

# The Quantum Critical higgs

Seung J. Lee



With B. Bellazzini, C. Csaki, J. Hubisz, J. Serra, J. Terning; arXiv:1511.08218

With C. Csaki, A. Parolini, Y. Shirman; work in progress

With C. Csaki, A. Parolini work in progress

with S. Biswas, M. Mandal, B. Fuks work in progress

with C. Csaki and M. Park work in progress



So, what's wrong with the SM higgs?



Weisskopf Phys. Rev.56 (1939) 72

# So, what's wrong with the SM higgs?



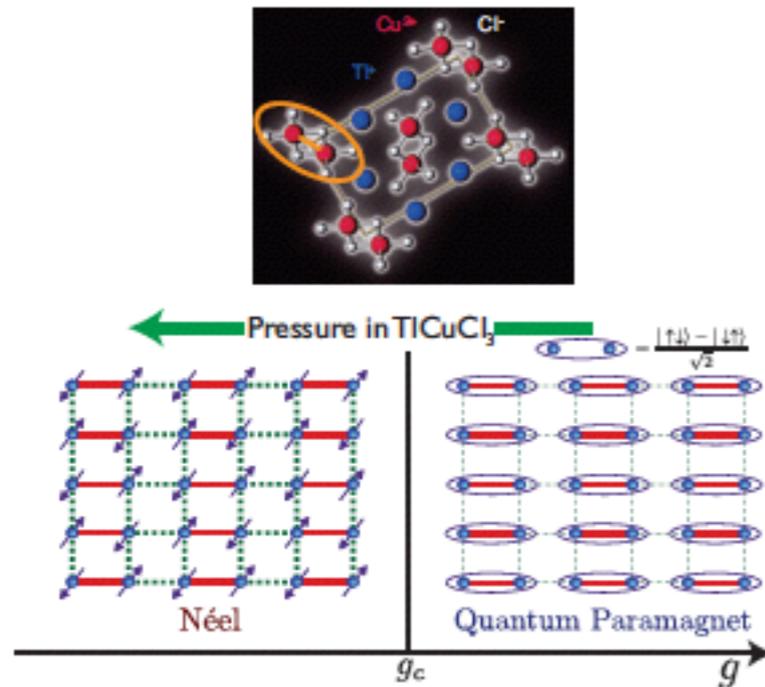
Weisskopf Phys. Rev.56 (1939) 72

# Post higgs Problems

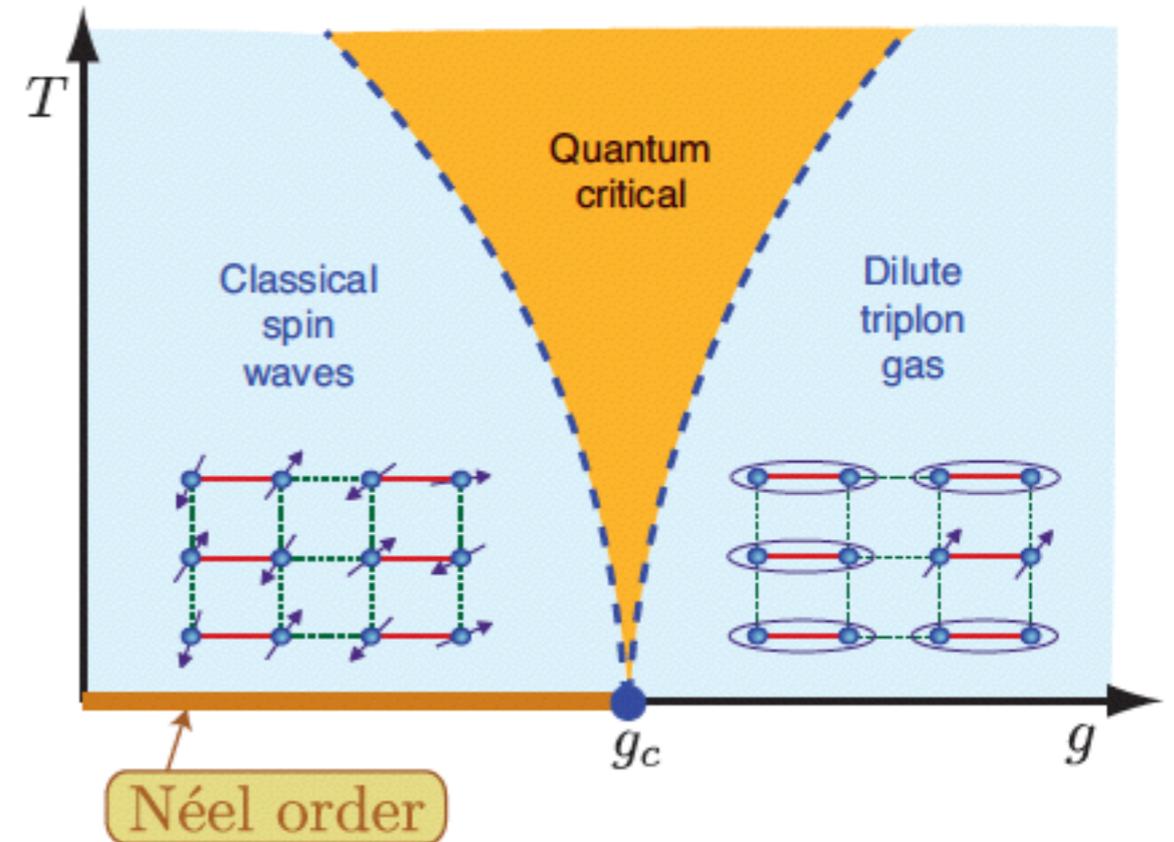
- ⑤ We see “what” is condensed
- ⑤ But we still don’t know “why”
- ⑤ Two problems:
  - Why anything is condensed at all
  - Why is the scale of condensation  $\sim \text{TeV} \ll M_{\text{pl}} = 10^{15} \text{TeV}$
- ⑤ Explanation most likely to be at  $\sim \text{TeV}$  scale because this is the relevant energy scale

# Post higgs Questions

Condensed matter systems can produce a light scalar by tuning the parameters close to a critical value where a continuous phase transition occurs.



Sachdev, arXiv:1102.4268

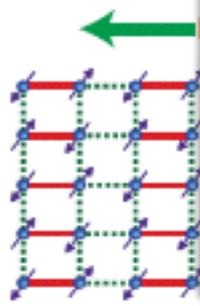


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@2nd order QPT, @ critical point, all masses vanish & the theory is scale invariant, characterized by the scaling dimensions of the field,

and at low energies we will see the universal behavior of some fixed point that constitutes the low-energy EFT.



Néel

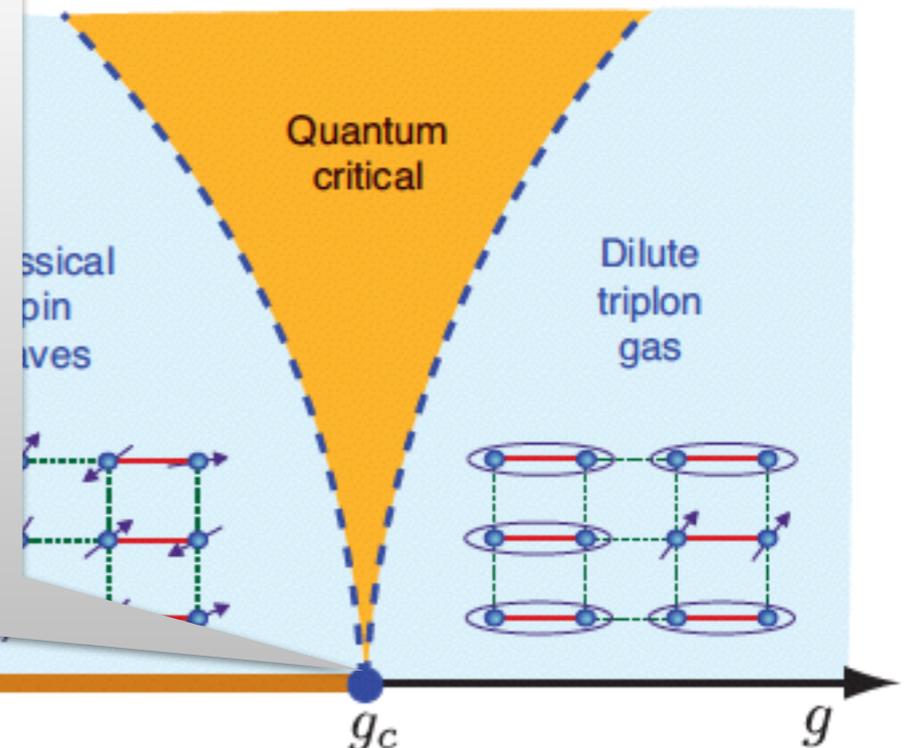
Quantum Paramagnet

$g_c$

$g$

Sachdev, arXiv:1102.4268

S.



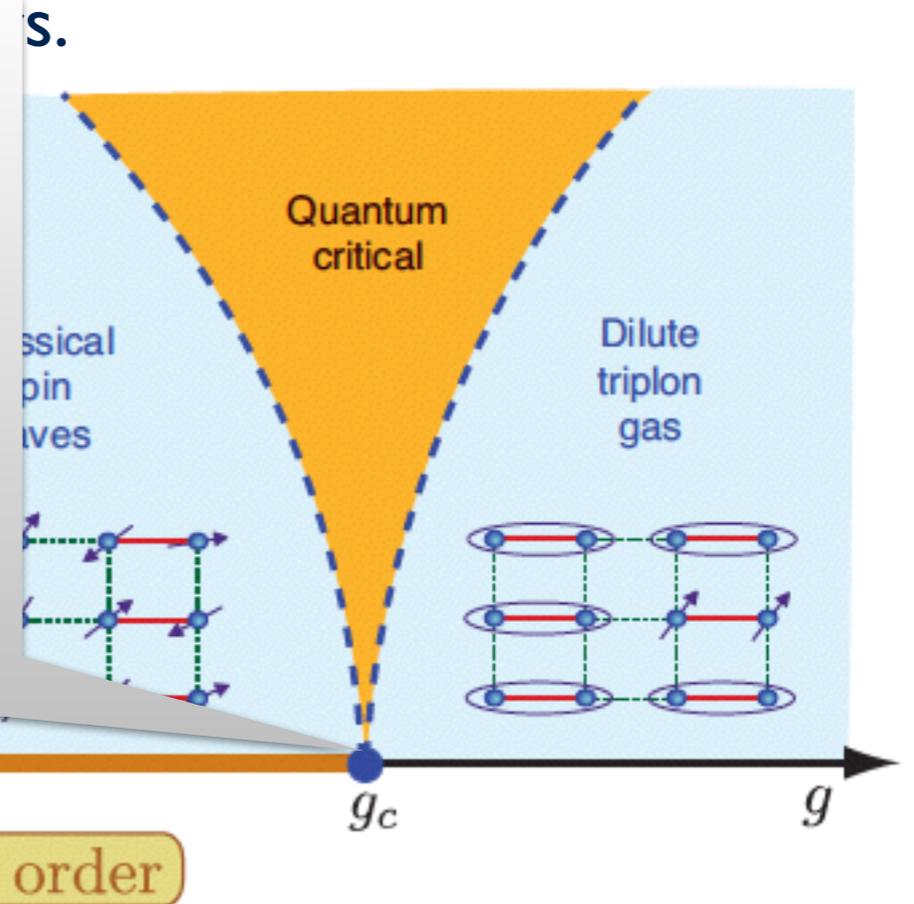
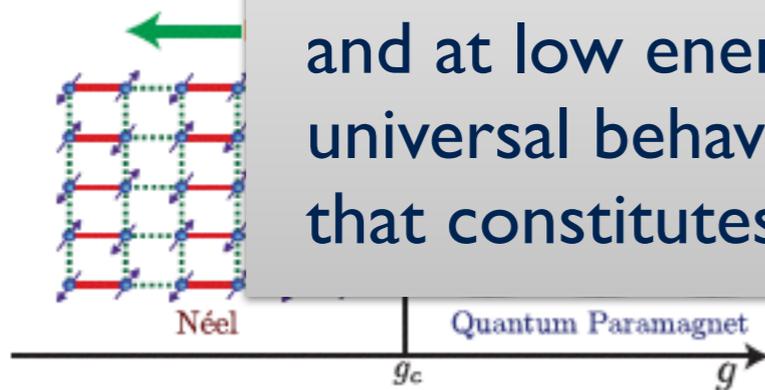
Néel order

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What is the nature of electroweak phase transition?

Does the underlying theory also have a QPT?

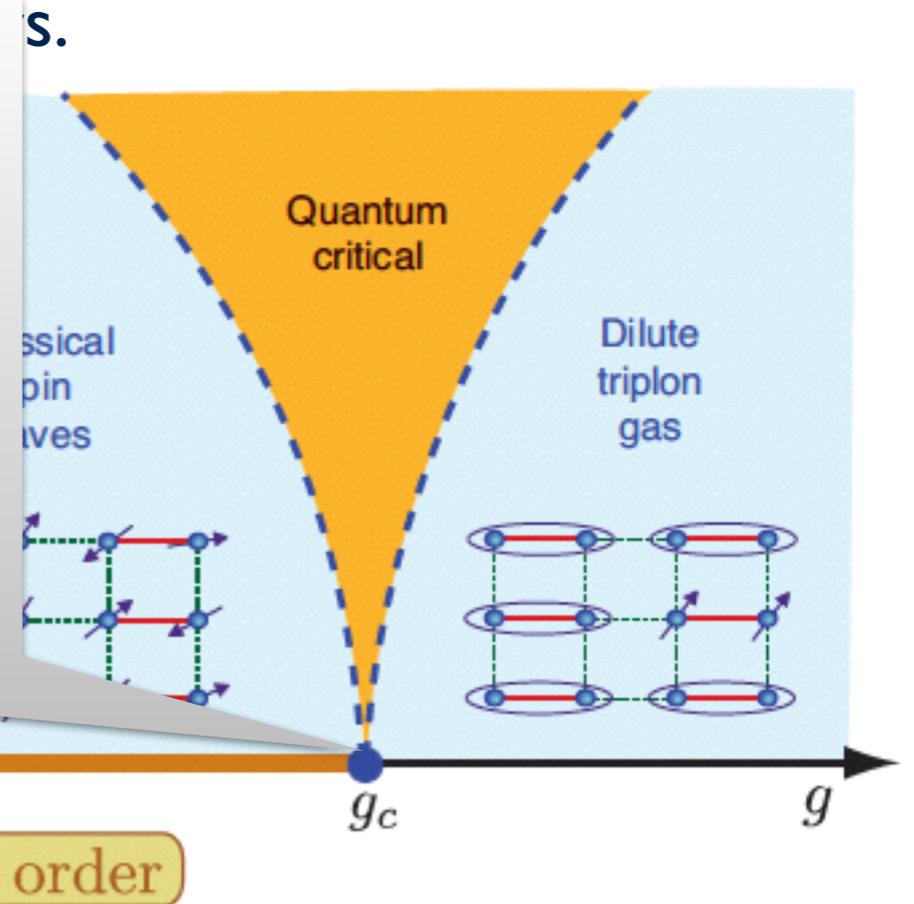
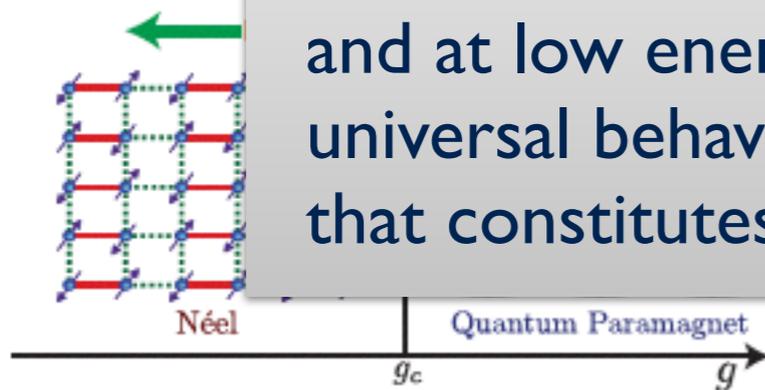
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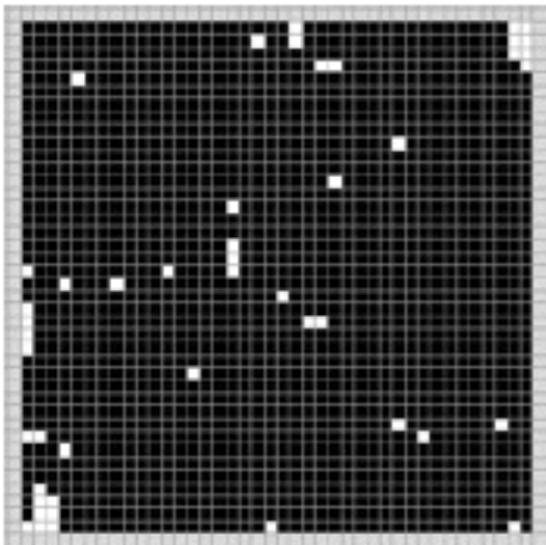
$$G(p) \sim \frac{i}{p^2} \quad \text{vs.} \quad G(p) \sim \frac{i}{(p^2)^{2-\Delta}} \quad \text{or} \quad G(p) \sim \frac{i}{(p^2 - \mu^2)^{2-\Delta}}$$

# Ising Model

$$H = -J \sum s(x)s(x+n)$$

$$s(x) = \pm 1$$

Low T



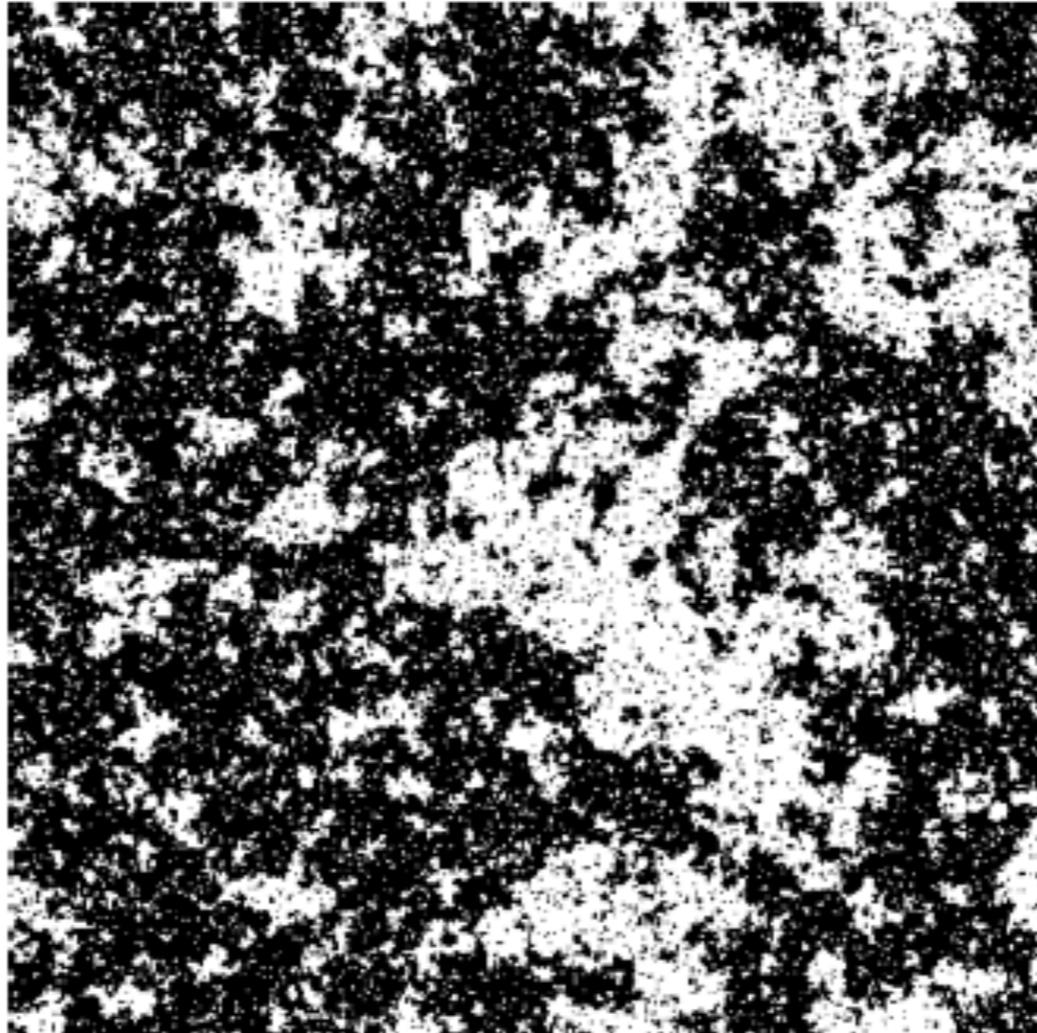
High T

$T_c$

$$\langle s(0)s(x) \rangle = e^{-|x|/\xi}$$

$$\text{at } T=T_c \quad \xi \rightarrow \infty$$

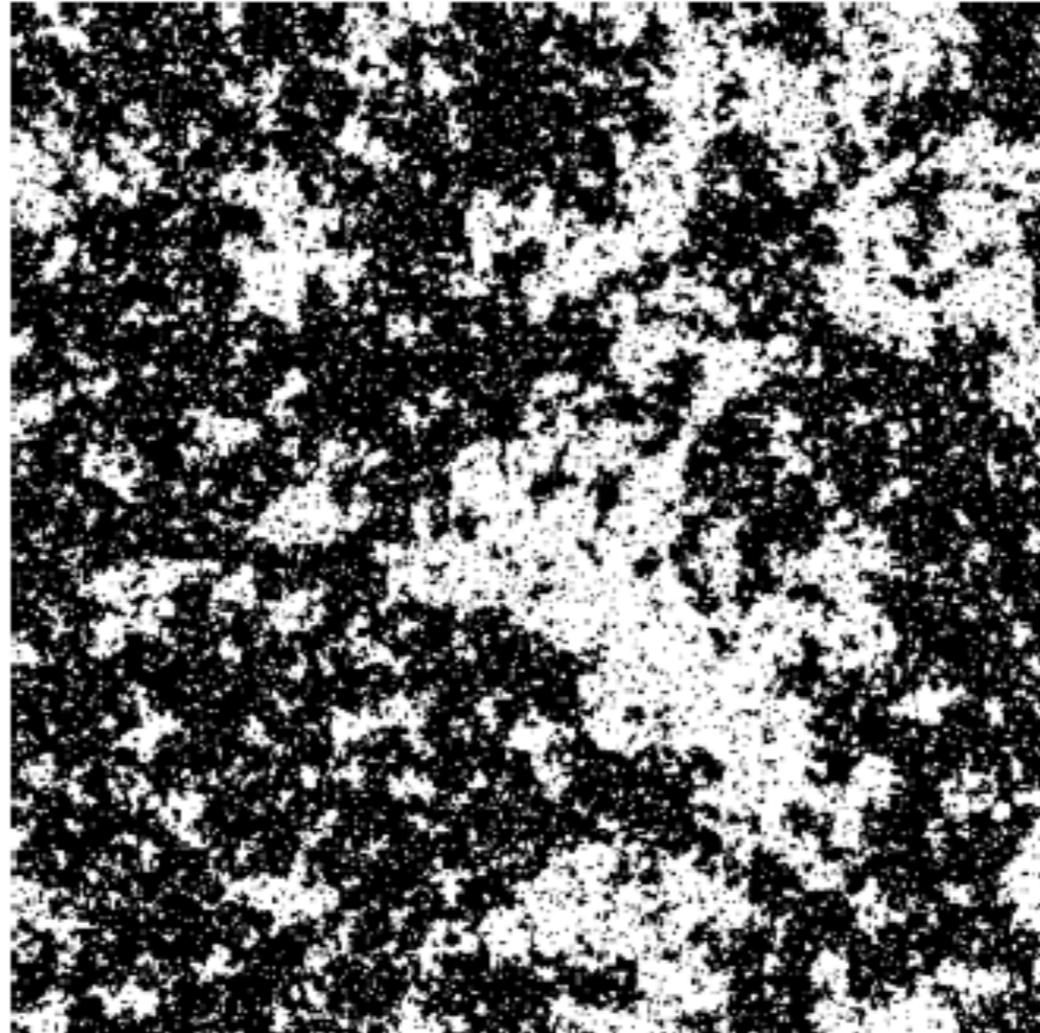
# Critical Ising Model is Scale Invariant



<http://bit.ly/2Dcrit>

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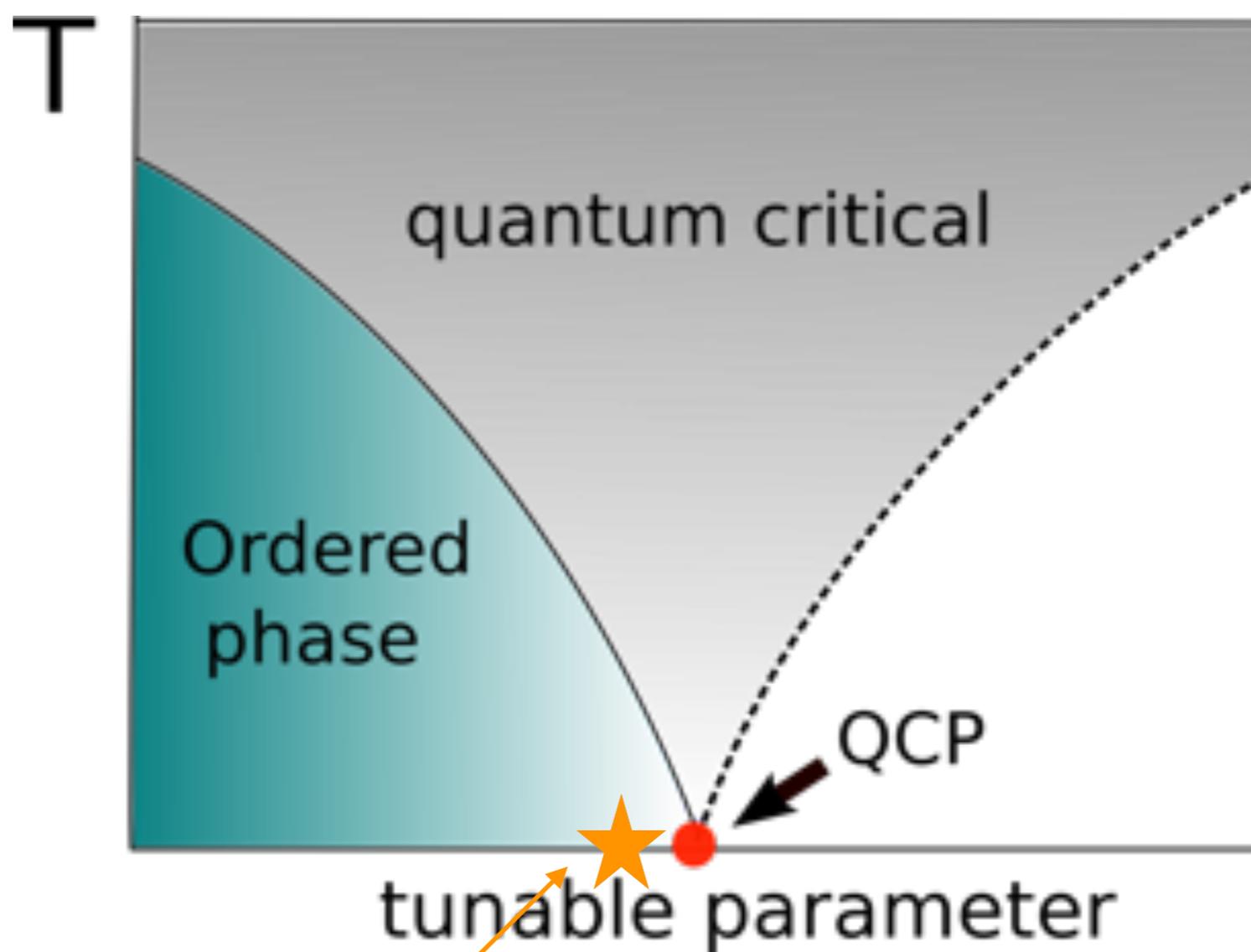


<http://bit.ly/2Dcrit>

$$\text{at } T=T_c \quad \langle s(0)s(x) \rangle \propto \frac{1}{|x|^{2\Delta-1}} = \int d^3p \frac{e^{ip \cdot x}}{|p|^{4-2\Delta}}$$

↑  
critical exponent

# Quantum Phase Transition



We are here

# The Quantum Critical higgs

- ❖ At a QPT the approximate scale invariant theory is characterized by [the scaling dimension  \$\Delta\$](#)  of the gauge invariant operators.

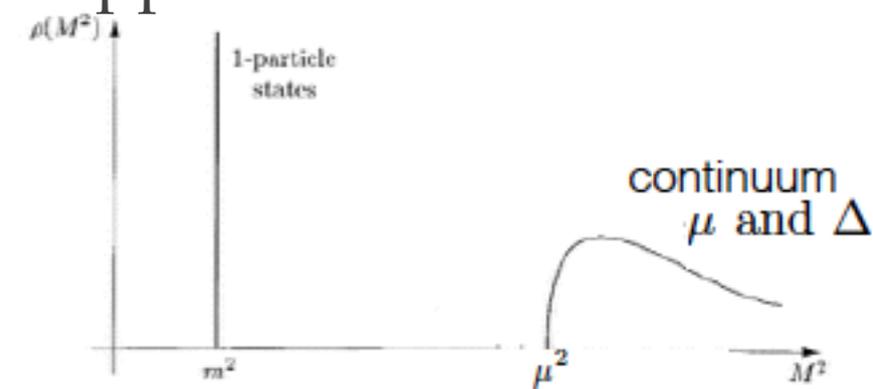
$$\text{SM: } \Delta = 1 + \mathcal{O}(\alpha/4\pi);$$

- ❖ We want to present a general class of theories describing a higgs field near a non-mean-field QPT.

- ❖ In such theories, in addition to the pole (Higgs), there can also be a higgs continuum, representing additional states associated with the dynamics underlying the QPT

$$G_h(p) = -\frac{i Z_h}{(\mu^2 - p^2 + i\epsilon)^{2-\Delta} - (\mu^2 - m_h^2)^{2-\Delta}}$$

- ❖ One result of the presence of the continuum will be the appearance of form factors in couplings of the Higgs to the SM particles.



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- ❖ We assume that the SM fermions, the massless gauge bosons, and the transverse parts of the W and Z are external to the CFT, that is elementary, while the Higgs ( $Z_{\text{long}}, W_{\text{long}}$ ) originates from or is mixed with the strong sector, corresponding to a theory with spontaneously or explicitly broken conformal symmetry.

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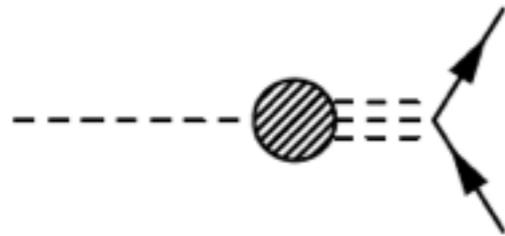
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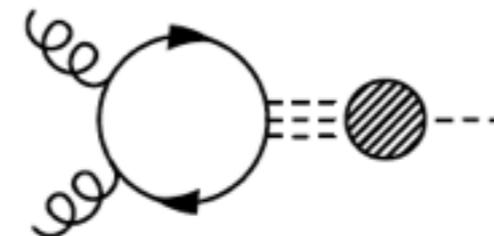
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  - => this strong sector is characterized by its n-point functions entering into form factors

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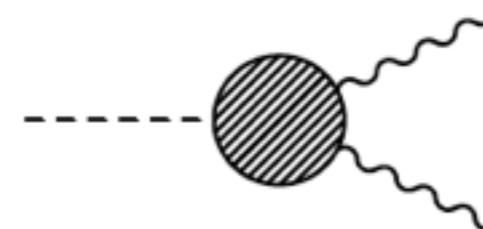
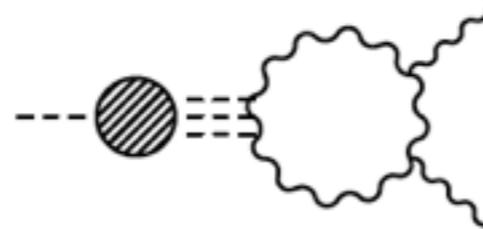
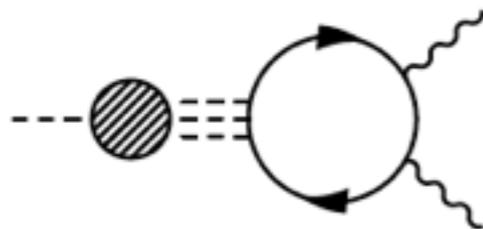
On-shell behavior: constant form factors (form factor reduces to a constant),)



$$\mathcal{M}_{\bar{f}fh} = \bar{u}_1^a F_{hff}(p_1 \cdot p_2; \mu) v_{2a} ,$$



$$\mathcal{M}_{ggh} = \left[ (\epsilon_1 \cdot p_2) (\epsilon_2 \cdot p_1) - \frac{m_h^2}{2} (\epsilon_1 \cdot \epsilon_2) \right] F_{ggh}(p_1 \cdot p_2 = m_h^2/2; \mu)$$

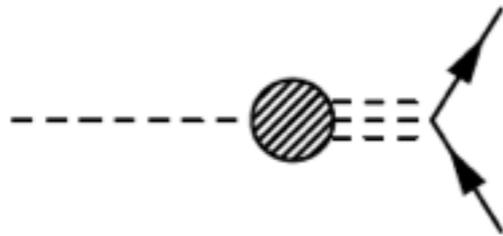


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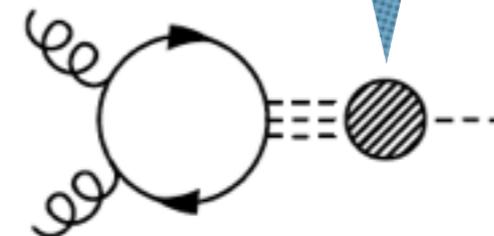
# Form Factors

Effects of the strong sector leading to the QPT are added via its n-point functions, leading to form factors

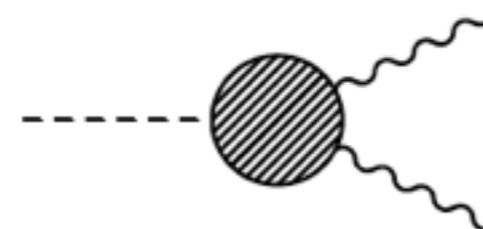
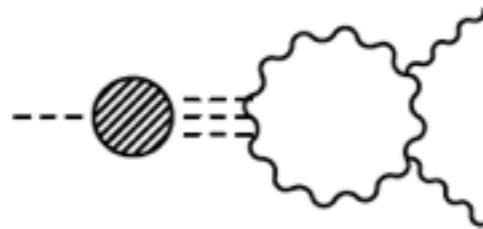
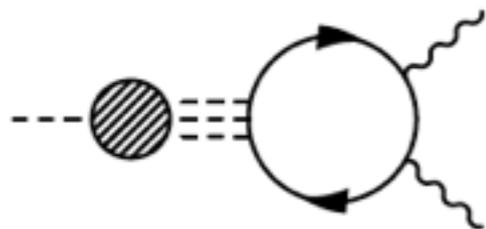
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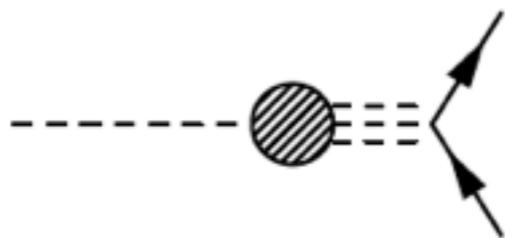


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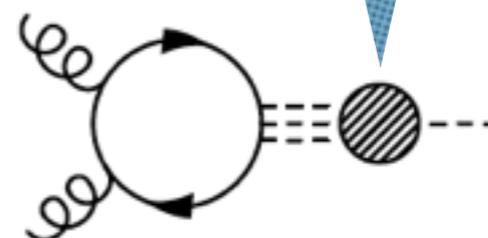
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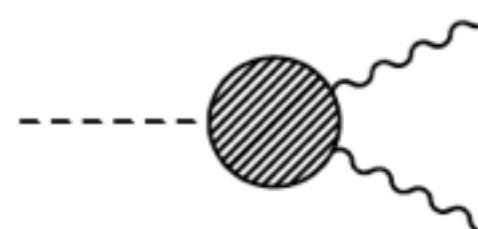
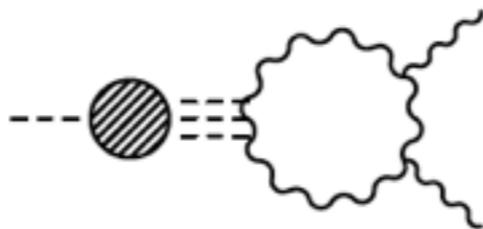
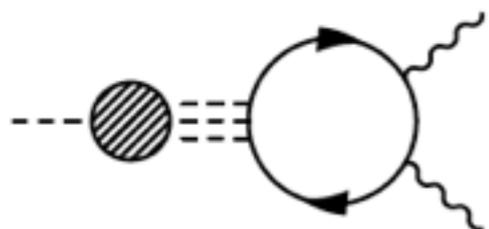


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represents the parametric dependence on the scale of conformal symmetry breaking,

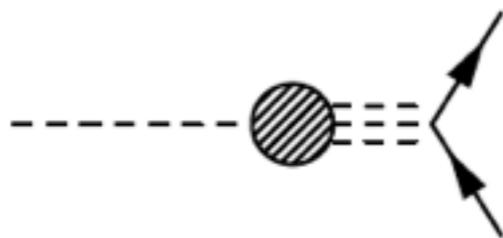


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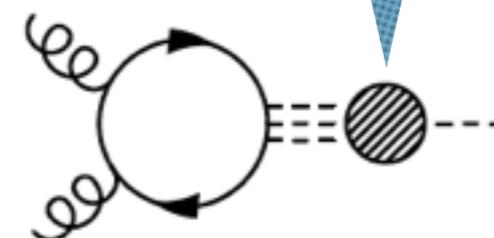
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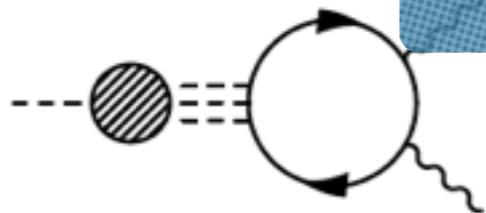
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represents the parametric dependence

restriction to on-shell states reduces the on-shell form factor to an effective coupling constant.

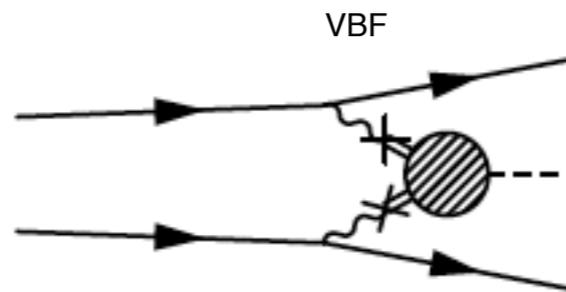


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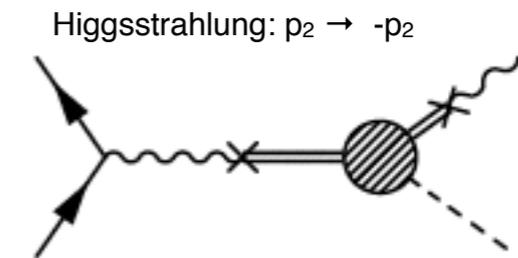
# Off-shell Form Factors for the Quantum Critical higgs

Off-shell behavior: nontrivial momentum dependent form factors

$$p_1^2 + p_2^2 = m_h^2 - 2p_1 \cdot p_2.$$



$$\mathcal{M}_{VBF} = J_1^\alpha G_{\alpha\mu}^V(p_1) J_2^\beta G_{\nu\beta}^V(p_2) F_{VVh}^{\mu\nu}(p_i; \mu) N_V$$



$$\mathcal{M}_{qq \rightarrow Vh} = J_I^\alpha G_{\alpha\mu}^V(p_1) \bar{\epsilon}_{2\nu} F_{VVh}^{\mu\nu}(p_1, -p_2; \mu) N_V$$

$$F_{VVh}^{\mu\nu}(p_i; \mu) = g^{\mu\nu} \Gamma_1 + (g^{\mu\nu} p_1 \cdot p_2 - p_2^\mu p_1^\nu) \Gamma_2 + (p_1^\mu p_1^\nu + p_2^\mu p_2^\nu) \Gamma_3 + (p_1^\mu p_1^\nu - p_2^\mu p_2^\nu) \Gamma_4 + p_1^\mu p_2^\nu \Gamma_5$$

$$\Gamma_i = \Gamma_i(p_1^2, p_2^2, p_1 \cdot p_2)$$

$$\Gamma_1^{(SM)} = 1 \text{ and } \Gamma_{i \neq 1}^{(SM)} = 0.$$

etc...

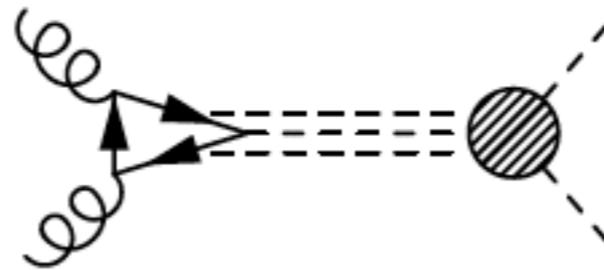
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$$p_1 \cdot p_2 = s/2$$



$$p_1 \cdot p_3 = (m_h^2 - t)/2$$

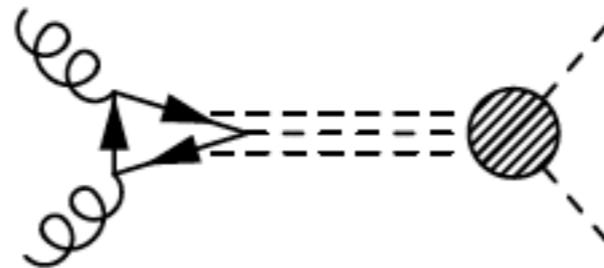
$$\begin{aligned} \mathcal{M}_{gghh} = & \left[ (\epsilon_1 \cdot p_2) (\epsilon_2 \cdot p_1) - (p_1 \cdot p_2) (\epsilon_1 \cdot \epsilon_2) \right] \Xi_1(p_1 \cdot p_2, p_1 \cdot p_3; \mu) \\ & + \epsilon_2 \cdot [(p_1 \cdot p_2)p_3 - (p_2 \cdot p_3)p_1] \epsilon_1 \cdot [(p_1 \cdot p_2)p_3 - (p_1 \cdot p_3)p_2] \Xi_2(p_1 \cdot p_2, p_1 \cdot p_3; \mu) \end{aligned}$$

Bose Symmetry:  $\Xi_i(p_1 \cdot p_2, p_1 \cdot p_3; \mu) = \Xi_i(p_1 \cdot p_2, p_2 \cdot p_3; \mu)$

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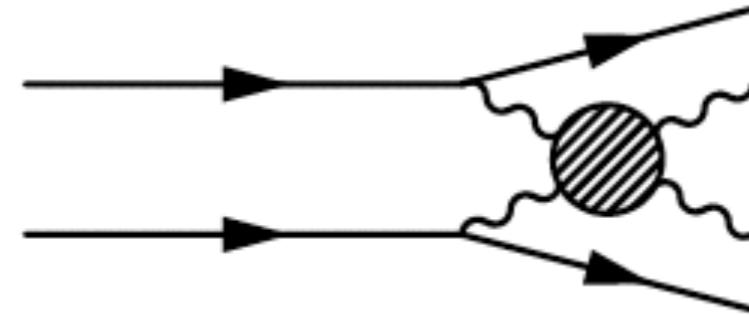
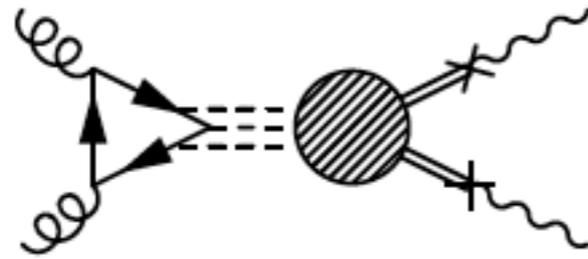
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suppressed in the large top mass limit in the SM

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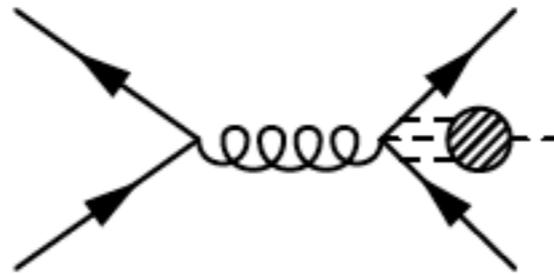
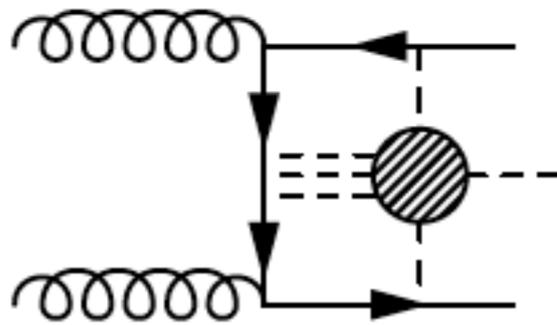


$$\mathcal{M}_{ggVV} = \epsilon_{1\mu}\epsilon_{2\nu} \left[ F_{ggVV}^{\mu\nu\rho\sigma}(p_i; \mu) + \hat{F}_{ggVV}^{\mu\nu\rho\sigma}(p_i; \mu) \right] \bar{\epsilon}_{3\rho}\bar{\epsilon}_{4\sigma}$$

$$F_{ggVV}^{\mu\nu\rho\sigma}(p_i; \mu) = [\sigma^{\mu\nu}(p_1 \cdot p_2) - p_1^\nu p_2^\mu] (\sigma^{\rho\sigma}\Theta_1 + p_1^\rho p_2^\sigma \Theta_2 + p_1^\sigma p_2^\rho \Theta_3) + [\sigma^{\mu\nu}\sigma^{\rho\sigma}(p_1 \cdot p_2) + \sigma^{\mu\nu}p_1^\rho p_2^\sigma - \sigma^{\mu\nu}p_2^\rho p_1^\sigma - \sigma^{\mu\nu}p_1^\sigma p_2^\rho] \Theta_4 + \sigma^{\mu\nu} [\sigma^{\rho\sigma}(p_1 \cdot p_2)(p_1 \cdot p_2) - p_1^\rho p_2^\sigma (p_1 \cdot p_2) + p_1^\sigma p_2^\rho (p_1 \cdot p_2) + p_2^\rho p_1^\sigma (p_1 \cdot p_2) - p_2^\sigma p_1^\rho (p_1 \cdot p_2)] \Theta_5 + p_1^\rho [\sigma^{\mu\nu}p_2^\sigma (p_1 \cdot p_2) + \sigma^{\mu\nu}p_2^\sigma (p_1 \cdot p_2) - \sigma^{\mu\nu}p_1^\sigma (p_1 \cdot p_2) - p_2^\sigma p_1^\rho] \Theta_6 + [\sigma^{\mu\nu}p_1^\rho p_2^\sigma (p_1 \cdot p_2) - \sigma^{\mu\nu}p_2^\rho p_1^\sigma (p_1 \cdot p_2) + \sigma^{\mu\nu}p_1^\sigma p_2^\rho (p_1 \cdot p_2) - \sigma^{\mu\nu}p_2^\sigma p_1^\rho (p_1 \cdot p_2)] \Theta_7 + [\sigma^{\mu\nu}p_2^\rho p_1^\sigma (p_1 \cdot p_2) - \sigma^{\mu\nu}p_1^\rho p_2^\sigma (p_1 \cdot p_2) + \sigma^{\mu\nu}p_2^\sigma p_1^\rho (p_1 \cdot p_2) - \sigma^{\mu\nu}p_1^\sigma p_2^\rho (p_1 \cdot p_2)] \Theta_8$$

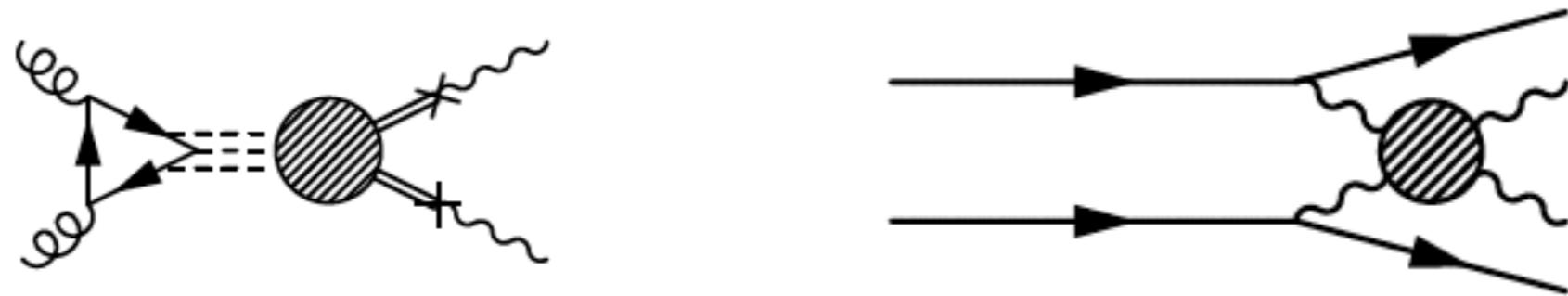
$$\hat{F}_{ggVV}^{\mu\nu\rho\sigma}(p_i; \mu) = p_1^\mu p_2^\nu p_3^\rho p_4^\sigma (\epsilon^{\mu\nu\rho\sigma} \Theta_9 + \epsilon^{\mu\nu\sigma\rho} \Theta_{10} + \epsilon^{\mu\nu\rho\sigma} \Theta_{11}) + \epsilon_1^\mu \epsilon_2^\nu \epsilon^{\mu\nu\rho\sigma} \epsilon^{\rho\sigma\mu\nu} \Theta_{12} + \epsilon_1^\mu \epsilon_2^\nu \epsilon^{\mu\nu\rho\sigma} \epsilon^{\rho\sigma\nu\mu} \Theta_{13}$$

$$\Theta_1 = \Theta_1(p_1 \cdot p_2, p_1 \cdot p_3), \quad \Theta_2' = \Theta_2'(p_1 \cdot p_2, p_1 \cdot p_3)$$



# Off-shell Form Factors for the Quantum Critical higgs

Off-shell behavior: nontrivial momentum dependent form factors

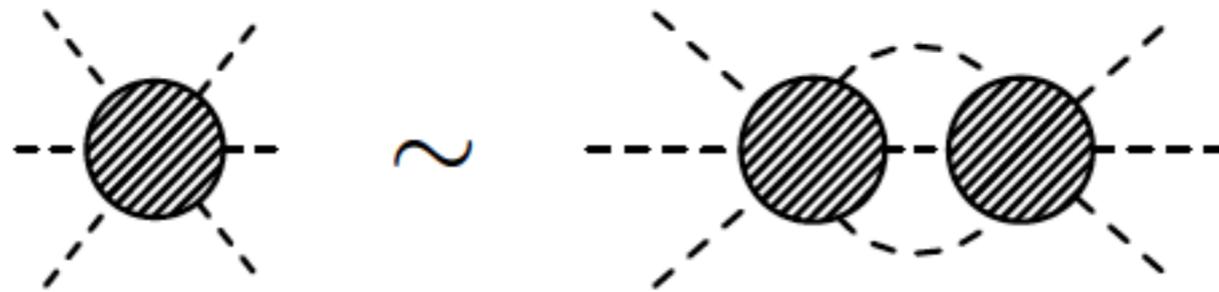


One can estimate from an EFT perspective, where Higgs is (the only) light degree of freedom surviving from the strongly coupled sector  
 $\Rightarrow$  can estimate the size of the N-point Higgs correlator by considering the effect of loops on its renormalization...

# Estimation of Form Factors

use low energy effective theory of 125 GeV resonance  
apply tenets of NDA below onset of cut/continuum:

$$\mathcal{L} = \frac{\alpha_n}{\mu^{n-4}} \phi^n$$



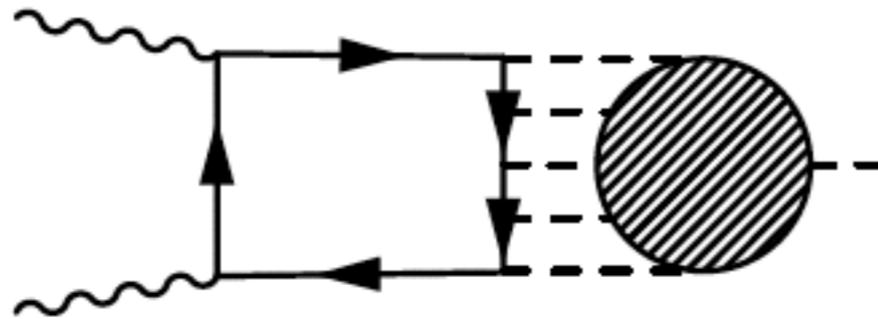
$$\alpha_n \sim (16\pi^2)^{n/2-1}$$

**Counting:**

$n/2-1$  loops cut off at IR scale and dimensional analysis

# Estimation of Form Factors

**If top quark is external to strong dynamics:**

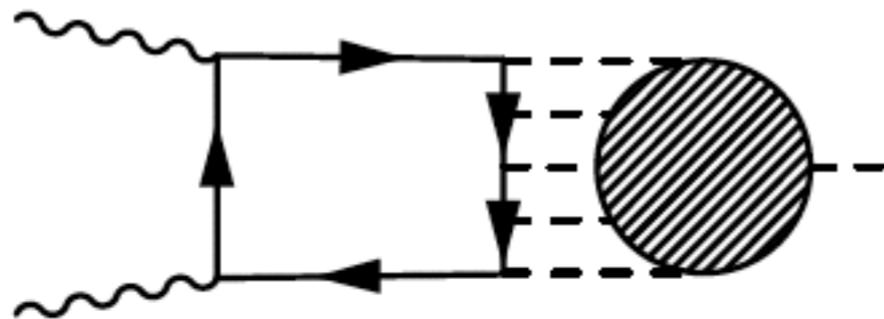


$$g_n^{tth} \sim 4\pi \left( \frac{\lambda_t}{4\pi} \right)^{n-1}$$

Gluon fusion process involves (perturbative) coupling of top quark to Higgs field

# Estimation of Form Factors

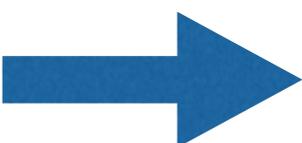
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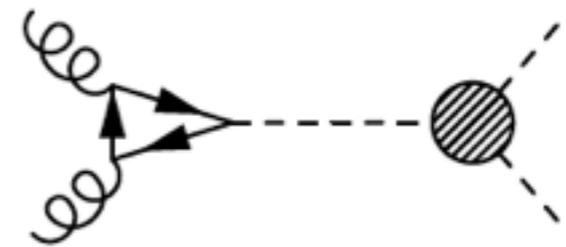


$$g_n^{tth} \sim 4\pi \left( \frac{\lambda_t}{4\pi} \right)^{n-1}$$

Gluon fusion process involves (perturbative) coupling of top quark to Higgs field

**e.g. double Higgs production through gluon fusion would be dominated by**

 dominant contribution comes from tree diagram



# Modeling the QCH: generalized free fields

- ❖ The upshot is that there is a QFT (CFT) with non-trivial dynamics, and the pole (physical Higgs) arises as a composite bound state of CFT similar to composite Higgs models

## Generalized Free Fields Polyakov, early '70s- skeleton expansions

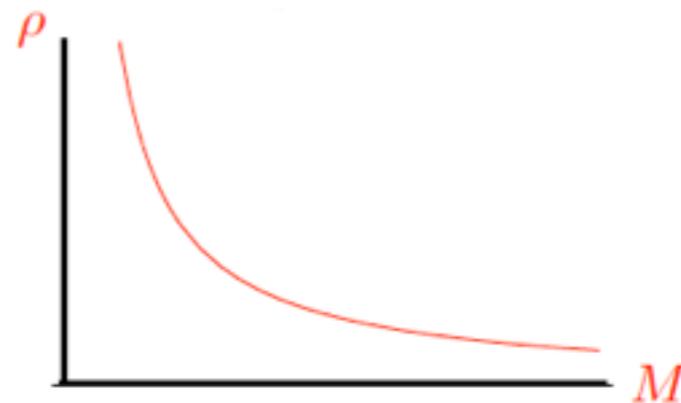
CFT completely specified by 2-point function - rest vanish

**Scaling - 2-point function:**  $G(p^2) = -\frac{i}{(-p^2 + i\epsilon)^{2-\Delta}}$

**Can be generated from:**  $\mathcal{L}_{\text{GFF}} = -\bar{h}^\dagger (\partial^2)^{2-\Delta} h$  Georgi  
hep-ph/0703260

Branch cut starting at origin - spectral density purely a continuum:

$$G(p) \sim \int_{\mu^2}^{\infty} dM^2 \frac{\rho(M^2)}{p^2 - M^2}$$



# Generalized Free Fields via AdS/CFT

- ❖ SO(4) global symmetry is gauged in the 5D bulk

Cacciapaglia, Marandella and Terning 08'  
Falkowski and Perez-Victoria 08'

$$S = \int d^4x dz \sqrt{g} \left[ |D_M H|^2 - \frac{1}{4g_4^2} W_{MN}^a{}^2 - \phi(z) |H|^2 + \mathcal{L}_{\text{int}}(H) \right] + \int d^4x \mathcal{L}_{\text{perturbative}}$$

$$ds^2 = a(z)^2 (\eta_{\mu\nu} dx^\mu dx^\nu - dz^2) \quad a(z) = \frac{R}{z} e^{-\frac{2}{3}\mu(z-R)}$$

$$G_h(R, R, p^2) = i\tilde{Z}_h \left[ \frac{\mu K_{1-\nu}(\mu R)}{R K_\nu(\mu R)} - \frac{\sqrt{\mu^2 - p^2} K_{1-\nu}(\sqrt{\mu^2 - p^2} R)}{R K_\nu(\sqrt{\mu^2 - p^2} R)} - M_0^2 \right]^{-1}$$

Soft wall terminates CFT with continuum, not set of KK modes

The bulk to brane propagator is then given by  $G_h(R, z, p^2) = a^{-\frac{3}{2}}(z)(z/R)^{\frac{1}{2}} \frac{K_\nu(\sqrt{\mu^2 - p^2} z)}{K_\nu(\sqrt{\mu^2 - p^2} R)}$

=> reduce to the previous propagator in the limit  $pR \ll 1$  :

$$G_h(p) = -\frac{i Z_h}{(\mu^2 - p^2 + i\epsilon)^{2-\Delta} - (\mu^2 - m_h^2)^{2-\Delta}} \quad Z_h = \frac{(2 - \Delta)}{(\mu^2 - m_h^2)^{\Delta-1}}$$

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obtain such propagator from a calculable model of this sort based on a Banks-Zaks fixed point in a supersymmetric QCD theory:  
Csaki, SL, Shirmanm, Parolini (in preparation)

# Generalized Free Fields via AdS/CFT

**new 5D model**  $\phi_{UV}(z) = a_{UV}^{-2} \left(\frac{R}{z}\right)^2 \left(m^2 - \frac{3z\mu_{UV}}{R^2}\right) \quad \nu^2 = 4 + m^2 R^2$

$$ds^2 = a(z)^2(dx^2 - dz^2)$$

$$a_{UV}(z) = \frac{R}{z} e^{\frac{2}{3}(R-z)\mu_{UV}}, \quad a_{IR}(z) = \frac{R_p}{z} e^{\frac{2}{3}(R_p-z)\mu_{IR}}$$

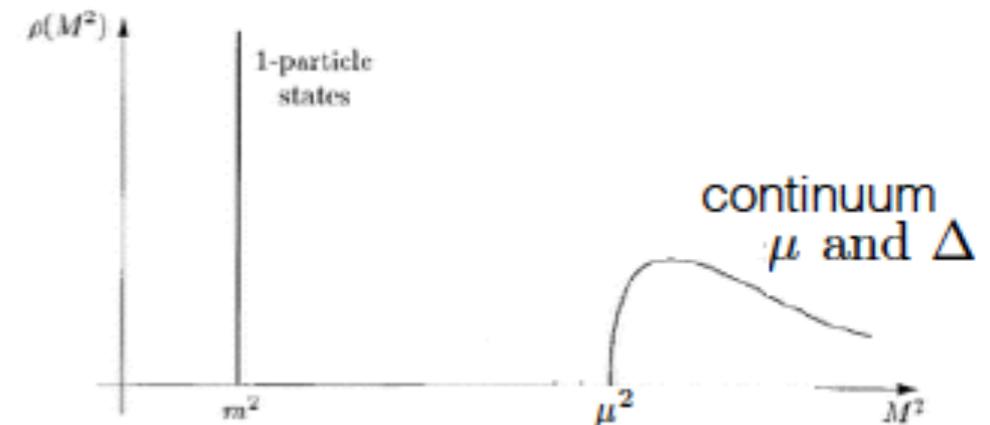
$$a(z)^{-4}(a(z)a''(z) - 2a'(z)^2) \leq 0$$

wec  
holographic a-theorem

Soft wall terminates CFT with continuum, not set of KK modes

**Generally:**

$$G_h(p^2) = \frac{i}{p^2 - m_h^2} + \int_{\mu^2}^{\infty} dM^2 \frac{\rho(M^2)}{p^2 - M^2}$$

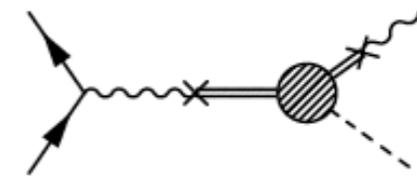
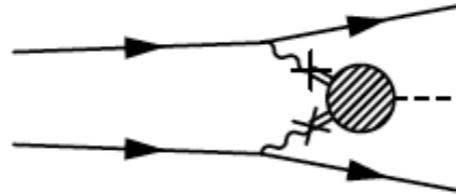


SM recovered in limits  $\mu \rightarrow \infty$  and/or  $\Delta \rightarrow 1$

obtain such propagator from a calculable model of this sort based on a Banks-Zaks fixed point in a supersymmetric QCD theory:  
Csaki, SL, Shirmanm, Parolini (in preparation)

# Direct Signals

❖ Form factors

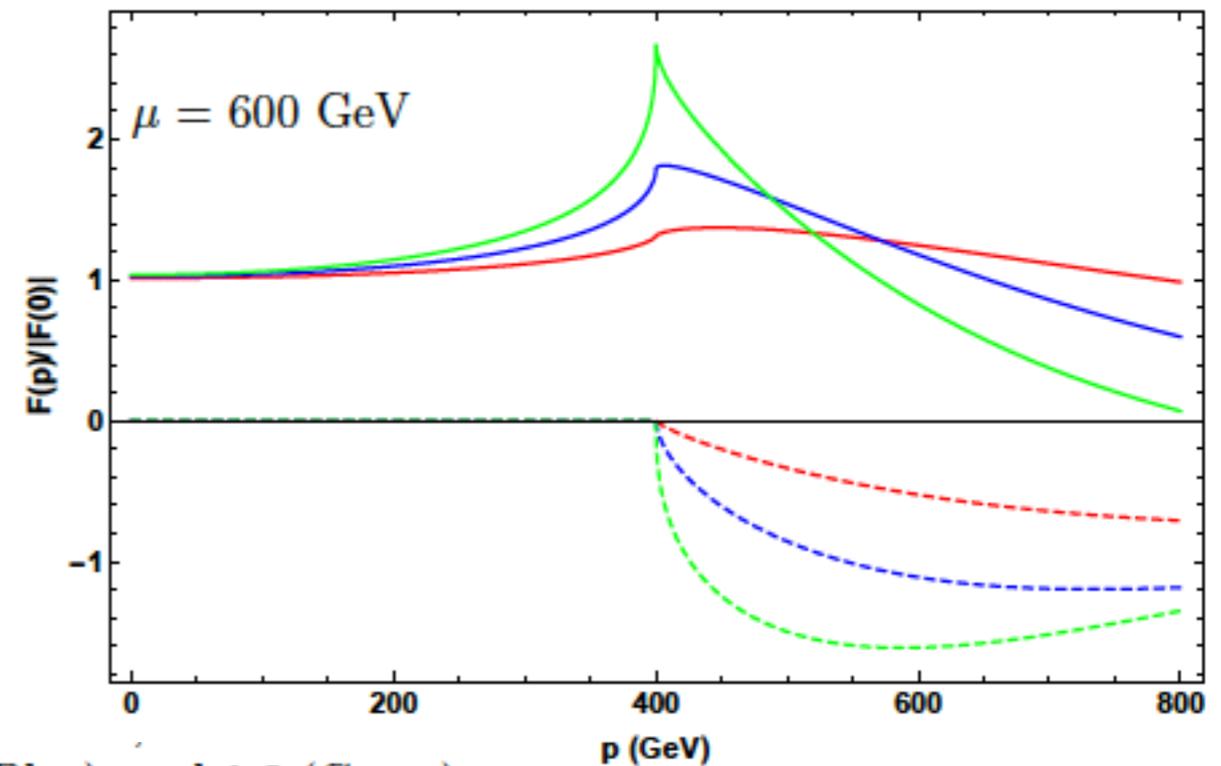
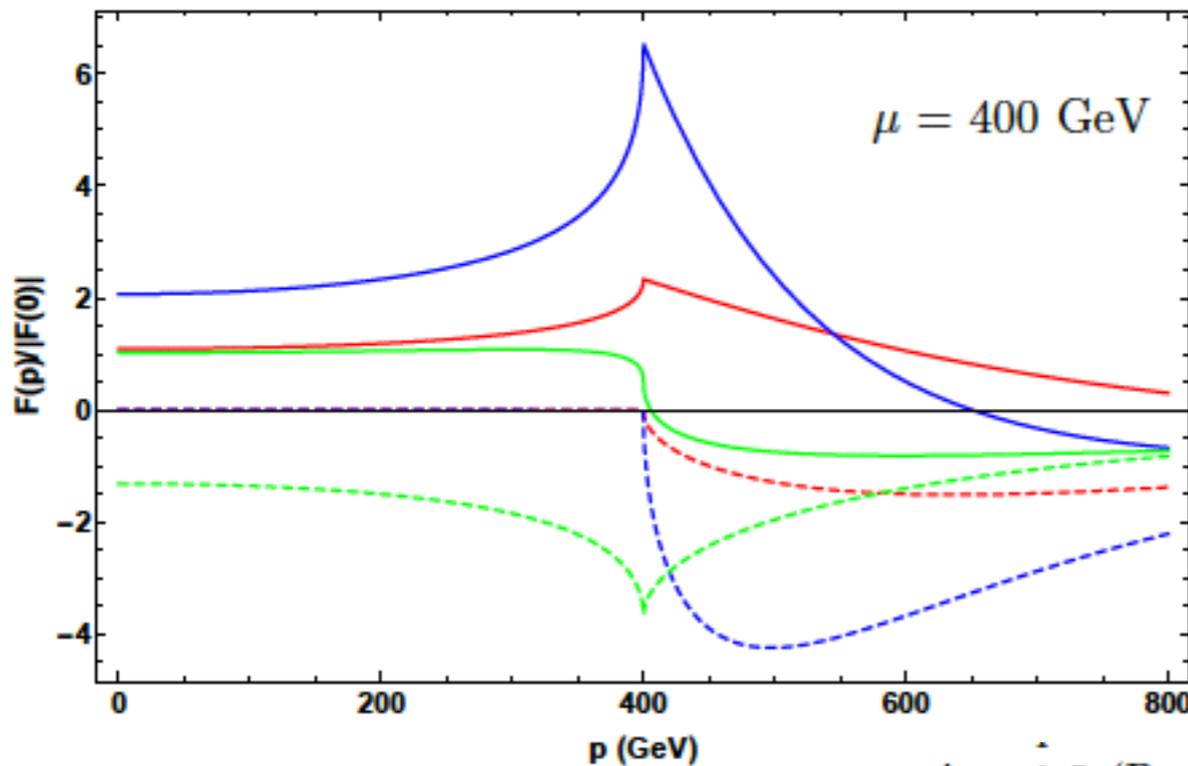


$$\mathcal{H} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ \mathcal{V} + h \end{pmatrix}$$

$$\mathcal{M}_{VBF} = J_1^\alpha G_{\alpha\mu}^V(p_1) J_2^\beta G_{\nu\beta}^V(p_2) F_{VVh}^{\mu\nu}(p_i; \mu) N_V$$

$$\mathcal{M}_{qq \rightarrow Vh} = J_I^\alpha G_{\alpha\mu}^V(p_1) \bar{e}_{2\nu} F_{VVh}^{\mu\nu}(p_1, -p_2; \mu) N_V$$

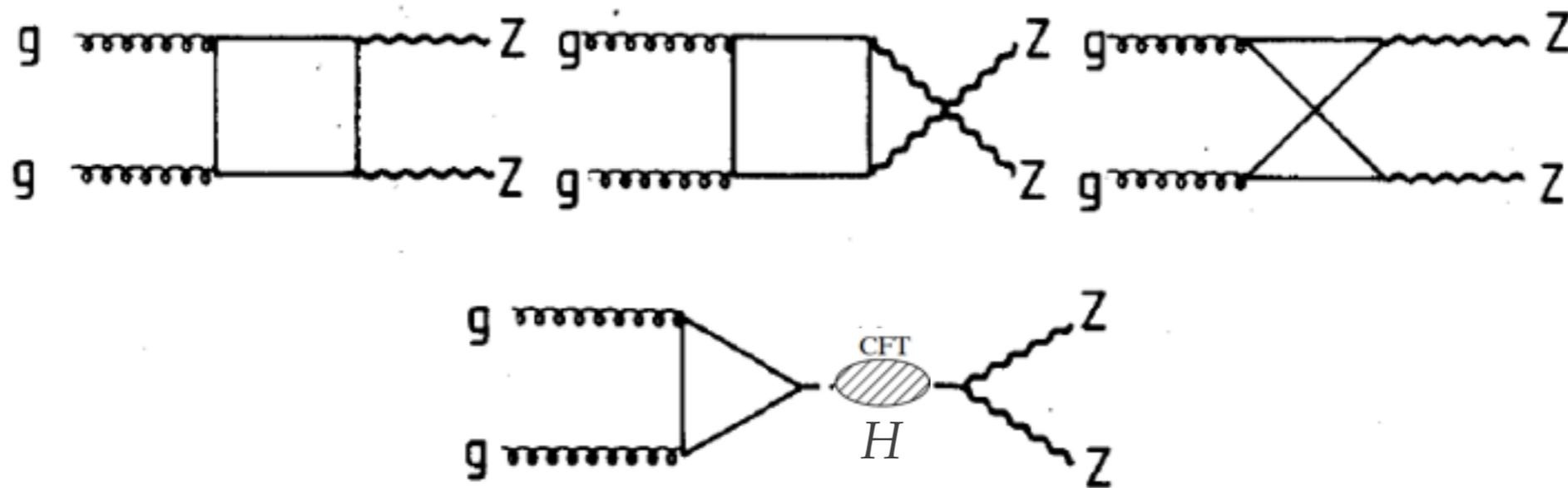
$$F_{VVh}^{ab} = 2 \frac{\mathcal{V}}{L M^2} \int_R^\infty dz a^2 \left( \frac{z}{R} \right) \frac{K_{2-\Delta}(\sqrt{\mu^2 - (p_1 + p_2)^2} z) K_{2-\Delta}(\mu z)}{K_{2-\Delta}(\sqrt{\mu^2 - (p_1 + p_2)^2} R) K_{2-\Delta}(\mu R)}$$



$\Delta = 1.2$  (Red),  $1.4$  (Blue), and  $1.6$  (Green)

# Direct Signals

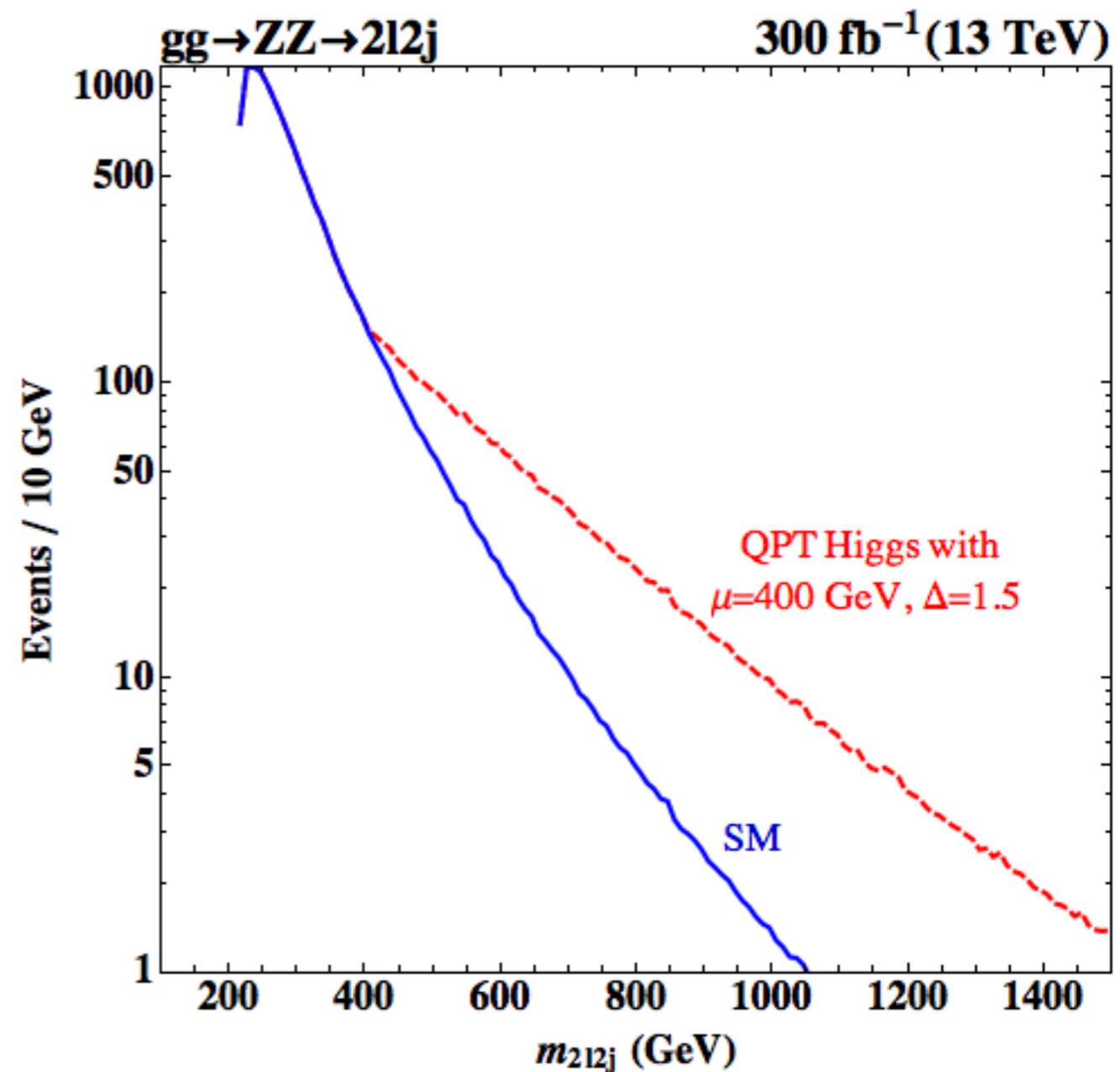
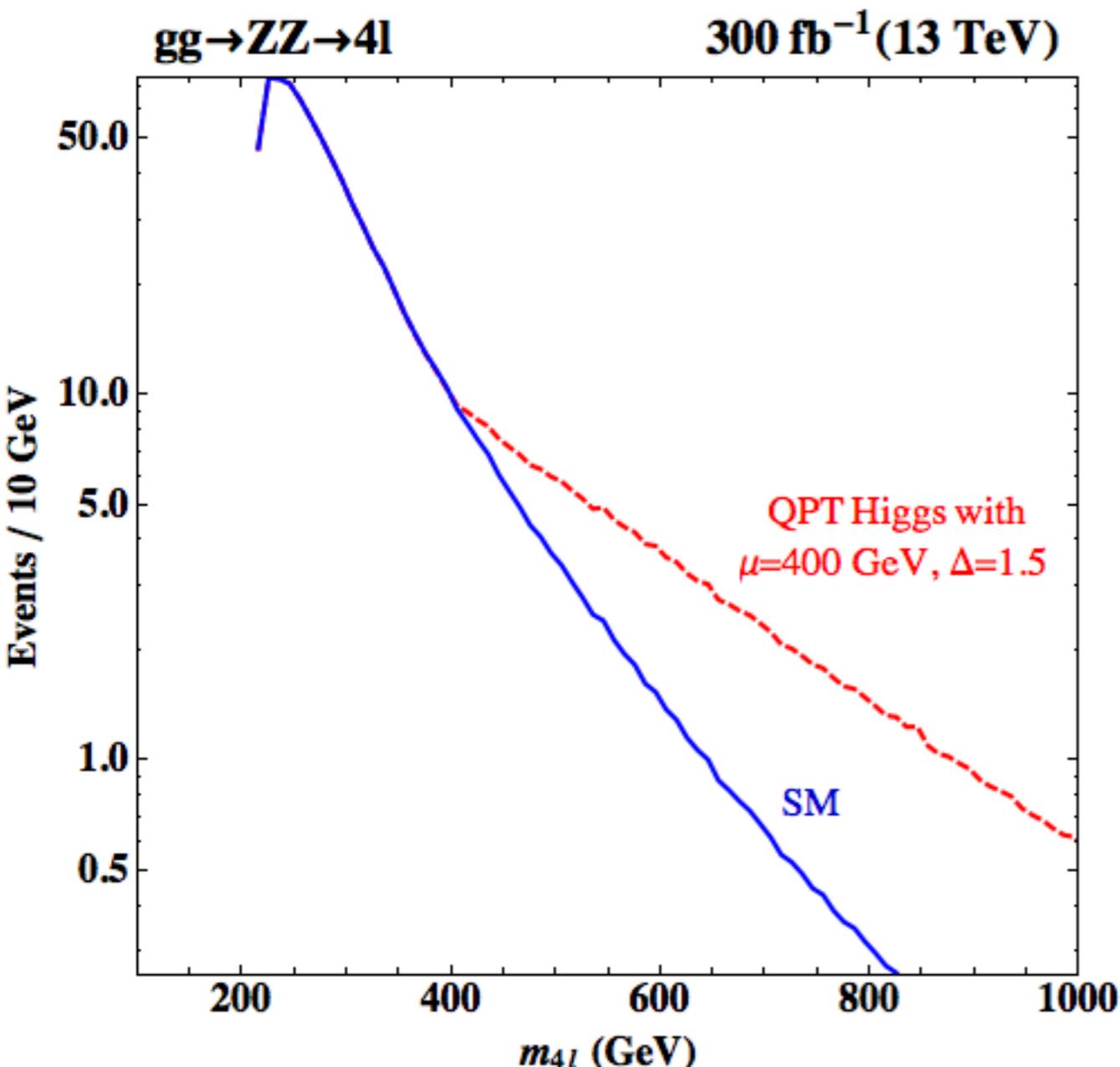
- ❖ Off-shell Higgs can be tested via interference.



sensitive to the  
modifications of the Higgs two-point function

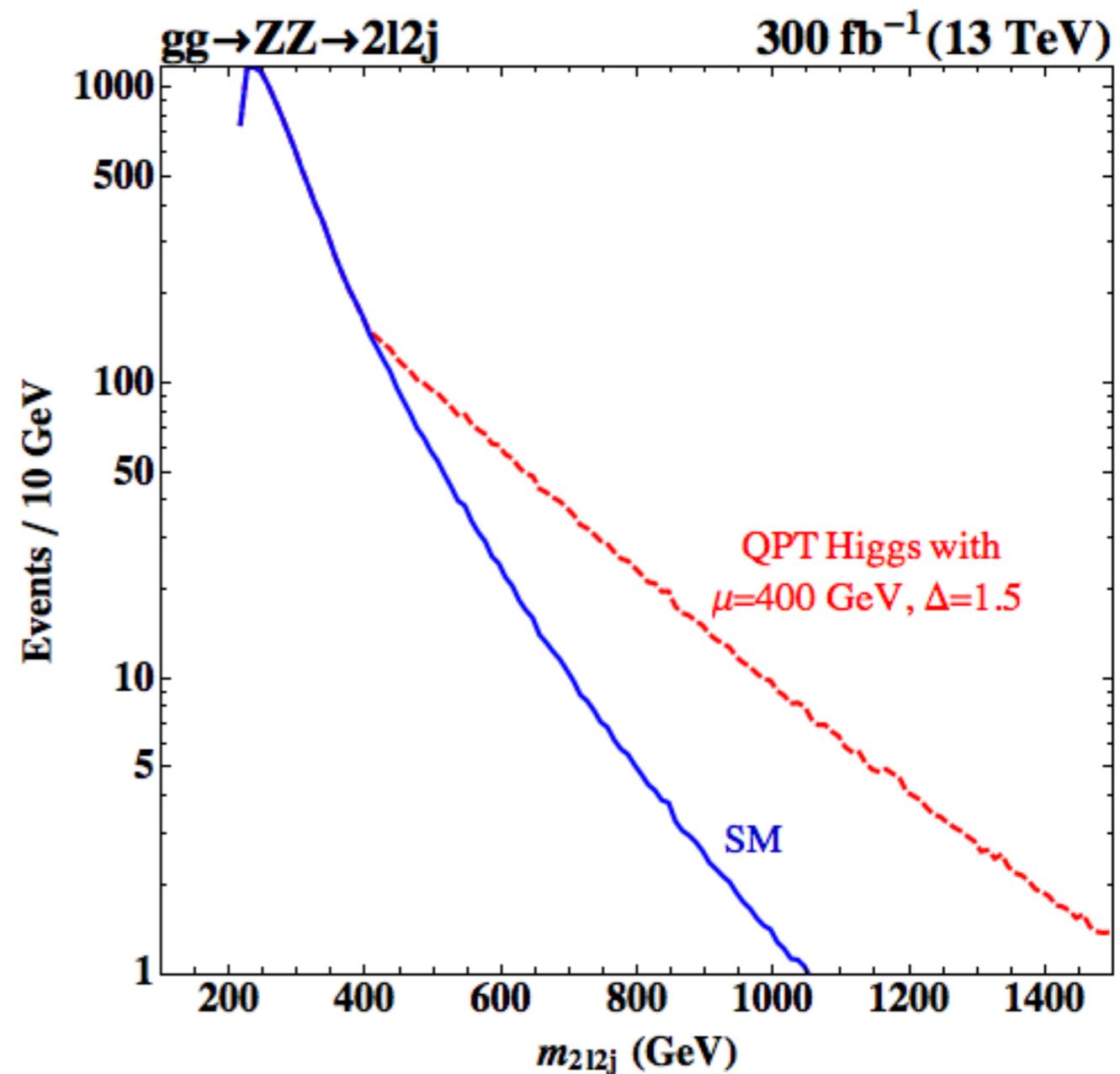
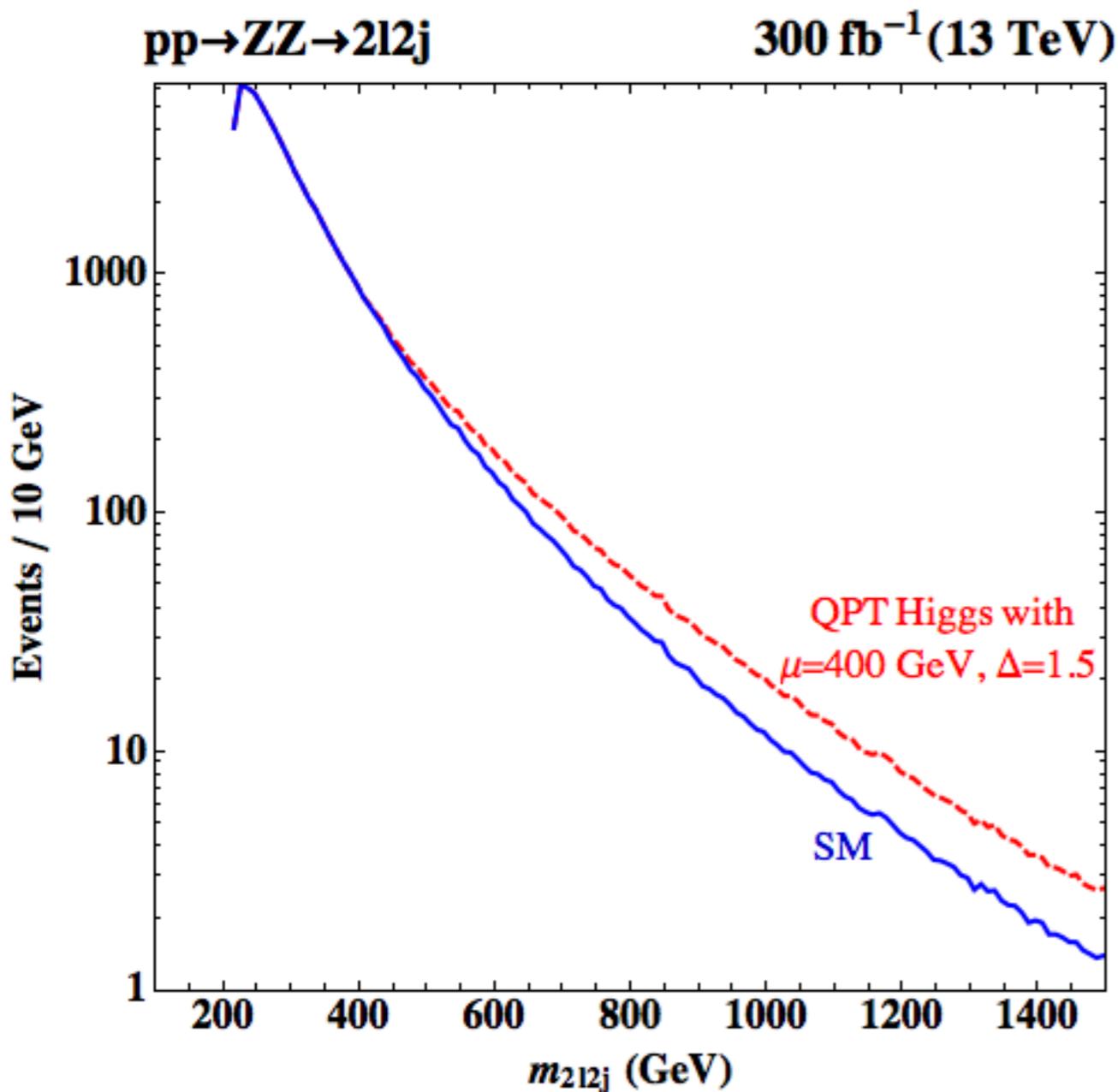
# Direct Signals

- ❖ Single Higgs production: Production of the cut modifies Higgs cross sections for energies above  $\mu \Rightarrow$  modifies any cross sections that involve the (tree-level) exchange of the components of Higgs



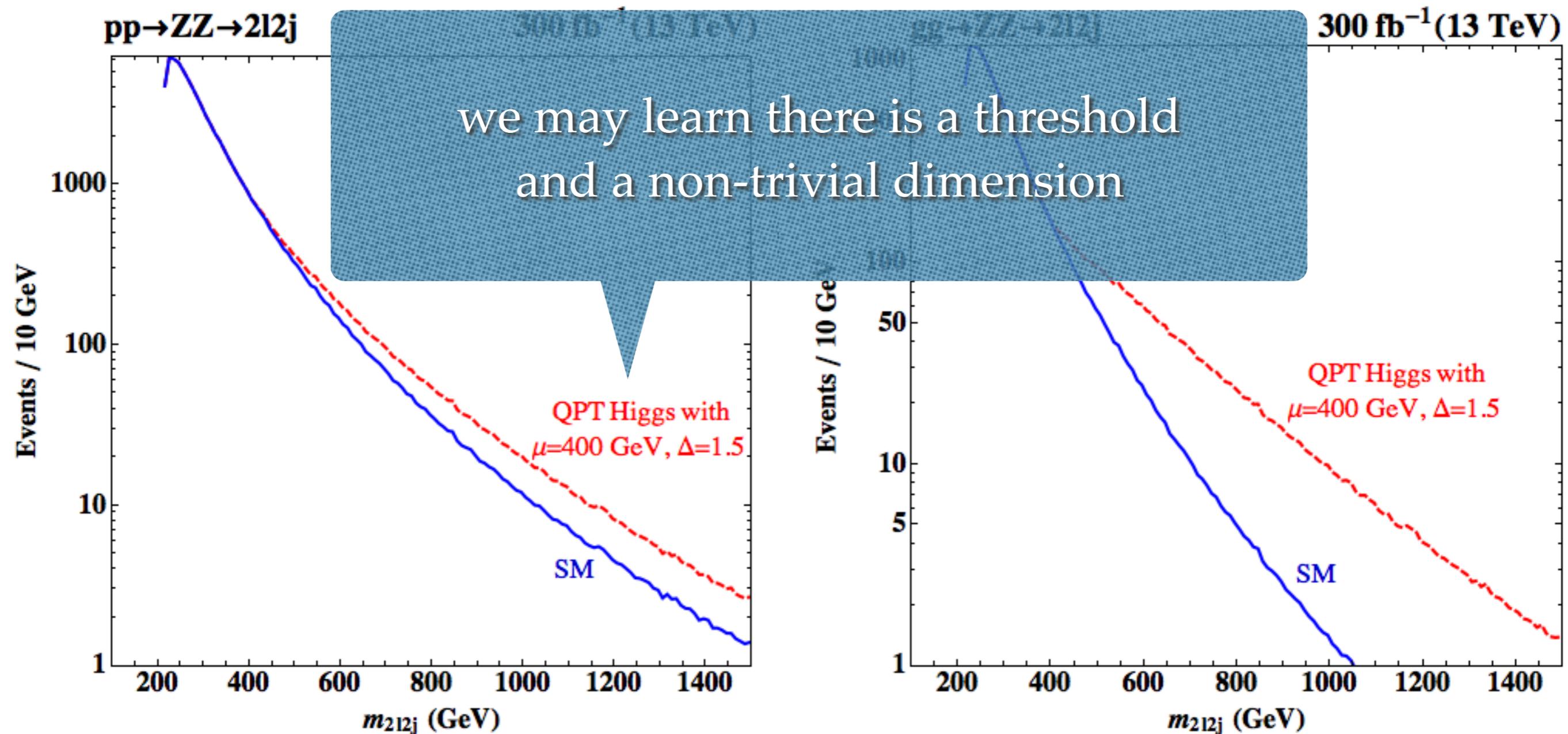
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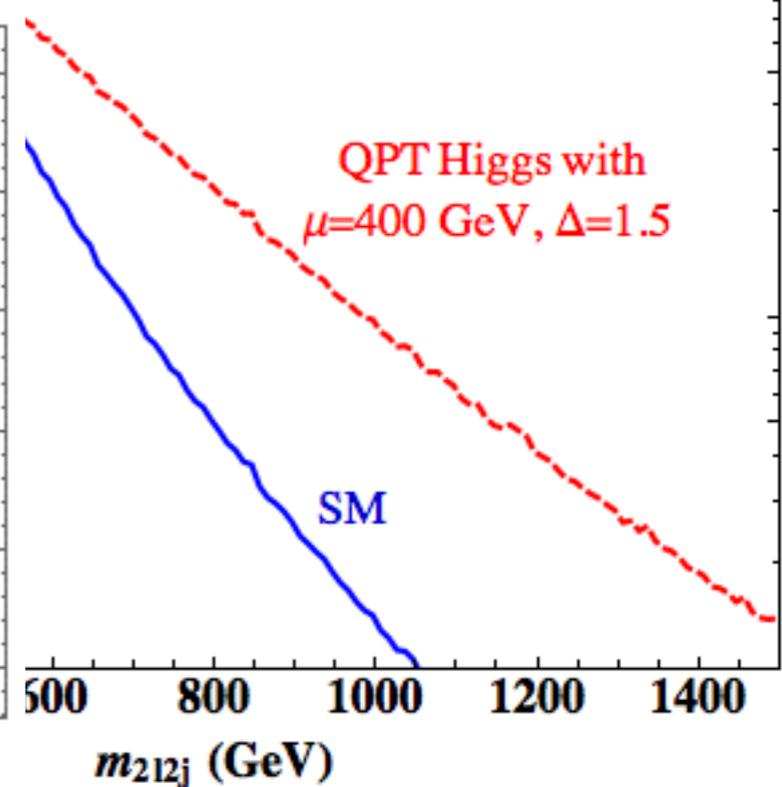
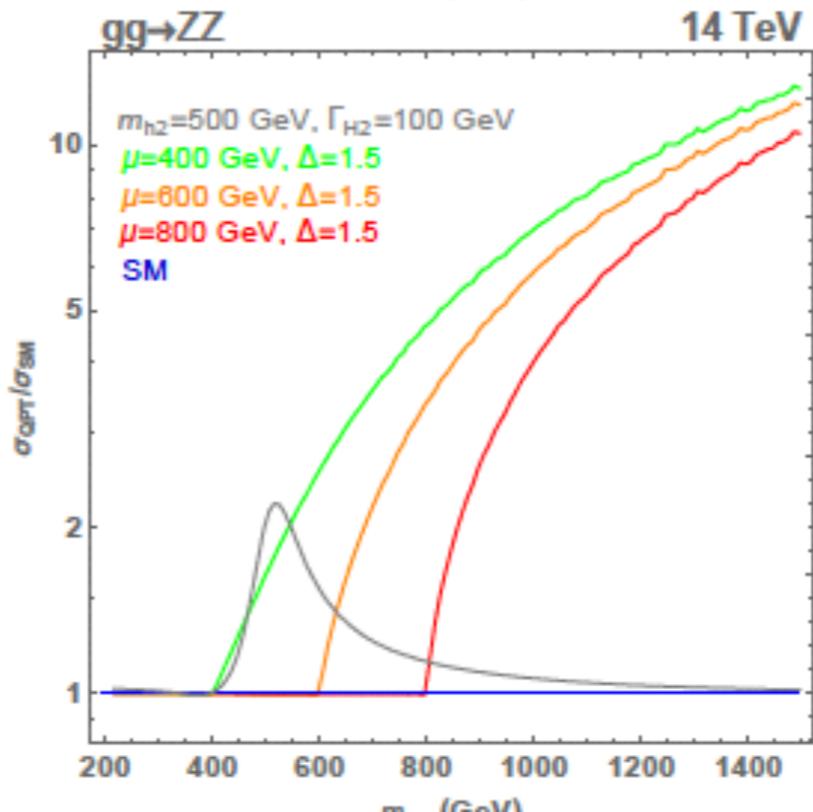
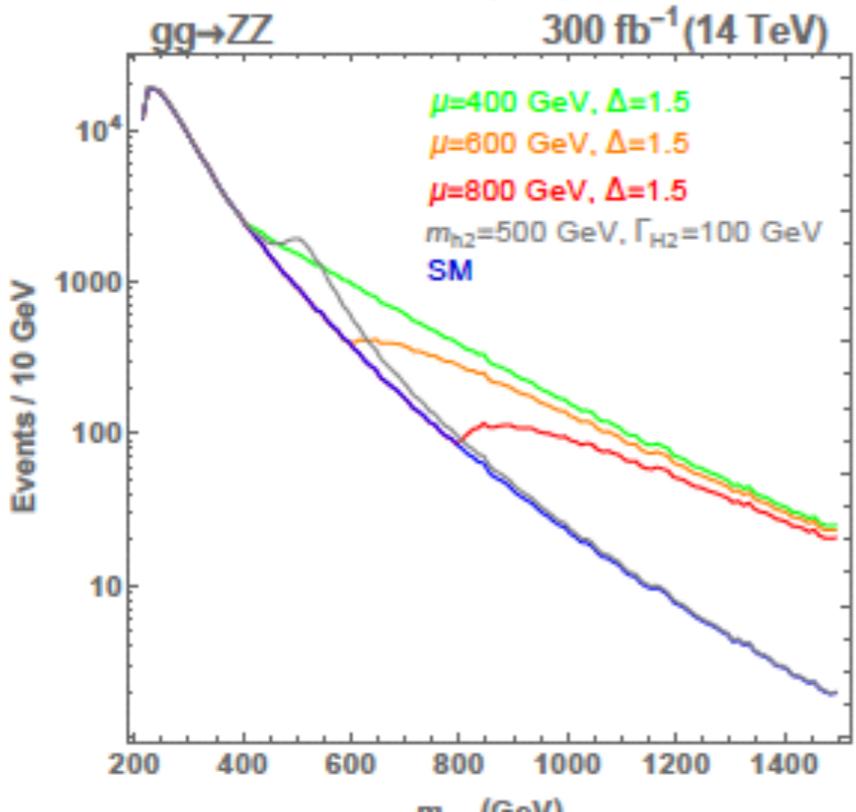
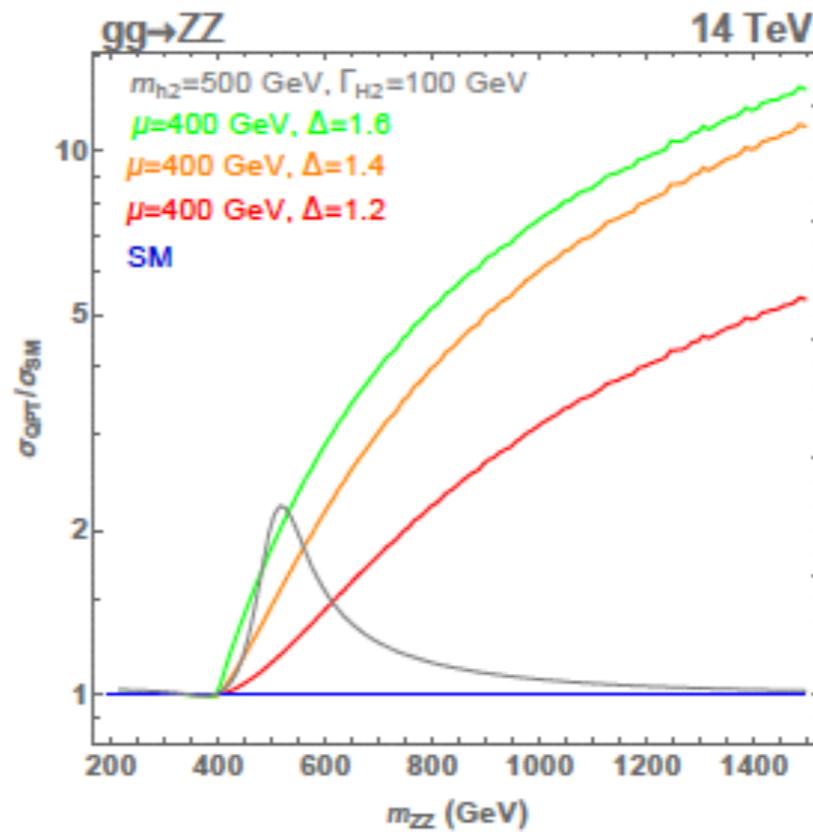
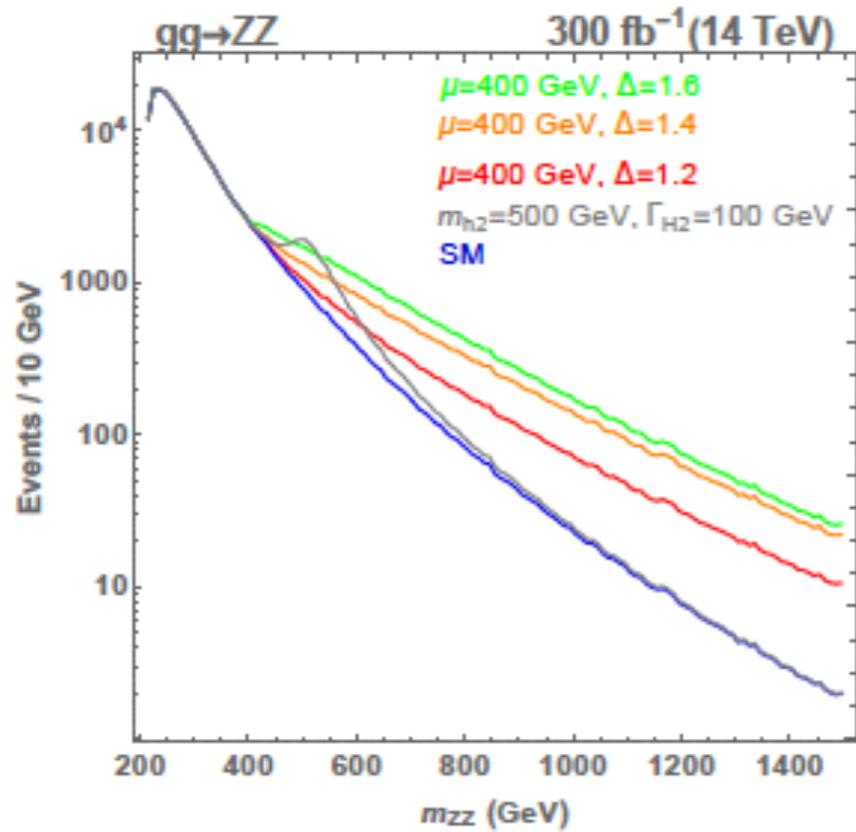
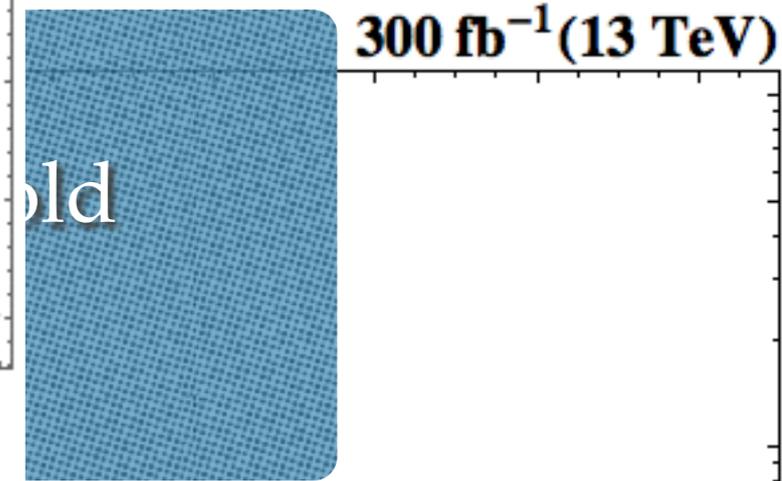
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# Direct Signals

Higgs cross sections for  
olve the (tree-level)



◇

1000

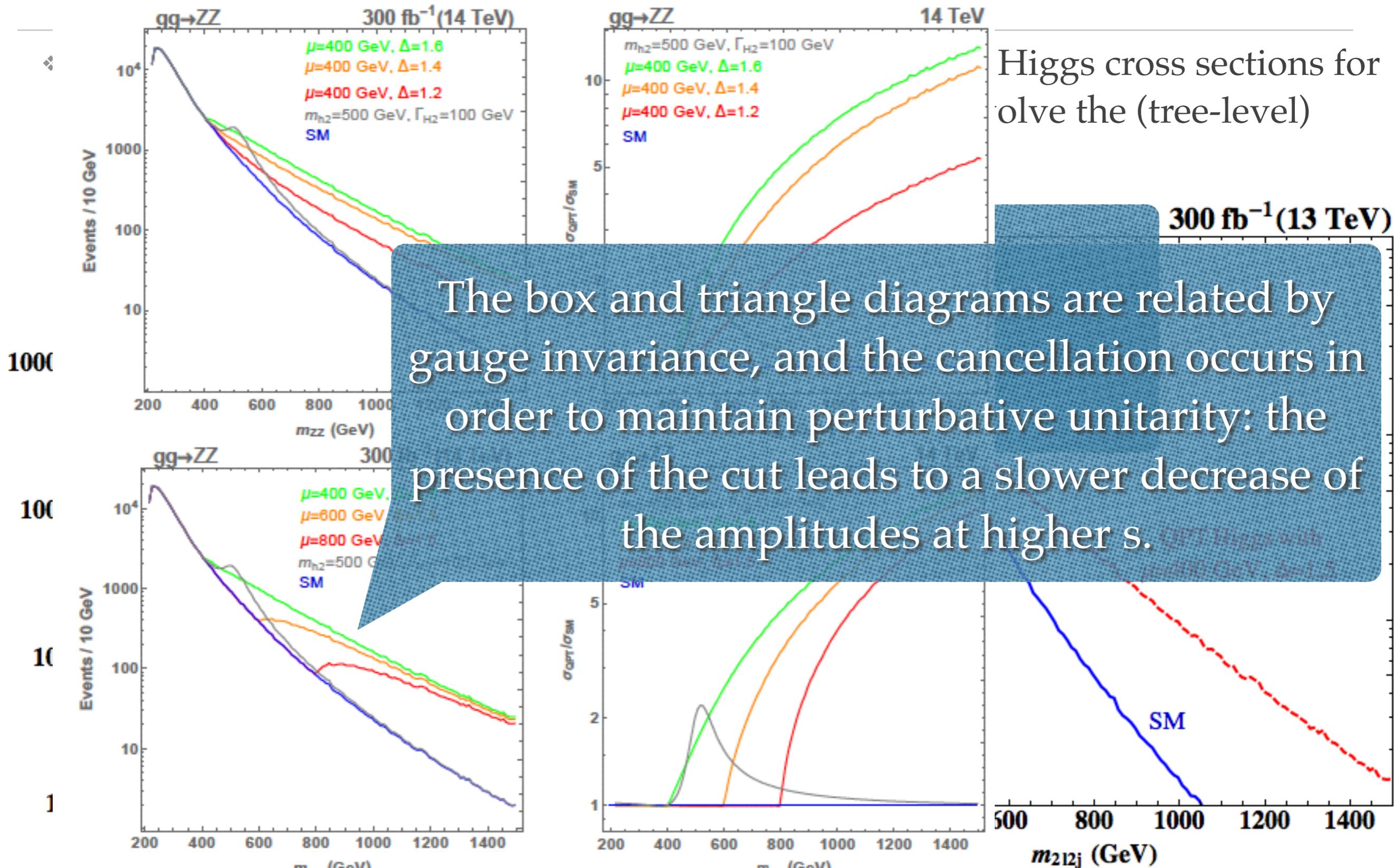
100

10

1

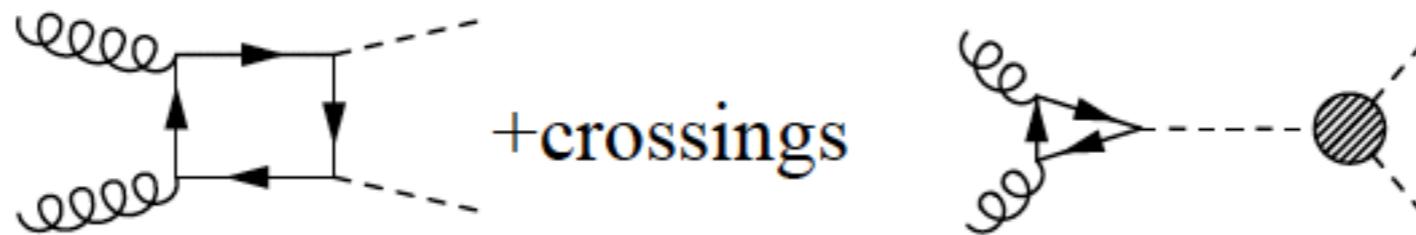
# Direct Signals

Higgs cross sections for  
olve the (tree-level)



# Direct Signals

- ❖ Double Higgs production



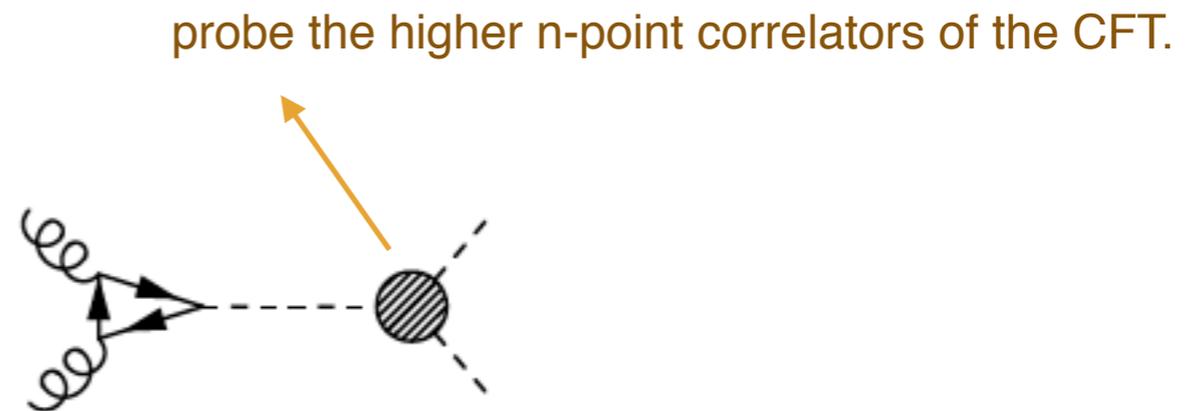
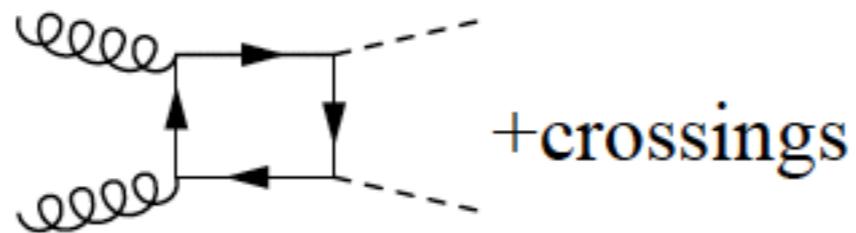
$$\frac{d\hat{\sigma}}{d\hat{t}} = \frac{\alpha_w^2 \alpha_s^2}{2^{15} \pi M_W^4 \hat{s}^2} (|\text{gauge1}|^2 + |\text{gauge2}|^2)$$

gauge1 = box + triangle (negative interference)

gauge2 = box (largest contribution)

# Direct Signals

❖ Double Higgs production



$$\frac{d\hat{\sigma}}{d\hat{t}} = \frac{\alpha_w^2 \alpha_s^2}{2^{15} \pi M_W^4 \hat{s}^2} (|\text{gauge1}|^2 + |\text{gauge2}|^2)$$

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# Direct Signals

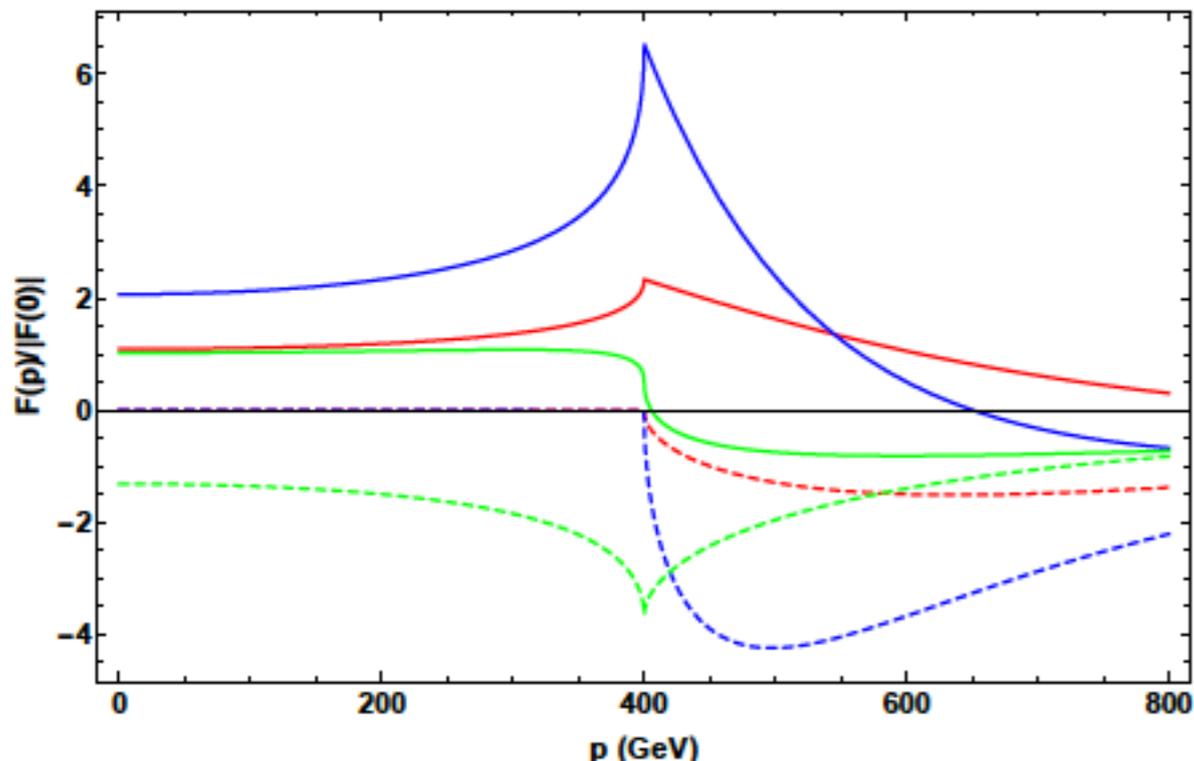
- Form factors for trilinear Higgs self coupling

$$\lambda_5(H^\dagger H)^2$$

$$F_{hhh} = \frac{\lambda_5}{L^2} \mathcal{V} \int_R^\infty dz \frac{1}{a} \left(\frac{z}{R}\right)^2 \frac{K_{2-\Delta}(\mu z)}{K_{2-\Delta}(\mu R)} \prod_{i=1}^3 \frac{K_{2-\Delta}(\sqrt{\mu^2 - p_i^2} z)}{K_{2-\Delta}(\sqrt{\mu^2 - p_i^2} R)}$$

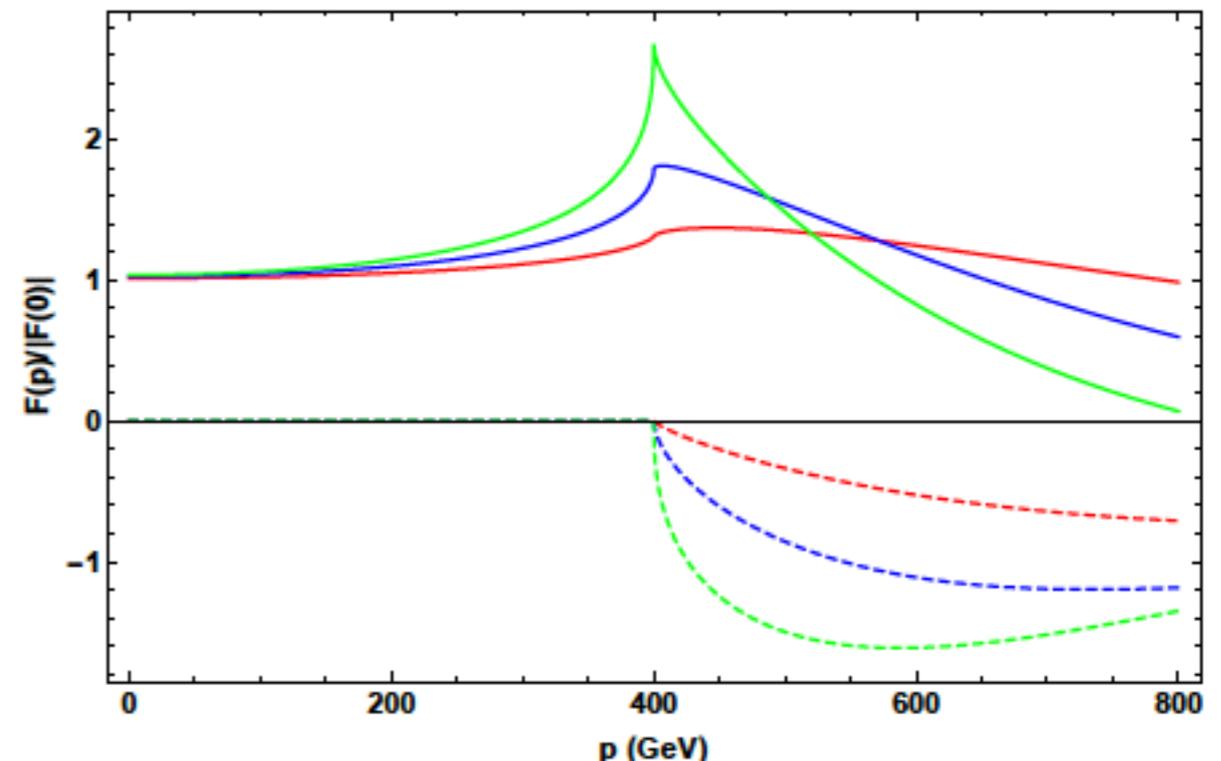
$$\mu = 400, \quad \Delta = 1.5,$$

Higgs momentum: 200 GeV (Red), 400 GeV (Blue), and 600 GeV (Green)



$$\mu = 400,$$

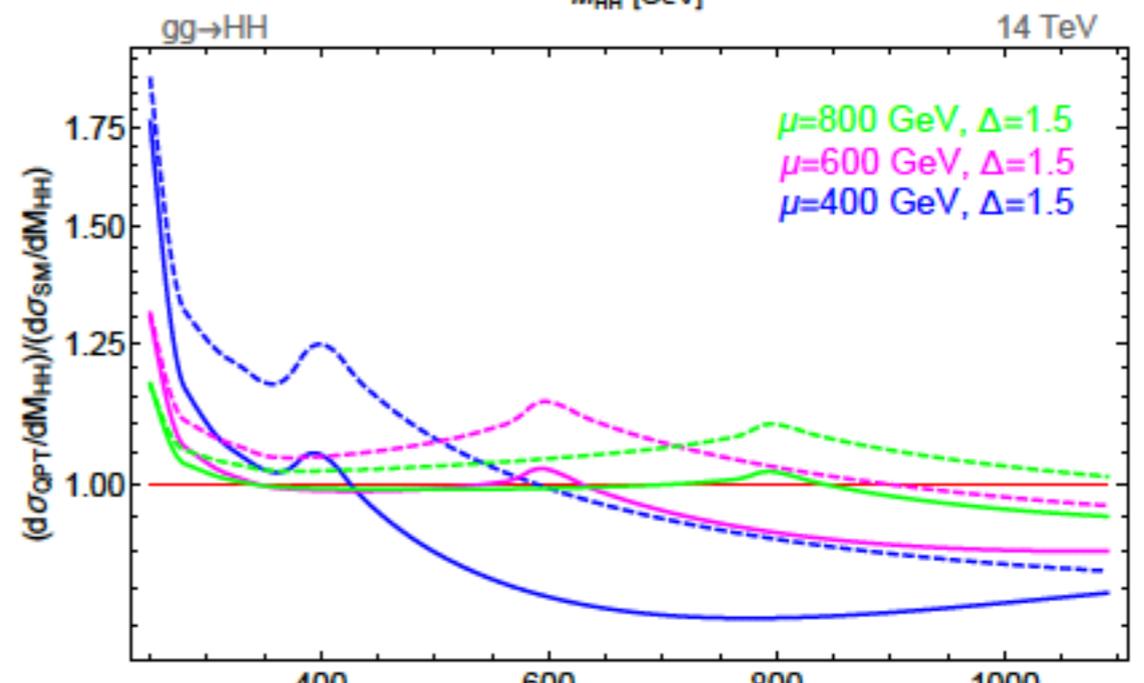
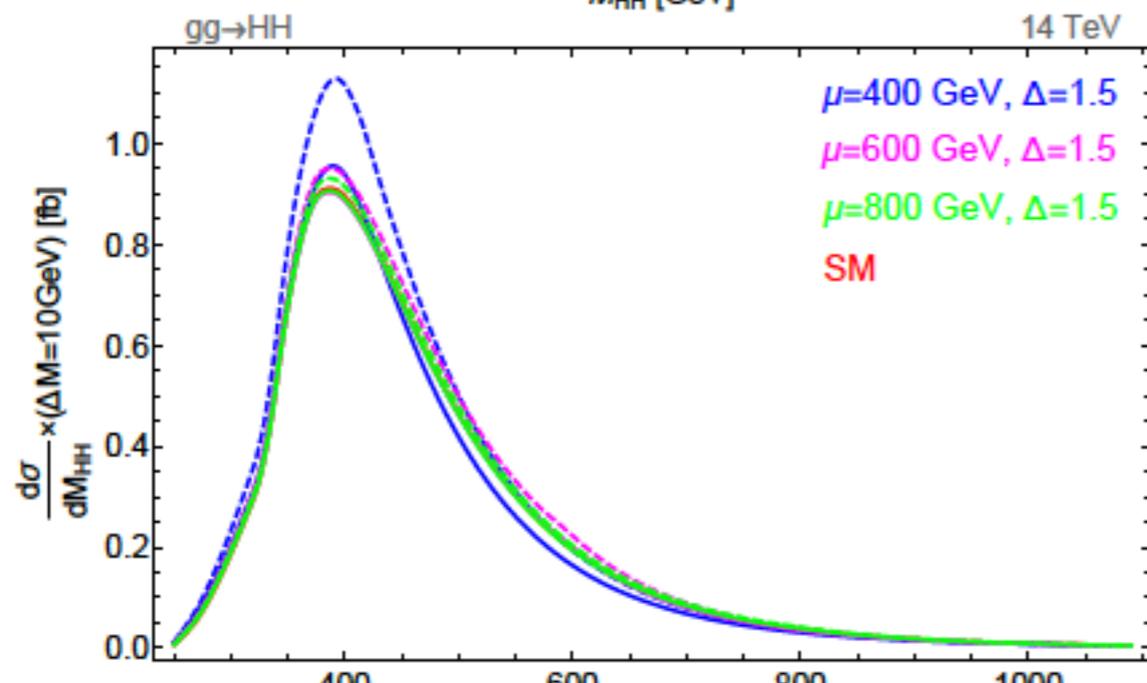
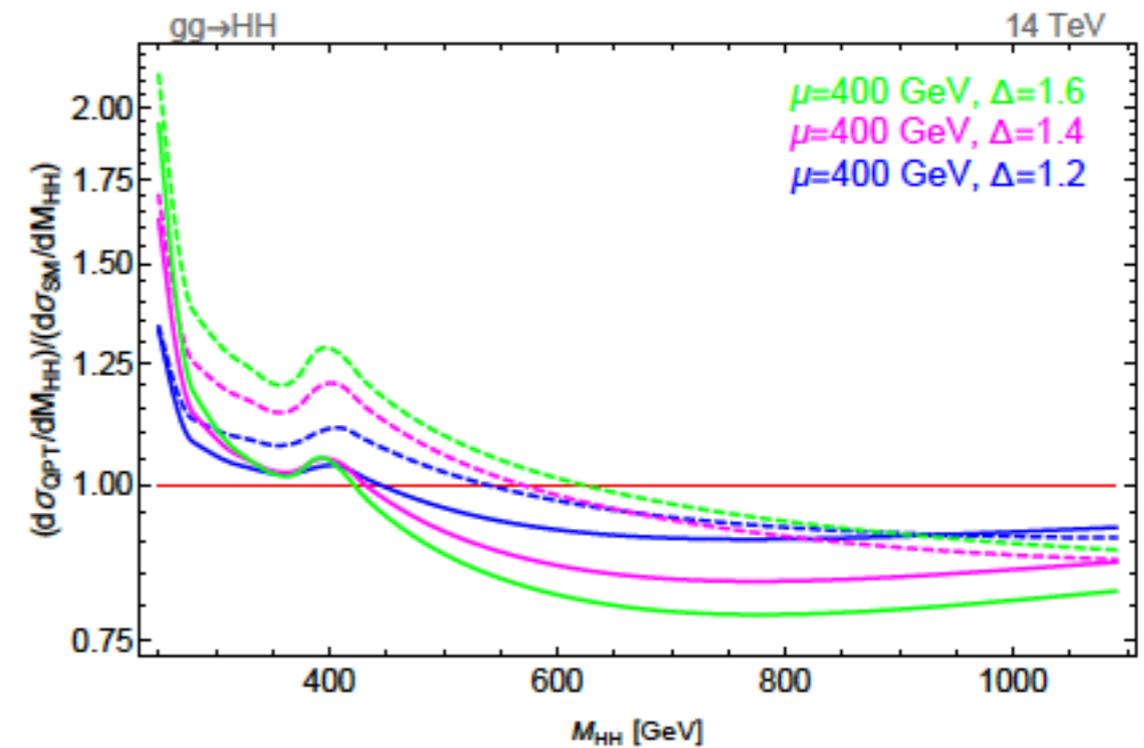
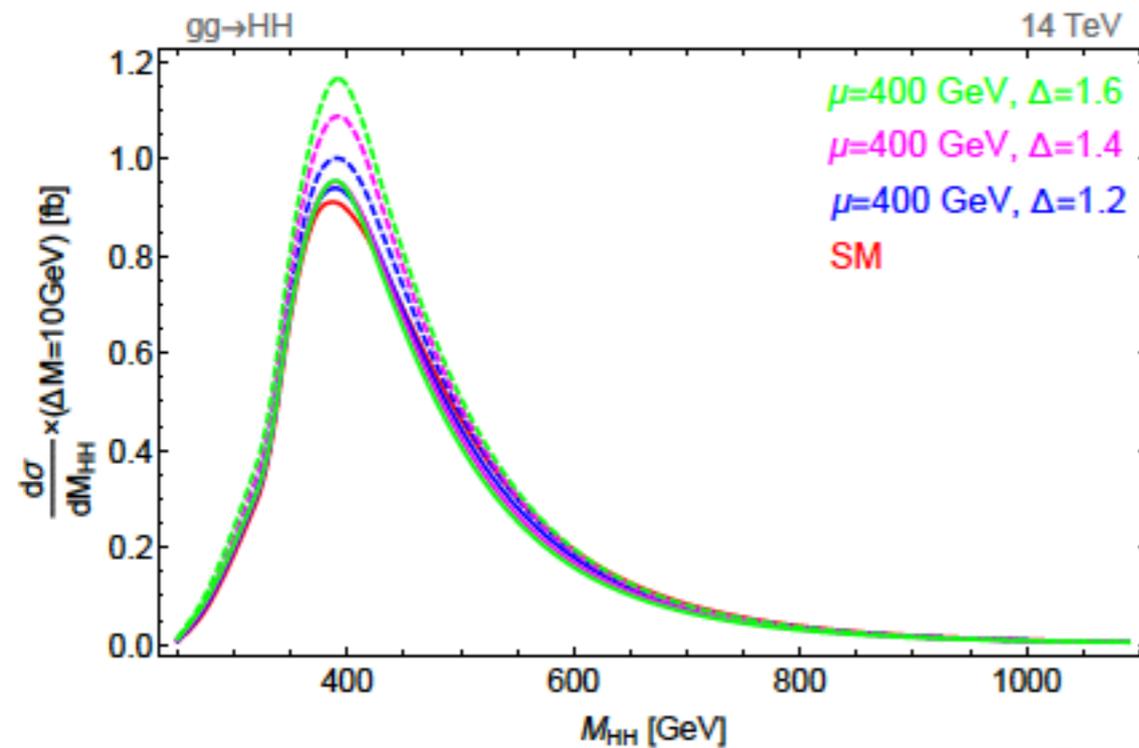
$\Delta = 1.2$  (Red) 1.4 (Blue), and 1.6 (Green).



# Direct Signals

## ❖ Double Higgs production

dashed lines correspond to the case where only the Higgs two-point function has non-trivial behavior inherited from a sector with strong dynamics.



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# Summary

---

- ❖ The Higgs sector may exhibit signs of quantum criticality with non-trivial non-mean-field behavior
  - ❖ CFT scaling may lead to a gapped continuum rather than a tower of new particles - depending on manner of IR exit of CFT
- ❖ Low-energy EFT for such a quantum critical Higgs shows that critical exponents can be extracted from the LHC measurements (and future colliders...)
  - ❖ Nontrivial momentum-dependent **form factors** for Higgs physics interesting for the future measurement.
- ❖ UV completions with non-trivial fixed point, where hierarchy problem is addressed possible.

Back-up

---

# AdS/CFT

---

$$\left\langle e^{\int d^4x \phi_0(x) \mathcal{O}(x)} \right\rangle_{\text{CFT}} \approx e^{-S_{5\text{Dgravity}}[\phi(x,z)|_{z=0}=\phi_0(x)]}$$

$$ds^2 = \frac{R^2}{z^2} (dx_\mu^2 - dz^2)$$

$\mathcal{O} \subset \text{CFT} \leftrightarrow \phi$  AdS<sub>5</sub> field

---

# AdS/CFT

---

$$ds^2 = \frac{R^2}{z^2} (dx_\mu^2 - dz^2)$$
$$z > \epsilon$$

$$S_{bulk} = \frac{1}{2} \int d^4x dz \sqrt{g} (g^{\alpha\beta} \partial_\alpha \phi \partial_\beta \phi + m^2 \phi^2)$$

$$\phi(p, z) = az^2 J_\nu(pz) + bz^2 J_{-\nu}(pz)$$

$$\Delta[\mathcal{O}] = 2 \pm \nu = 2 \pm \sqrt{4 + m^2 R^2}$$

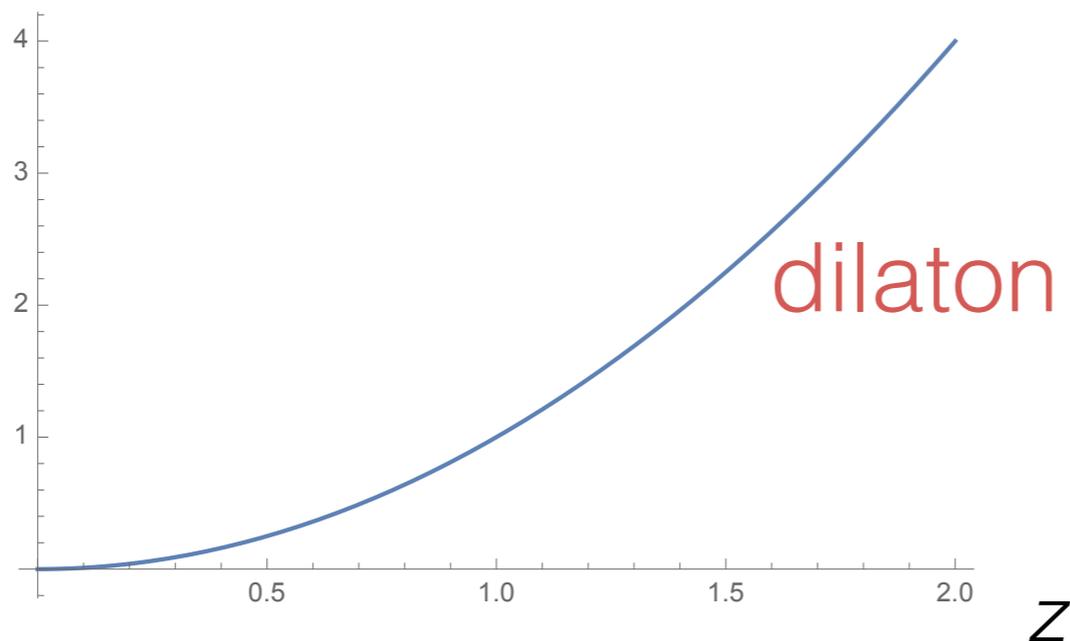
$$\langle \mathcal{O}(p) \mathcal{O}(p) \rangle \propto \frac{\delta^{(4)}(p+p')}{(2\pi)^2} (p^2)^{\Delta-2}$$

---

# broken CFT

---

- ❖ Randall Sundrum 2 (only UV brane and bulk): cuts from 0 (CFT)
- ❖ RS1: putting IR cutoff at TeV
- ❖ New type of IR cutoff (soft wall) gives rise to a different phenomenology



---

# broken CFT by IR cutoff

---

$$S_{\text{int}} = \frac{1}{2} \int d^4x dz \sqrt{g} \phi \mathcal{H}^\dagger \mathcal{H}$$

$$\phi = \mu z^2$$

$$z^5 \partial_z \left( \frac{1}{z^3} \partial_z \mathcal{H} \right) - z^2 (p^2 - \mu^2) \mathcal{H} - m^2 R^2 \mathcal{H} = 0$$

$$\langle \mathcal{O}(p) \mathcal{O}(p) \rangle \propto \frac{\delta^{(4)}(p+p')}{(2\pi)^2} (p^2 - \mu^2)^{\Delta-2}$$

$$[\partial^2 - \mu^2]^{2-\Delta} \delta(x-y)$$

---

# Non-local operators

---

$$\mathcal{L}_{\mathcal{H}} = -\mathcal{H}^\dagger [D^2 + \mu^2]^{2-\Delta} \mathcal{H} + \mu^{4-2\Delta} \mathcal{H}^\dagger \mathcal{H} - V(|\mathcal{H}|)$$
$$[\partial^2 - \mu^2]^{2-\Delta} \delta(x - y)$$

similar to SCET!

---

# Non-local operators

---

$$\mathcal{L}_{\mathcal{H}} = -\mathcal{H}^\dagger [D^2 + \mu^2]^{2-\Delta} \mathcal{H} + \mu^{4-2\Delta} \mathcal{H}^\dagger \mathcal{H} - V(|\mathcal{H}|)$$

$$[\partial^2 - \mu^2]^{2-\Delta} \delta(x - y)$$

$$W(x, y) = P \exp \left[ -igT^a \int_x^y A_\mu^a d\omega^\mu \right]$$

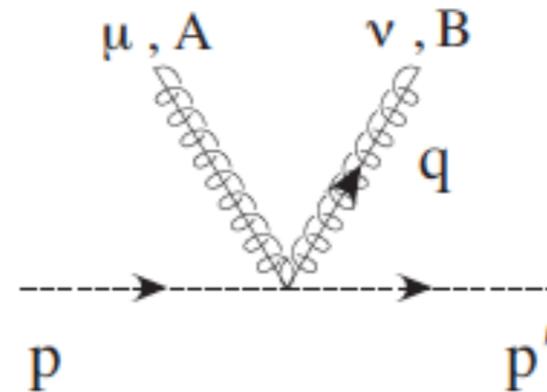
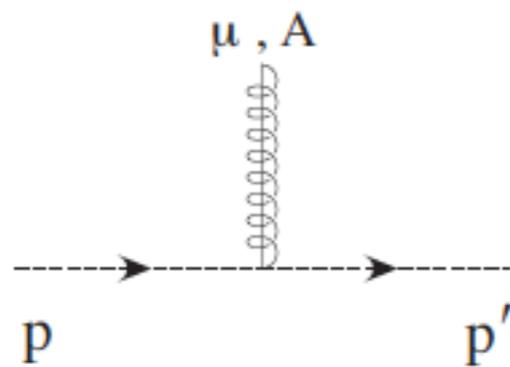
similar to SCET!

# Non-local operators

$$\mathcal{L}_{\mathcal{H}} = -\mathcal{H}^\dagger [D^2 + \mu^2]^{2-\Delta} \mathcal{H} + \mu^{4-2\Delta} \mathcal{H}^\dagger \mathcal{H} - V(|\mathcal{H}|)$$

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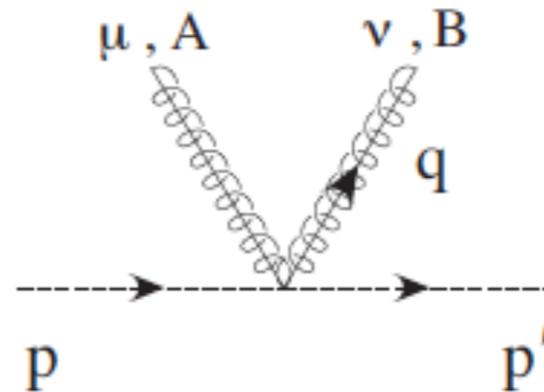
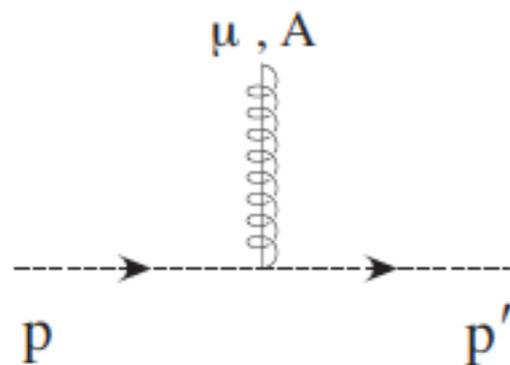
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- ❖ e.g. for the trilinear interaction in momentum space:  $\mathcal{H}^\dagger(p+q) A_\mu^a(q) \mathcal{H}(p) \Gamma^{\mu,a}(p, q)$

$$\Gamma^{\mu,a}(p, q) = gT^a (2p^\mu + q^\mu) F(p, q) ,$$

$$F(p, q) = -\frac{(\mu^2 - (p+q)^2)^{2-\Delta} - (\mu^2 - p^2)^{2-\Delta}}{2p \cdot q + q^2}$$

similar to SCET!