Spectroscopy and Decay Properties of The D Meson

by

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Motivation

Discovery of new exited state of D meson through experiment by BABAR, BELLE and CLEO have ignite interest towards the spectroscopy of this mesons.,

State	Compared state
$D(2550)$	2^1S_0
$D(2600)$	2^3S_1
$D(2750)$	$1D_2$
$D(2760)$	1^3D_3

Quite recently LHCb has been able to discover and measure the properties of these excited state has reignite interest into these heavy-Light meson.

Study of D meson carry special interest as it is a hadron with two open flavour (c; $\bar{u}$ or $\bar{d}$) which restricts its decay via strong interactions.

For any successful attempt to understand these states, it is require satisfactorily prediction of the mass spectra as well as their decay properties.

For better predictions of decay widths, many models have incorporated additional contributions such as radiative and higher order QCD corrections.

Motivation

Quarkonia, Mesons with one heavy and another light quark/anti quark – semi relativistic system

Applicability Coulomb plus linear potential in the regime of heavy-light quark mesons (semi-relativistic scheme) has always been questionable.

To test the applicability of such a Coulomb plus linear potential model, and would like to study how far one can extend this formulation by including the kinematic relativistic corrections within the Hamiltonian for the *D* meson.

Theoretical Framework

Mass spectrum

Hamitonian

K.E. Part

P.E. Part

$$\mathbf{H} = \sqrt{\mathbf{p}^2 + \mathbf{m}_Q^2} + \sqrt{\mathbf{p}^2 + \mathbf{m}_{\bar{q}}^2} + \mathbf{V(r)}$$

Heavy Light

Expanded using

Binomial theorem

$$\mathrm{K.E.} = \frac{\mathrm{p}^2}{2}\left(\frac{1}{m_Q} + \frac{1}{m_{\bar{q}}}\right) - \frac{\mathrm{p}^4}{8}\left(\frac{1}{m_Q^3} + \frac{1}{m_{\bar{q}}^3}\right) + \mathcal{O}(\mathrm{p}^6)$$

$$V(r) = -\frac{\alpha_c}{r} + Ar^{\upsilon} + V_0$$

$$\alpha_c = (4/3)\alpha_s(M^2)$$

$$\alpha_s(M^2) = \frac{4\pi}{(11 - \frac{2}{3}n_f)\ln\frac{M^2 + M_B^2}{\Lambda^2}}$$

QCD Coupling Constant

1. A.M. Badalian, Phys. Rev. D70, (2004)
2. Y.A. Simonov, Physics of Atomic Nuclei 58, 107 (1995)

Theoretical Framework

Mass spectrum

Gaussian Wave function in Position Space

$$R_{nl}(\mu, r) = \mu^{3/2}\left(\frac{2\,(n-1)!}{\Gamma\,(n+l+1/2)}\right)^{1/2}(\mu r)^l \times e^{-\mu^2 r^2/2} L_{n-1}^{l+1/2}(\mu^2 r^2)$$

Gaussian Wave function in Momentum Space

$$R_{nl}(\mu, p) = \frac{(-1)^n}{\mu^{3/2}}\left(\frac{2\,(n-1)!}{\Gamma\,(n+l+1/2)}\right)^{1/2}\left(\frac{p}{\mu}\right)^l \times e^{-p^2/2\mu^2} L_{n-1}^{l+1/2}\left(\frac{p^2}{\mu^2}\right)$$

Variational parameter

Laguerre polynomial

We employ Ritz variational scheme for expectation values of the Hamiltonian as $H\psi = E\psi$

For the chosen value of ν, μ is determined for each state using the varial theorem

Hwang D.S. *Phys. Rev. D*, 1997, *55*, 6944-6951

The spin averaged mass for the ground state is computed and matched with experimental value using the equation

$$M_{SA} = M_P + \frac{3}{4}(M_V - M_P)$$

Theoretical Framework

Mass spectrum

Introduces separately " Spin-dependent part of the usual one gluon exchange potential (OGEP) between the quark anti quark " for computing the hyperfine and spin-orbit splitting of the low-lying S-states (splitting of the nL levels), in the Hamiltonian

$$V_{SD}(\mathbf{r}) = \left(\frac{\mathbf{L}\cdot\mathbf{S_Q}}{2m_Q^2} + \frac{\mathbf{L}\cdot\mathbf{S_{\bar{q}}}}{2m_{\bar{q}}^2}\right)\left(-\frac{dV^{(0)}(r)}{rdr} + \frac{8}{3}\alpha_S\frac{1}{r^3}\right) +$$

$$\frac{4}{3}\alpha_S\frac{1}{m_Q m_{\bar{q}}}\frac{\mathbf{L}\cdot\mathbf{S}}{r^3} + \frac{4}{3}\alpha_S\frac{2}{3m_Q m_{\bar{q}}}\mathbf{S_Q}\cdot\mathbf{S_{\bar{q}}}4\pi\delta(\mathbf{r})$$

$$+\frac{4}{3}\alpha_S\frac{1}{m_Q m_{\bar{q}}}\left\{3(\mathbf{S_Q}\cdot\mathbf{n})(\mathbf{S_{\bar{q}}}\cdot\mathbf{n}) - (\mathbf{S_Q}\cdot\mathbf{S_{\bar{q}}})\right\}\frac{1}{r^3}, \quad \mathbf{n} = \frac{\mathbf{r}}{r}$$

Relativistic corrections to potential

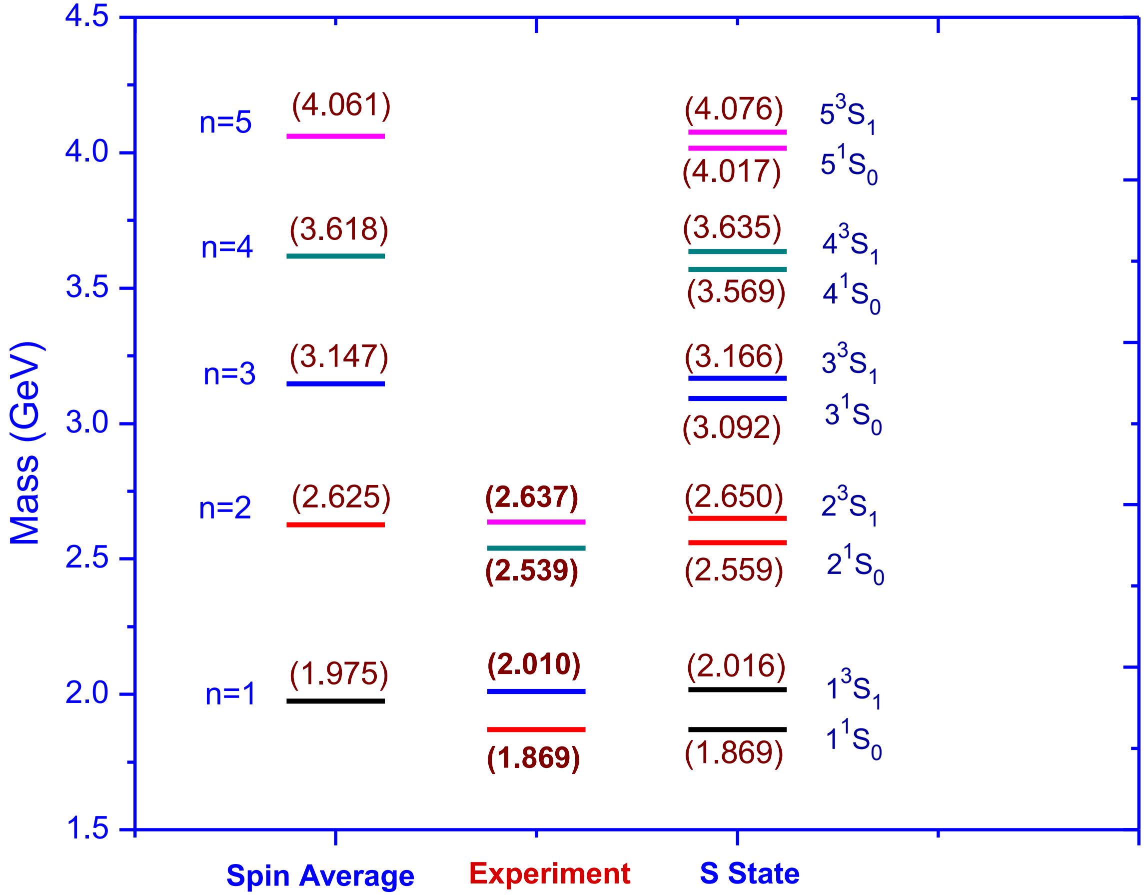
Spin orbital interaction

Spin-spin interaction part

Stands for tensor interaction

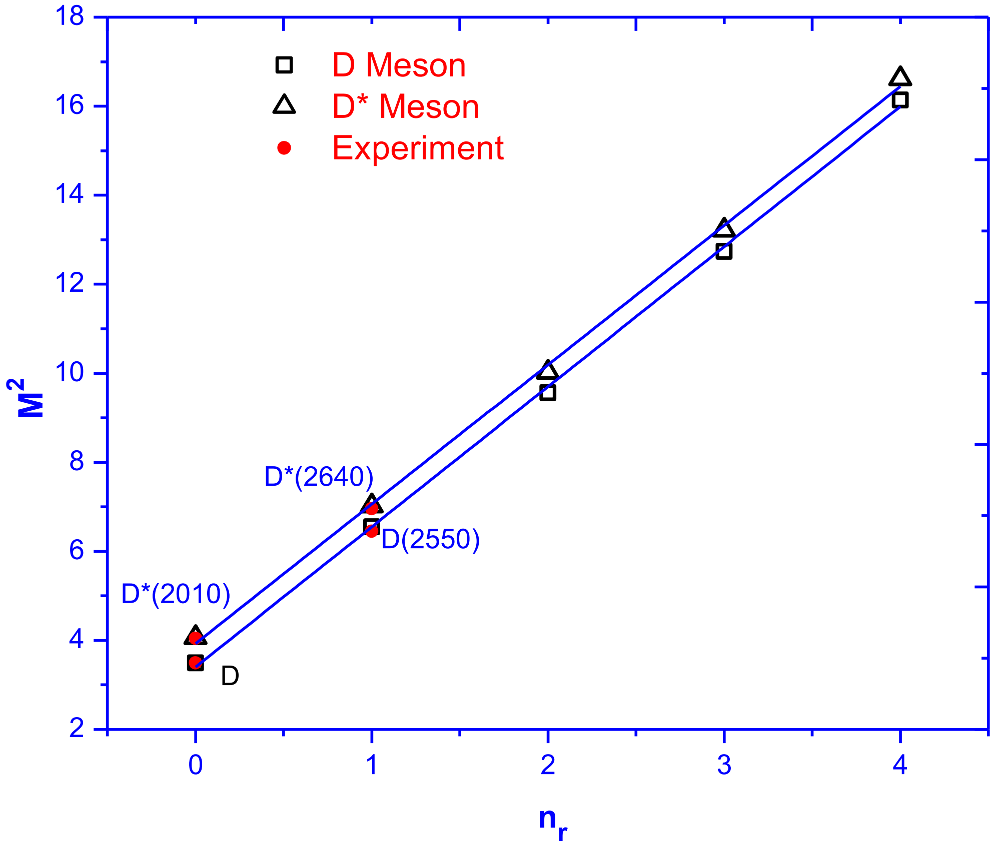
Mass Spectra

Results



Regge Trajectories

The (n_r, M^2) Regge trajectories for pseudoscalar and vector mesons



$$n_r = \beta M^2 + \beta_0$$

β =Slopes, β_0= Intercepts

The fitted parameters of the (n_r, M^2) Regge trajectories

Meson	β	β_0
D	0.318 ± 0.004	-1.079 ± 0.048
D*	0.319 ± 0.005	-1.253 ± 0.061

Straight lines obtained by a χ^2 fit of the calculated values. The calculated heavy light meson masses fit nicely to the linear trajectories, trajectories are almost parallel and equidistant.

Theoretical Framework

Decay Constants

- Important parameters in study of leptonic or non-leptonic weak decay processes
- We compute the decay constants using the relation

$$f^2_{P/V} = \frac{12\left|\psi_{P/V}(0)\right|^2}{M_{P/V}}\bar{C}^2(\alpha_S)$$

Ground state wave function at the origin

QCD correction factor

$$\bar{C}^2(\alpha_S) = 1 - \frac{\alpha_S}{\pi}\left[2 - \frac{m_Q - m_{\bar{q}}}{m_Q + m_{\bar{q}}}\ln\frac{m_Q}{m_{\bar{q}}}\right]$$

Decay Constants

Results

Pseudoscalar decay constants of meson (in GeV)

		1S	2S	3S	4S
fp	This work	0.295	0.201	0.166	0.145
fpcor		0.195	0.133	0.110	0.096
	[1]	0.206±0.008 ±0.002			
	[2]	0.207±0.004			
	[3]	0.230±0.025			
	[4]	0.234			

Vector decay constants of meson (in GeV)

		1S	2S	3S	4S
fv	This work	0.308	0.204	0.168	0.146
fvcor		0.204	0.135	0.111	0.097

[5]	$0.223^{+0.023}_{-0.019}$
[6]	0.234
[7]	0.237

1. D. Asner et al. (Heavy Flavor Averaging Group) (2010),
2. E. Follana, (HPQCD and UKQCD Collabos.), Phys. Rev. Lett.100, (2008)
3. G. Cvetic,, Phys. Lett.B596, 84 (2004)
4. D. Ebert, Phys. Lett. B635,93 (2006)
5. C. Albertus,,Phys. Rev. D 71(11), 113006 (2005)
6. K. Bowler et al. (UKQCD Collaboration), Nucl.Phys.B619, 507 (2001)
7. A. Abd El-Hady, Phys. Rev. D 58(1), 014007 (1998)

Theoretical Framework
Leptonic Branching Fraction

The leptonic branching fractions for the meson are obtained using the formula

$$\mathrm{BR} = \Gamma \times \tau$$

Γ= Leptonic decay width

Life time τ_D

$$\Gamma(\mathrm{D}^+ \to \mathrm{l}^+\nu_\mathrm{l}) = \frac{\mathrm{G}_\mathrm{F}^2}{8\pi}\mathrm{f}_\mathrm{D}^2\,|\mathrm{V}_{\mathrm{cu}}|^2\,\mathrm{m}_\mathrm{l}^2\left(1-\frac{\mathrm{m}_\mathrm{l}^2}{\mathrm{M}_\mathrm{D}^2}\right)^2\mathrm{M}_\mathrm{D}$$

Calculated values of decay constants

Calculated Masses of mesons

Results

Leptonic Branching Fraction

$D^+ \longrightarrow \tau^+ n_\tau$ $BR_\tau \times 10^{-3}$	$D^+ \longrightarrow \mu^+ n_\mu$ $BR_\mu \times 10^{-4}$	$D^+ \longrightarrow e^+ n_e$ $BR_e \times 10^{-9}$
0.901	3.42	8.10
$< 1.2 \times 10^{-3}$ [1]	$(3.82 \pm 0.33) \times 10^{-4}$ [1]	$< 8.8 \times 10^{-6}$ [1]

1. K. Olive, P.D. Group, (2014)

Radiative Leptonic Branching Fraction

The radiative leptonic decay width Γ for the meson are obtained using the formula

C.D. Lu, Phys. Lett. B562, 75 (2003)

$$\Gamma(D^- \to \gamma l \bar{\nu}) = \frac{\alpha G_F^2 \left|V_{cd}\right|^2}{2592\pi^2} f_{D^-}^2 m_{D^-}^3 \left[x_d + x_c\right],$$

Where, $x_d = \left(3 - \frac{m_{D^-}}{m_d}\right)^2, \quad x_c = \left(3 - 2\frac{m_{D^-}}{m_c}\right)^2$

Results

	fp		fpcort	
ν	$\Gamma x10^{-18}$	BR x 10^{-6}	$\Gamma x10^{-18}$	BR x 10^{-6}
1	4.81	7.67	2.10	3.35
C.D. Lu, Phys. Lett. B562, 75 (2003)			2.9	4.6

Mixing Parameters

The neutral D mesons mix with their antiparticles leading to oscillations between the mass eigenstates and the time evolution doublet is described by the Schrodinger equation

$$i\frac{d}{dt}\begin{pmatrix} D_q \\ \bar{D}_q \end{pmatrix} = \left[\begin{pmatrix} M_{11}^q & M_{12}^{q\star} \\ M_{12}^q & M_{11}^q \end{pmatrix} - \frac{i}{2}\begin{pmatrix} \Gamma_{11}^q & \Gamma_{12}^{q\star} \\ \Gamma_{12}^q & \Gamma_{11}^q \end{pmatrix}\right]\begin{pmatrix} D_q \\ \bar{D}_q \end{pmatrix}$$

The light and heavy mass eigenstates have a mass difference $\triangle m_q = 2\,|M_{12}^q|$

and Total decay width difference $\triangle \Gamma_q = 2\,|\Gamma_{12}^q|$

Expressions for the off-diagonal elements of the mass and the decay matrices are

$$M_{12} = -\frac{G_F^2 m_W^2 \eta_D m_{D_q} B_{D_q} f_{D_q}^2}{12\pi^2} S_0\left(m_s^2/m_W^2\right)\left(V_{us}^\star V_{cs}\right)^2$$

$$\Gamma_{12} = \frac{G_F^2 m_c^2 \eta'_D m_{D_q} B_{D_q} f_{D_q}^2}{8\pi}\left[\left(V_{us}^\star V_{cs}\right)^2\right]$$

Where,

G_F =Fermi constant, m_{Dq}= D^0 Mass, V_{us} & V_{cs} = CKM matrix elements

m_W =W boson mass, B_{Dq}=Bag parameter, f_{Dq}=Weak decay constant,

M_c = Mass of the c quark, η_D , η'_D = Glunoic correction to the oscillation

Mixing Parameters

The integrated oscillation rate (χ_q) is the probability of observing a $\overline{D}$ meson in a jet initiated by a $\bar{c}$ quark.

$$\chi_q = \frac{x_q^2 + y_q^2}{2\left(x_q^2 + 1\right)} \quad \text{Where,} \quad x_q = \frac{\Delta m_q}{\Gamma_q} = \Delta m_q \tau_{B_q} \quad \text{and} \quad y_q = \frac{\Delta\Gamma_q}{2\Gamma_q} = \frac{\Delta\Gamma_q \tau_{B_q}}{2}$$

The time-integrated mixing rate is $R_M \simeq \frac{1}{2}(x_q^2 + y_q^2)$

Results

Mixing Parameters of D Meson

ν	Δm_q	x_q	y_q	χ_q	R_M
1	1.76×10^{-15}	1.83×10^{-3}	10.62×10^{-3}	5.81×10^{-5}	0.58×10^{-4}
[1]		(0.80 ± 0.29) %	(0.33 ± 0.24)%		$0.864\pm0.311\times10^{-4}$
[3]		$1.6\pm2.3\pm1.2\pm0.8$			$0.13^{\pm0.22}_{\pm0.20}\times10^{-3}$ [2]
					$1.3\pm2.69\times10^{-4}$ [4]

1. P. del Amo Sanchez et al. (The BABAR Collaboration), Phys.Rev.Lett. 105, 081803 (2010),
2. U. Bitenc et al., Phys. Rev. D 77, 112003 (2008)
3. L.M. Zhang et al., Phys. Rev. Lett. 99, 131803 (2007)
4. A.J. Schwartz, arXiv:0911:1464[hep-ex] (2009)

Summary

- The experimental masses and other theoretical estimates are very close to our estimated mass spectra.

- The calculated heavy light meson masses fit nicely to the linear trajectories, trajectories are almost parallel and equidistant.

- The pseudoscalar and vector decay constants are calculated with and without QCD correction. We compared our results with other theoretical model predictions. Our results with QCD correction are slightly lower than with other predictions.

- Leptonic and radiative leptonic branching fractions are evaluated using spectroscopic parameters of these mesons and the obtained results are compared with PDG values and with C.D. Lu, Phys. Lett. B562, 75 (2003).

- Finally using the predicted results we have estimated the mixing parameter values, which shows reasonably close agreement with experimental values.

for your attention