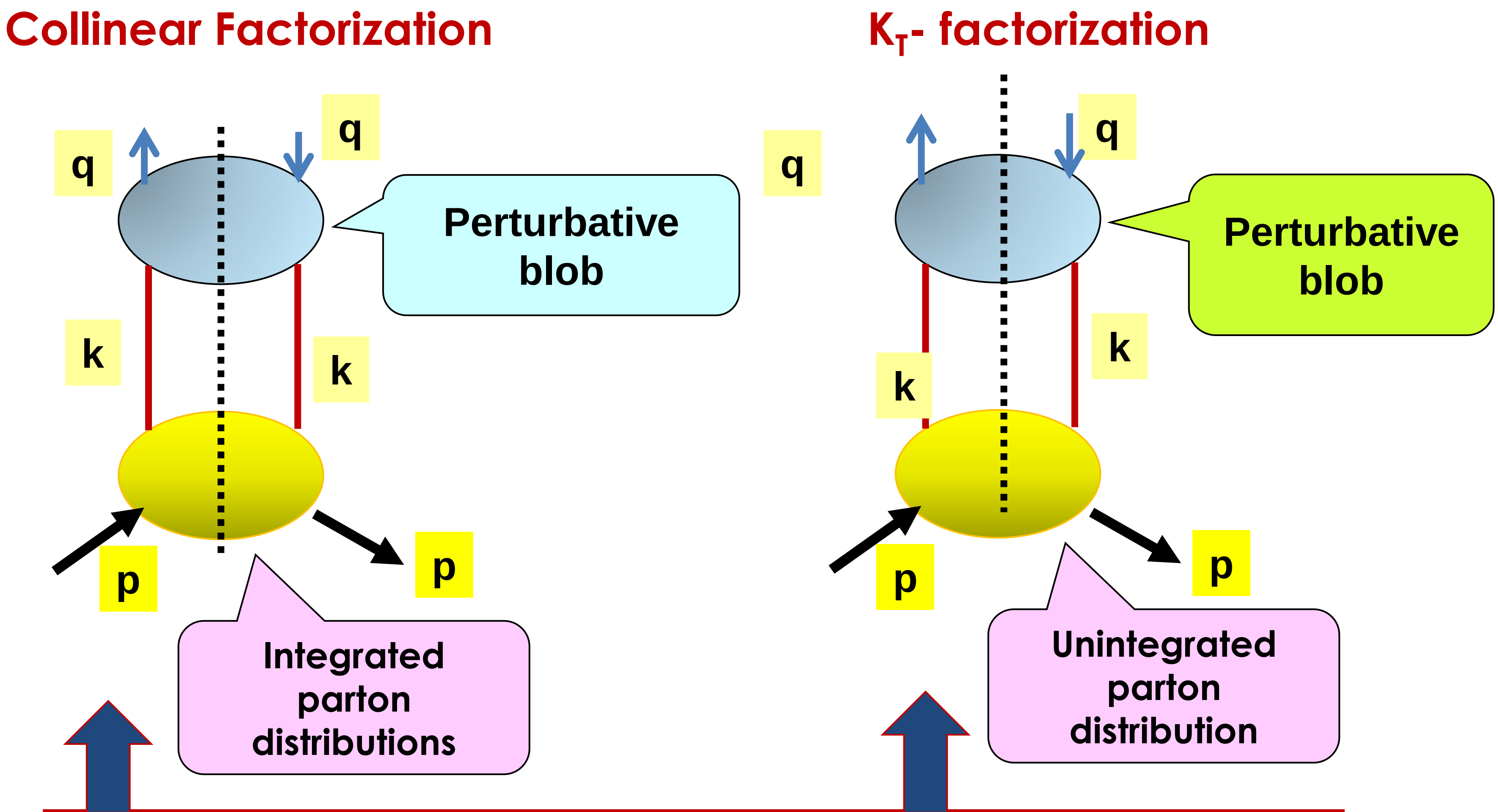
These two conventional forms of factorization were introduced from different considerations and are used for different perturbative approaches

Recently we suggested a new, more general kind of factorization:

Basic Factorization

We showed how to reduce it step-by-step to K_T and Collinear Factorizations, keeping the non-perturbative inputs in a general form

Conventional illustrations of Factorizations



Pictures look identically but formulae differ

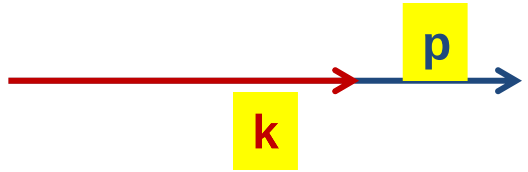
NB Standard Feynman diagram technique cannot be applied to these graphs

Different Factorizations imply different parameterizations of momenta of the connecting partons

$$\vec{k} = \beta\ \vec{p} \qquad (0 < \beta < 1)$$

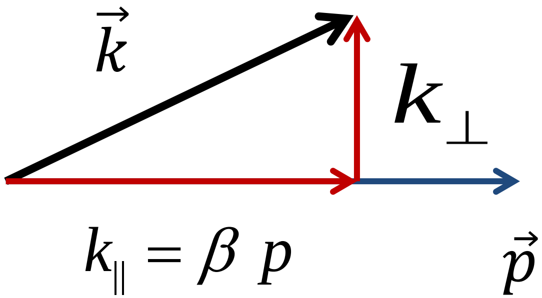
momentum fraction

Collinear Factorization



$$\vec{k} = \beta\ \vec{p} + \vec{k}_{\perp}$$

K_T -Factorization



Factorization representation for parton distributions

Collinear Factorization $\vec{k} = \beta \, \vec{p}$

factorization scale

$$D_{col}(x,q^2) = \int_x^1 \frac{d\beta}{\beta} D_{col}^{(pert)}(x/\beta, q^2/\mu^2)\, \varphi\,(\beta,\mu^2)$$

integrated parton distribution

Perturbative terms are calculated with evolution equations

Parton distributions found from phenomenological considerations

K_T-Factorization $\vec{k} = \beta \, \vec{p} + \vec{k}_\perp$

$$D_{KT}(x,q^2) = \int \frac{d\beta}{\beta} \frac{dk_\perp^2}{k_\perp^2} D_{KT}^{(pert)}(x/\beta, q^2/k_\perp^2)\, \Phi(\beta,k_\perp^2)$$

Unintegrated parton distribution

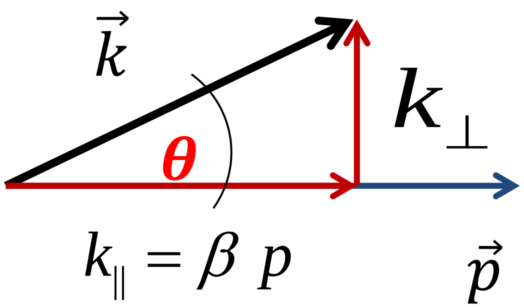
Actual situation is more involved: $\mathbf{k} = [k_0, k_x, k_y, k_z]$
and all components of $\mathbf{k}$ should be accounted for

For instance, all of them are present in Sudakov representation

$$k = \alpha q + \beta p + k_\perp$$ so that

$$d^4\mathbf{k} = d^2 k_{||} d^2 k_\perp = (s/2) d\alpha d\beta d^2 k_\perp \approx \pi s\, d\alpha d\beta k_\perp d k_\perp$$

Kinematical contents of α and β



$\theta \ll 1$

$$\alpha \approx \frac{\omega}{4E}\theta^2 \qquad \beta \approx \frac{\omega}{E}$$

parton energy

hadron energy

$\alpha << \beta$

by this reason the α -dependence is often neglected compared to the β -dependence

When α -dependence is taken into account, we arrive at Basic Factorization

new integration

perturbative part

non-perturbative input

$$A(S_q, S_h, w, q^2) = \int \frac{d\beta}{\beta} \, dk^2_{\perp} \, d\alpha \, A^{(pert)}(S_q, w\beta, q^2, k^2) \left(\frac{k^2_{\perp}}{k^2 k^2} \right) T \, (S_h, w\alpha, k^2)$$

parton spin

hadron spin

In expressions for parton-hadron amplitudes integration over momentum k covers the whole phase space and it should yield a finite result

However, the integrand has singularities:

HANDLING THE SINGULARITIES :

A: IR and UV singularities of the perturbative amplitude $A^{(pert}$

IR singularities are regulated by k^2 and therefore $A^{(pert)}$ is IR stable as long as k^2 is not equal to zero .
UV singularities in Pert QCD are known to be absorbed by redefinitions of the couplings and masses.

B: However, after substitution of $A^{(pert)}$ and T into the convolution, the problem of IR and UV singularities appears once again

$$A(S_q, S_h, w, q^2) = \int \frac{d\beta}{\beta}\, dk^2 \;\; d\alpha\, A^{(pert)}(S_q, w\beta, q^2, k^2) \left(\frac{k^2{}_{\perp}}{k^2 k^2}\right) T\,(S_h, w\alpha, k^2)$$

Integration over k^2 runs through the point $k^2 = 0$ and there is no reason to introduce a new IR cut-off
Integration over α may yield a diverging result at large $|\alpha|$

WAY OUT: input T should kill both IR and UV divergences in order to ensure IR and UV stability of the factorization convolutions

IR stability: $T \sim (k^2)^{1+\eta}$ at small k^2, with $\eta > 0$

UV stability $T \sim |\alpha|^{-\kappa}$ at large $|\alpha|$, with $\kappa > 0$

So, integrability of factorization convolutions leads to theoretical restrictions on models for non-perturbative inputs T

Any model for input T in the parton-hadron scattering amplitudes must satisfy the following constraints:

(i) Input T should respect the IR and UV stability restrictions

(ii) It should have non-zero imaginary part in the s-channel in order to apply the Optical theorem

(iii) Model should ensure the step-by-step reductions of Basic Factorization to other forms of factorization.

In particular, the input in K_T – factorization should have a sharp-peaked form . This ensures reducing to Collinear Factorization

First of all, we fix the spinor part of the input for quark-hadron amplitudes

ASSUMPTION

$$\hat{T} = (\hat{p} + m_h)\, T_U - (\hat{p} + m_h)\, \gamma_5 \hat{S}\, T_S$$

Hadron mass

Hadron spin

Invariant amplitude for polarized hadron

Invariant amplitude for unpolarized hadron

Such a representation obeys Conformity: When the hadron is replaced by an elementary fermion, $\hat{T}$ is replaced by $\hat{\rho}$

For gluon-hadron amplitudes , we choose the inputs in the following form :

$$T_{\lambda\rho} = \left(2p_\lambda p_\varrho - k_\lambda p_\varrho - pk\, g_{\lambda\rho}\right) T_U + i\, m_h \epsilon_{\lambda\varrho\tau\sigma} k_\tau S_\sigma\, T_S$$

Invariant unpolarized amplitude

Hadron spin

Invariant amplitude for polarized hadron

All such invariant amplitudes are scalars

$$T_{U,S} = T_{U,S}\left(s_1, k^2, M^2\right)$$

Invariant energy

Quark virtuality

$$s_1 = (p-k)^2 = w\alpha + k^2 + M^2$$

In order to fix $T_{U,S}$ we use **RESONANCE MODEL**

MOTIVATION FOR THE RESONANCE MODEL

After emitting the active quark from the hadron, the set of remaining partons is unstable, so it can be described through resonances.
It satisfies the requirements of integrability

In what follows we skip the subscripts U,S

$$T = R(k^2) Z_n(s_1) \qquad Z_n(s_1) = \prod_{r=1}^{r=n} \frac{1}{(s_1 - M_r{}^2 + i\,\Gamma_r)}$$

$n = 2,3,\ldots$

$$T = \tilde{R}(k^2) \left[\frac{1}{s_1 - M_1{}^2 + i\Gamma_1} - \frac{1}{s_1 - M_2{}^2 + i\Gamma_2} \right]$$

$$s_1 = (p-k)^2 = w\alpha + k^2 + M^2$$

Transition from Basic factorization to K_T- factorization

Integration over α and replacement of k^2 by ${k_\perp}^2$

$$T_{KT} = R({k_\perp}^2)\left[\frac{1}{\zeta - {\mu_1}^2 + i\Gamma_1} + \frac{1}{\zeta - {\mu_2}^2 + i\Gamma_2} + \frac{1}{\zeta + {\mu_1}^2 + i\Gamma_1} + \frac{1}{\zeta + {\mu_2}^2 + i\Gamma_2}\right]$$

where $\zeta \ll {k_\perp}^2/\beta$ so we choose $\zeta = \xi\, {k_\perp}^2/\beta$

with $\xi \ll 1$

Transition from Basic factorization to K_T- factorization leads to

$$T_{KT} = R\left(k_\perp{}^2\right)[T_R + T_B]$$

$$T_R = \frac{1}{k_\perp{}^2/\beta - \mu_1{}^2 + i\Gamma_1} + \frac{1}{k_\perp{}^2/\beta \; - \mu_2{}^2 + i\Gamma_2}$$

within the resonance region

$$T_B = \frac{1}{k_\perp{}^2/\beta + \mu_1{}^2 + i\Gamma_1} + \frac{1}{k_\perp{}^2/\beta + \mu_2{}^2 + i\Gamma_2}$$

outside the resonance region and therefore it can be regarded as background

Applying the Optical theorem, we arrive at the input for parton distributions:

$$D_{KT} = R\left(k_{\perp}{}^{2}\right)\left[D_R + D_B\right]$$

Resonance contribution

$$D_R = \frac{1}{\left(k_{\perp}{}^{2}/\beta - \mu_1{}^{2}\right)^{2} + \Gamma_1{}^{2}} + \frac{1}{\left(k_{\perp}{}^{2}/\beta - \mu_2{}^{2}\right)^{2} + \Gamma_2{}^{2}}$$

$$D_B = \frac{1}{\left(k_{\perp}{}^{2}/\beta + \mu_1{}^{2}\right)^{2} + \Gamma_1{}^{2}} + \frac{1}{\left(k_{\perp}{}^{2}/\beta + \mu_2{}^{2}\right)^{2} + \Gamma_2{}^{2}}$$

Background contribution

Specifying the factor *R*.

The only rigorous requirement on *R*: the IR stability requires that

at small $k_\perp^2$

$$R(k_\perp^2) \le (k_\perp^2)^\eta$$

In many papers *R* is chosen in the exponential/Gaussian form:

$$R(k_\perp^2) = R_1(k_\perp^2) \equiv e^{-\lambda k_\perp^2}$$

Violates the IR stability

K. Golec-Biernat, M. Wustoff;
Jon Pumplin

$$R(k_\perp^2) = R_2(k_\perp^2) \equiv (k_\perp^2)^\eta e^{-\lambda k_\perp^2}$$

Agrees with the IR stability

H. Jung
A.V. Lipatov, G.I. Lykasov, A.A. Grinyuk, N.P. Zotov

Minimal Resonance Model

$$D_{KT} = R\left(k_{\perp}{}^{2}\right)[D_R + D_B]$$

$$R\left(k_{\perp}{}^{2}\right) = \left(k_{\perp}{}^{2}\right)^{\eta} e^{-\lambda k_{\perp}{}^{2}}$$

$$D_R = \frac{1}{\left(k_{\perp}{}^{2}/\beta - \mu_1{}^{2}\right)^{2} + \Gamma_1{}^{2}}$$

Resonance contribution

$$D_B = \frac{1}{\left(k_{\perp}{}^{2}/\beta + \mu_1{}^{2}\right)^{2} + \Gamma_1{}^{2}}$$

Background contribution

CONCLUSIONS

We obtained the most general kind of QCD factorization.
We call it Basic Factorization
Basic Factorization can be reduced first to K_T- and then to Collinear Factorizations

Imposing the requirements of IR and UV stability on the convolutions in Basic Factorization allowed us to impose general restrictions on the non-perturbative inputs for parton distributions, without specifying the inputs

Motivated by the simple observation that the ensemble of quarks and gluons in a hadron becomes unstable after the hadron emits an active parton(s) and therefore can be described through resonances, we suggested a model for non-perturbative inputs to the factorization convolutions
We call it Resonance Model. We have constructed it for Single-Parton Scattering but a generalization on Multi-Parton Scattering is easy to obtain.
This model can universally describe the inputs to parton-hadron amplitudes, parton distributions, DIS structure functions, etc., and can universally be used for the polarized and unpolarized hadrons