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Chapter 4

FUNDAMENTAL LEPTONS AS COMPOSITIONS OF MASSLESS PREONS: AN ALTERNATIVE TO HIGGS MECHANISM

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Abstract

Within the framework of the confinement mechanism proposed earlier by the author in quantum chromodynamics (QCD) the problem of masses for fundamental leptons in particle physics is discussed for muon and τ-lepton. It is shown that the observed parameters of the mentioned leptons such as their masses and magnetic moments can be obtained in a preon model dynamically due to a preon gauge interaction. The radii of fundamental leptons are also estimated. Under the circumstances preons might be massless in virtue of existence of the nonzero chiral limit for the preon interaction energy.

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1. Introduction and Preliminary Remarks

As is known, at present the situation with discovering Higgs boson remains obscure. In addition, the discovery of Higgs boson will not completely resolve the puzzle of origin of masses in particle physics. E.g., the question will remain about the nature of the miscellaneous coupling constants describing interactions of Higgs boson with fundamental fermions (quarks and leptons). Thereupon there makes sense to consider other possibilities to generate masses of the latter particles.

For a long time a cardinal approach to the problem of masses is connected with the preon models (see, e.g., [1] and references therein). Under this approach quarks, leptons and

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gauge vector bosons are suggested to be composed of stable spin-1/2 preons, for example, existing in three flavours (preon trinity) and being combined according to simple rules. The main theoretical objection to preon theories is the mass paradox which arises by virtue of the Heisenberg's uncertainty principle. Scattering experiments have shown [2] that quarks and leptons are point-like up to the scales of order 10^{-3} fm which corresponds to a preon mass of order 197 GeV (due to the uncertainty principle) if the preon is confined to a box of such a size, i.e. its mass will approximately be 0.4×10^5 times greater than, e.g., that of d-quark.

Thus, the preon models are faced with a mass paradox: how could quarks or electrons be made of smaller particles that would have masses of many orders of magnitude greater than the fundamental fermion masses? The paradox might be resolved by the rather dubious postulate about a large binding force between preons cancelling their mass-energies.

The appearance and evolution of quantum chromodynamics (QCD) point out the more physically acceptable way of overcoming these obstacles. Indeed, according to the point of view of QCD, light objects (such as light quarks) can be locked up at small distances (of order 1 fm) in virtue of a confinement mechanism that entails forming mesons and baryons. So long as the quark interaction is described by SU(3)-gauge theory then supposing that preons also interact with each other through a gauge field (pregluons) we can borrow the confinement mechanism of QCD to apply it for preons, naturally, in a reformulated form.

The trouble, however, consists in that at present no generally accepted quark confinement mechanism exists that would be capable to calculate a number of nonperturbative parameters characterizing hadrons (masses, radii, decay constants and so on) appealing directly to quark and gluon degrees of freedom related to QCD-Lagrangian. At best there are a few scenarios (directly not connected to QCD-Lagrangian) of confinement that restrict themselves mainly to qualitative considerations with small possiblities of concrete calculation. In view of it in [3, 4, 5] a confinement mechanism has been proposed which was based on the *unique* family of compatible nonperturbative solutions for the Dirac-Yang-Mills system directly derived from QCD-Lagrangian. The word *unique* should be understood in the strict mathematical sense. Let us write down arbitrary SU(3)-Yang-Mills field as the SU(3)-algebra Lie valued 1-form $A = A_\mu dx^\mu = A^a_\mu \lambda_a dx^\mu$ (λ_a are the known Gell-Mann matrices, $\mu = t, r, \vartheta, \varphi$, $a = 1, ..., 8$ and we use the ordinary set of local spherical coordinates r, ϑ, φ for spatial part of the flat Minkowski spacetime).

In fact in [3, 4, 5] (see also Appendix C in [11] and Appendix in [12]) the following uniqueness theorem was proved:

The unique exact spherically symmetric (nonperturbative) solutions (depending only on r and r^{-1}) of SU(3)-Yang-Mills equations in Minkowski spacetime consist of the family of form:

Electric colour field part:

$$\mathcal{A}_{1t} \equiv A^3_t + \frac{1}{\sqrt{3}}A^8_t = -\frac{a_1}{r} + A_1 \, , \mathcal{A}_{2t} \equiv -A^3_t + \frac{1}{\sqrt{3}}A^8_t = -\frac{a_2}{r} + A_2 \, ,$$

$$\mathcal{A}_{3t} \equiv -\frac{2}{\sqrt{3}}A^8_t = \frac{a_1 + a_2}{r} - (A_1 + A_2) \, ,$$

Magnetic colour field part:

$$\mathcal{A}_{1\varphi} \equiv A^3_\varphi + \frac{1}{\sqrt{3}} A^8_\varphi = b_1 r + B_1 \, , \mathcal{A}_{2\varphi} \equiv -A^3_\varphi + \frac{1}{\sqrt{3}} A^8_\varphi = b_2 r + B_2 \, ,$$

$$\mathcal{A}_{3\varphi} \equiv -\frac{2}{\sqrt{3}} A^8_\varphi = -(b_1 + b_2) r - (B_1 + B_2) \qquad (1)$$

with the real constants a_j, A_j, b_j, B_j parametrizing the family. Besides in [4, 5] (see also [6]) it was shown that the above unique confining solutions (1) satisfy the so-called Wilson confinement criterion [7, 8].

Now, having postulated interaction between two quarks as (1) (r is distance between quarks) and inserting solution (1) into the Dirac equation $\mathcal{D}\Psi = \mu_0 \Psi$ (where the explicit form of the Dirac operator $\mathcal{D}$ in arbitrary curvilinear coordinates on Minkowski spacetime can be found in [3, 4, 5]), we obtain that the meson wave functions $\Psi = (\Psi_1, \Psi_2, \Psi_3)$ are given by the unique nonperturbative modulo square integrable solutions of the mentioned Dirac equation in the confining SU(3)-field of (1) with the four-dimensional Dirac spinors Ψ_j representing the jth colour component of the meson, so Ψ may describe the relative motion (relativistic bound states) of two quarks in mesons and is at $j = 1, 2, 3$ (with Pauli matrix σ_1)

$$\Psi_j = e^{-i\omega_j t} \psi_j \equiv e^{-i\omega_j t} r^{-1} \begin{pmatrix} F_{j1}(r) \Phi_j(\vartheta, \varphi) \\ F_{j2}(r) \sigma_1 \Phi_j(\vartheta, \varphi) \end{pmatrix} , \qquad (2)$$

with the 2D eigenspinor $\Phi_j = \begin{pmatrix} \Phi_{j1} \\ \Phi_{j2} \end{pmatrix}$ of the Euclidean Dirac operator $\mathcal{D}_0$ on the unit sphere $\mathbb{S}^2$. An explicit form of Φ_j can be found, e.g., in [3, 4, 5].

The relations (1)–(2) give grounds to build a confinement mechanism for mesons and quarkonia which was successfully applied to the description of both the heavy quarkonia spectra (see [9] and references to early papers therein) and all the mesons from pseudoscalar nonet [10, 11] (and references therein). Recently in the papers [13, 14] this confinement mechanism has been extended over vector mesons while [15] does so over baryons.

If the interaction among preons is described by a QCD-like theory based on, e.g., SU(3)-group [1] then such a theory should also manifest the similar confinement mechanism to generate masses for particles composed from preons. This signifies that preons might possess small masses or be just massless and, as a result, mass paradox would be removed. In present paper we just want to explore one such a possibility by the example of muon μ^+ and τ-lepton τ^- taking into account the constructions of Ref. [1]. Clearly, the results obtained will hold true also for the conforming antiparticles: μ^- and τ^+ with obvious changes.

It should be noted, however, that in accordance with Ref. [1] the main building blocks of that model are a trinity of preons α, β, δ in notations of [1] so τ^--lepton is represented by combination $\alpha\beta\beta$ while μ^+, respectively, by that of $\alpha\alpha\delta$, i.e., we deal with three-body problem. In this situation it is natural to use the confinement mechanism for baryons [15] for modelling the mentioned leptons when replacing a trinity of quarks by that of preons. Besides we take that an interaction law for a pair of preons is given by (1) (r is now a distance between preons) while the parameters a_j, b_j, B_j in (1) now describe a pregluonic field and we may put $A_j = 0$ as in QCD [3, 4, 5].

Section 2 contains a survey of main relations underlying description of any premesons (the two-body problem) as well as a construction of the possible wave functions of leptons (the three-body problem) in our approach which allows us to obtain such characteristics as the radii and magnetic moments of leptons in an explicit analytic form. Section 3 uses the obtained relations to give numerical results for the case of leptons under exploration. Section 4 is devoted to concluding remarks. At last, Appendix recalls a few facts about tensorial products of Hilbert spaces necessary to construct the leptonic wave functions.

Throughout the paper we employ the Heaviside-Lorentz system of units with $\hbar = c = 1$, unless explicitly stated otherwise, so the (preon) gauge coupling constant g and the (preon) strong coupling constant α_{ps} are connected by relation $g^2/(4\pi) = \alpha_{ps}$. Further we shall denote $L_2(F)$ the set of the modulo square integrable complex functions on any manifold F furnished with an integration measure, then $L_2^n(F)$ will be the n-fold direct product of $L_2(F)$ endowed with the obvious scalar product while $\dagger$ and $*$ stand, respectively, for Hermitian and complex conjugation.

When calculating we apply the relations 1 $\mathrm{GeV}^{-1} \approx 0.1973269679$ fm, 1 $\mathrm{s}^{-1} \approx 0.658211915 \times 10^{-24}$ GeV.

2. Specification of Main Relations

Let us describe the main relations necessary for further considerations. The given relations are modelled on those used in our QCD-considerations with obvious changes.

2.1. Two-body Wave Functions

The two-body wave functions describing a pair of preons (premeson) interacting according to the solution (1) are given by (2).

In this situation, if a premeson is composed of preons $q_{1,2}$ with different flavours then the energy spectrum of the premeson will be given by $\epsilon = m_{q_1} + m_{q_2} + \omega$ with the current preon masses m_{q_k} (rest energies) of the corresponding preons and an interaction energy ω. On the other hand at $j = 1, 2, 3$ the energy ω_j is quantized [3, 4, 5] according to

$$\omega_j = \omega_j(n_j, l_j, \lambda_j) =$$

$$\frac{\Lambda_j g^2 a_j b_j + (n_j + \alpha_j)\sqrt{(n_j^2 + 2n_j\alpha_j + \Lambda_j^2)\mu_0^2 + g^2 b_j^2 (n_j^2 + 2n_j\alpha_j)}}{n_j^2 + 2n_j\alpha_j + \Lambda_j^2} \tag{3}$$

with the gauge coupling constant g while μ_0 is a mass parameter and one should consider it to be the reduced mass which is equal to $m_{q_1} m_{q_2}/(m_{q_1} + m_{q_2})$ with the current preon masses m_{q_k} (rest energies) of the corresponding preons forming a premeson, $a_3 = -(a_1 + a_2)$, $b_3 = -(b_1 + b_2)$, $B_3 = -(B_1 + B_2)$, $\Lambda_j = \lambda_j - gB_j$, $\alpha_j = \sqrt{\Lambda_j^2 - g^2 a_j^2}$, $n_j = 0, 1, 2, ...$, while $\lambda_j = \pm(l_j + 1)$ are the eigenvalues of Euclidean Dirac operator $\mathcal{D}_0$ on a unit sphere with $l_j = 0, 1, 2, ...$

In line with the above we should have $\omega = \omega_1 = \omega_2 = \omega_3$ in energy spectrum $\epsilon = m_{q_1} + m_{q_2} + \omega$ for the premeson and this at once imposes two conditions on parameters a_j, b_j, B_j when choosing some value for ϵ at the given current preon masses m_{q_1}, m_{q_2}.

The general form of the radial parts of (2) can be found, e.g., in Refs. [5, 9, 10, 11] and within the given paper we need only the radial parts of (2) at $n_j = 0$ (the ground state) that are

$$F_{j1} = C_j P_j r^{\alpha_j} e^{-\beta_j r}\left(1 - \frac{gb_j}{\beta_j}\right), P_j = gb_j + \beta_j,$$

$$F_{j2} = iC_j Q_j r^{\alpha_j} e^{-\beta_j r}\left(1 + \frac{gb_j}{\beta_j}\right), Q_j = \mu_0 - \omega_j \quad (4)$$

with $\beta_j = \sqrt{\mu_0^2 - \omega_j^2 + g^2 b_j^2}$, while C_j is determined from the normalization condition $\int_0^\infty (|F_{j1}|^2 + |F_{j2}|^2)dr = \frac{1}{3}$. The corresponding eigenspinors of (2) with $\lambda_j = \pm 1$ ($l_j = 0$) are

$$\lambda_j = -1 : \Phi = \frac{C}{2}\begin{pmatrix} e^{i\frac{\vartheta}{2}} \\ e^{-i\frac{\vartheta}{2}} \end{pmatrix} e^{i\varphi/2}, \text{ or } \Phi = \frac{C}{2}\begin{pmatrix} e^{i\frac{\vartheta}{2}} \\ -e^{-i\frac{\vartheta}{2}} \end{pmatrix} e^{-i\varphi/2},$$

$$\lambda_j = 1 : \Phi = \frac{C}{2}\begin{pmatrix} e^{-i\frac{\vartheta}{2}} \\ e^{i\frac{\vartheta}{2}} \end{pmatrix} e^{i\varphi/2}, \text{ or } \Phi = \frac{C}{2}\begin{pmatrix} -e^{-i\frac{\vartheta}{2}} \\ e^{i\frac{\vartheta}{2}} \end{pmatrix} e^{-i\varphi/2} \quad (5)$$

with the coefficient $C = 1/\sqrt{2\pi}$ (for more details, see Refs. [3, 4, 5]). In what follows for distinctness we take all eigenvalues λ_j of the Euclidean Dirac operator $\mathcal{D}_0$ on the unit 2-sphere $\mathbb{S}^2$ equal to 1.

2.2. The Root-mean-square Radius and Magnetic Moment

With the help of the premeson wave functions of ground state it is easy to calculate the root-mean-square radius of the premeson in the form ($d^3x = r^2 \sin\vartheta dr d\vartheta d\varphi$)

$$< r >= \sqrt{\int r^2 \Psi^\dagger \Psi d^3x} = \sqrt{\sum_{j=1}^{3} \int r^2 \Psi_j^\dagger \Psi_j d^3x} = \sqrt{\sum_{j=1}^{3} \frac{2\alpha_j^2 + 3\alpha_j + 1}{6\beta_j^2}} \quad (6)$$

that is in essence a radius of confinement and is a function of parameters of pregluonic field from (1) and the current preon masses.

Also we can define magnetic form factor $F(Q^2)$ (Q is the momentum transfer) for a premeson which entails *the anomalous magnetic moment* for the premeson by the relation (for more details see [15])

$$\mu_a = F(Q^2 = 0) = \frac{2}{3}\frac{|q|}{q}\sum_{j=1}^{3}\frac{2\alpha_j + 1}{6\beta_j} \cdot \frac{\mathcal{P}_j \mathcal{Q}_j}{\mathcal{P}_j^2 + \mathcal{Q}_j^2} \quad (7)$$

with the premeson electric charge q and $\mathcal{P}_j = P_j(1 - gb_j/\beta_j)$, $\mathcal{Q}_j = O_j(1 + gb_j/\beta_j)$.

2.3. The Wave Functions of Leptons

Let us reformulate the results of Ref. [15] for baryons and quarks, accordingly, replacing them by leptons and preons. In accordance with [1] fundamental leptons are composed from three preons q_1, q_2, q_3, generally speaking, with different flavours, so let us denote distances between (q_1, q_2), (q_1, q_3), (q_2, q_3), respectively, as r_1, r_2, r_3, then a lepton can be represented in the schematic form shown in Fig. 1. As a consequence, we are faced with a relativistic 3-body problem. Under the circumstances the main task is to obtain an

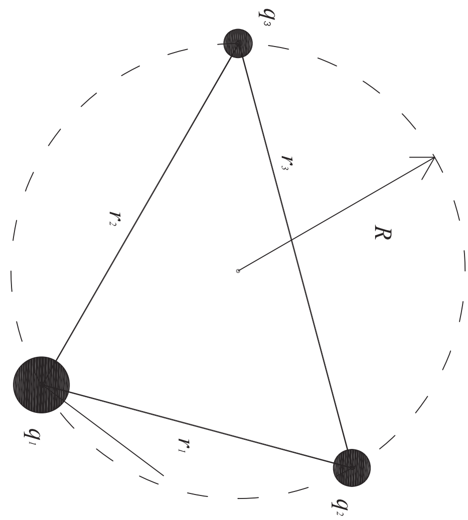

Figure 1. Schematic sketch of a lepton according to [1].

interaction Hamiltonian H of the 3-preon system if taking that every two preons interact through the pregluonic field described by solution (1), in general, with different constants a_j, b_j, B_j for each pair of preons. Then energy levels of a lepton will be given by relation $\epsilon = m_{q_1} + m_{q_2} + m_{q_3} + \omega$ with the current preon masses m_{qk} (rest energies) of the preons constituting the lepton while the interaction energy ω should be subject to equation $H\psi = \omega\psi$ with a leptonic wave function ψ describing relative motion (bound state) of three preons. Further on the analogy of the baryon case [15] it should be noted that a pregluon condensate (a classical SU(3)-pregluonic field) should be mainly concentrated near the axis connecting preons. Under the circumstances we may represent a lepton composed from three pairs of the interacting preons in the schematic form of Fig. 2, where the shaded areas schematically stand for soft pregluons between preons.

As a consequence, we may suppose main interaction of preons in a lepton to be concentrated along the axes joining preons in each pair while one may practically neglect possible interaction of any pair of preons as a whole with third remaining preon. When approaching all three preons the latter interaction should not increase due to asymptotical freedom (which exists for any SU(N)-gauge theory) so the given picture may in fact hold true at the scales of leptons. In what follows we consider that is the case and then we can construct the lepton wave functions in the next way.

In view of the above it is natural to characterize interaction of each pair of preons in a lepton by a Hamiltonian and, accordingly, relative motion of preons in the pair by spherical coordinates r_i, ϑ_i, φ_i. Then we can introduce the Hamiltonians $H^{(i)} = (H_j^{(i)})$ with colour index $j = 1, 2, 3$. In their turn, $H^{(i)}$ will depend on $a_j^{(i)}$, $b_j^{(i)}$,$B_j^{(i)}$, i. e., the conforming parameters of solution (1) describing interaction of the ith pair of preons while μ_0 will be the reduced mass of the corresponding preon pair. Each $H^{(i)}$ acts in Hilbert space $L_2^{12}(\mathbb{R}^3)$ which possesses the basis consisting of the eigenfunctions of $H^{(i)}$ and having the form of

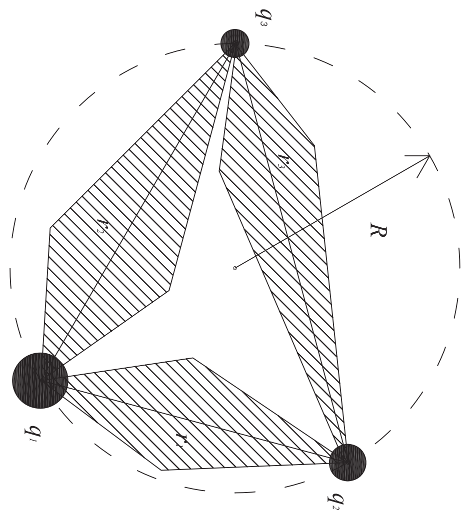

Figure 2. Schematic sketch of preon interactions in a lepton.

(2) (with the corresponding change of notations and omitting the phase factor $e^{-i\omega_j t}$). In what follows we denote such a basis by $\psi^{(i)} = (\psi_j^{(i)})$. In virtue of smallness of lepton sizes we may put the time variables t for each preon pair to be equal, i. e., $t_1 = t_2 = t_3$ and then the stationary states of a lepton can be chosen in the form of tensorial product of $\psi^{(i)}$

$$\psi = \psi^{(1)} \otimes \psi^{(2)} \otimes \psi^{(3)} , \tag{8}$$

so ψ will be a basis in the Hilbert space $\mathcal{H} = L_2^{12}(\mathbb{R}^3) \otimes L_2^{12}(\mathbb{R}^3) \otimes L_2^{12}(\mathbb{R}^3)$ and also the eigenfunctions for operator in $\mathcal{H}$ (with identical operator I)

$$H = H^{(1)} \otimes I \otimes I + I \otimes H^{(2)} \otimes I + I \otimes I \otimes H^{(3)} \tag{9}$$

with the eigenvalues $\omega = \omega_j^{(1)} + \omega_j^{(2)} + \omega_j^{(3)}$, where $\omega_j^{(i)}$ is an eigenvalue for operator $H_j^{(i)}$ (see the explicit form of H in [15] and Appendix A for more details concerning the tensorial products of Hilbert spaces), i.e.,

$$H\psi = H(\psi^{(1)} \otimes \psi^{(2)} \otimes \psi^{(3)}) = \omega\psi = (\omega_j^{(1)} + \omega_j^{(2)} + \omega_j^{(3)})\psi . \tag{10}$$

It is clear that operator H is just the sought interaction Hamiltonian for preons in a lepton. As a result, the energy levels of the lepton will be given by relation $\epsilon = m_{q_1} + m_{q_2} + m_{q_3} + \omega$ with the current preon masses m_{q_k} (rest energies) of the preons constituting the lepton and the interaction energy ω of (10).

2.4. Chiral Limit

There is an interesting limit for relation (3) – the chiral one, i.e., the situation when $m_{q1}, m_{q2} \to 0$ which entails $\mu_0 \to 0$ and (3) reduces to (at $j = 1, 2, 3$)

$$(\omega_j)_{\text{chiral}} = \frac{\Lambda_j g^2 a_j b_j + (n_j + \alpha_j) g |b_j| \sqrt{n_j^2 + 2n_j\alpha_j}}{n_j^2 + 2n_j\alpha_j + \Lambda_j^2} , \tag{3'}$$

which mathematically signifies that the Dirac equation in the field (1) possesses a nontrivial spectrum of bound states even for massless fermions. Physically this gives us a possible approach to the problem of chiral symmetry breaking in QCD [10]: in chirally symmetric world masses of mesons are fully determined by the confining SU(3)-gluonic field between (massless) quarks and not equal to zero. Accordingly chiral symmetry is a sufficiently rough approximation holding true only when neglecting the mentioned SU(3)-gluonic field between quarks and no additional mechanism of the spontaneous chiral symmetry breaking connected to the so-called Goldstone bosons is required. As a result, e.g., masses of mesons from pseudoscalar nonet have a purely gluonic contribution and we considered it in Ref. [11] more in details.

One can note that for being the nonzero chiral limit of (3) the crucial role belongs to the colour magnetic field linear in r [parameters $b_{1,2}$ from solution (1)] inasmuch as chiral limit is equal exactly to zero when $b_{1,2} = 0$. On the contrary, when parameters $a_{1,2}$ of the Coulomb colour electric part of solution (1) are equal to zero, the chiral limit may be nonzero at $b_{1,2} \neq 0$, as is seen from $(3')$ except for the case $n_j = 0$ when both parts of SU(3)-gluonic field (1) are important for confinement and mass generation in chiral limit.

2.5. Choice of Preon Masses and the Gauge Coupling Constant

Obviously, we should choose a few quantities that are the most important from the physical point of view to characterize leptons under consideration and then we should evaluate the given quantities within the framework of our approach. In the circumstances let us settle on the ground state energy (mass) of lepton, the root-mean-square radius of it and the magnetic moment. All three magnitudes are essentially nonperturbative ones, and can be calculated only by nonperturbative techniques.

It is evident for employing the above relations we have to assign some values to preon masses and the (preon) gauge coupling constant g. It should be noted the following. As was remarked in subsection 2.4, the Dirac equation in the field (1) possesses a nontrivial spectrum of bound states even for massless fermions. As a result, mass of any premeson remains nonzero in chiral limit when masses of preons $m_q \to 0$ and premeson masses will only be expressed through the parameters of the confining SU(3)-pregluonic field of (1) (see relation $(3')$).

We can here only say that, in the preon case, where we know nothing about possible preon masses, we can put $m_q = 0$. Under the circumstances we shall use relations $(3')$ at $n_j = 0 = l_j$ so energy (mass) of premeson is given by $m_{mes} = m_{q1} + m_{q2} + \omega = \omega$ with $\omega = \omega_j(0, 0, \lambda_j)$ for any $j = 1, 2, 3$ whereas

$$\omega = \frac{g^2 a_1 b_1}{\Lambda_1} = \frac{g^2 a_2 b_2}{\Lambda_2} = \frac{g^2 a_3 b_3}{\Lambda_3} = m_{mes} \, . \tag{11}$$

Further one should evidently take into account that at $m_q = 0$ for leptons under investigation from physical point of view we have all the grounds of considering the interaction in pairs $q_1 q_2$, $q_1 q_3$, $q_2 q_3$ (see Fig. 1) approximately to be the same and, as a result, the corresponding parameters $a_j^{(i)}$, $b_j^{(i)}$ $B_j^{(i)}$ of pregluonic field from solution (1) describing interaction of the ith pair of preons should be equal: $a_j^{(i)} = a_j$, $b_j^{(i)} = b_j$, $B_j^{(i)} = B_j$. Then, in accordance with (10) and (11) we obtain the leptonic masses as

$$m = 3(m_q + \omega) = 3\omega\,, \tag{12}$$

where $\omega = m/3$ is given by (11) while $m = 105.658367$ MeV, 1776.82 MeV for the leptons under consideration [16].

As to the gauge coupling constant $g = \sqrt{4\pi\alpha_{ps}}$, it should be noted that in QCD recently some attempts have been made to generalize the standard formula for the usual strong coupling constant $\alpha_s = \alpha_s(Q^2) = 12\pi/[(33-2n_f)\ln(Q^2/\Lambda^2)]$ (n_f is number of quark flavours) holding true at the momentum transfer $\sqrt{Q^2} \to \infty$ to the whole interval $0 \leq \sqrt{Q^2} \leq \infty$. Since preons are suggested to be described by a QCD-like theory then we can employ one such a generalization used in Refs. [17, 18] which we have already discussed elsewhere (for more details see [9, 10]) to obtain $g \approx 6.16465, 2.75451$ necessary for our further computations at the mass scale of the corresponding leptons.

2.6. Lepton Radius

Obviously, in virtue of the above assumptions it is natural for leptons under exploration to put $< r_1 >^2 = < r_2 >^2 = < r_3 >^2 = r_0^2$, where $< r_i >^2$ conforms to the corresponding preon pair (see Fig. 1). But then simple geometric considerations give rise to the lepton radius $< R > = r_0/\sqrt{3}$ which entails the expression

$$< R > = \frac{1}{\sqrt{3}}\sqrt{\sum_{j=1}^{3}\frac{2\alpha_j^2+3\alpha_j+1}{6\beta_j^2}} \tag{13}$$

according to (6).

2.7. Magnetic Moment

Also it is natural to define the lepton magnetic form factor by the relation $F_M(Q^2) = \frac{1}{3}\sum\limits_{i=1}^{3} F_i(Q^2)$, where form factor $F_i(Q^2)$ corresponds to ith preon pair (see Fig. 1), which gives rise to the expression for the lepton magnetic moment in virtue of (7)

$$\mu_a = F_M(Q^2=0) = \frac{2}{3}\frac{|q|}{q}\sum_{j=1}^{3}\frac{2\alpha_j+1}{6\beta_j}\cdot\frac{\mathcal{P}_j\mathcal{Q}_j}{\mathcal{P}_j^2+\mathcal{Q}_j^2}\,,$$

$$\mathcal{P}_j = P_j(1-gb_j/\beta_j)\,,\ \mathcal{Q}_j = O_j(1+gb_j/\beta_j) \tag{14}$$

while q is electric charge of ith preon pair so that, e.g., for τ^--lepton $|q|/q = -1$ in accordance with [1].

3. Numerical Results

Now we should consider the equations (12)–(14) as a system which should be solved compatibly when $m_q = 0$ MeV with m and g adduced above for each lepton while

$\mu_a = 0.404249 \times 10^{-3}$ 1/MeV, 0.237637×10^{-4} 1/MeV according to [16]. At last the possible values of $< R >$ can be estimated from the following considerations. Classical radius of muon $r_\mu = m_e r_e/m \sim 0.0136$ fm while that of τ-lepton $r_\tau = m_e r_e/m \sim 0.810417 \times 10^{-3}$ fm, where classical electron radius $r_e \approx 2.818$ fm and electron mass $m_e \approx 0.511$ MeV [16]. On the other hand, the Compton wave length for muon and τ-lepton is, respectively, $\lambda_c \approx 1.867594337$ fm, 0.111 fm. But, as is known [19], from the point of view of quantum electrodynamics (QED) the photon condensate [huge number of (virtual) photons] described by Coulomb law exists only at $r >> \lambda_c$ while at $r < \lambda_c$ one may only speak about single photons rather than about condensate, i.e., the field in classical sense. It is clear, however, the preon confinement should be realized at the scales $r < \lambda_c$ on the analogy of QCD. Indeed, e.g., for charged pions with masses of order 140 MeV we have $\lambda_c \sim 1.41$ fm while the radius of pions (radius of confinement) is of order 0.6 fm [16]. So that it is reasonable to take $< R >\sim (0.7 - 0.8)$ fm for muon and (0.07-0.08) fm for τ-lepton. As to electron, the Compton wave length for electron $\lambda_c \approx 386$ fm and classical electron radius $r_e \approx 2.818$ fm [16]. For comparison a confinement radius for proton is of order 0.8 fm [16]. So at present we have not any reasonable estimate for a size of electron at our disposal and, as a consequence, we did not include electron in our considerations within the framework of the given paper.

At last, as was remarked in Subsection 2.1, while computing for distinctness we took all eigenvalues λ_j of the Euclidean Dirac operator $\mathcal{D}_0$ on the unit 2-sphere $\mathbb{S}^2$ equal to 1. The results of numerical compatible solving of equations (12)–(14) are gathered in Tables 1 and 2.

Table 1. Gauge coupling constant, reduced mass μ_0 and parameters of the confining SU(3)-pregluonic field for fundamental leptons.

Particle	g	μ_0 (MeV)	a_1	a_2	b_1 (GeV)	b_2 (GeV)	B_1	B_2
$\mu^+ = \alpha\alpha\delta$	6.16465	0	0.0163468	-0.00513040	0.103875	-0.0136027	-0.135	0.150
$\tau^- = \alpha\beta\beta$	2.75451	0	-0.0789020	0.15250	-4.30040	0.934868	-1.2150	-0.300

Table 2. Theoretical and experimental masses, magnetic moments and radii for fundamental leptons.

Particle	Theoret. (MeV)	Experim. (MeV)	Theoret. μ_a (1/MeV)	Experim. μ_a (1/MeV)	Theoret. $< R >$ (fm)	Experim. $< R >$ (fm)
$\mu^+ = \alpha\alpha\delta$	$m = 3\omega_j(0,0,1)$ $= 105.658$	105.658367	0.390919 $\times 10^{-3}$	0.40424 $\times 10^{-3}$	0.754968	–
$\tau^- = \alpha\beta\beta$	$m = 3\omega_j(0,0,1)$ $= 1776.82$	1776.82	-0.237637 $\times 10^{-4}$	-0.240386 $\times 10^{-4}$	0.0769993	–

4. Conclusion

Of course, the tables 1 and 2 correspond only to one of possible solutions for system (12)–(14). Further specification of such solutions requires knowledge of the exact preon masses, if any, and the (preon) gauge coupling constant g. At present the given knowledge is absent as well as the existence of preons themselves is the open question. But our results shows that the mechanisms of generating masses dynamically for fundamental leptons can be realized. The similar considerations can be conducted for both electron and quarks and also for all types of neutrinos if the latter would possess masses.

The possibility of generating massive particles from *massless* ones should be specially emphasized. As was noted in subsection 2.4, this remarkable feature is obligatory to the colour magnetic field of solution (1) which in essence provides two preons (quarks) with confinement [13, 14, 15].

Finally, one should make a remark about the role of gravity. In accordance with [1] the preons are supposed to be the spin-1/2 particles. On the other hand, quanta of gravitational field, gravitons, should be the spin-2 particles when being massless themselves. So far, however, gravitons have not been detected (see, e.g., recent review [20]). As a result, massless preons are not gravitons but, as we have seen above, they could dynamically generate masses even being massless themselves. Under the situation, then perhaps the gravity is a statistical concept like entropy or temperature, only defined for gravitational effects of matter in bulk and not for effects of individual elementary particles [20]. As a consequence, then the gravitational field is not a local field like the electromagnetic field which implies that the gravitational field at a point in spacetime does not exist, either as a classical or as a quantum field.

Appendix

In subsection 2.3 we built the three-body leptonic wave functions as tensorial products from the two-body wave functions of premesons. The latter, however, are represented by triplets ψ of $4D$-Dirac spinors [see, e.g., (2)], so ψ belongs to the Hilbert space $\mathcal{H} = L_2^{12}(\mathbb{R}^3)$. Under the situation we here recall some facts about tensorial products of Hilbert spaces necessary for building the lepton wave functions. For more information see, e.g., Ref. [21] and references therein.

For two Hilbert spaces $\mathcal{H}_1$ and $\mathcal{H}_2$, by definition, their tensorial product $\mathcal{H}_1 \otimes \mathcal{H}_2$ consists from every possible linear combinations of elements of the form $\vec{x}_1 \otimes \vec{x}_2$, where $\vec{x}_1 \in \mathcal{H}_1$, $\vec{x}_2 \in \mathcal{H}_2$. The obvious relations take place

$$(\alpha\vec{x}_1 + \beta\vec{x}_2) \otimes \vec{y} = \alpha(\vec{x}_1 \otimes \vec{y}) + \beta(\vec{x}_2 \otimes \vec{y}) \,, \tag{A.1}$$

$$x \otimes (\alpha\vec{y}_1 + \beta\vec{y}_2) = \alpha(\vec{x} \otimes \vec{y}_1) + \beta(\vec{x} \otimes \vec{y}_2) \tag{A.2}$$

with arbitrary complex numbers α, β.

Scalar product in $\mathcal{H}_1 \otimes \mathcal{H}_2$ is defined by the equality

$$(\vec{x}_1 \otimes \vec{y}_1, \vec{x}_2 \otimes \vec{y}_2) = (x_1, x_2)_1 (y_1, y_2)_2 \,, \tag{A.3}$$

where $(x_1, x_2)_1$, $(y_1, y_2)_2$ are the scalar products, respectively, in $\mathcal{H}_1$ and $\mathcal{H}_2$. Under the situation, if $\{\vec{e}_i\}$ is an orthonormal basis in $\mathcal{H}_1$ and $\{\vec{f}_j\}$ is an orthonormal basis in $\mathcal{H}_2$ then $\vec{h}_{ij} = \vec{e}_i \otimes \vec{f}_j$ is an orthonormal basis in $\mathcal{H}_1 \otimes \mathcal{H}_2$.

Further, if A is a linear operator in $\mathcal{H}_1$ and B is a linear operator in $\mathcal{H}_2$ then tensorial (Kronecker) product $A \otimes B$ acts in $\mathcal{H}_1 \otimes \mathcal{H}_2$ and is defined by relation $(A \otimes B)(\vec{x}_1 \otimes \vec{x}_2) = A\vec{x}_1 \otimes B\vec{x}_2$ with the properties

$$A \otimes (\alpha B_1 + \beta B_2) = \alpha A \otimes B_1 + \beta A \otimes B_2 \,, \tag{A.4}$$

$$(\alpha A_1 + \beta A_2) \otimes B = \alpha A_1 \otimes B + \beta A_2 \otimes B \,, \tag{A.5}$$

$$A_1 A_2 \otimes B_1 B_2 = (A_1 \otimes B_1)(A_2 \otimes B_2) \,. \tag{A.6}$$

In a similar way, the Kronecker sum of A and B is defined by $A \oplus B = A \otimes I + I \otimes B$ with identical operator I. Under the circumstances, let λ_i be the eigenvalues A and μ_j be those of B (listed according to multiplicity). Then the eigenvalues of $A \otimes B$ are $\lambda_i \mu_j$ while those of $A \oplus B$ will be $\lambda_i + \mu_j$.

All the above is directly generalized to the arbitrary number of Hilbert spaces. For example, the operator $A_1 \otimes I \otimes I + I \otimes A_2 \otimes I + I \otimes I \otimes A_3$ has the eigenvalues $\lambda_i + \mu_j + \nu_k$ in space $\mathcal{H}_1 \otimes \mathcal{H}_2 \otimes \mathcal{H}_3$ if λ_i, μ_j, ν_k are the eigenvalues of A_1, A_2 and A_3, respectively, in $\mathcal{H}_1, \mathcal{H}_2, \mathcal{H}_3$.

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