Impact Factors for Reggeon-Gluon Transitions in the Infrared Region

V.S Fadin

Budker Institute of Nuclear Physics
Novosibirsk

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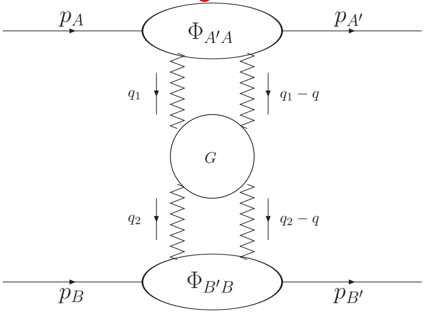
Introduction

In the BFKL (Balitsky–Fadin–Kuraev–Lipatov) approach $2 \to 2$ scattering amplitudes are given by the convolution:

$$\Phi_{A'A} \otimes G \otimes \Phi_{B'B}.$$

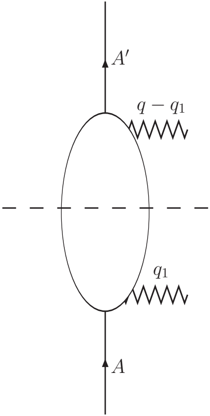
(here s-channel from left to right, t-channel from up to down).



$$\hat{\mathcal{G}} = e^{Y\hat{\mathcal{K}}},$$

$\hat{\mathcal{K}}$ is the BFKL kernel, Y is the total rapidity $(Y = \ln(s/s_0))$,
and the amplitude (more precisely, its imaginary part)

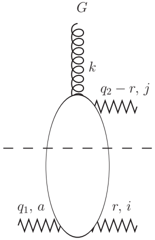
Particle-particle impact factors $\Phi_{A'A}$ describe transitions of particles $A \to A'$ due to interaction with reggeized gluons (*t*-channel from left to right, *s*-channel from down to up).



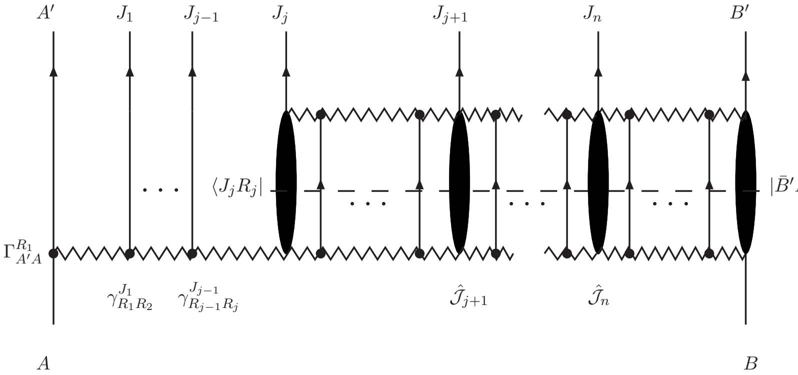
The dashed line denotes discontinuity of the reggeon-particle scattering amplitude. In fact, the impact factor is given by the integral over squared invariant mass of particles on this discontinuity.

Introduction

The impact factors for Reggeon-gluon transitions describe transitions of Reggeons(Reggeized gluons) into ordinary gluons due to interaction with Reggeized gluons.



These impact factors enter the s_i discontinuities of multi-Regge amplitudes similar to as particle impact factors enter the discontinuities of elastic amplitudes



Applications of these discontinuites

— For the proof of the Reggeized form of multi-particle amplitudes.

There is an infinite set of the bootstrap relations which follow from the requirement of compatibility of the s_i-channel unitarity with the Reggeized form of multi-particle amplitudes.

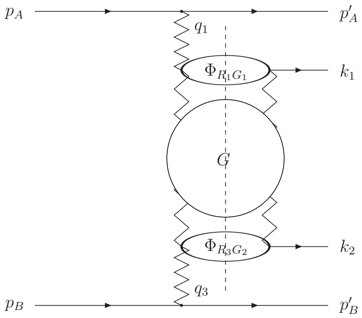
Fulfillment of these relations guarantees the multi–Regge form of scattering amplitudes.

All bootstrap relations are fulfilled if several bootstrap conditions imposed on the Reggeon vertices and the trajectory are satisfied.

All these conditions are proved to be satisfied in the NLO.

— For a simple demonstration of violation of the BDS ansatz M^{BDS} for MHV amplitudes in N=4 SYM in the planar limit.

Energy behaviour of the discontinuity of $A_{2\to4}$ in the s_2- channel (s_2 is the invariant mass square of produced gluons)

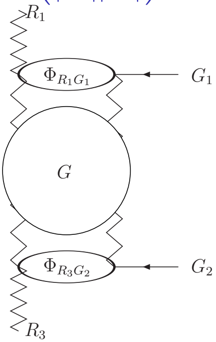


is not given by the Regge factor $\left(\frac{s_2}{|\vec{k}_1||\vec{k}_2|}\right)^{\omega(t_2)}$, as in the BDS ansatz.

Introduction

Instead, it is determined by the matrix element

$$\langle G_1 R_1 | e^{\hat{K}\ln\left(\frac{s_2}{|\vec{k}_1||\vec{k}_2|}\right)} | G_2 R_3 \rangle = \left(\frac{s_2}{|\vec{k}_1||\vec{k}_2|}\right)^{\omega(t_2)} \langle G_1 R_1 | e^{\hat{K}_m \ln\left(\frac{s_2}{|\vec{k}_1||\vec{k}_2|}\right)} | G_2 R_3$$



where $\hat{K}_m = \hat{K} - \omega(t)$ – modified BFKL kernel.

Introduction

—For the check of validity of hypotheses about the reminder function R to the BDS ansatz:

Factorization hypothesis, which states the correct amplitude can be presented as the product M^{BDS} times the remainder function R, where M^{BDS} contains all infrared divergencies and R depends only on the anharmonic ratios of kinematic invariants. This property is called dual conformal (Möbius) invariance.

This hypothesis is not proved.

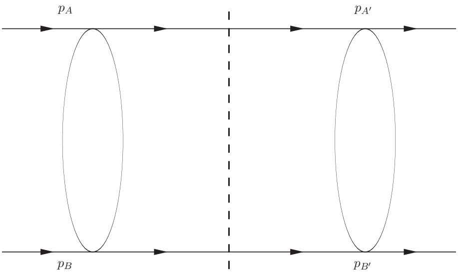
the hypothesis that the remainder functions are given by expectation values of Wilson loops in N=4 SYM.

This hypothesis is also not proved.

To check these hypotheses we performed the calculation of the matrix element $\langle G_1 R_1 | e^{\hat{K}_m \ln\left(\frac{s_2}{|\vec{k}_1||\vec{k}_2|}\right)} | G_2 R_3 \rangle$.

Introduction

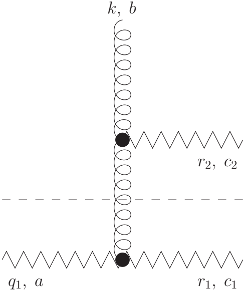
– —For development of the BFKL approach in the NNLLA.
In the simplest two-particle intermediate state:



In imaginary parts, one $\ln s$ is lost. Products of imaginary and real parts in the unitarity relations cancel due to summation of contributions complex conjugated to each other. Due to this, imaginary parts don't play any role in the NLLA.
But they become important in the NNLLA.

NLO impact factors

In the Born approximation only gluon can be in the intermediate state



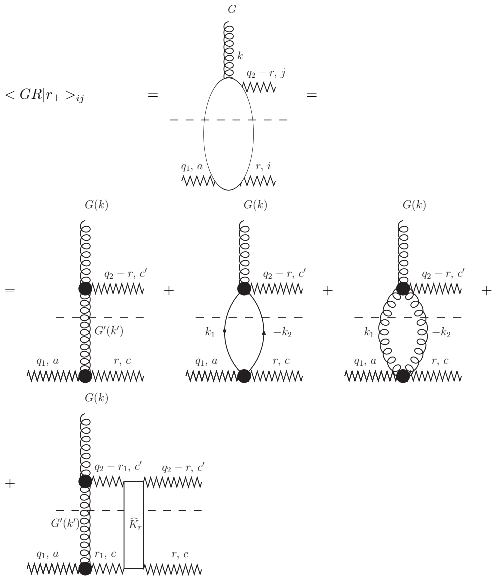
$$\langle GR_1|\mathcal{G}_1\mathcal{G}_2\rangle^{(B)} = 2g^2\delta(\vec{q}_1 - \vec{k} - \vec{r}_1 - \vec{r}_2)\left(T^aT^b\right)_{c_1c_2} \vec{e}^{\,*}\vec{C}_1,$$

$\vec{e}^{\,*}$ is the conjugated transverse part of the polarization vector $e(k)$ in the gauge $e(k)p_2 = 0$ with the lightcone vector p_2 close to the vector p_B,

$$\vec{C}_1 \;=\; \vec{q}_1 - (\vec{q}_1 - \vec{r}_1)\frac{\vec{q}_1^{\,2}}{(\vec{q}_1 - \vec{r}_1)^2}.$$

NLO impact factors

In the NLO



The NLO impact factor contains gluon, fermion and scalar contributions. These contributions were found
M.G. Kozlov, A.V. Reznichenko, V.S. F. 2012,
M.G. Kozlov, A.V. Reznichenko, V.S. F. 2013,
for Yang-Mills theories with any number of fermions and scalars in arbitrary representations of the gauge group.

The In general, the impact factors contain two parts with different colour structure.

$$\langle GR_1|\mathcal{G}_1\mathcal{G}_2\rangle = g^2\delta(\vec{q}_1-\vec{k}-\vec{r}_1-\vec{r}_2)\vec{e}^{\,*}\Bigg[\left(T^aT^b\right)_{c_1c_2}\Bigg(2\vec{C}_1$$

$$+\bar{g}^2\vec{\Phi}_1(\vec{q}_1,\vec{k};\vec{r}_1,\vec{r}_2)\Bigg)+\frac{1}{N_c}Tr\left(T^{c_2}T^aT^{c_1}T^b\right)\;\bar{g}^2\vec{\Phi}_2(\vec{q}_1,\vec{k};\vec{r}_1,\vec{r}_2)\Bigg].$$

NLO impact factors

In the planar N=4 SYM

$$\vec{\Phi}_1 = \vec{C}_1\left(\ln\left(\frac{(\vec{q}_1-\vec{r}_1)^2}{\vec{k}^{\,2}}\right)\ln\left(\frac{\vec{r}_2^{\,2}}{\vec{k}^{\,2}}\right)+\ln\left(\frac{(\vec{q}_1-\vec{r}_1)^2\vec{q}_1^{\,2}}{\vec{k}^{\,4}}\right)\ln\left(\frac{\vec{r}_1^{\,2}}{\vec{q}_1^{\,2}}\right)\right.$$

$$\left.-4\frac{(\vec{k}^{\,2})^{\epsilon}}{\epsilon^2}+6\zeta(2)\right)+\vec{C}_2\left(\ln\left(\frac{\vec{k}^{\,2}}{\vec{r}_2^{\,2}}\right)\ln\left(\frac{(\vec{q}_1-\vec{r}_1)^2}{\vec{r}_2^{\,2}}\right)\right.$$

$$\left.+\ln\left(\frac{\vec{q}_2^{\,2}}{\vec{q}_1^{\,2}}\right)\ln\left(\frac{\vec{k}^{\,2}}{\vec{q}_2^{\,2}}\right)\right)-2\left[\vec{C}_1\times\left(\left[\vec{k}\times\vec{r}_2\right]I_{\vec{k},\vec{r}_2}-\left[\vec{q}_1\times\vec{r}_1\right]I_{\vec{q}_1,-\vec{r}_1}\right)\right]$$

$$\left.+2\left[\vec{C}_2\times\left(\left[\vec{k}\times\vec{r}_2\right]I_{\vec{k},\vec{r}_2}+\left[q_1\times\vec{k}\right]I_{\vec{q}_1,-\vec{k}}\right)\right]\right\}.$$

NLO impact factors

Here $\bar{g}^2 = g^2\Gamma(1-\epsilon)/(4\pi)^{2+\epsilon}$,

$$\vec{C}_2 \;=\; \vec{q}_1 - \vec{k}\frac{\vec{q}_1^{\,2}}{\vec{k}^{\,2}},$$

$\Gamma(x)$ is the Euler gamma-function, $\zeta(n)$ is the Riemann zeta-function ($\zeta(2) = \pi^2/6$), $\left[\vec{a}\times c\left[\vec{b}\times\vec{c}\right]\right]$ is a double vector product, and

$$I_{\vec{p},\vec{q}} = \int_0^1 \frac{dx}{(\vec{p}+x\vec{q})^2}\ln\left(\frac{\vec{p}^{\,2}}{x^2\vec{q}^{\,2}}\right)\;,\quad I_{\vec{p},\vec{q}} = I_{-\vec{p},-\vec{q}} = I_{\vec{q},\vec{p}} = I_{\vec{p},-\vec{p}-\vec{q}}\;.$$

The NLO correction $\vec{\Phi}_1$ is obtained after huge cancellations between gluon, fermion and scalar contributions. In particular, solely due to these cancellations only two vector structures ($\vec{C}_1$ and $\vec{C}_2$) remain; each of the contributions separately contains three independent vector structures.

Problems with infrared behaviour

The impact factors $\vec{\Phi}_i(\vec{q}_1, \vec{k}; \vec{r}_1, \vec{r}_2)$ were obtained with the accuracy up to terms vanishing at $\epsilon \to 0$.
Unfortunately, using $\vec{\Phi}_i(\vec{q}_1, \vec{k}; \vec{r}_1, \vec{r}_2)$ for calculation of discontinuities does not provide such accuracy for the last ones.
The reason is the measure

$$d^{2+2\epsilon} r_{1\perp} d^{2+2\epsilon} r_{1\perp} / (r_{1\perp}^2 r_{2\perp}^2) \delta(q_{2\perp} - r_{1\perp} - r_{2\perp})$$

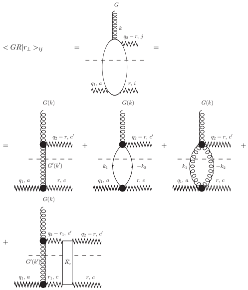
for integration over Reggeon momenta, which is singular at $\epsilon \to 0$. To keep in the discontinuities all terms nonvanishing in the limit $\epsilon \to 0$ one has to calculate $\vec{\Phi}_i(\vec{q}_1, \vec{k}; \vec{r}_1, \vec{r}_2)$ in the region of small $|\vec{r}_i|$ (infrared region) more accurately.
In fact, greater accuracy is required only in the region of small $|\vec{r}_2|$, because in the limit $|\vec{r}_1| \to 0$ $\vec{\Phi}_i(\vec{q}_1, \vec{k}; \vec{r}_1, \vec{r}_2)$ turn to be zero.
In contrast, in the limit $|\vec{r}_2| \to 0$ $\vec{\Phi}_i(\vec{q}_1, \vec{k}; \vec{r}_1, \vec{r}_2)$ not only do not vanish but have logarithmic singularities $\ln \vec{r}_2^{\,2}$.

Problems with infrared behaviour

But these singularities are artificial.
They appear from wrong order of taking limits $\epsilon \to 0$ and $|\vec{r}_2| \to 0$.
The terms $\ln \vec{r}_2^{\,2}$ appear when the limit $\epsilon \to 0$ is taken first.
When the limit $|\vec{r}_2| \to 0$ is taken at nonzero ϵ, the impact factors have to turn to zero.
Therefore, to keep in the discontinuities all terms non-vanishing in the limit $\epsilon \to 0$ one has not to expand $(\vec{r}_2^{\,2})^{\epsilon}$ in powers of ϵ and keep the terms of order ϵ in the region of small $\vec{r}_2^{\,2}$.

Problems with infrared behaviour

In the NLO, the impact factor contains contributions of two types: virtual ones, which are obtained from the one-loop corrections to the Reggeon vertices and the gluon trajectory, and real contributions arising from production of two real particles.



Infrared behaviour of the real corrections

The real contributions to $\vec{\Phi}_i(\vec{q}_1, \vec{k}; \vec{r}_1, \vec{r}_2)$ are calculated at small $|\vec{r}_2|$ exactly in ϵ.
For $\vec{\Phi}_1(\vec{q}_1, \vec{k}; \vec{r}_1, \vec{r}_2)$ in the bootstrap scheme they are V.S. F., R. Fiore, 2015,

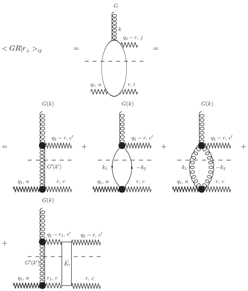
$$\vec{\Phi}_1(\vec{q}_1, \vec{k}; \vec{r}_1, \vec{r}_2)^{real}_* = 4(\vec{r}_2^{\,2})^{\epsilon}\frac{\Gamma^2(1+\epsilon)}{\Gamma(1+2\epsilon)}\left[\vec{C}_2\left(\frac{1}{2\epsilon^2}+\frac{(\psi(1)-\psi(1+2\epsilon))}{\epsilon}\right.\right.$$

$$\left.\left.-\frac{\Gamma(1+2\epsilon)}{\Gamma(4+2\epsilon)}\left(\frac{a_1(1+\epsilon)}{\epsilon}+a_2\right)\right)+\frac{2\Gamma(1+2\epsilon)}{\Gamma(4+2\epsilon)}\frac{\vec{r}_2(\vec{r}_2\vec{C}_2)}{\vec{r}_2^{\,2}}a_2\right],$$

$$a_1 = 11+7\epsilon-2(1+\epsilon)a_f-\frac{a_s}{2}, \quad a_2 = 1+\epsilon-a_f+\frac{a_s}{2}.$$

a_f and a_s are the fermion and scalar group coefficients. For $N=4$ SYM, the coefficients a_1 and a_2 vanish.

The virtual corrections contribute only to $\vec{\Phi}_1(\vec{q}_1, \vec{k}; \vec{r}_1, \vec{r}_2)$. They are expressed in terms of the gluon trajectory, the gluon-gluon-Reggeon vertex and the the Reggeon-Reggeon-gluon vertex.



For the trajectory and the gluon-gluon-Reggeon vertex exact in ϵ expressions are known, as well as for fermion and scalar contributions to the Reggeon-Reggeon-gluon vertex.
The only missing piece is the gluon contribution to the Reggeon-Reggeon-gluon vertex,
which must be known with $\mathcal{O}(\epsilon)$ accuracy.
It can be obtained by different ways.
One can use the gluon production vertex in $N = 4$ SYM which was computed through to $\mathcal{O}(\epsilon^2)$
V. Del Duca, C. Duhr and E. W. Nigel Glover, 2009
in terms of hypergeometric functions.
Another possibility is to use a nice result
M.G. Kozlov, R.N. Lee, 2015
presenting exact in ϵ expression for ”pentagon” as one-dimentiional integral.

Infrared behaviour of the virtual corrections

There is also exact in ϵ result
V. S. Fadin, R. Fiore and A. Papa, 2000
presenting the gluon contribution in terms of four $2+\epsilon$ -dimensional integrals

$$\mathcal{I}_3 = \int \frac{d^{2+2\epsilon}l}{\pi^{1+\epsilon}\Gamma(1-\epsilon)} \frac{1}{\vec{l}^2(\vec{q}_1-\vec{l})^2(\vec{q}_2-\vec{l})^2} ,$$

$$\mathcal{L}_3 = \int \frac{d^{1+2\epsilon}l}{\pi^{3+\epsilon}\Gamma(1-\epsilon)} \frac{1}{\vec{l}^2(\vec{l}-\vec{q}_1)^2(\vec{l}-\vec{q}_2)^2} \ln\left(\frac{(\vec{l}-\vec{q}_1)^2(\vec{l}-\vec{q}_2)^2}{\vec{k}^2\,\vec{l}^2}\right) ,$$

$$\mathcal{I}_{4B}=\int_0^1 \frac{dx}{x} \int \frac{d^{2+2\epsilon}l}{\pi^{1+\epsilon}\Gamma(1-\epsilon)} \left[\frac{1-x}{[x\vec{l}^2+(1-x)(\vec{l}-\vec{q}_1)^2](\vec{l}-(1-x)\vec{k})^2} - \frac{1}{(\vec{l}-\vec{q}_1)^2(\vec{l}-\vec{k})^2}\right],$$

Infrared behaviour of the virtual corrections

$$\mathcal{I}_5 = \int_0^1 \frac{dx}{1-x} \int \frac{d^{2+1\epsilon} l}{\pi^{1+\epsilon}\Gamma(1-\epsilon)} \frac{1}{\vec{l}^2[(1-x)\vec{l}^2 + x(\vec{l}-\vec{q}_1)^2]}$$

$$\times \left(\frac{x^2}{(\vec{l}-x\vec{k})^2} - \frac{1}{(\vec{l}-\vec{k})^2} \right) .$$

We have chosen the third way.
With the $\mathcal{O}(\epsilon)$ accuracy the integrals can be expressed in terms of polylogarithms.
But it seems that such representation is not the best one for use of the impact factors.
Most probably, the representation in terms of the $2+\epsilon$ -dimensional integrals is the best one.

Scheme dependence of the impact factors

As it is known, NLO corrections are scheme dependent. The scheme used in the first calculations of the NLO impact factors was adjusted for simplifying the verification of the bootstrap conditions (we call it bootstrap scheme). It is different from the standard scheme defined in
V.S. F., R. Fiore, M.G. Kozlov, A.V. Reznichenko, 2006.
The impact factors in these schemes are connected by the transformation

$$\langle GR_1| = \langle GR_1|_s - \langle GR_1|^{(B)}\widehat{\mathcal{U}}_k,$$

where subscript s means the standard scheme and the the operator $\widehat{\mathcal{U}}_k$ is defined by the matrix elements

$$\langle \mathcal{G}_1'\mathcal{G}_2'|\widehat{\mathcal{U}}_k|\mathcal{G}_1\mathcal{G}_2\rangle = \frac{1}{2}\ln\left(\frac{\vec{k}^{\,2}}{(\vec{r}_1-\vec{r}_1{}')^2}\right)\langle \mathcal{G}_1'\mathcal{G}_2'|\widehat{\mathcal{K}}_r^B|\mathcal{G}_1\mathcal{G}_2\rangle.$$

Here $\widehat{\mathcal{K}}_r^B$ is the part of the LO BFKL kernel related with the real gluon production:

Scheme dependence of the impact factors

$$\langle \mathcal{G}_1' \mathcal{G}_2' | \widehat{\mathcal{K}}_r^B | \mathcal{G}_1 \mathcal{G}_2 \rangle = \delta(\vec{r}_1{}' + \vec{r}_2{}' - \vec{r}_1 - \vec{r}_2) \frac{g^2}{(2\pi)^{D-1}} T^i_{c_1 c_1'} T^i_{c_2' c_2}$$

$$\times \left(\frac{\vec{r}_1{}^2 \vec{r}_2{}'^{\,2} + \vec{r}_2{}^2 \vec{r}_1{}'^{\,2}}{\vec{l}^{\,2}} - \vec{q}_2{}^2 \right),$$

where $\vec{l} = \vec{r}_1 - \vec{r}_1{}' = \vec{r}_2{}' - \vec{r}_2, \quad \vec{q}_2 = \vec{r}_1 + \vec{r}_2 = \vec{r}_1{}' + \vec{r}_2{}'$.
The transformation contains the integral

$$\vec{\mathcal{I}}_1 = \int \frac{d\vec{l}}{\Gamma(1-\epsilon)\pi^{1+\epsilon}} \vec{C}_1{}' \frac{1}{\vec{r}_1{}'^{\,2} \vec{r}_2{}'^{\,2}} \left(\frac{\vec{r}_1{}^2 \vec{r}_2{}'^{\,2} + \vec{r}_2{}^2 \vec{r}_1{}'^{\,2}}{\vec{l}^{\,2}} - \vec{q}_2{}^2 \right) \ln\left(\frac{\vec{k}^{\,2}}{\vec{l}^{\,2}} \right),$$

where

$$\vec{C}_1{}' \;=\; \vec{q}_1 - \vec{q}_1{}^2 \frac{(\vec{q}_1 - \vec{r}_1{}')}{(\vec{q}_1 - \vec{r}_1{}')^2}.$$

Scheme dependence of the impact factors

This integral was calculated also in the limit $\epsilon \to 0$. The result is not applicable at small $\vec{r}_2$, where it is

$$\vec{\mathcal{I}}_1 = \left(\vec{q}_1 - \vec{k}\frac{\vec{q}_1^{\,2}}{\vec{k}^{\,2}}\right)\left[-\ln^2\left(\frac{\vec{r}_2^{\,2}}{\vec{k}^{\,2}}\right) + 2\frac{(\vec{k}^{\,2})^{\epsilon}}{\epsilon^2} - 2\zeta(2)\right].$$

It must be changed to

$$\vec{\mathcal{I}}_1 = \left(\vec{q}_1 - \vec{k}\frac{\vec{q}_1^{\,2}}{\vec{k}^{\,2}}\right)\frac{\Gamma^2(1+\epsilon)}{\Gamma(1+2\epsilon)}(\vec{r}_2^{\,2})^{\epsilon}\left[\frac{2}{\epsilon^2} - \frac{2}{\epsilon}\left(\psi(1) - \psi(1-\epsilon)\right.\right.$$

$$\left.\left.+\psi(1+\epsilon) - \psi(1+2\epsilon) + \ln\left(\frac{\vec{r}_2^{\,2}}{\vec{k}^{\,2}}\right)\right)\right].$$

Summary

- The impact factors for Reggeon-gluon transition are an inherent part of the discontinuities of multi-Regge amplitudes.
- The impact factors were calculated in Yang-Mills theories with any number of fermions and scalars in arbitrary representations of the gauge group with the accuracy up to terms vanishing at $\epsilon = 0$, $\epsilon - (D-4)/2$, D being the space-time dimension.
- To keep in the discontinuities the terms not vanishing at $\epsilon = 0$, the impact factors must be known with higher accuracy in the region of Reggeon momenta tending to zero.
- Calculation of real NLO corrections to the impact factors with this accuracy is performed.
- Calculation of virtual corrections is nearing completion.