Leptonic B-decays with Emission of a Photon in the Final State

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Motivation

- $B_{(s)} \to \gamma l^+ l^- \quad (b \to s l^+ l^-)$
- FCNCs are forbidden at tree level in the SM and BSM physics can be visible through contributions of new particles in the loops
- There are some descrepances between experimental values and predictions of the SM:

  $\frac{Br(B^+ \to K^+ \mu^+ \mu^-)}{Br(B^+ \to K^+ e^+ e^-)} \approx 0.75$ (SM value is 1) at 2.6σ

  $Br(B_s \to \mu^+ \mu^-) \approx 0.75$ of the SM value, but here it's 1σ effect only

- $B_s \to \gamma \ell^+ \ell^-$ contains an additional α_{em} in the branching ratio but doesn't have the chirality constraint and thus is comparable to $B_s \to \mu^+ \mu^-$

Effective Theory of B-physics

Wilson expansion:

$$H_{eff} = \frac{G_F}{\sqrt{2}} V_{tb} V_{tq}^* \sum_i c_i(\mu) O_i(\mu)$$

- W, t are integrated out
- c-quarks are dynamical

Buras,1995.

Effective Theory of B-physics

Effective Hamiltonian for $b \to d\ell^+\ell^-$ is

$$H_{\text{eff}}^{b\to d\ell^+\ell^-} = \frac{G_F}{\sqrt{2}}\frac{\alpha_{\text{em}}}{2\pi}V_{tb}V_{tq}^*\Big[-2im_b\frac{C_{7\gamma}(\mu)}{q^2}\cdot\bar{d}\sigma_{\mu\nu}q^\nu\left(1+\gamma_5\right)b\cdot\bar{\ell}\gamma^\mu\ell+$$

$$+C_{9V}^{\text{eff}}(\mu,q^2)\cdot\bar{d}\gamma_\mu\left(1-\gamma_5\right)b\cdot\bar{\ell}\gamma^\mu\ell+C_{10A}(\mu)\cdot\bar{d}\gamma_\mu\left(1-\gamma_5\right)b\cdot\bar{\ell}\gamma^\mu\gamma_5\ell\Big]$$

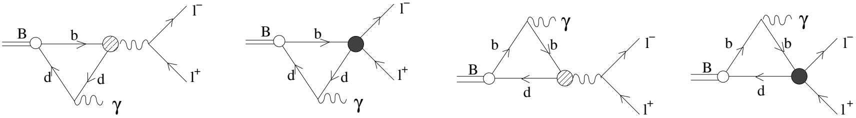
The amplitude

$$A_{B\to\gamma l^+l^-} = \left\langle\gamma(k,\,\epsilon),\,\ell^+(p_1),\,\ell^-(p_2)\left|H_{\text{eff}}^{b\to d\ell^+\ell^-}\right|B(p)\right\rangle$$

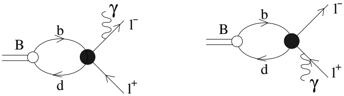
takes into account nonperturbative QCD effects related to B-mesons.

Contributions

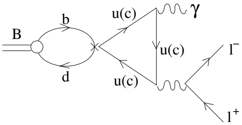
$O_{7\gamma}$, O_{9V}, O_{10AV}: dashed circles denote the $b \to d\gamma$ operator $O_{7\gamma}$ and solid circles denote the $b \to d\ell^+\ell^-$ operators O_{9V} and O_{10AV}:



Bremsstrahlung:



Weak annihilation diagrams:



Amplitude

$$A_\mu^{(1)} = \langle \gamma(k,\, \epsilon),\, \ell^+(p_1),\, \ell^-(p_2) \left| H_{\rm eff}^{b\to d\ell^+\ell^-} \right| B(p)\rangle = \frac{G_F}{\sqrt{2}}\, V_{tb} V_{tq}^* \,\frac{\alpha_{\rm em}}{2\pi}\, e\, \epsilon_\alpha^*$$

$$\times \left[\frac{2\, C_{7\gamma}(\mu)}{q^2}\, m_b \left(\varepsilon_{\mu\alpha\xi\eta} p_\xi k_\eta\, F_{TV}(q^2,0) - i\left(g_{\mu\alpha}\, pk - p_\alpha k_\mu\right) F_{TA}(q^2,0)\right) \bar\ell(p_2)\gamma_\mu \ell(-p_1) \right.$$

$$C_{9V}^{\rm eff}(\mu, q^2) \left(\varepsilon_{\mu\alpha\xi\eta} p_\xi k_\eta \frac{F_V(q^2)}{M_B} - i\left(g_{\mu\alpha}\, pk - p_\alpha k_\mu\right) \frac{F_A(q^2)}{M_B} \right) \bar\ell(p_2)\gamma_\mu \ell(-p_1) +$$

$$\left. C_{10A}(\mu) \left(\varepsilon_{\mu\alpha\xi\eta} p_\xi k_\eta \frac{F_V(q^2)}{M_B} - i\left(g_{\mu\alpha}\, pk - p_\alpha k_\mu\right) \frac{F_A(q^2)}{M_B} \right) \bar\ell(p_2)\gamma_\mu \gamma_5 \ell(-p_1) \right].$$

Form Factors

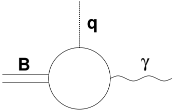
$$\langle \gamma(k,\,\epsilon)|\bar{d}\gamma_\mu\gamma_5 b|B(p)\rangle = i\,e\,\epsilon^*_\alpha\,(g_{\mu\alpha}\,pk - p_\alpha k_\mu)\,\frac{F_A(q^2)}{M_B},$$

$$\langle \gamma(k,\,\epsilon)|\bar{d}\gamma_\mu b|B(p)\rangle = e\,\epsilon^*_\alpha\,\epsilon_{\mu\alpha\xi\eta}p_\xi k_\eta\,\frac{F_V(q^2)}{M_B},$$

$$\langle \gamma(k,\,\epsilon)|\bar{d}\sigma_{\mu\nu}\gamma_5 b|B(p)\rangle\,(p-k)^\nu = e\,\epsilon^*_\alpha\,[g_{\mu\alpha}\,pk - p_\alpha k_\mu]\,F_{TA}(q^2,0),$$

$$\langle \gamma(k,\,\epsilon)|\bar{d}\sigma_{\mu\nu} b|B(p)\rangle\,(p-k)^\nu = i\,e\,\epsilon^*_\alpha\epsilon_{\mu\alpha\xi\eta}p_\xi k_\eta\,F_{TV}(q^2,0).$$

Form factors: constraints in QCD



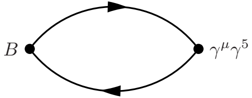
$$F_i(q^2, k^2), \ \ 4m_\ell^2 \le q^2 \le M_B^2$$

- Electromagnetic gauge invariance $\Longrightarrow$ constraints at $q^2 = 0$ (q^2 is the invariant mass of the lepton pair)
- Large Energy Effective Theory (LEET) $\Longrightarrow$ $F_A(E_\gamma) \approx F_V(E_\gamma) \approx F_{TA}(E_\gamma) \approx F_{TV}(E_\gamma)$ for $E_\gamma >> \Lambda_{QCD}$, $E_\gamma = \frac{M_B^2 - q^2}{2M_B}$
- Locations of meson singularities at $q^2 \ge M_B^2 \Longrightarrow$ constraints at $q^2 = M_B^2$
- Form factors related to photon emission by heavy quark are $1/m_Q$ suppressed compared to those with γ-emission by light quark

Relativistic Quark Model

We make use of dispersion approach based on constituent quark picture: All hadron observables are given by dispersion representations in terms of the hadron relativistic wave functions and the spectral densities of Feynman diagrams with constituent quarks in the loops.

- Decay constants: $f_B = \int ds \phi_B(s) \rho(s)$



- Meson-meson form factors:
  $F_{M_1 \to M_2}(q^2) = \int ds_1 \phi_1(s_1) ds_2 \phi_2(s_2) \Delta(s_1, s_2, q^2)$
- Meson-photon transition form factors: $F(q^2, k^2) = \int ds \phi(s) \frac{ds' \Delta(s, s', q_2^2)}{s' - q_1^2}$
- Wave function ($\phi(s) \sim e^{-\frac{k^2}{2\beta^2}}$) is normalized by the condition that electromagnetic form factor is 1 at $q^2 = 0$: $F_{el}(q^2 = 0) = 1$

Relativistic Quark Model

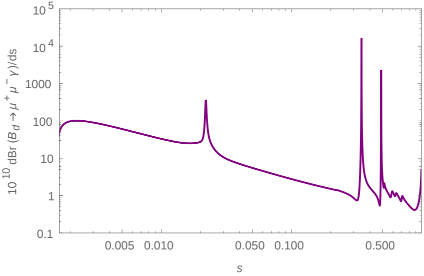
In leading and subleading order in $1/m_Q$:

- The meson-meson transition form factors satisfy constraints from HQET for heavy-to-heavy transitions
- The meson-meson and meson-photon form factors satisfy constraints from LEET for heavy-to-light transitions

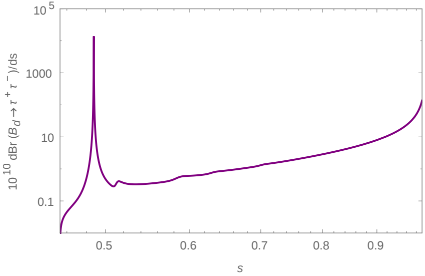
Fixing parameters:

- We use a simple Gaussian parametrization for $\phi(s)$ ($\phi(s) \sim e^{-\frac{k^2}{2\beta^2}}$)
- Constituent quark masses and the parameter of the wave function are fixed by the condition that meson decay constants reproduce well-known results from lattice QCD and sum rules.

Numerical Estimates



$B_d^0 \to \gamma\mu^+\mu^-$



$B_d^0 \to \gamma\tau^+\tau^-$

Numerical Estimates

$lepton$	e	μ	τ
$Br\,(B_d \to \ell^+\ell^-\gamma)\ \times\ 10^{10}$	5.41	2.22	3.27
$Br\,(B_s \to \ell^+\ell^-\gamma)\ \times\ 10^{9}$	20.0	13.5	11.3
$Br\,(B_d \to \ell^+\ell^-\gamma)\ \times\ 10^{10}$ [1]	1.01	0.66	3.39
$Br\,(B_s \to \ell^+\ell^-\gamma)\ \times\ 10^{9}$ [1]	3.30	2.16	11.6
$Br\,(B_s \to \ell^+\ell^-\gamma)\ \times\ 10^{9}$ [2]	20	12	–

[1] G.Q.Geng, C.C.Lih and W.M.Zhang, *Phys. Rev.* D **62**, 074017 (2000).

[2] Yu.Dincer and L.M.Sehgal, *Phys. Lett.* B **521**, 7 (2001)

Summary

- We obtained predictions for the branching ratios of $B_{d,s} \to \gamma l^+ l^-$ decays in the Standard Model
- We used reliable form factors that satisfy all known QCD constraints.
- We took into accout contributions of light vector resonances.

Prospects:

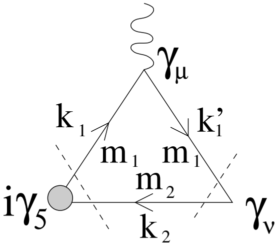
- B to four leptons decays
- Similar non-SM semileptonic decays

Thank you for your attention!

BACK-UP

Relativistic Quark Model

- Form-factors are represented in the form of dispersion integrals over the invariant mass s.



$$A^{\alpha\mu}(q^2) \sim \varepsilon^{\mu\nu pq} \int_{(m_1+m_2)^2}^{\infty} ds \frac{\phi(s)}{(s-q^2)} \rho(s, m_1^2, m_2^2)$$

Form Factors

- For the form factors F_V and F_A the result of the direct calculation in the relativistic dispersion approach was used.

$$\frac{1}{M_B} F_V^{(1)}(q^2) = -\frac{\sqrt{N_c}}{4\pi^2} \int\limits_{(m_1+m_2)^2}^{\infty} \frac{ds \phi_B(s)}{(s-q^2)} \rho_+(s, m_1, m_2).$$

$$\frac{1}{M_B} F_A^{(1)}(q^2) = \frac{\sqrt{N_c}}{4\pi^2} \int\limits_{(m_1+m_2)^2}^{\infty} \frac{ds\ \phi_B(s)}{(s-q^2)} \left(\rho_+(s, m_1, m_2) + 2\frac{m_1 - m_2}{s - q^2} \rho_{k_\perp^2}(s, m_1, m_2) \right),$$

$$\rho_+(s, m_1, m_2) = (m_2 - m_1) \frac{\lambda^{1/2}(s, m_1^2, m_2^2)}{s} + m_1 \log\left(\frac{s + m_1^2 - m_2^2 + \lambda^{1/2}(s, m_1^2, m_2^2)}{s + m_1^2 - m_2^2 - \lambda^{1/2}(s, m_1^2, m_2^2)} \right),$$

$$\rho_{k_\perp^2}(s, m_1, m_2) = \frac{s + m_1^2 - m_2^2}{2s} \lambda^{1/2}(s, m_1^2, m_2^2) - m_1^2 \log\left(\frac{s + m_1^2 - m_2^2 + \lambda^{1/2}(s, m_1^2, m_2^2)}{s + m_1^2 - m_2^2 - \lambda^{1/2}(s, m_1^2, m_2^2)} \right).$$