LFV and EDM

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Outline

- Introduction.
- Bounds on new physics from LFV and EDMs.
- Implications for physics beyond the Standard Model.
- Supersymmetric models: correlations among processes.
- LFV and neutrino masses: SUSY see-saw.
- Conclusions.

Introduction

Lepton flavour violation is a very powerful tool to probe physics beyond the Standard Model.

In the Standard Model

$$U(3)_{e_R} \times U(3)_L \longrightarrow U(1)_e \times U(1)_\mu \times U(1)_\tau$$

Charged lepton masses

Family lepton numbers and total lepton number are strictly conserved.

Consistent with experiments searching for neutrinoless double beta decay and rare lepton decays, but not with neutrino oscillation experiments.

In the Standard Model with massive neutrinos

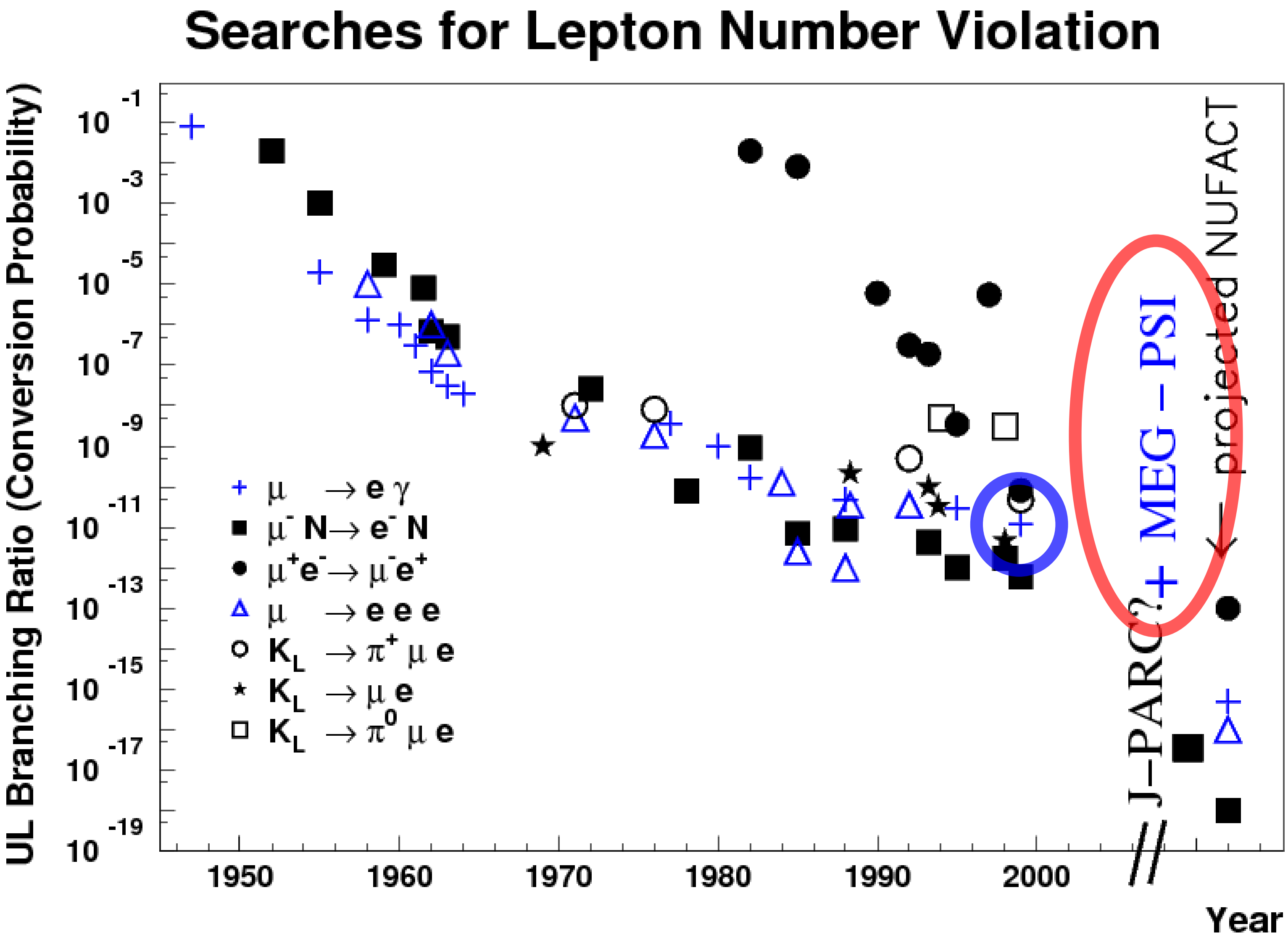
$$U(3)_{e_R} \times U(3)_L \longrightarrow U(1)_{\text{lep}}$$ Dirac Mass

$$U(3)_{e_R} \times U(3)_L \longrightarrow \text{nothing}$$ Majorana Mass

LFV already discovered! Next challenge: discover LFV in the charged lepton sector

The predictions for the rare lepton decays are $BR(\mu \to e\gamma) \simeq 10^{-57}$, $BR(\tau \to \mu\gamma) \simeq 10^{-54}$, $BR(\tau \to e\gamma) \simeq 10^{-57}$,

Well consistent with experiments searching for rare (charged) lepton decays.



Searches for Lepton Number Violation
UL Branching Ratio (Conversion Probability)
10^-1
10^-3
10^-5
10^-7
10^-9
10^-11
10^-13
10^-15
10^-17
10^-19
1950
1960
1970
1980
1990
2000
Year
μ → e γ
μ⁻ N → e⁻ N
μ⁺e⁻ → μ⁻e⁺
μ → e e e
K_L → π⁺ μ e
K_L → μ e
K_L → π⁰ μ e
J–PARC?
MEG – PSI
← projected NUFACT

Äystö et al.

Bounds on new physics from μ→eγ

Lowest dimension operator which induces μ→eγ

$$-\mathcal{L} = m_\mu \bar{\mu}(f_{M1}^{\mu e} + \gamma_5 f_{E1}^{\mu e})\sigma^{\mu\nu} e F_{\mu\nu} + \text{h.c.}$$

The rate for the rare muon decay is:

$$\text{BR}(\mu \to e\gamma) = \frac{96\pi^3\alpha}{G_F^2}(|f_{E1}^{\mu e}|^2 + |f_{M1}^{\mu e}|^2)$$

The present experimental bound BR(μ→eγ)$<1.2\times10^{-11}$ gives:

$$|f_{E1}^{\mu e}|, |f_{M1}^{\mu e}| \lesssim 10^{-12}\text{GeV}^{-2}$$

Naively,

$$f^{\mu e} \sim \frac{1}{\Lambda^2} \longrightarrow \Lambda \gtrsim 300\text{TeV}$$

In most models the contact interaction arises as a result of quantum effects (new particles interacting with the muon and the electron circulating in loops).

$$f^{\mu e} \sim \frac{\theta_{\mu e}^2 \alpha}{\Lambda^2}$$

Then, the present bound on BR($\mu \to e\gamma$) requires

$$\Lambda \gtrsim 20\text{TeV} \quad \text{if} \quad \theta_{\mu e} \sim \frac{1}{\sqrt{2}}$$

$$\theta_{\mu e} \lesssim 0.01 \quad \text{if} \quad \Lambda \sim 300\text{GeV}$$

A large mass scale for the new particles and/or small coupling between the electron or muon with the new particles.

Rare tau decays

Complementary probe of lepton flavour violation.

Until very recently, not as interesting as $\mu\to e\gamma$ for constraining models.

PDG 2004

$$\mathrm{BR}(\tau \to e\gamma) \leq 2.7 \times 10^{-6} \qquad \mathrm{CL} = 90\%$$
$$\mathrm{BR}(\tau \to \mu\gamma) \leq 1.1 \times 10^{-6} \qquad \mathrm{CL} = 90\%$$

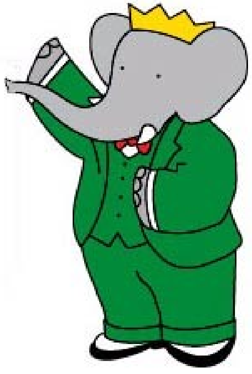
The experimental bound $\mathrm{BR}(\tau\to\mu\gamma)<1.1\times10^{-6}$ yields:

$\Lambda \gtrsim 800\mathrm{GeV}$ if $\theta_{\tau\mu} \sim \frac{1}{\sqrt{2}}$ (compare to 20TeV!)

$\theta_{\tau\mu} \lesssim 0.3$ if $\Lambda \sim 300\mathrm{GeV}$

Impressive experimental progress in the last years!!

BELLE



PDG 2004

$$\mathrm{BR}(\tau \to e\gamma) \leq 2.7 \times 10^{-6} \qquad \mathrm{CL} = 90\%$$

$$\mathrm{BR}(\tau \to \mu\gamma) \leq 1.1 \times 10^{-6} \qquad \mathrm{CL} = 90\%$$

PDG 2005

$$\mathrm{BR}(\tau \to e\gamma) \leq 3.9 \times 10^{-7} \qquad \mathrm{CL} = 90\%$$

$$\mathrm{BR}(\tau \to \mu\gamma) \leq 3.1 \times 10^{-7} \qquad \mathrm{CL} = 90\%$$

PDG 2006

$$\mathrm{BR}(\tau \to e\gamma) \leq 1.1 \times 10^{-7} \qquad \mathrm{CL} = 90\%$$

$$\mathrm{BR}(\tau \to \mu\gamma) \leq 6.8 \times 10^{-8} \qquad \mathrm{CL} = 90\%$$

September 2009

$$\mathrm{BR}(\tau \to e\gamma) \leq 1.1 \times 10^{-7} \qquad \mathrm{CL} = 90\%$$ BaBar

$$\mathrm{BR}(\tau \to \mu\gamma) \leq 4.5 \times 10^{-8} \qquad \mathrm{CL} = 90\%$$ Belle

Projected sensitivity of present B-factories

$$\mathrm{BR}(\tau \to e\gamma) \sim 10^{-8}$$

$$\mathrm{BR}(\tau \to \mu\gamma) \sim 10^{-8}$$

The present experimental bounds on the rare tau decays yield:

From $\tau \rightarrow e\gamma$ $\quad \Lambda \gtrsim 1300\text{GeV}$ if $\theta_{\tau e} \sim \frac{1}{\sqrt{2}}$

$\theta_{\tau e} \lesssim 0.2$ if $\Lambda \sim 300\text{GeV}$

From $\tau \rightarrow \mu\gamma$ $\quad \Lambda \gtrsim 1700\text{GeV}$ if $\theta_{\tau \mu} \sim \frac{1}{\sqrt{2}}$

$\theta_{\tau \mu} \lesssim 0.1$ if $\Lambda \sim 300\text{GeV}$

fairly stringent constraints

Other LFV processes

Three body decays:

$BR(\mu^- \to e^- e^+ e^-) < 1.0\times10^{-12}$

$BR(\mu^- \to e^- \gamma\gamma) < 7.2\times10^{-11}$

$BR(\tau^- \to e^- e^+ e^-) < 3.6\times10^{-8}$

$BR(\tau^- \to e^- \mu^+ \mu^-) < 3.7\times10^{-8}$

$BR(\tau^- \to e^+ \mu^- \mu^-) < 2.3\times10^{-8}$

$BR(\tau^- \to \mu^- e^+ e^-) < 2.7\times10^{-8}$

$BR(\tau^- \to \mu^+ e^- e^-) < 2.0\times10^{-8}$

$BR(\tau^- \to \mu^- \mu^+ \mu^-) < 3.2\times10^{-8}$

$BR(\tau^- \to e^- \pi^+ \pi^-) < 1.2\times10^{-7}$

...

μ-e conversion in nuclei: $\Gamma(\mu^- Ti \to e^- Ti)/\Gamma(\mu^- Ti \to all) < 4.3\times10^{-12}$

Rare K decays: $\Gamma(K_L \to \mu e)/\Gamma(K_L \to all) < 4.7\times10^{-12}$

$\Gamma(K^+ \to \pi^+ e^- \mu^+)/\Gamma(K^+ \to all) < 1.3\times10^{-11}$

Rare Z boson decays: $BR(Z \to \mu e) < 1.7\times10^{-6}$

$BR(Z \to \tau e) < 9.8\times10^{-6}$

$BR(Z \to \tau\mu) < 1.2\times10^{-5}$

Electric Dipole Moments

Electric Dipole Moments exist when parity (P) and time reversal (T) are violated (Landau'57). Weak interactions with CP violation can induce EDMs.

Very strong experimental bounds:

Present: $d_e < 1.6\ 10^{-27}$ e cm at 90% c.l.

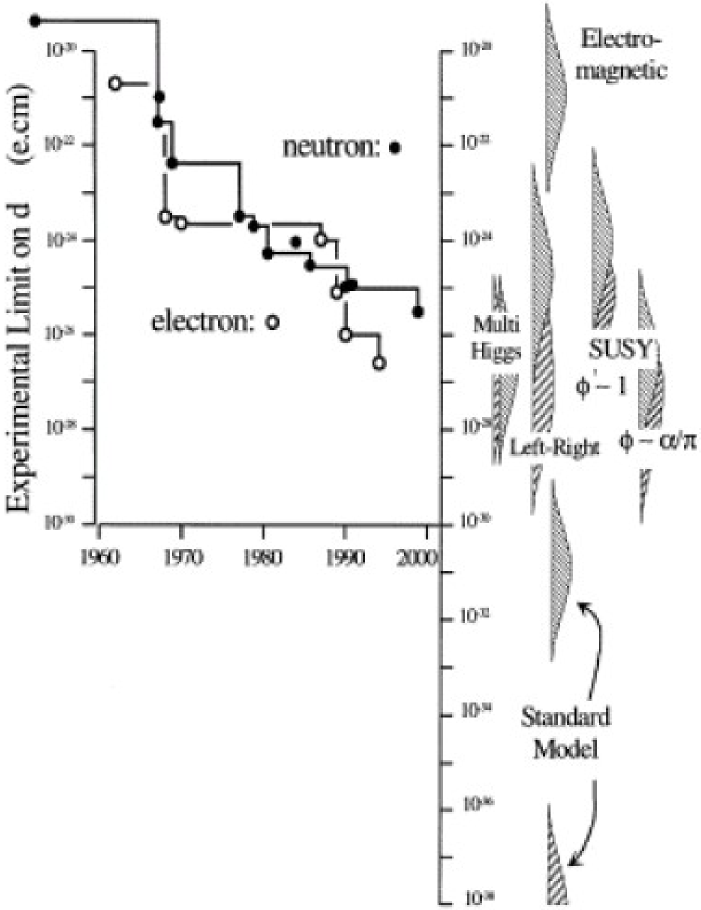
Regan et al. (Berkeley group)

Projected: $d_e \lesssim 10^{-29}\ (10^{-31})$ e cm

De Mille et al. (Yale group)

$d_e \lesssim 10^{-35}$ e cm

Lamoreaux et al. (LANL group)

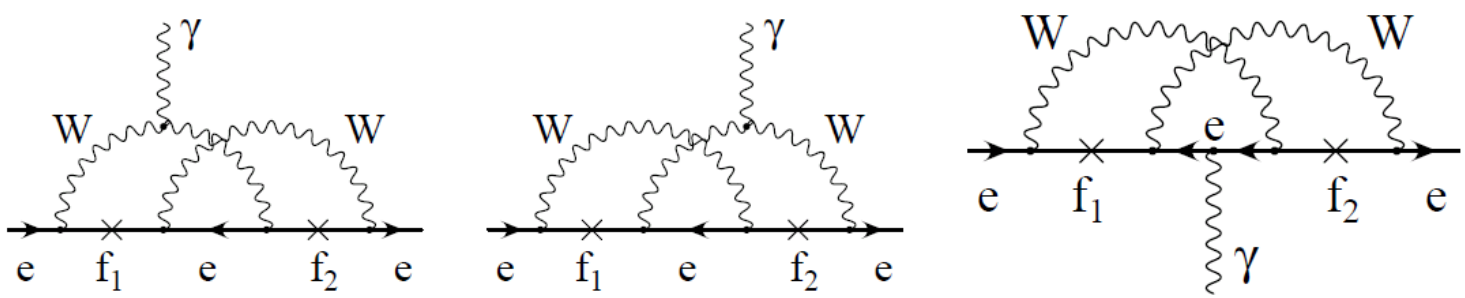


This effect is lepton number (and family lepton number) conserving, thus it could already appear in the Standard Model

- It does indeed! The CKM phase in the quark sector induces a lepton EDM at the four loop level

$$d_e \sim 10^{-38} \text{ e cm}$$

- If neutrinos are Majorana particles



$$d_e \sim \mathcal{O}(m_e m_\nu^2 G_F^2) \sim 10^{-48} \text{ e cm}$$

- Sensitivity of the electron EDM to new physics

$$d_e \sim e \times \frac{m_e}{\Lambda^2} \sim 10^{-23} \text{ e cm} \times \left(\frac{1 \text{ TeV}}{\Lambda}\right)^2 \quad \Rightarrow \Lambda \gtrsim 70 \text{ TeV} \quad (\text{for } \mathcal{O}(1) \text{ phases})$$

Implications for Physics BSM

DRAMATIC! Many extensions of the Standard Model postulate new particles at the electroweak scale (hierarchy problem, "WIMP miracle", cosmic ray anomalies...)

Recall: the present bound on BR($\mu\to e\gamma$) requires

$$\Lambda \gtrsim 20\text{TeV} \quad \text{if} \quad \theta_{\mu e} \sim \frac{1}{\sqrt{2}}$$

$$\theta_{\mu e} \lesssim 0.01 \quad \text{if} \quad \Lambda \sim 300\text{GeV}$$

The present bound on the electron EDM requires

$$d_e \sim e \times \frac{m_e}{\Lambda^2} \sim 10^{-23}\ \text{e cm} \times \left(\frac{1\ \text{TeV}}{\Lambda}\right)^2 \Rightarrow \Lambda \gtrsim 70\ \text{TeV} \quad \left(\text{for } \mathcal{O}(1) \text{ phases}\right)$$

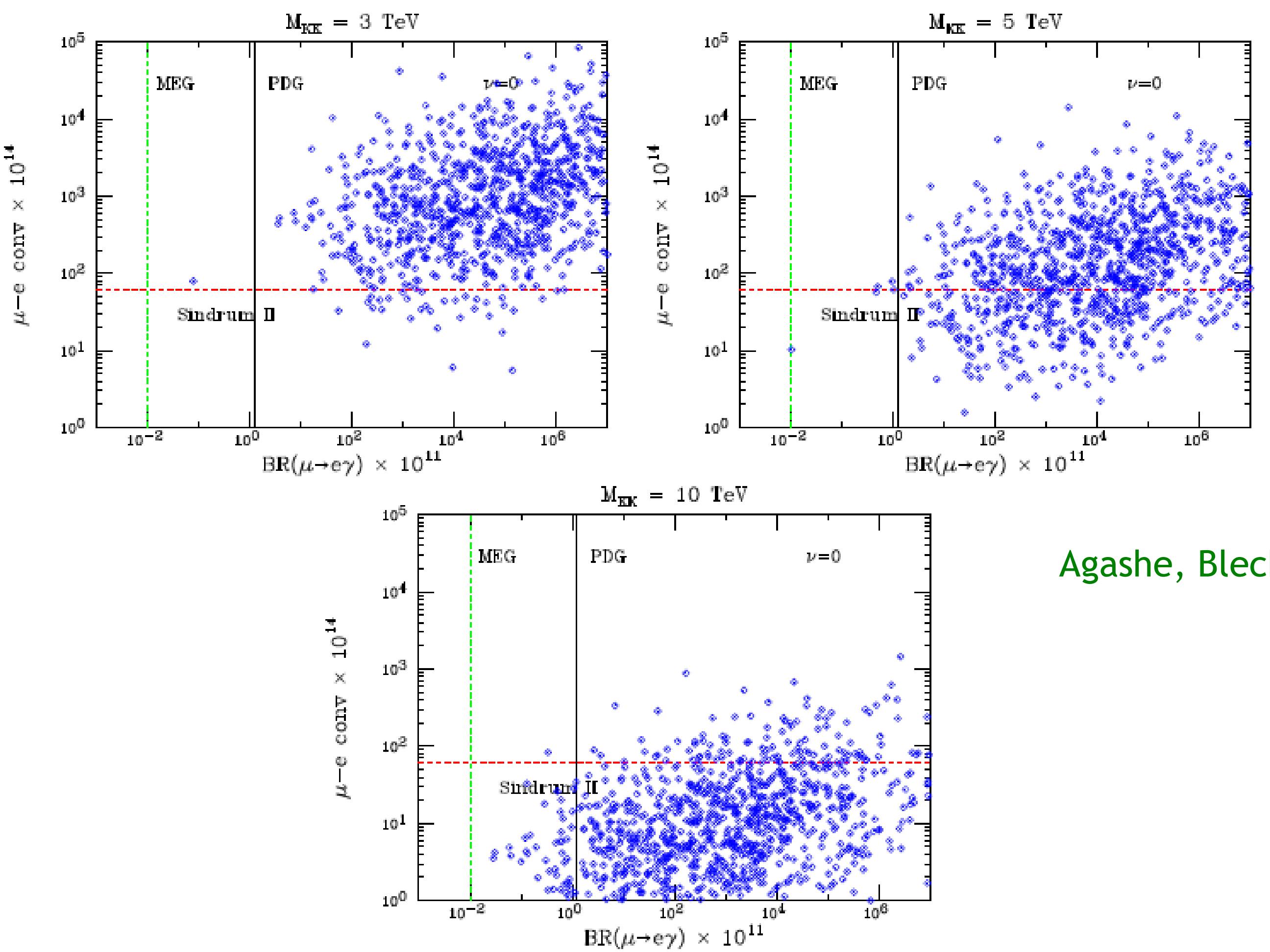
Very stringent constraints on models. Or on the positive side, detection might be around the corner.

Implications for Physics BSM

- Models with Extra dimensions
- Little Higgs models (with T-parity)
- Supersymmetry
- See-saw models

Implications for Physics BSM

- Models with Extra dimensions

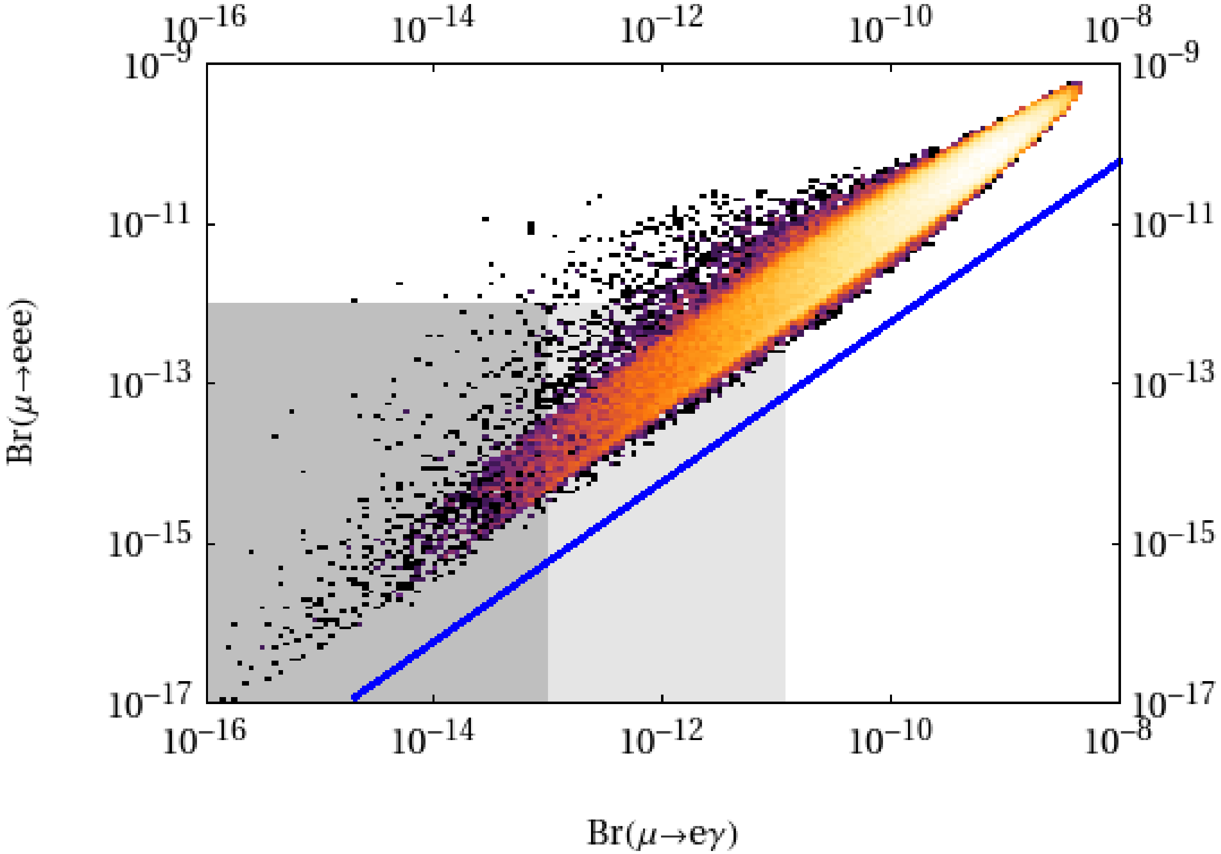


Agashe, Blechman, Petriello

“Anarchic” Randall-Sundrum model

Implications for Physics BSM

- Little Higgs models (with T-parity)



Blanke et al.

Mirror lepton masses between 300 GeV-1.5 TeV
Generic angles and phases

Implications for Physics BSM

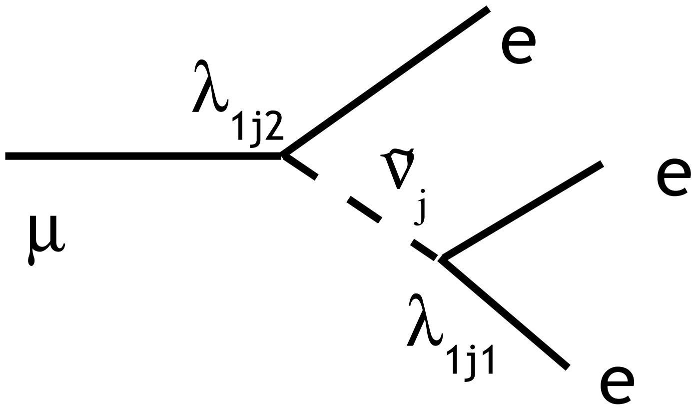
- Models with Extra dimensions
- Little Higgs models
- Supersymmetry
- See-saw models

Supersymmetry

Many attractive features. However, SUSY has in flavour and CP its Achiles' heel. Even the minimal model introduces many new sources of flavour and CP violation.

1- Flavour and CP are badly violated at the renormalizable level:

$$W_{MSSM} = \mathbf{Y}^e_{ij} e^c_{Ri} L_j H_d + \mathbf{Y}^d_{ij} d^c_{Ri} Q_j H_d + \mathbf{Y}^u_{ij} u^c_{Ri} Q_j H_u + \mu H_u H_d + \frac{1}{2}\lambda_{ijk} L_i L_j e^c_k + \lambda'_{ijk} L_i Q_j d^c_k + \frac{1}{2}\lambda''_{ijk} u^c_i d^c_j d^c_k + \mu'_i L_i H_u.$$



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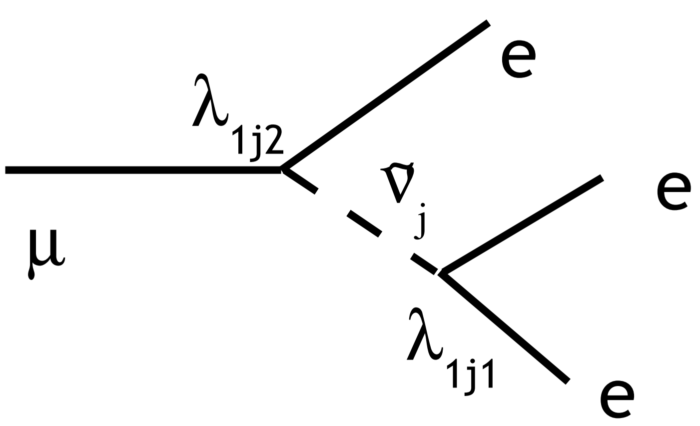
$$\frac{1}{2}\lambda_{ijk} L_i L_j e^c_k + \lambda'_{ijk} L_i Q_j d^c_k + \frac{1}{2}\lambda''_{ijk} u^c_i d^c_j d^c_k + \mu'_i L_i H_u.$$



$\lvert\lambda_{1j1}\lambda_{1j2}\rvert < 7 \times 10^{-7}$	From $\mu \to 3e$
$\lvert\lambda_{231}\lambda_{131}\rvert < 7 \times 10^{-7}$	From $\mu \to 3e$
$\lvert\lambda_{231}\lambda_{232}\rvert < 5.3 \times 10^{-6}$	From μTi $\to e$Ti at one loop
$\lvert\lambda_{232}\lambda_{132}\rvert < 8.4 \times 10^{-6}$	From μTi $\to e$Ti at one loop
$\lvert\lambda_{233}\lambda_{133}\rvert < 1.7 \times 10^{-5}$	From μTi $\to e$Ti at one loop
$\lvert\lambda_{122}\lambda'_{211}\rvert < 4.0 \times 10^{-8}$	From μTi $\to e$Ti at tree level
$\lvert\lambda_{132}\lambda'_{311}\rvert < 4.0 \times 10^{-8}$	From μTi $\to e$Ti at tree level
$\lvert\lambda_{121}\lambda'_{111}\rvert < 4.0 \times 10^{-8}$	From μTi $\to e$Ti at tree level
$\lvert\lambda_{231}\lambda'_{311}\rvert < 4.0 \times 10^{-8}$	From μTi $\to e$Ti at tree level

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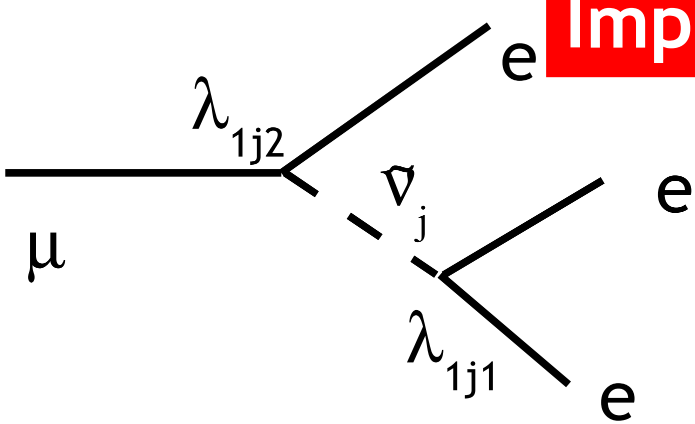
$$W_{MSSM} = \mathbf{Y}^e_{ij} e^c_{Ri} L_j H_d + \mathbf{Y}^d_{ij} d^c_{Ri} Q_j H_d + \mathbf{Y}^u_{ij} u^c_{Ri} Q_j H_u + \mu H_u H_d +$$
$$\frac{1}{2}\lambda_{ijk} L_i L_j e^c_k + \lambda'_{ijk} L_i Q_j d^c_k + \frac{1}{2}\lambda''_{ijk} u^c_i d^c_j d^c_k + \mu'_i L_i H_u .$$

Impose R-parity conservation



$\|\lambda_{231}\lambda_{232}\|$	$< 5.3 \times 10^{-6}$	From μTi $\rightarrow e$Ti at one loop
$\|\lambda_{232}\lambda_{132}\|$	$< 8.4 \times 10^{-6}$	From μTi $\rightarrow e$Ti at one loop
$\|\lambda_{233}\lambda_{133}\|$	$< 1.7 \times 10^{-5}$	From μTi $\rightarrow e$Ti at one loop
$\|\lambda_{122}\lambda'_{211}\|$	$< 4.0 \times 10^{-8}$	From μTi $\rightarrow e$Ti at tree level
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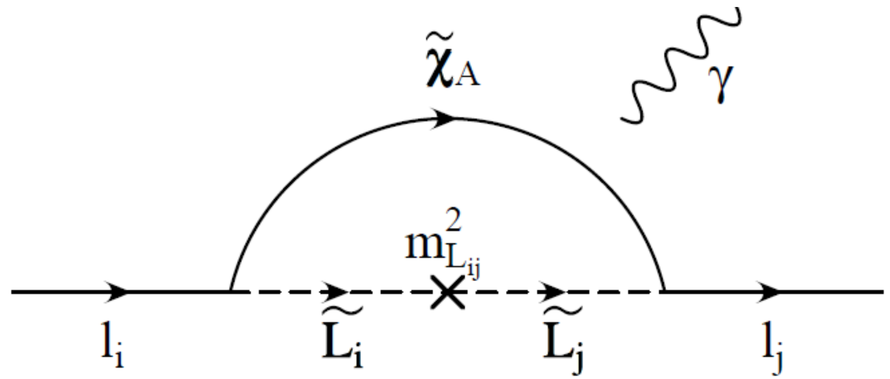
$\|\lambda_{231}\lambda'_{311}\|$	$< 4.0 \times 10^{-8}$	From μTi $\rightarrow e$Ti at tree level

2- Soft SUSY breaking terms in general violate flavour

$$-\mathcal{L}_{\rm soft}^{\rm lep} = (\mathbf{m}_L^2)_{ij}\widetilde{L}_i^*\widetilde{L}_j + (\mathbf{m}_e^2)_{ij}\widetilde{e}_{Ri}^*\widetilde{e}_{Rj} + \left(\mathbf{A}_{eij}\widetilde{e}_{Ri}^*\widetilde{L}_j H_d + \text{h.c.}\right)$$



Many possibilities for the origin of the lepton flavour violation:
LL, RR, RL, LR

Very stringent constraints from the non-observation of $l_i \to l_j \gamma$

e.g.

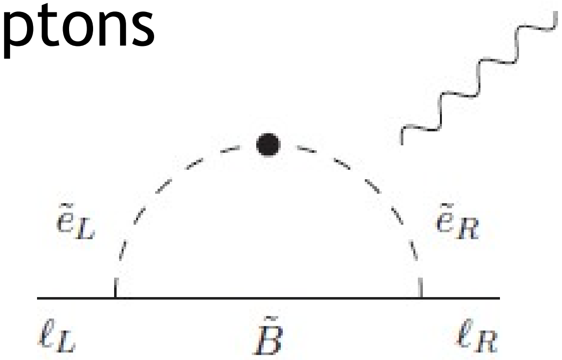
$$(\mathbf{m}_L^2)_{12}/m_S^2 < 3\times 10^{-4}$$
$$(\mathbf{m}_L^2)_{13}/m_S^2 < 0.09$$
$$(\mathbf{m}_L^2)_{23}/m_S^2 < 0.09$$

(for m_S=400GeV and tanβ=10)

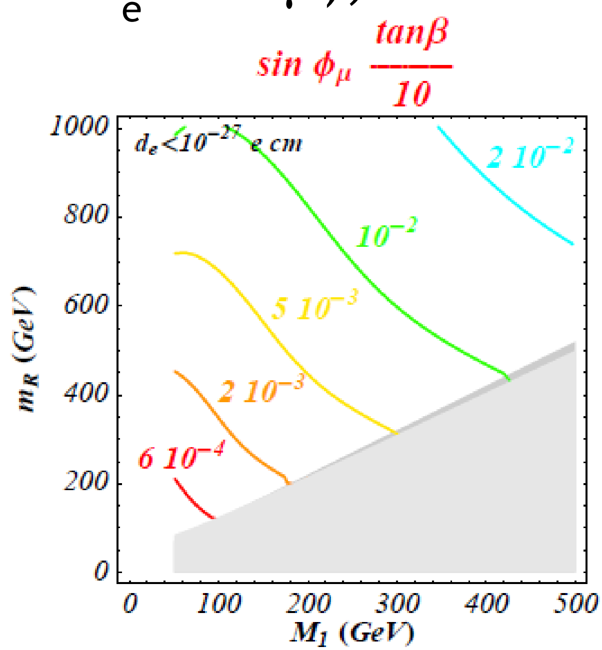
Possible explanation: messenger sector does not distinguish among flavours (gravity mediation, gauge mediation, gaugino mediation)

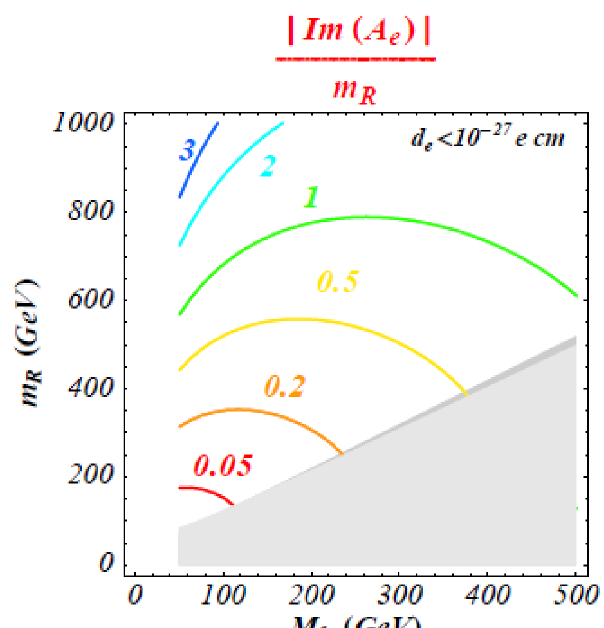
3- SUSY parameters introduce new sources of CP violation

The dominant contribution is a one loop diagram involving the bino and the sleptons



In SUSY scenarios (even with R-parity conserved and flavour blind messengers) there are at least two physical phases (the overall phases of A_e and μ), which contribute to the EDMs





Masina, Savoy

Correlations in SUSY scenarios

If SUSY exists in Nature, there are good chances to observe rare decays in the near future. But, how to tell that it is SUSY?

- Correlations between rare processes violating the same family number:

  $\mu\to e\gamma$, $\mu\to eee$, μ Ti$\to$e Ti

  $\tau\to\mu\gamma$, $\tau\to\mu ee$...

- Correlations between rare processes violating different family numbers

  Is there any model independent correlation between $\mu\to e\gamma$ and $\tau\to\mu\gamma$?

Correlations between processes violating the same flavours

Both processes are proportional to the same order parameter of the flavour symmetry breaking $\Rightarrow$ correlation

ratio	MSSM (dipole)	MSSM (Higgs)
$\frac{Br(\mu^-\to e^-e^+e^-)}{Br(\mu\to e\gamma)}$	$\sim 6\cdot 10^{-3}$	$\sim 6\cdot 10^{-3}$
$\frac{Br(\tau^-\to e^-e^+e^-)}{Br(\tau\to e\gamma)}$	$\sim 1\cdot 10^{-2}$	$\sim 1\cdot 10^{-2}$
$\frac{Br(\tau^-\to \mu^-\mu^+\mu^-)}{Br(\tau\to \mu\gamma)}$	$\sim 2\cdot 10^{-3}$	$0.06\ldots0.1$
$\frac{Br(\tau^-\to e^-\mu^+\mu^-)}{Br(\tau\to e\gamma)}$	$\sim 2\cdot 10^{-3}$	$0.02\ldots0.04$
$\frac{Br(\tau^-\to \mu^-e^+e^-)}{Br(\tau\to \mu\gamma)}$	$\sim 1\cdot 10^{-2}$	$\sim 1\cdot 10^{-2}$
$\frac{Br(\tau^-\to e^-e^+e^-)}{Br(\tau^-\to e^-\mu^+\mu^-)}$	~ 5	$0.3\ldots0.5$
$\frac{Br(\tau^-\to \mu^-\mu^+\mu^-)}{Br(\tau^-\to \mu^-e^+e^-)}$	~ 0.2	$5\ldots10$
$\frac{R(\mu\mathrm{Ti}\to e\mathrm{Ti})}{Br(\mu\to e\gamma)}$	$\sim 5\cdot 10^{-3}$	$0.08\ldots0.15$

Blanke et al.

Correlations between processes violating the same flavours

Both processes are proportional to the same order parameter of the flavour symmetry breaking ⇒ correlation

ratio	MSSM (dipole)	MSSM (Higgs)	LHT
$\frac{Br(\mu^-\to e^-e^+e^-)}{Br(\mu\to e\gamma)}$	$\sim 6\cdot 10^{-3}$	$\sim 6\cdot 10^{-3}$	$0.02\ldots 1$
$\frac{Br(\tau^-\to e^-e^+e^-)}{Br(\tau\to e\gamma)}$	$\sim 1\cdot 10^{-2}$	$\sim 1\cdot 10^{-2}$	$0.04\ldots 0.4$
$\frac{Br(\tau^-\to \mu^-\mu^+\mu^-)}{Br(\tau\to \mu\gamma)}$	$\sim 2\cdot 10^{-3}$	$0.06\ldots 0.1$	$0.04\ldots 0.4$
$\frac{Br(\tau^-\to e^-\mu^+\mu^-)}{Br(\tau\to e\gamma)}$	$\sim 2\cdot 10^{-3}$	$0.02\ldots 0.04$	$0.04\ldots 0.3$
$\frac{Br(\tau^-\to \mu^-e^+e^-)}{Br(\tau\to \mu\gamma)}$	$\sim 1\cdot 10^{-2}$	$\sim 1\cdot 10^{-2}$	$0.04\ldots 0.3$
$\frac{Br(\tau^-\to e^-e^+e^-)}{Br(\tau^-\to e^-\mu^+\mu^-)}$	~ 5	$0.3\ldots 0.5$	$0.8\ldots 2.0$
$\frac{Br(\tau^-\to \mu^-\mu^+\mu^-)}{Br(\tau^-\to \mu^-e^+e^-)}$	~ 0.2	$5\ldots 10$	$0.7\ldots 1.6$
$\frac{R(\mu\mathrm{Ti}\to e\mathrm{Ti})}{Br(\mu\to e\gamma)}$	$\sim 5\cdot 10^{-3}$	$0.08\ldots 0.15$	$10^{-3}\ldots 10^{2}$

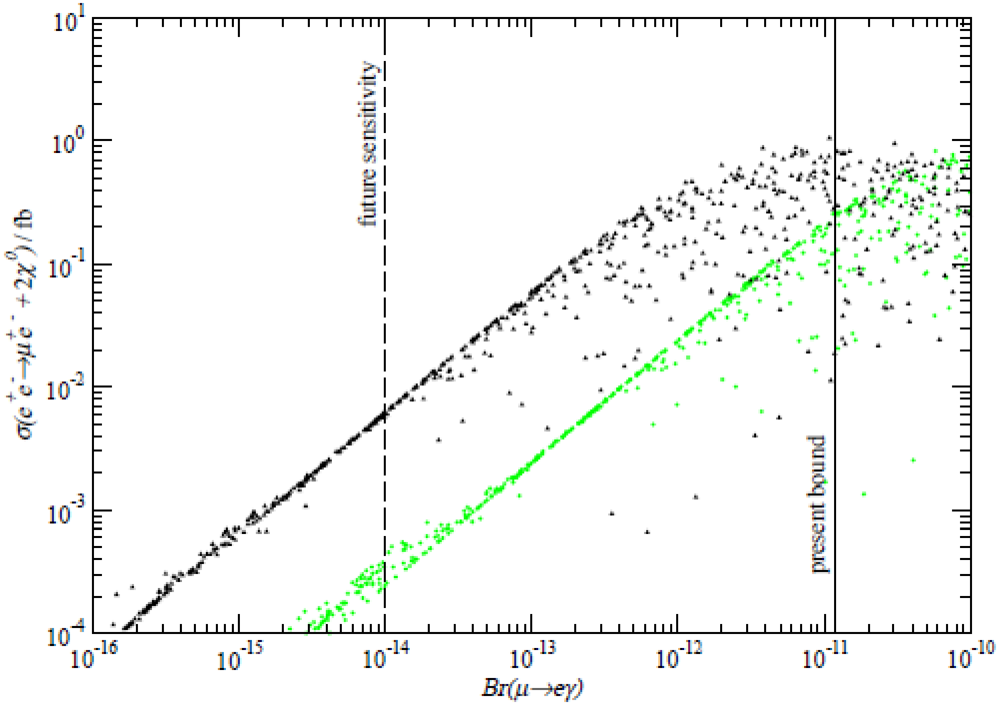
Blanke et al.

Possibility to distinguish among models

Correlations between processes violating the same flavours

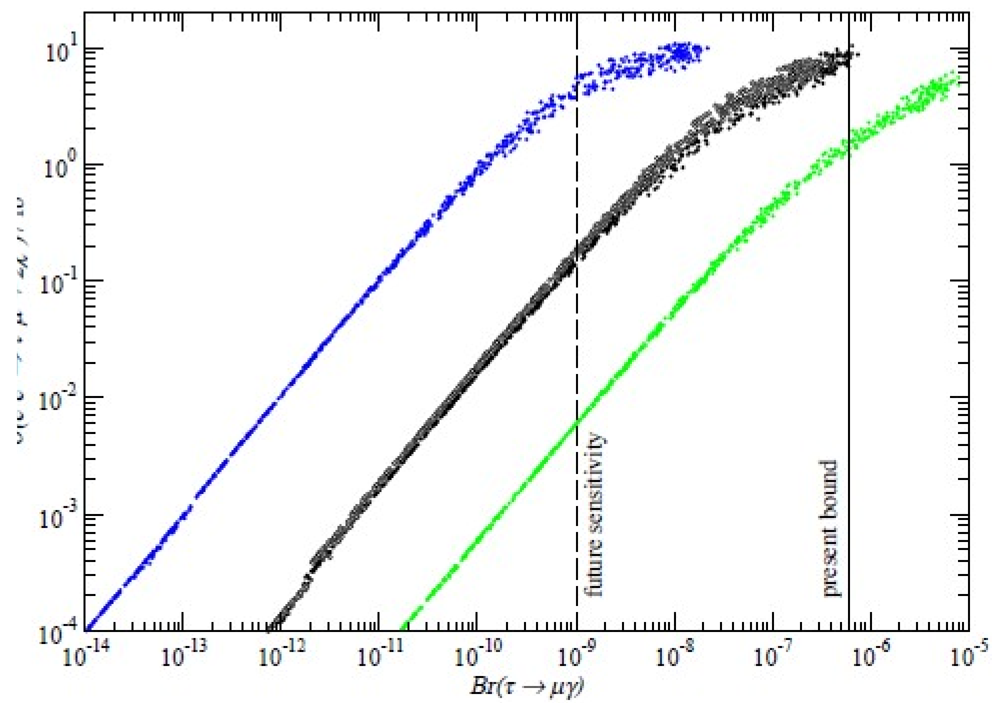
Correlations also at colliders, e.g.

$\sigma(e^+e^- \to \mu^+e^- + 2\,\chi^0)$ and BR($\mu \to e\gamma$)



Deppisch et al.

$\sigma(e^+e^- \to \tau^+\mu^- + 2\,\chi^0)$ and BR($\tau \to \mu\gamma$)



Correlations between processes violating different flavours

Less clear in a model independent way.

Rationale: If $\tau \to \mu\gamma$ is observed, the tau and muon family numbers are necessarily broken. However, the electron family symmetry might still be preserved. Then, the processes $\tau \to e\gamma$ and $\mu \to e\gamma$ might have vanishing rates.

Correlations between processes violating **different** flavours

Less clear in a model independent way.

Rationale: If $\tau \to \mu\gamma$ is observed, the tau and muon family numbers are necessarily broken. However, the electron family symmetry might still be preserved. Then, the processes $\tau \to e\gamma$ and $\mu \to e\gamma$ might have vanishing rates.

If $\tau \to \mu\gamma$ and $\tau \to e\gamma$ are both observed, *all* the family lepton numbers are broken. The rate for $\mu \to e\gamma$ is necessarily non-vanishing.

$$\mathrm{BR}(\mu \to e\gamma) \gtrsim C \times \mathrm{BR}(\tau \to \mu\gamma)\mathrm{BR}(\tau \to e\gamma)$$

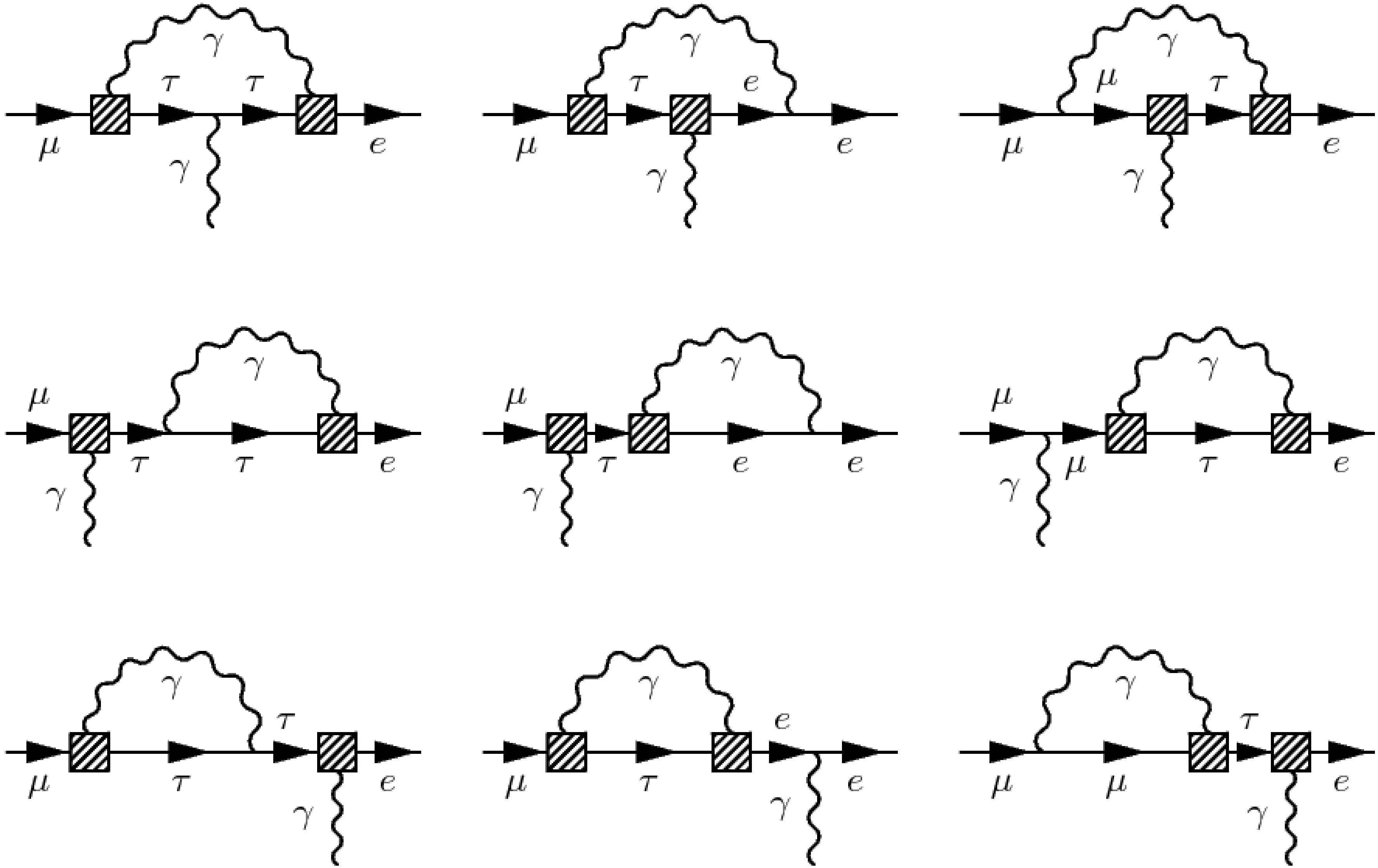
AI, Simonetto

A simple example

The Feynman diagrams for the dipole transitions $\tau \to l\gamma$ are

Even if the dipole transition $\mu \to e\gamma$ does not exist at tree level, there is no symmetry which forbids this transition. It will arise at the quantum level

$$BR(\mu \to e\gamma) \simeq C \times BR(\tau \to \mu\gamma) BR(\tau \to e\gamma)$$



In the effective theory approach, the calculation yields

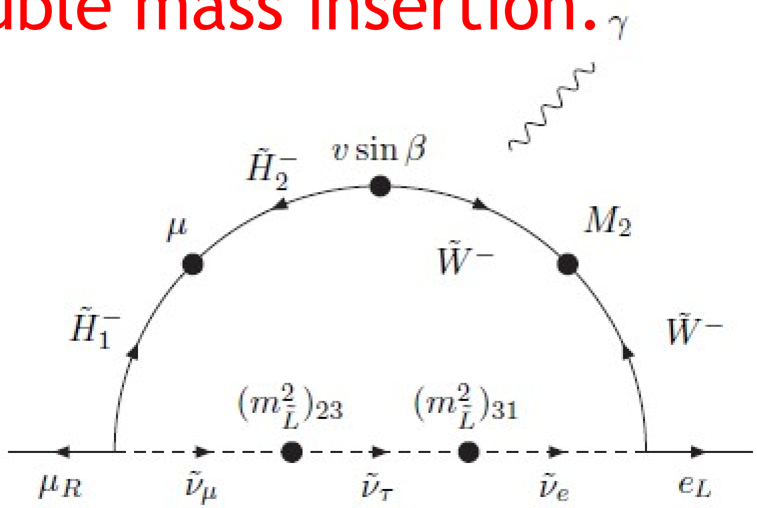
$$\mathrm{BR}(\mu \to e\gamma) \quad \gtrsim \quad 4 \times 10^{-23} \left(\frac{\mathrm{BR}(\tau \to \mu\gamma)}{4.5 \times 10^{-8}} \right) \left(\frac{\mathrm{BR}(\tau \to e\gamma)}{1.1 \times 10^{-7}} \right)$$

Not very useful in practice, but completely model independent

Minimal Supersymmetric Standard Model

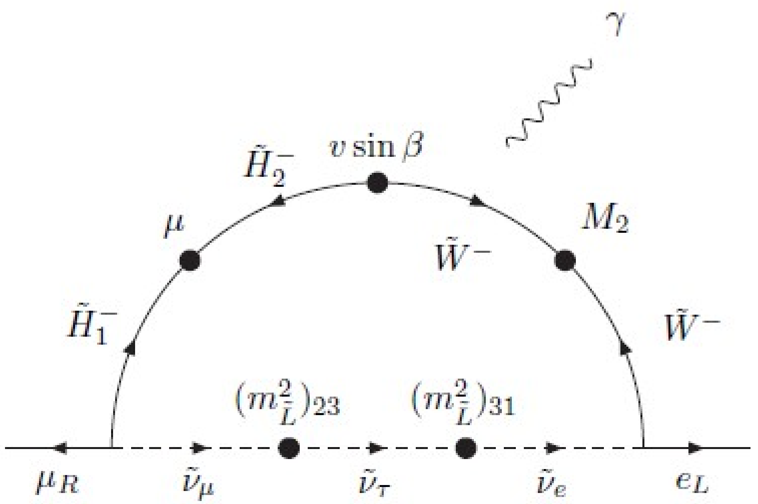
To derive correlations between the different rare processes, assume the worst case for the detection of $\mu \to e\gamma$, namely all $(m_L^2)_{12}$, $(m_e^2)_{12}$, A_{e12} are equal to zero.

Still, if the rates for $\tau \to \mu\gamma$ and $\tau \to e\gamma$ do not vanish, a non-vanishing rate for $\mu \to e\gamma$ will be generated at one loop via a double mass insertion.



Again, we find $\mathrm{BR}(\mu \to e\gamma) \gtrsim C \times \mathrm{BR}(\tau \to \mu\gamma)\mathrm{BR}(\tau \to e\gamma)$

The process $\mu \to e\gamma$ can be generated by 16 combinations of mass insertions:



LL for $\tau\mu$

LL for τe

But also LL-LR, RR-RL, RL-LR, etc

The 16 combinations can be classified in four classes (same dependence with $\tan\beta$, the fermion masses and the overall size of the scalar masses)

- Class I : LL-LL, RR-RR

$$\mathrm{BR}(\mu \to e\gamma) \gtrsim 9\times 10^{-10} \left(\frac{\widetilde{m}}{200\ \mathrm{GeV}}\right)^4 \left(\frac{\tan\beta}{10}\right)^{-2} \left(\frac{\mathrm{BR}(\tau \to \mu\gamma)}{4.5 \times 10^{-8}}\right) \left(\frac{\mathrm{BR}(\tau \to e\gamma)}{1.1 \times 10^{-7}}\right)$$

- Class II : LL-RR, RR-LL, LR-RR, RR-LR, RL-LL, LL-RL

$$\mathrm{BR}(\mu \to e\gamma) \gtrsim 3\times 10^{-7} \left(\frac{\widetilde{m}}{200\ \mathrm{GeV}}\right)^4 \left(\frac{\tan\beta}{10}\right)^{-2} \left(\frac{\mathrm{BR}(\tau \to \mu\gamma)}{4.5 \times 10^{-8}}\right) \left(\frac{\mathrm{BR}(\tau \to e\gamma)}{1.1 \times 10^{-7}}\right)$$

- Class III : LL-LR, LR-LL, RR-RL, RL-RR, LR-LR, RL-RL

$$\mathrm{BR}(\mu \to e\gamma) \gtrsim 5 \times 10^{-14} \left(\frac{\tan\beta}{10}\right)^{2} \left(\frac{\mathrm{BR}(\tau \to \mu\gamma)}{4.5 \times 10^{-8}}\right) \left(\frac{\mathrm{BR}(\tau \to e\gamma)}{1.1 \times 10^{-7}}\right)$$

- Class IV : LR-RL, RL-LR

$$\mathrm{BR}(\mu \to e\gamma) \gtrsim 2 \times 10^{-11} \left(\frac{\tan\beta}{10}\right)^{2} \left(\frac{\mathrm{BR}(\tau \to \mu\gamma)}{4.5 \times 10^{-8}}\right) \left(\frac{\mathrm{BR}(\tau \to e\gamma)}{1.1 \times 10^{-7}}\right)$$

Same origin for the LFV in the τ–μ and the τ-e sectors

- Class I : LL-LL, RR-RR

$$\mathrm{BR}(\mu \to e\gamma) \gtrsim 9\times 10^{-10} \left(\frac{\widetilde{m}}{200\,\mathrm{GeV}}\right)^4 \left(\frac{\tan\beta}{10}\right)^{-2} \left(\frac{\mathrm{BR}(\tau \to \mu\gamma)}{4.5\times 10^{-8}}\right) \left(\frac{\mathrm{BR}(\tau \to e\gamma)}{1.1\times 10^{-7}}\right)$$

- Class II : LL-RR, RR-LL, LR-RR, RR-LR, RL-LL, LL-RL

$$\mathrm{BR}(\mu \to e\gamma) \gtrsim 3\times 10^{-7} \left(\frac{\widetilde{m}}{200\,\mathrm{GeV}}\right)^4 \left(\frac{\tan\beta}{10}\right)^{-2} \left(\frac{\mathrm{BR}(\tau \to \mu\gamma)}{4.5\times 10^{-8}}\right) \left(\frac{\mathrm{BR}(\tau \to e\gamma)}{1.1\times 10^{-7}}\right)$$

- Class III : LL-LR, LR-LL, RR-RL, RL-RR, LR-LR, RL-RL

$$\mathrm{BR}(\mu \to e\gamma) \gtrsim 5\times 10^{-14} \left(\frac{\tan\beta}{10}\right)^{2} \left(\frac{\mathrm{BR}(\tau \to \mu\gamma)}{4.5\times 10^{-8}}\right) \left(\frac{\mathrm{BR}(\tau \to e\gamma)}{1.1\times 10^{-7}}\right)$$

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- Class I : LL-LL, RR-RR

See-saw mechanism
SU(5) GUTs

$$\mathrm{BR}(\mu \to e\gamma) \gtrsim 9\times 10^{-10} \left(\frac{\widetilde{m}}{200\ \mathrm{GeV}}\right)^4 \left(\frac{\tan\beta}{10}\right)^{-2} \left(\frac{\mathrm{BR}(\tau \to \mu\gamma)}{4.5\times 10^{-8}}\right) \left(\frac{\mathrm{BR}(\tau \to e\gamma)}{1.1\times 10^{-7}}\right)$$

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- Class III : LL-LR, LR-LL, RR-RL, RL-RR, LR-LR, RL-RL

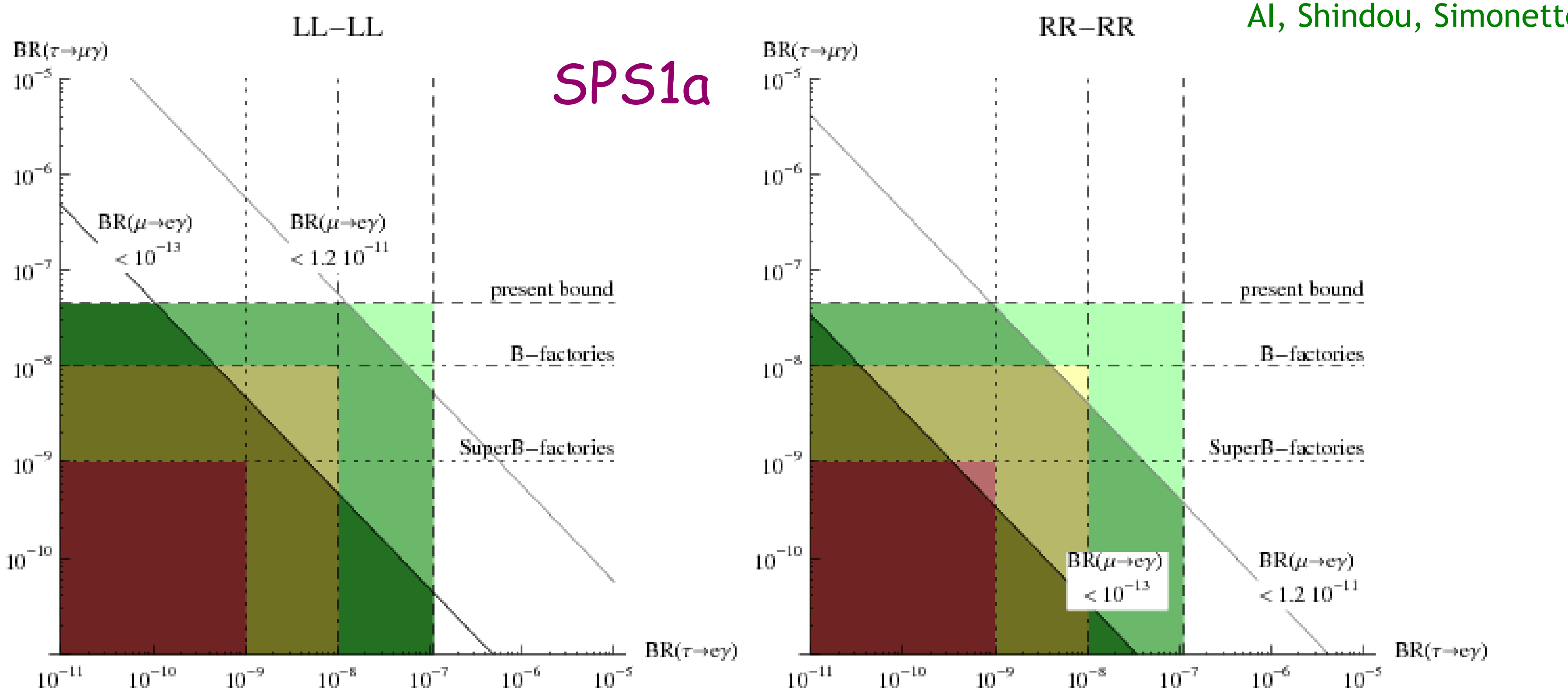
$$\mathrm{BR}(\mu \to e\gamma) \gtrsim 5\times 10^{-14} \left(\frac{\tan\beta}{10}\right)^{2} \left(\frac{\mathrm{BR}(\tau \to \mu\gamma)}{4.5\times 10^{-8}}\right) \left(\frac{\mathrm{BR}(\tau \to e\gamma)}{1.1\times 10^{-7}}\right)$$

- Class IV : LR-RL, RL-LR

$$\mathrm{BR}(\mu \to e\gamma) \gtrsim 2\times 10^{-11} \left(\frac{\tan\beta}{10}\right)^{2} \left(\frac{\mathrm{BR}(\tau \to \mu\gamma)}{4.5\times 10^{-8}}\right) \left(\frac{\mathrm{BR}(\tau \to e\gamma)}{1.1\times 10^{-7}}\right)$$

theoretical constraints on the rare tau decays in the MSSM

AI, Shindou, Simonetto



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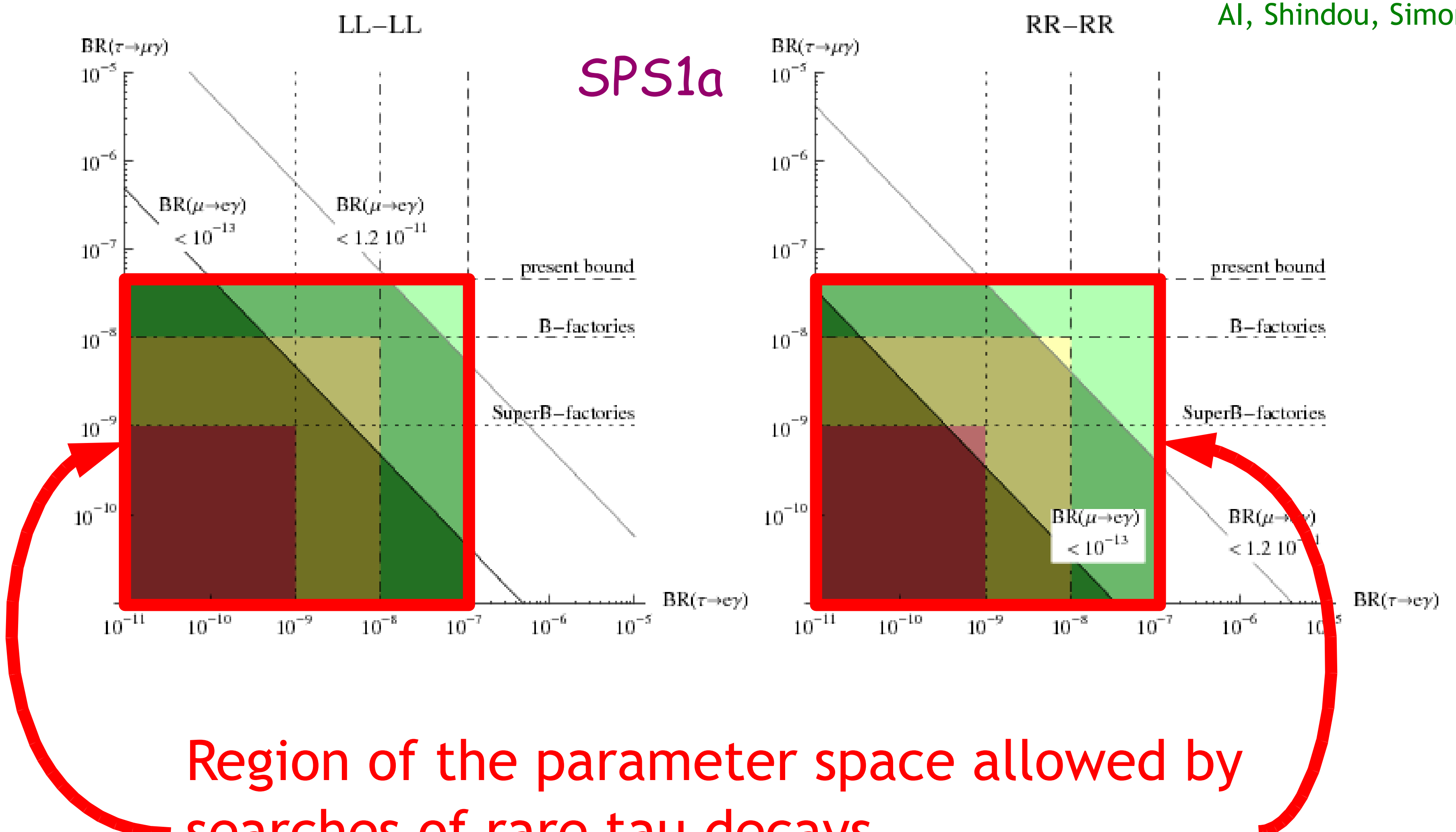
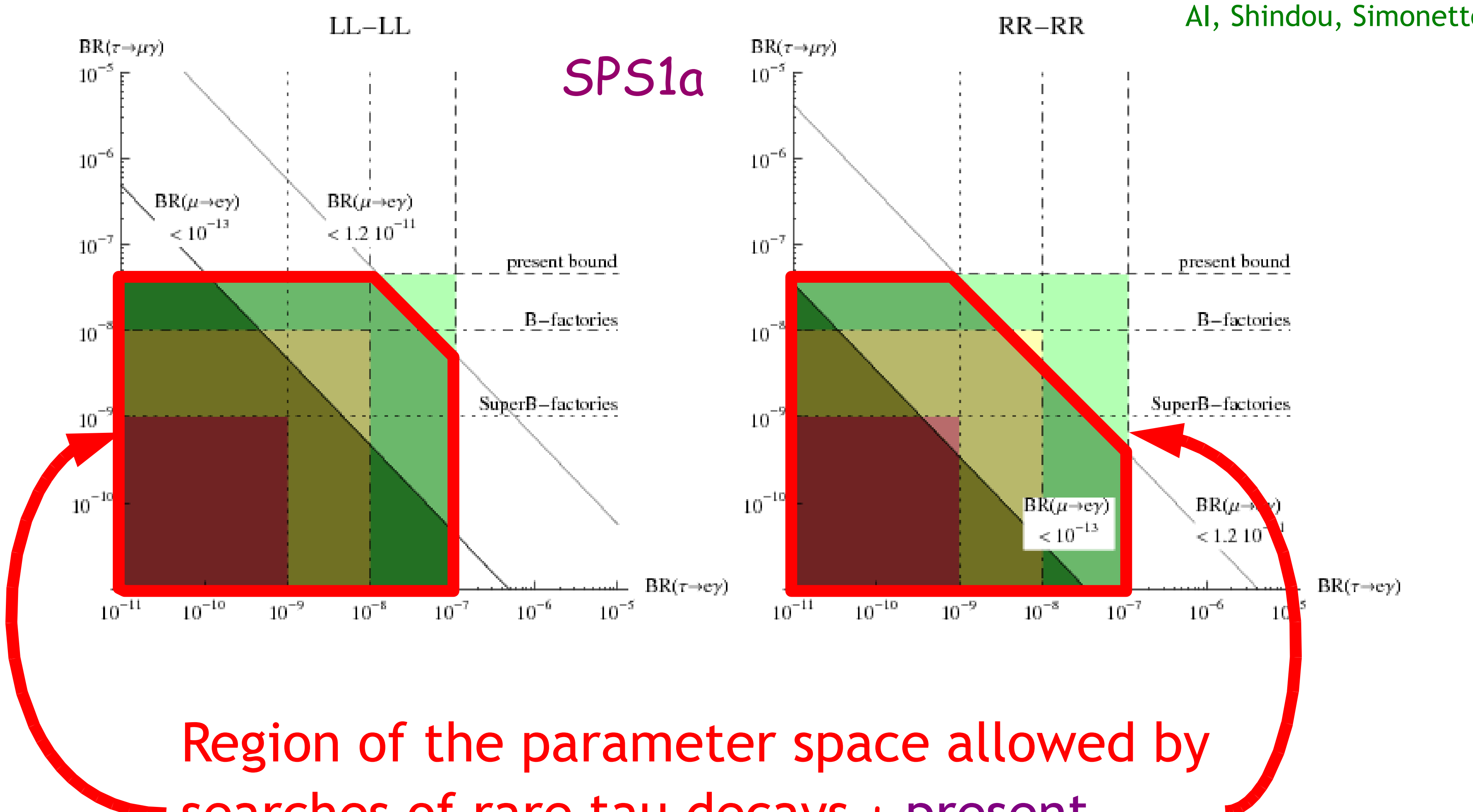


Region of the parameter space allowed by searches of rare tau decays

theoretical constraints on the rare tau decays in the MSSM

AI, Shindou, Simonetto

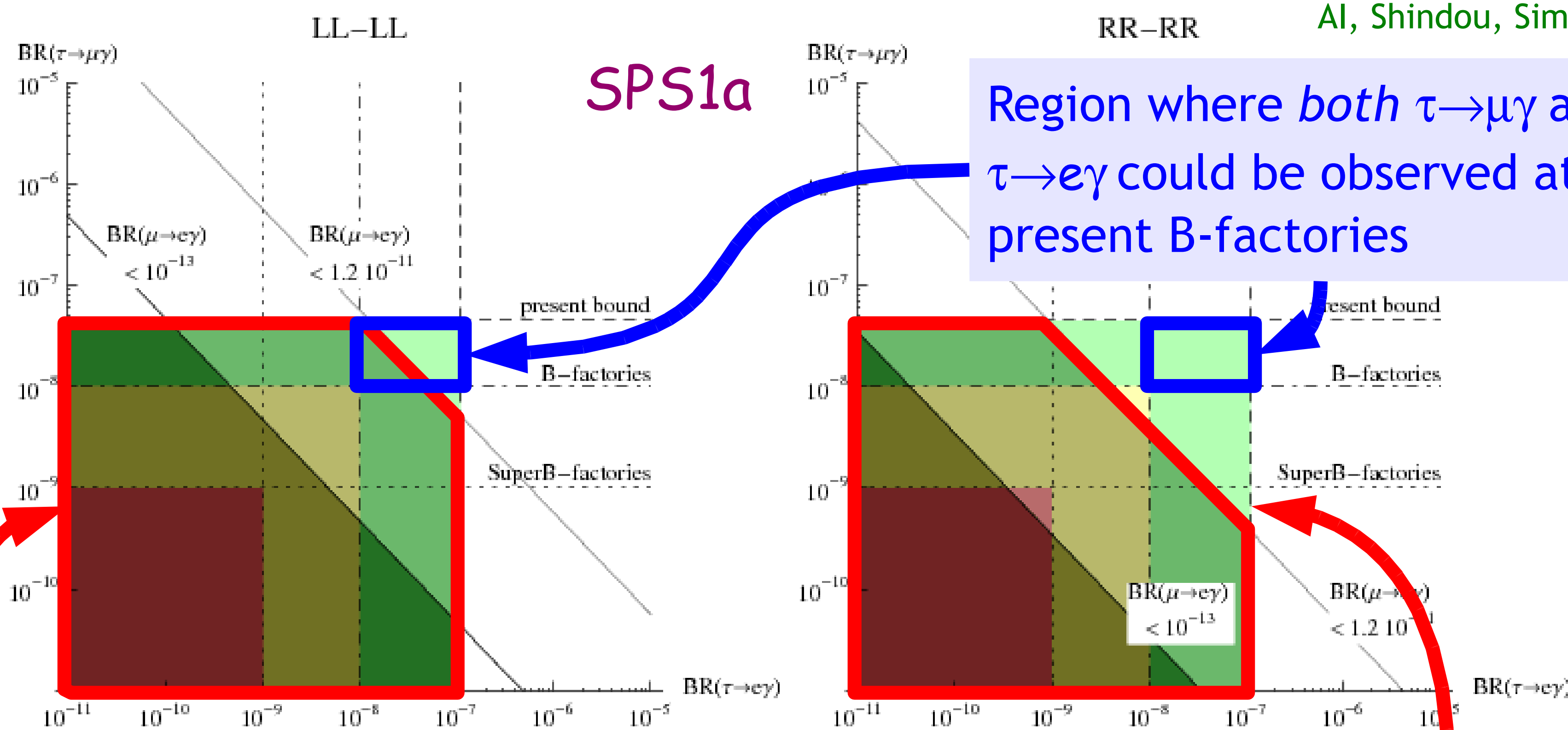


Region of the parameter space allowed by searches of rare tau decays + present bound on $\mu \to e\gamma$

theoretical constraints on the rare tau decays in the MSSM

Al, Shindou, Simonetto

SPS1a

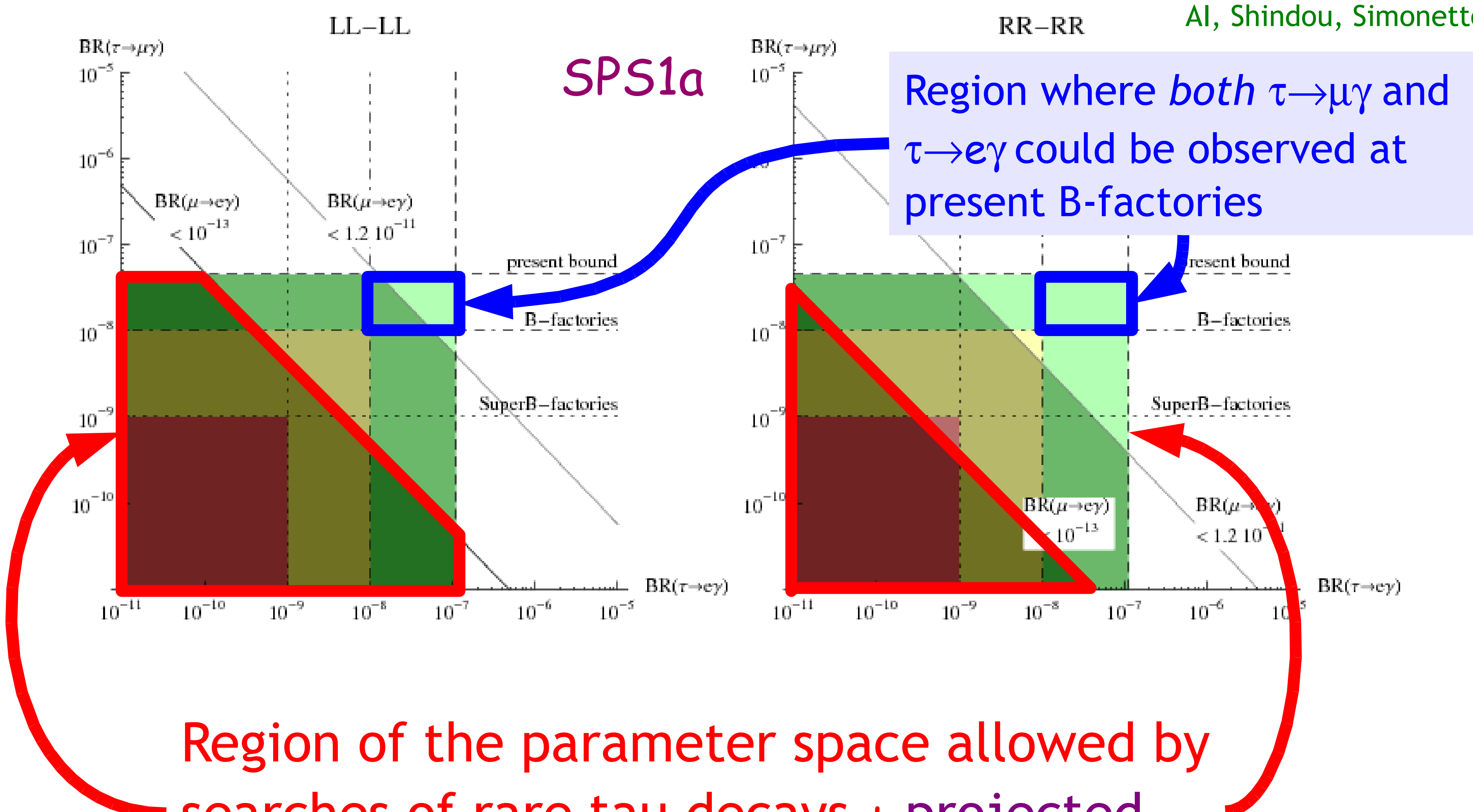


Region where *both* $\tau \to \mu\gamma$ and $\tau \to e\gamma$ could be observed at present B-factories

Region of the parameter space allowed by searches of rare tau decays + present bound on $\mu \to e\gamma$

theoretical constraints on the rare tau decays in the MSSM

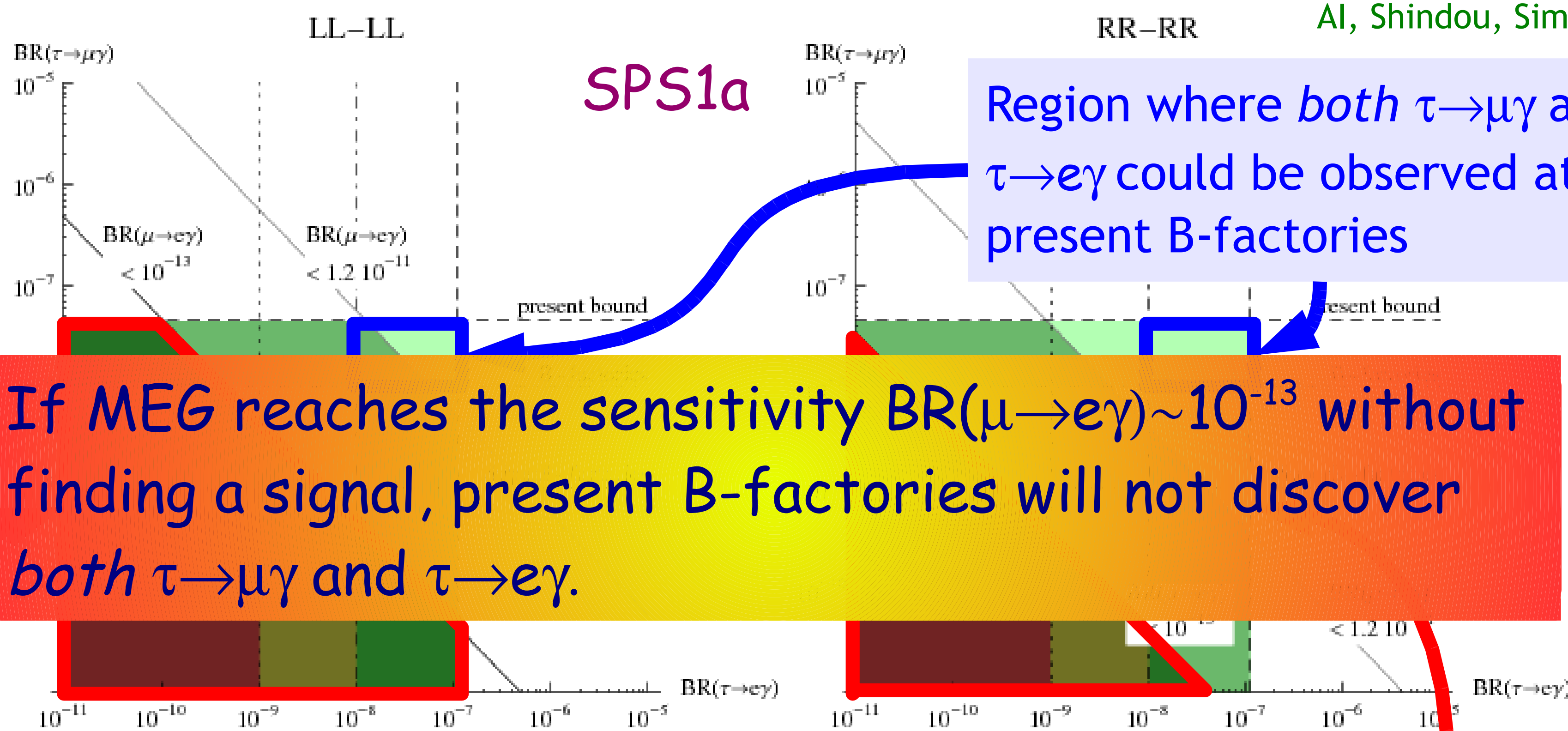
AI, Shindou, Simonetto



Region of the parameter space allowed by searches of rare tau decays + projected bound on $\mu \to e\gamma$ (BR<10^{-13})

theoretical constraints on the rare tau decays in the MSSM

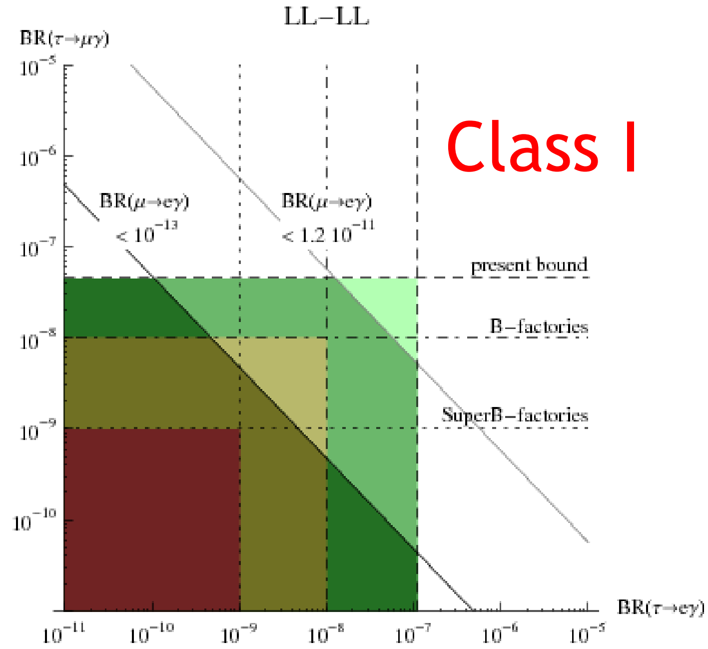
Al, Shindou, Simonetto

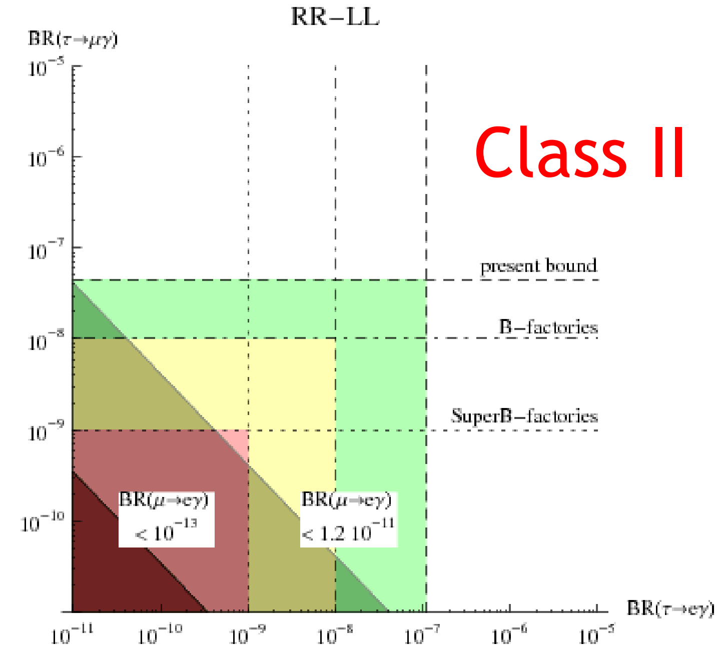


Region where *both* $\tau\to\mu\gamma$ and $\tau\to e\gamma$ could be observed at present B-factories

If MEG reaches the sensitivity BR($\mu\to e\gamma$)$\sim 10^{-13}$ without finding a signal, present B-factories will not discover *both* $\tau\to\mu\gamma$ and $\tau\to e\gamma$.

Region of the parameter space allowed by searches of rare tau decays + projected bound on $\mu\to e\gamma$ (BR<10^{-13})


LL–LL
Class I
BR($\tau\to\mu\gamma$)
BR($\tau\to e\gamma$)
BR($\mu\to e\gamma$) $< 10^{-13}$
BR($\mu\to e\gamma$) $< 1.2\ 10^{-11}$
present bound
B–factories
SuperB–factories
10^{-5} 10^{-6} 10^{-7} 10^{-8} 10^{-9} 10^{-10}
10^{-11} 10^{-10} 10^{-9} 10^{-8} 10^{-7} 10^{-6} 10^{-5}


RR–LL
Class II
BR($\tau\to\mu\gamma$)
BR($\tau\to e\gamma$)
BR($\mu\to e\gamma$) $< 10^{-13}$
BR($\mu\to e\gamma$) $< 1.2\ 10^{-11}$
present bound
B–factories
SuperB–factories
10^{-5} 10^{-6} 10^{-7} 10^{-8} 10^{-9} 10^{-10}
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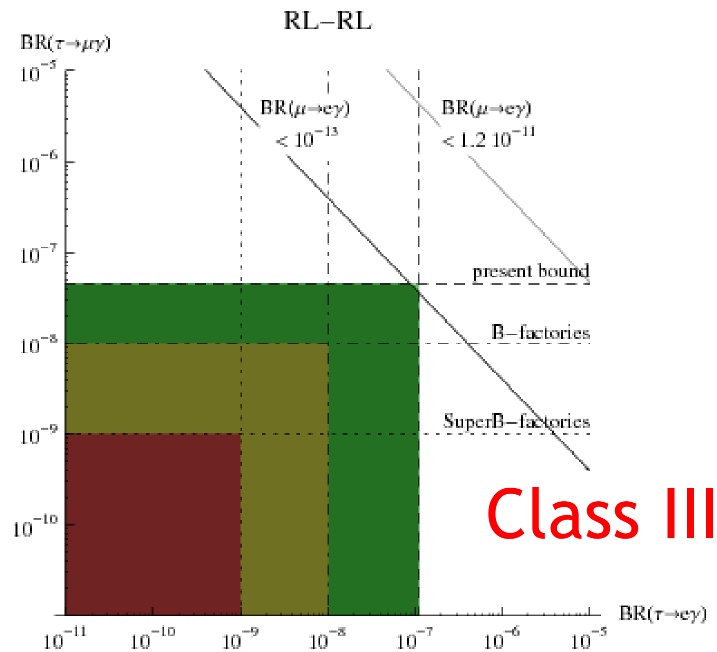
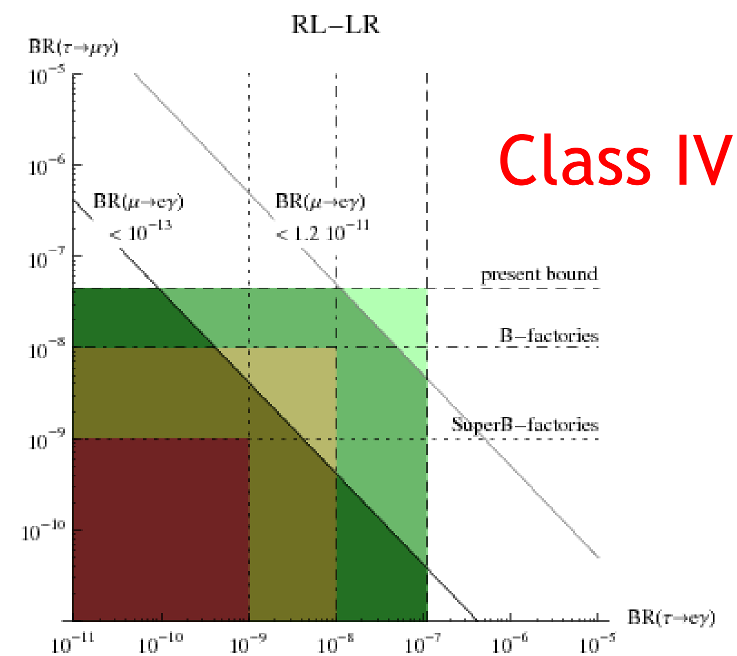
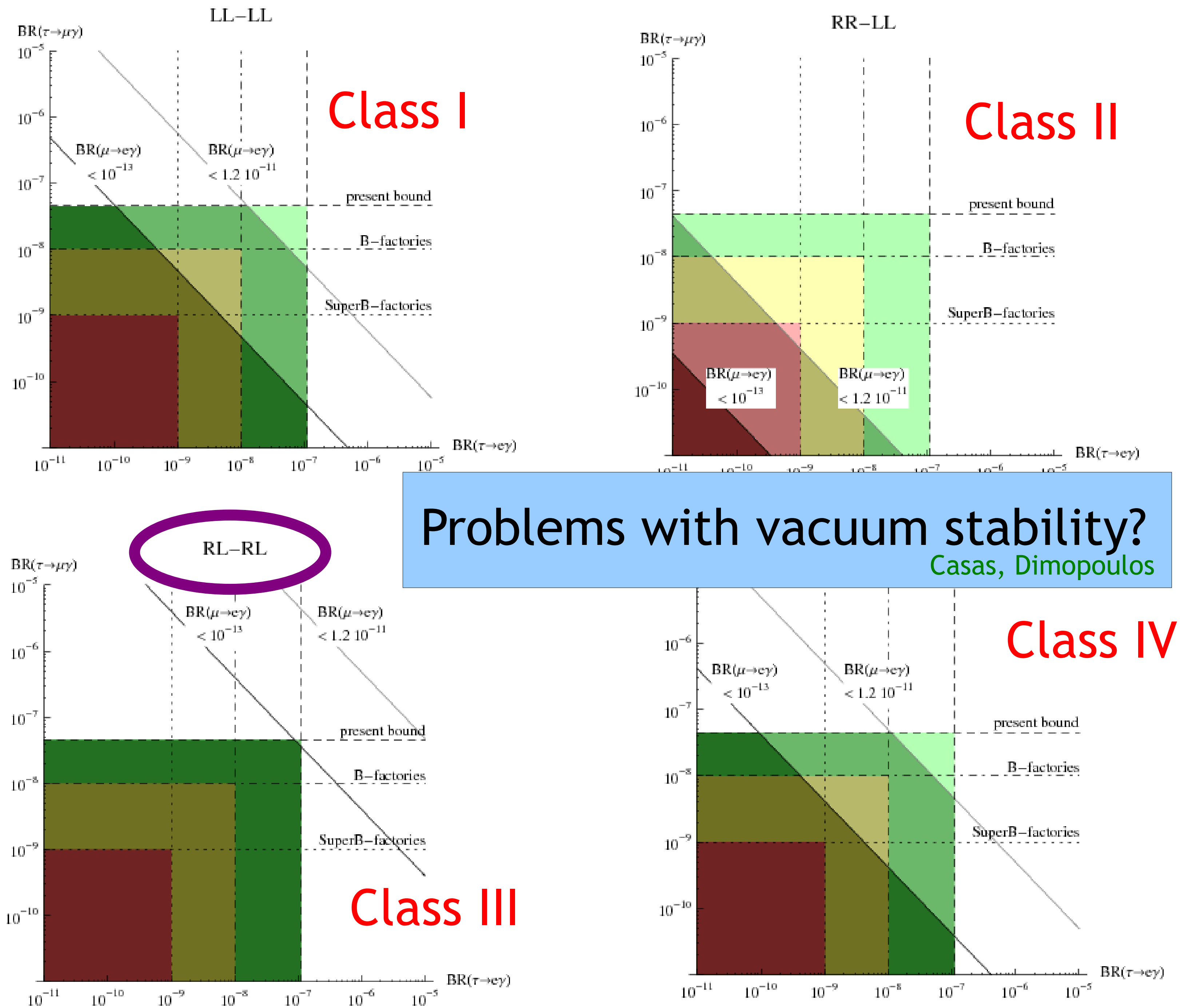

RL–RL
Class III
BR($\tau\to\mu\gamma$)
BR($\tau\to e\gamma$)
BR($\mu\to e\gamma$) $< 10^{-13}$
BR($\mu\to e\gamma$) $< 1.2\ 10^{-11}$
present bound
B–factories
SuperB–factories
10^{-5} 10^{-6} 10^{-7} 10^{-8} 10^{-9} 10^{-10}
10^{-11} 10^{-10} 10^{-9} 10^{-8} 10^{-7} 10^{-6} 10^{-5}

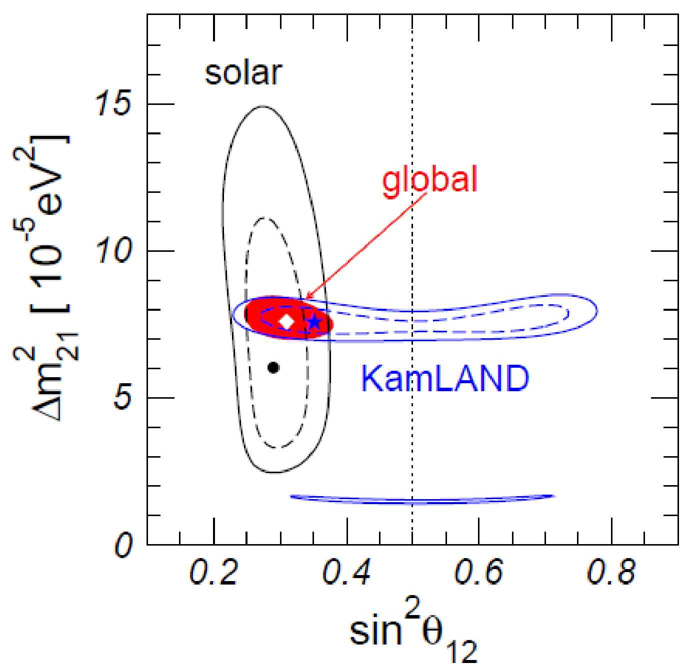

RL–LR
Class IV
BR($\tau\to\mu\gamma$)
BR($\tau\to e\gamma$)
BR($\mu\to e\gamma$) $< 10^{-13}$
BR($\mu\to e\gamma$) $< 1.2\ 10^{-11}$
present bound
B–factories
SuperB–factories
10^{-5} 10^{-6} 10^{-7} 10^{-8} 10^{-9} 10^{-10}
10^{-11} 10^{-10} 10^{-9} 10^{-8} 10^{-7} 10^{-6} 10^{-5}

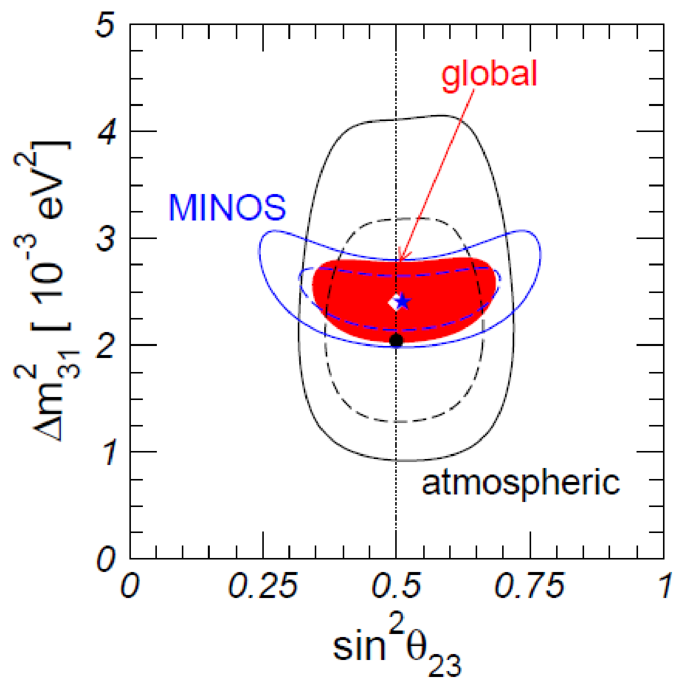


LL−LL
Class I
RR−LL
Class II
RL−RL
Class III
Class IV
BR(τ→μγ)
BR(τ→eγ)
BR(μ→eγ) < 10^−13
BR(μ→eγ) < 1.2 10^−11
present bound
B−factories
SuperB−factories
Problems with vacuum stability?
Casas, Dimopoulos

LFV and neutrino masses

Evidence of lepton flavour violation:

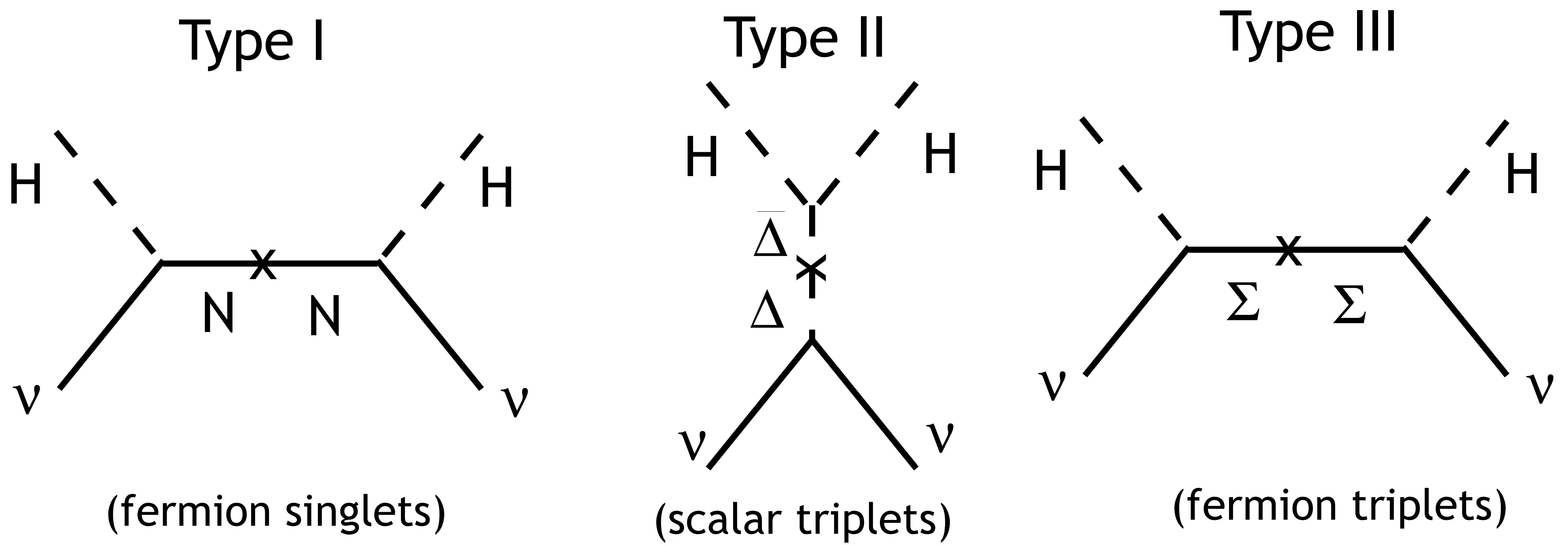




What are the implications for charged lepton flavour violation?

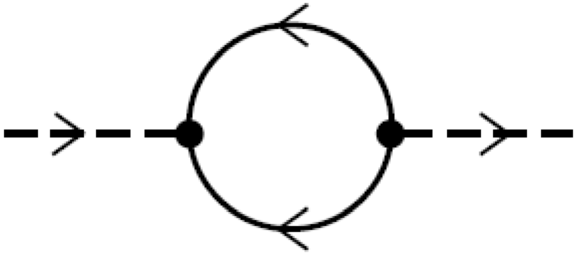
LFV in see-saw models

The smallness of neutrino masses can be very elegantly explained introducing new heavy degrees of freedom:



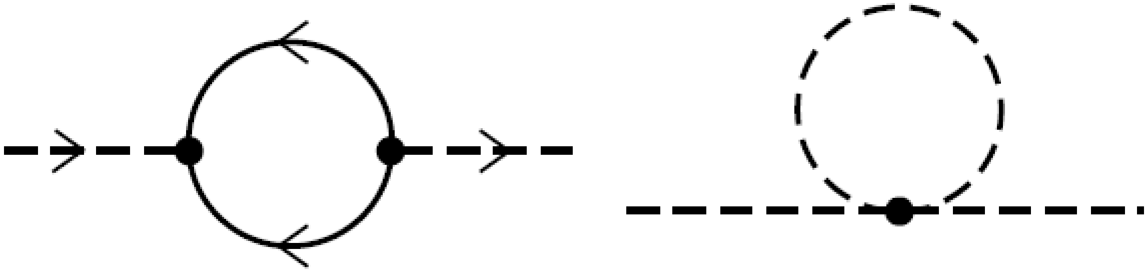
The new degrees of freedom induce LFV processes, with rates suppressed by the large mass scale of the new particles.
Good agreement with experiments, but the model is unnatural...

The Higgs doublet interacts with heavy degrees of freedom



$$\delta m_H^2 \sim \frac{1}{16\pi^2} Y^2 M^2$$

SUSY see-saw



$$\delta m_H^2 \sim \frac{1}{16\pi^2} Y^2 \log \frac{M^2}{\Lambda^2}$$

SUSY solves the hierarchy problem of the see-saw mechanism, but introduces yet another flavour problem...

Type I see-saw model

Consider the scenario with least number of new sources of LFV:

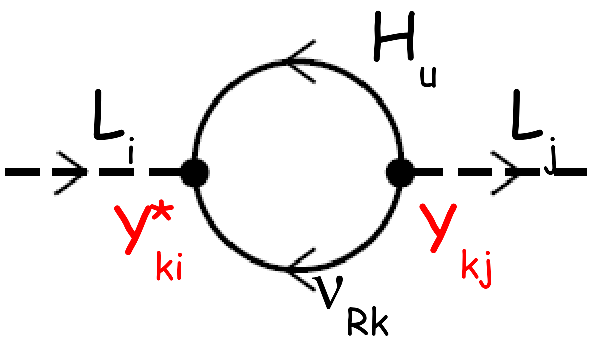
- R-parity conserved:

$$W_{\rm lep} = e^c_{Ri}\mathbf{Y}_{eij}L_jH_d + \nu^c_{Ri}\mathbf{Y}_{\nu ij}L_jH_u - \frac{1}{2}\nu^c_{Ri}\mathbf{M}_{ij}\nu^c_{Rj}$$

$$W^{\rm eff}_{\rm lep} = e^c_{Ri}\mathbf{Y}_{eij}L_jH_d + \frac{1}{2}\left(\mathbf{Y}_\nu^T\mathbf{M}^{-1}\mathbf{Y}_\nu\right)_{ij}(L_iH_u)(L_jH_u)$$

- Flavour blind mediation mechanism: no LFV in the soft terms at the cut-off scale.

If the particles responsible for neutrino masses are lighter than the mediation scale, quantum corrections will generate flavour violating terms in the slepton sector: Borzumati, Masiero



$$\left(\delta\mathbf{m}^2_L\right)_{ij} \simeq -\frac{1}{8\pi^2}(3m_0^2+|A_0|^2)(\mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu)_{ij}\log\left(\frac{\Lambda}{M_{\rm maj}}\right)$$

$$\left(\delta\mathbf{m}^2_e\right)_{ij} \simeq 0\ ,$$

$$(\delta\mathbf{A}_e)_{ij} \simeq \frac{-3}{8\pi^2}A_0\mathbf{Y}_e(\mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu)_{ij}\log\left(\frac{\Lambda}{M_{\rm maj}}\right)\ ,$$

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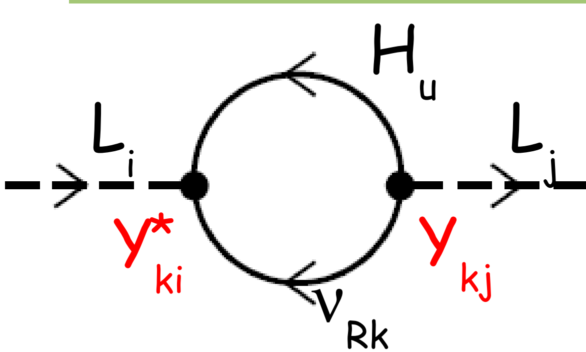

$$(\delta \mathbf{m}_L^2)_{ij} \simeq -\frac{1}{8\pi^2}(3m_0^2+|A_0|^2)(\mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu)_{ij}\log\left(\frac{\Lambda}{M_{\rm maj}}\right)$$

$$(\delta \mathbf{m}_e^2)_{ij} \simeq 0\,,$$

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Logarithmic dependence with M

The calculation of the rate is, however, full of uncertainties

Back of the envelope calculation of BR($l_i \to l_j \gamma$):

$$\mathrm{BR}(\ell_j \to \ell_i \gamma) \simeq \frac{\alpha^3}{G_F^2} \frac{|(\mathbf{m}_L^2)_{ij}|^2}{m_S^8} \tan^2\beta \, \mathrm{BR}(\ell_j \to \ell_i \nu_j \bar{\nu}_i)$$

$$(\mathbf{m}_L^2)_{ij} \simeq \mathbf{m}_L^2(\Lambda)_{ij} - \frac{1}{8\pi^2}(3m_0^2 + |A_0|^2)(\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu)_{ij} \log\left(\frac{\Lambda}{M_{\mathrm{maj}}}\right)$$

Cut-off scale?
Flavour structure of the soft terms at the cut-off scale?
soft-SUSY parameters?
tanβ?
Size and flavour structure of the Yukawa couplings?
Right-handed neutrino masses?

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Cut-off scale?

Flavour structure of the soft terms at the cut-off scale?

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Right-handed neutrino masses?

Connection LFV and neutrino masses

From neutrino oscillation experiments we know:

parameter	best fit	2σ	3σ
Δm^2_{21} [10^{-5}eV^2]	$7.65^{+0.23}_{-0.20}$	7.25–8.11	7.05–8.34
$\|\Delta m^2_{31}\|$ [10^{-3}eV^2]	$2.40^{+0.12}_{-0.11}$	2.18–2.64	2.07–2.75
$\sin^2\theta_{12}$	$0.304^{+0.022}_{-0.016}$	0.27–0.35	0.25–0.37
$\sin^2\theta_{23}$	$0.50^{+0.07}_{-0.06}$	0.39–0.63	0.36–0.67
$\sin^2\theta_{13}$	$0.01^{+0.016}_{-0.011}$	≤ 0.040	≤ 0.056

Is there a *model independent* connection between neutrino parameters and lepton flavour violation?

Connection LFV and neutrino masses

From neutrino oscillation experiments we know:

parameter	best fit	2σ	3σ
Δm^2_{21} $[10^{-5}\text{eV}^2]$	$7.65^{+0.23}_{-0.20}$	7.25–8.11	7.05–8.34
$\lvert\Delta m^2_{31}\rvert$ $[10^{-3}\text{eV}^2]$	$2.40^{+0.12}_{-0.11}$	2.18–2.64	2.07–2.75
$\sin^2\theta_{12}$	$0.304^{+0.022}_{-0.016}$	0.27–0.35	0.25–0.37
$\sin^2\theta_{23}$	$0.50^{+0.07}_{-0.06}$	0.39–0.63	0.36–0.67
$\sin^2\theta_{13}$	$0.01^{+0.016}_{-0.011}$	≤ 0.040	≤ 0.056

Is there a *model independent* connection between neutrino parameters and lepton flavour violation?

NO

The see-saw Lagrangian has 12+6 new parameters.
Neutrino observations at most can fix 6+3 parameters.
Still, there are 6+3 free parameters.

There are, compatible with the observed neutrino parameters, an infinite set of Yukawa couplings!

Casas, AI

Right-handed neutrino masses

$$\mathbf{Y}_\nu = \frac{1}{\langle H_u^0 \rangle} \sqrt{D_M}\, R\, \sqrt{D_m}\, U_{\rm lep}^\dagger$$

"Fixed" by experiments

$$R = \begin{pmatrix} \hat{c}_2\hat{c}_3 & -\hat{c}_1\hat{s}_3 - \hat{s}_1\hat{s}_2\hat{c}_3 & \hat{s}_1\hat{s}_3 - \hat{c}_1\hat{s}_2\hat{c}_3 \\ \hat{c}_2\hat{s}_3 & \hat{c}_1\hat{c}_3 - \hat{s}_1\hat{s}_2\hat{s}_3 & -\hat{s}_1\hat{c}_3 - \hat{c}_1\hat{s}_2\hat{s}_3 \\ \hat{s}_2 & \hat{s}_1\hat{c}_2 & \hat{c}_1\hat{c}_2 \end{pmatrix}$$

Changing R and the right-handed neutrino masses, any $Y^\dagger Y$ can be obtained.

In fact, there is a one-to-one correspondence between

$$\{Y,M\} \longleftrightarrow \{\mathcal{M},\, Y^\dagger Y\}$$

Davidson, AI

High-energy parameters of the see-saw Lagrangian

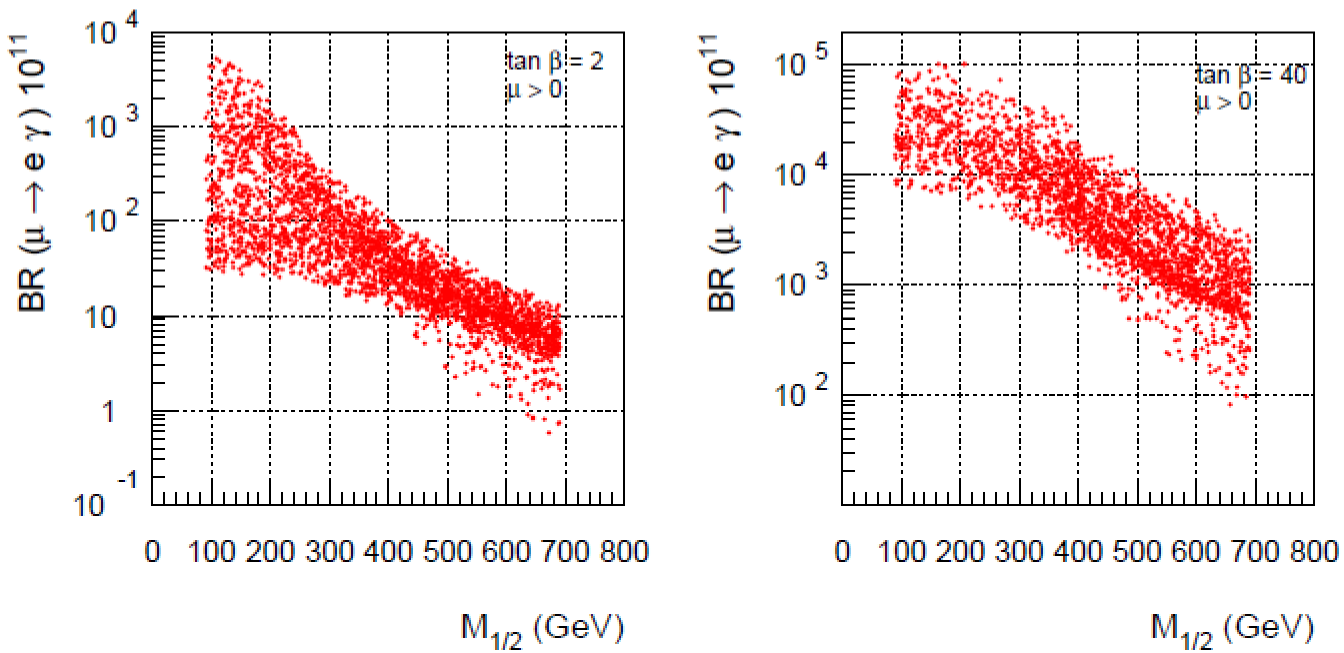
Low energy observables: neutrino mass matrix, BR($l_i \to l_j \gamma$), EDMs

From a *model independent* perspective, the type-I see-saw can accommodate anything at low energies!! **No predictions**

Top-down approach

Example 1: SO(10) inspired model. Mixing angles in the Yukawa couplings as the leptonic mixing matrix

$$(\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu)_{21} = y_t^2 U_{\mu 3} U_{e3} + y_c^2 U_{\mu 2} U_{e2} + \mathcal{O}(y_u^2)$$

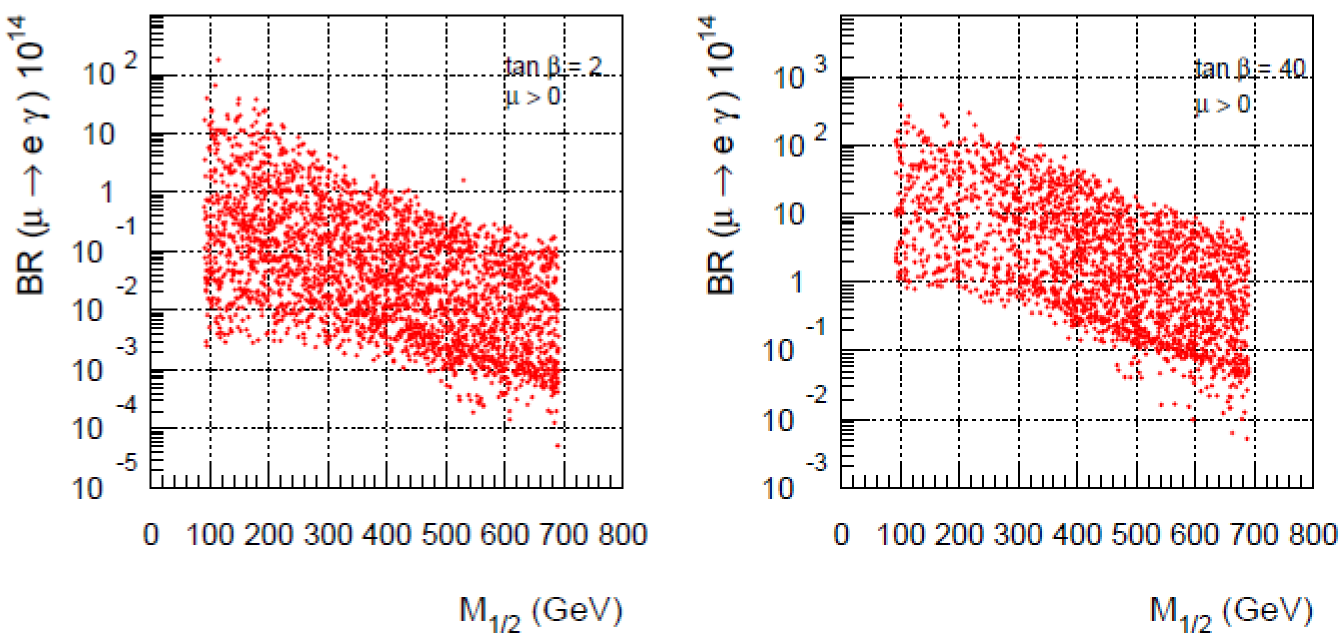


Masiero, Vempati, Vives

Top-down approach

Example 2: SO(10) inspired model. Mixing angles in the Yukawa couplings as the CKM matrix

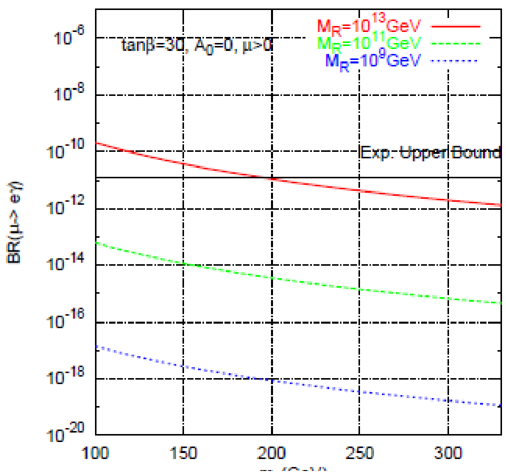
$$(\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu)_{21} = y_t^2 V_{td} V_{ts} + \mathcal{O}(y_c^2)$$

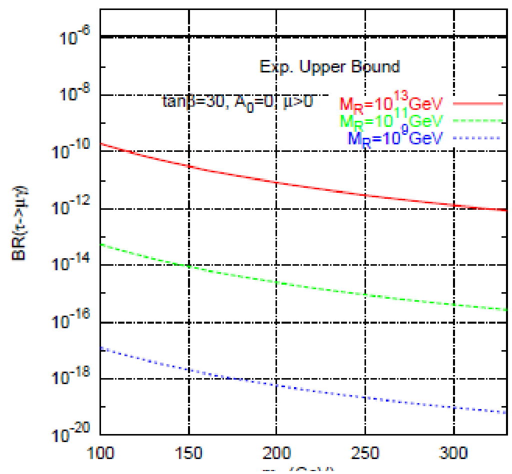


Masiero, Vempati, Vives

“Semi botton-up” approach

Incorporate the low energy neutrino constraints, but make assumptions about the high energy parameters





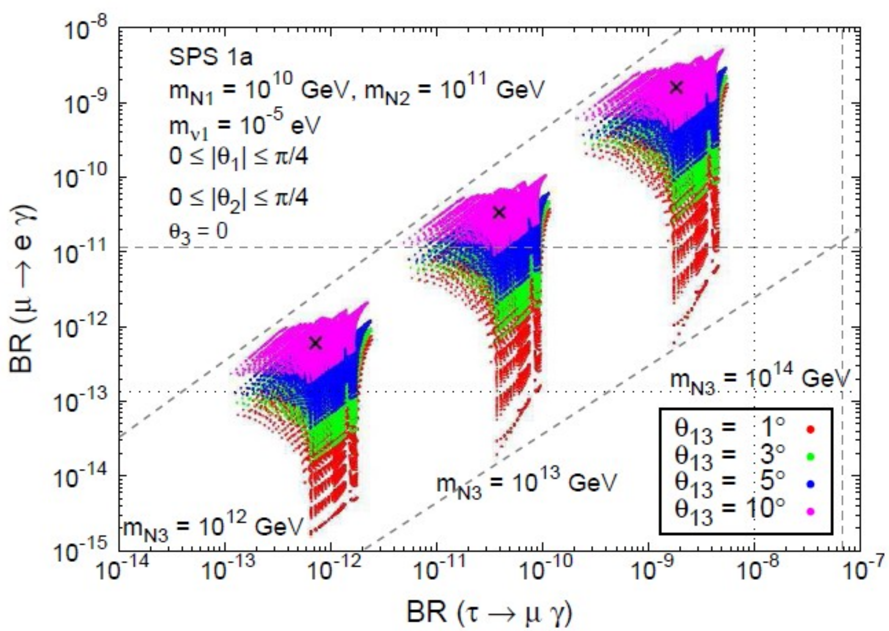
Petcov, Profumo, Takanishi, Yaguna

No CP violation

Right-handed neutrinos degenerate

θ_{13}=0.1

light neutrinos hierarchical



Antusch, Arganda, Herrero, Teixeira

One angle in the complex matrix R fixed

Two RH neutrino masses fixed

light neutrinos hierarchical

Is this a dead-end? Is it impossible to test the SUSY see-saw?

List of unknowns:

- Cut-off scale?
- Flavour structure of the soft terms at the cut-off scale?
- soft-SUSY parameters?
- $\tan\beta$?
- Size and flavour structure of the Yukawa couplings?
- Right-handed neutrino masses?

Remarkably, under some well motivated assumptions, it is possible to derive predictions for the LFV processes, in the form of lower bounds.

- Absence of tunings
- Hierarchical neutrino Yukawa couplings
- Leptogenesis as the origin of the baryon asymmetry

Prediction I.

Assume the worst case for the detection of $\mu \to e\gamma$, namely R-parity is conserved and all $(\mathbf{m}_L^2)_{12}$, $(\mathbf{m}_e^2)_{12}$, $\mathbf{A}_{e12}$ are equal to zero at low energies

- $(\mathbf{m}_L^2)_{12}$, $(\mathbf{m}_e^2)_{12}$, $\mathbf{A}_{e12}$ vanish at high energies (no LFV in the soft terms at tree level)

AND

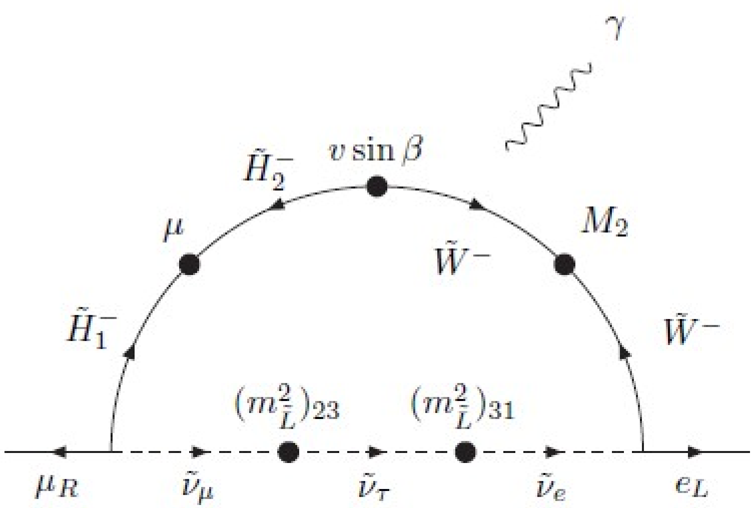
- $(Y_\nu^\dagger Y_\nu)_{12}=0$ (no LFV in the soft terms at one loop level)

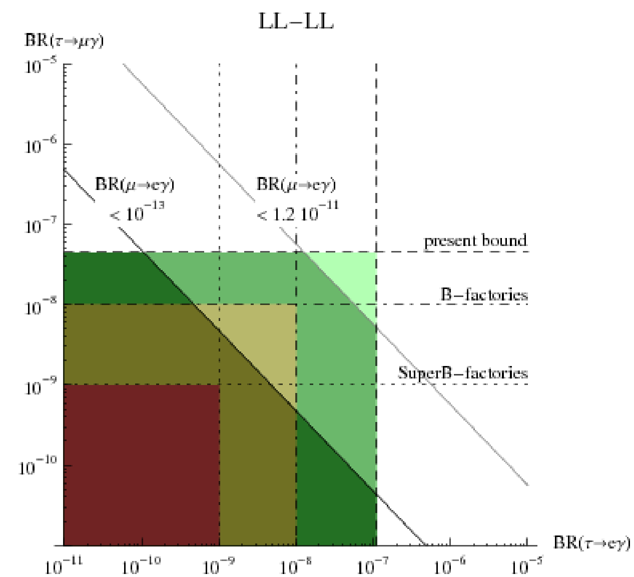
In the absence of cancellations, all the allowed models which lead to vanishing $(Y_\nu^\dagger Y_\nu)_{12}$ lead to

$$\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu \sim \begin{pmatrix} \times & 0 & \times \\ 0 & \times & \times \\ \times & \times & \times \end{pmatrix}$$

Implications:

- Off-diagonal soft terms are generated at one loop level in the 13 and 23 sectors. A non vanishing rate for $\mu \to e\gamma$ will be induced through the double mass insertion





$$BR(\mu \to e\gamma) \simeq C \times BR(\tau \to \mu\gamma) BR(\tau \to e\gamma)$$

In the absence of cancellations, all the allowed models which lead to vanishing $(Y_\nu^\dagger Y_\nu)_{12}$ lead to

$$\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu \sim \begin{pmatrix} \times & 0 & \times \\ 0 & \times & \times \\ \times & \times & \times \end{pmatrix}$$

Implications:

• Off-diagonal soft terms are generated at one loop level in the 13 and 23 sectors. A non vanishing rate for $\mu \to e\gamma$ will be induced through the double mass insertion

new effect

• That structure for $Y_\nu^\dagger Y_\nu$ is likely to hold at the cut-off scale. Off-diagonal soft terms are generated at two loop level in the 12 sector. Another contribution to BR($\mu \to e\gamma$)

AI, Simonetto

$$(\mathbf{m}_L^2)_{21}(M_{\rm maj}) \simeq \left(\frac{1}{16\pi^2}\right)^2 m_S^2 (\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu)_{32}^* (\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu)_{31} \log\left(\frac{M_{\rm X}}{M_{\rm maj}}\right)^2$$

Again, $BR(\mu \to e\gamma) \simeq C \times BR(\tau \to \mu\gamma) BR(\tau \to e\gamma)$

In the worst case neutrino scenario, where $\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu \sim \begin{pmatrix} \times & 0 & \times \\ 0 & \times & \times \\ \times & \times & \times \end{pmatrix}$

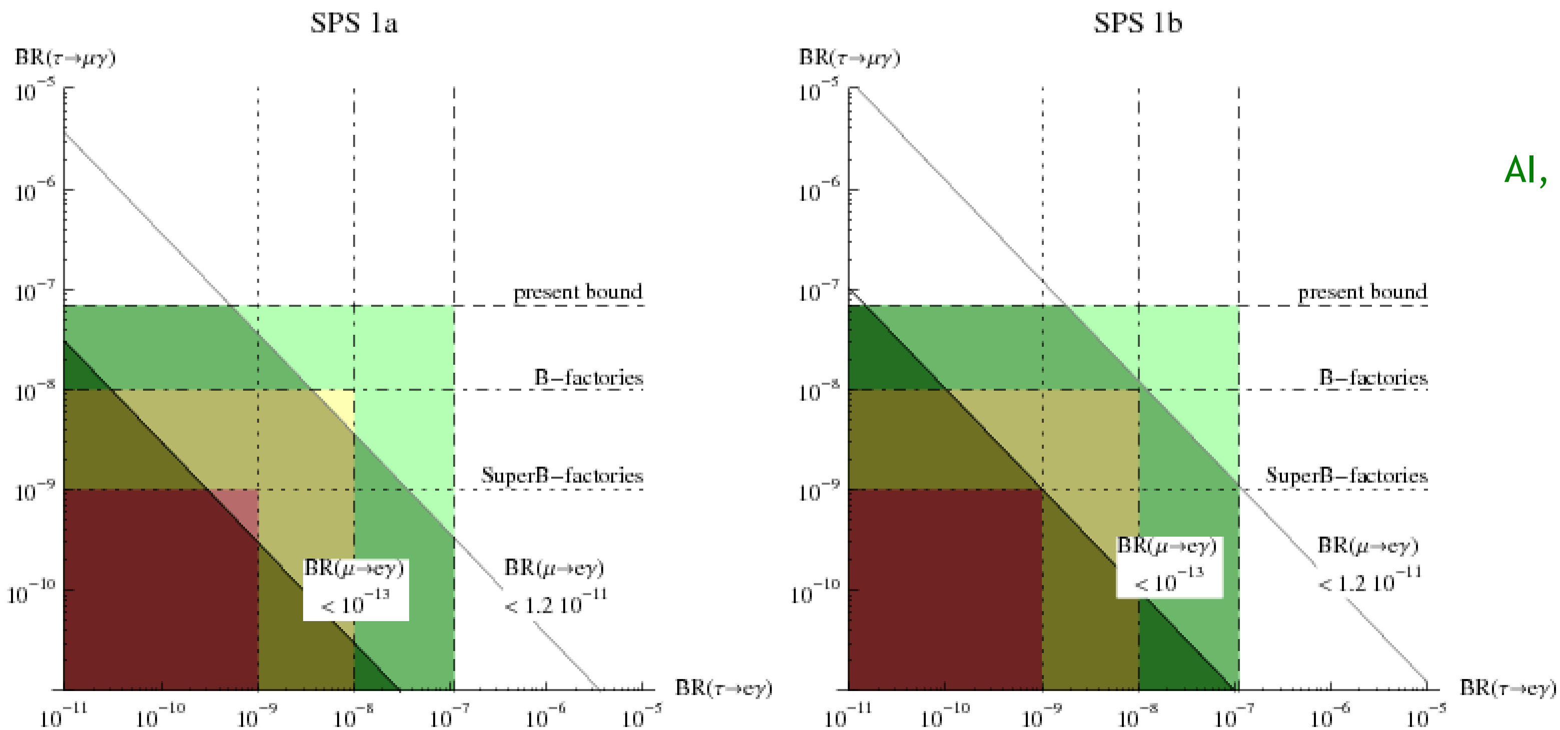
$$BR(\mu \to e\gamma) \simeq C \times BR(\tau \to \mu\gamma) BR(\tau \to e\gamma)$$

In any other scenario, with $(Y_\nu^\dagger Y_\nu)_{12} \neq 0$,

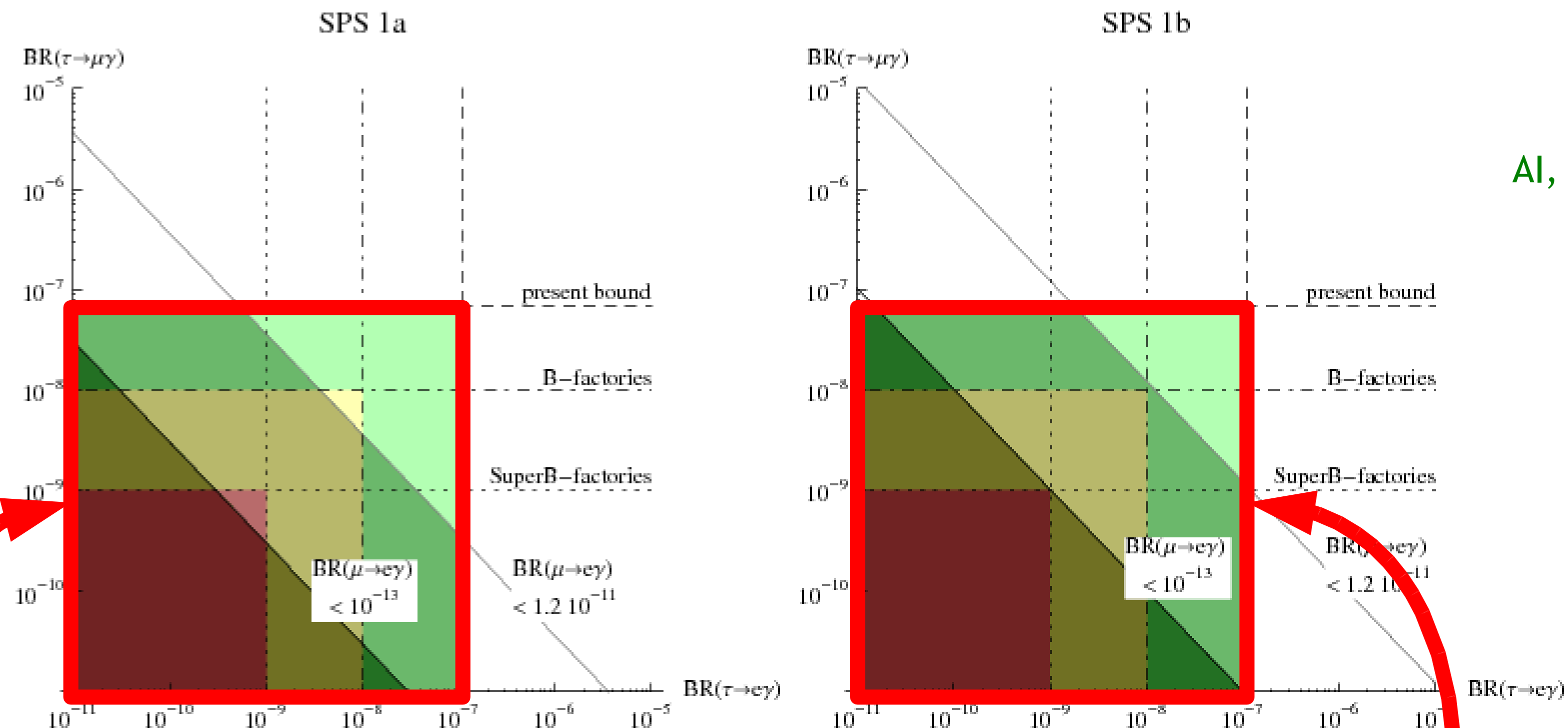
$$\mathrm{BR}(\mu \to e\gamma) \gtrsim C \times \mathrm{BR}(\tau \to \mu\gamma)\mathrm{BR}(\tau \to e\gamma)$$

C depends just on SUSY parameters and is independent of see-saw parameters

This bound holds for all the see-saw models that reproduce the low energy data. The only assumption is the absence of cancellations.

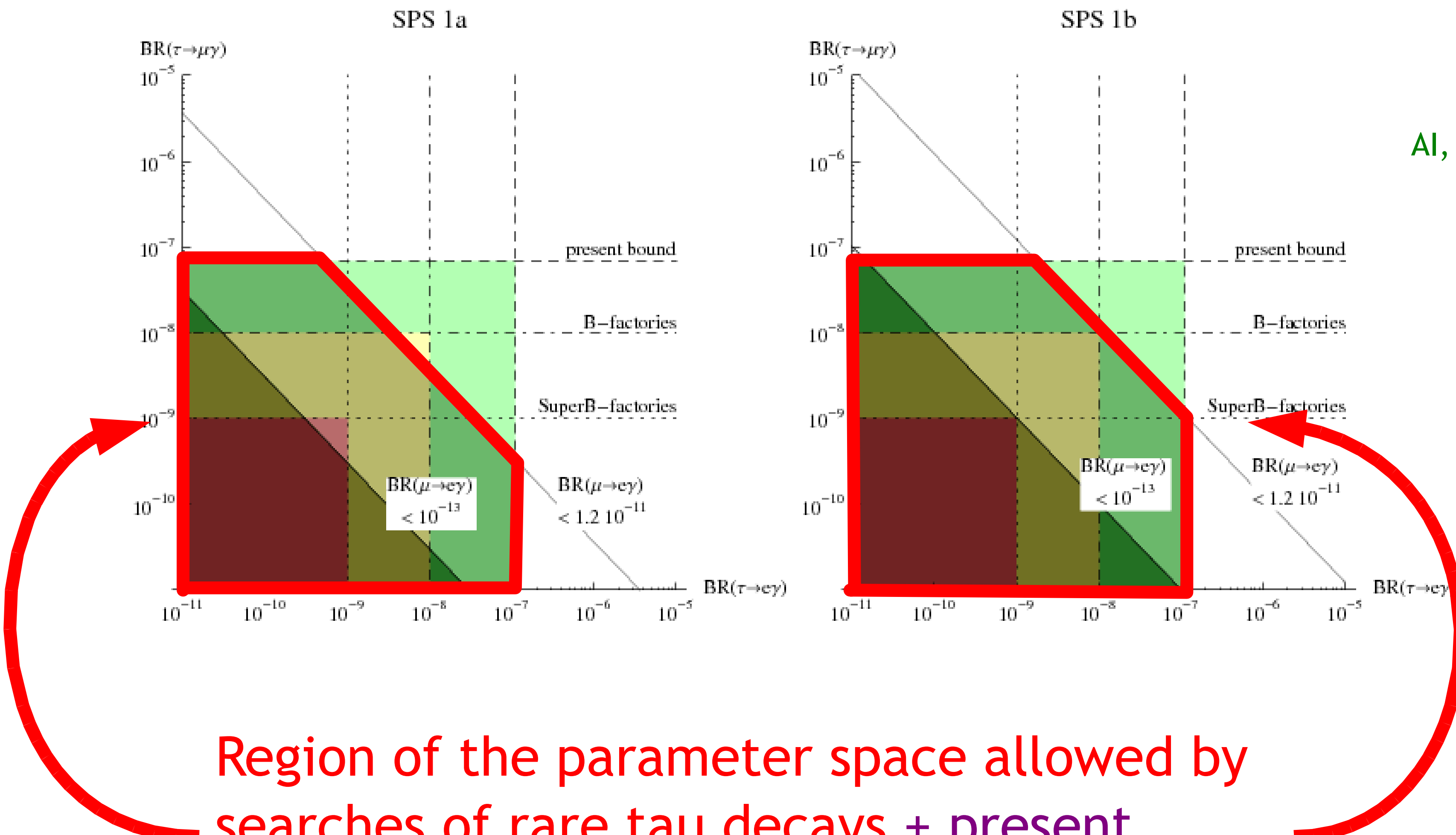


AI, Simonetto



AI, Simonetto

Region of the parameter space allowed by searches of rare tau decays



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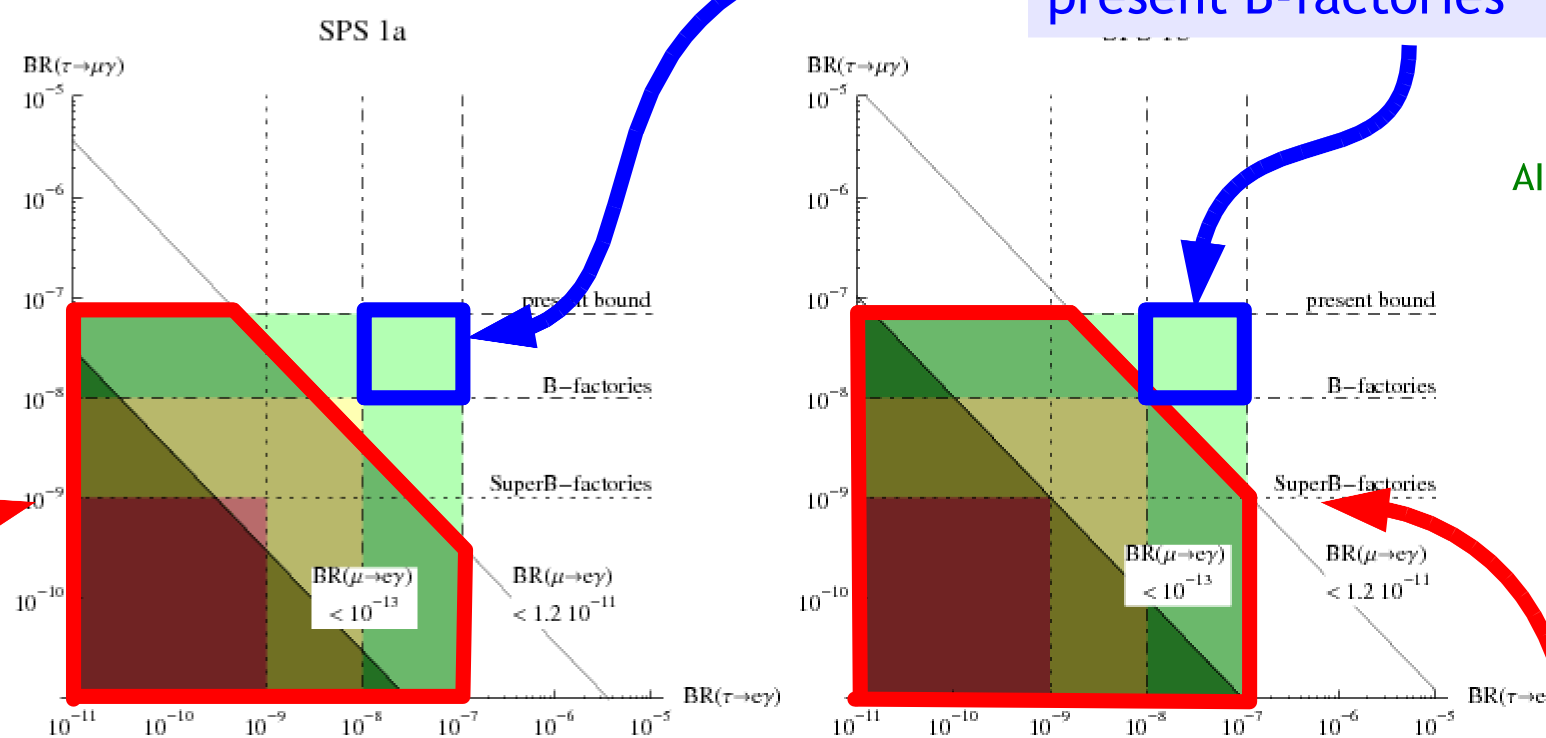
Region of the parameter space allowed by searches of rare tau decays + present bound on $\mu \to e\gamma$ ($BR < 1.2 \times 10^{-11}$)

Region where *both* $\tau \to \mu\gamma$ and $\tau \to e\gamma$ could be observed at present B-factories

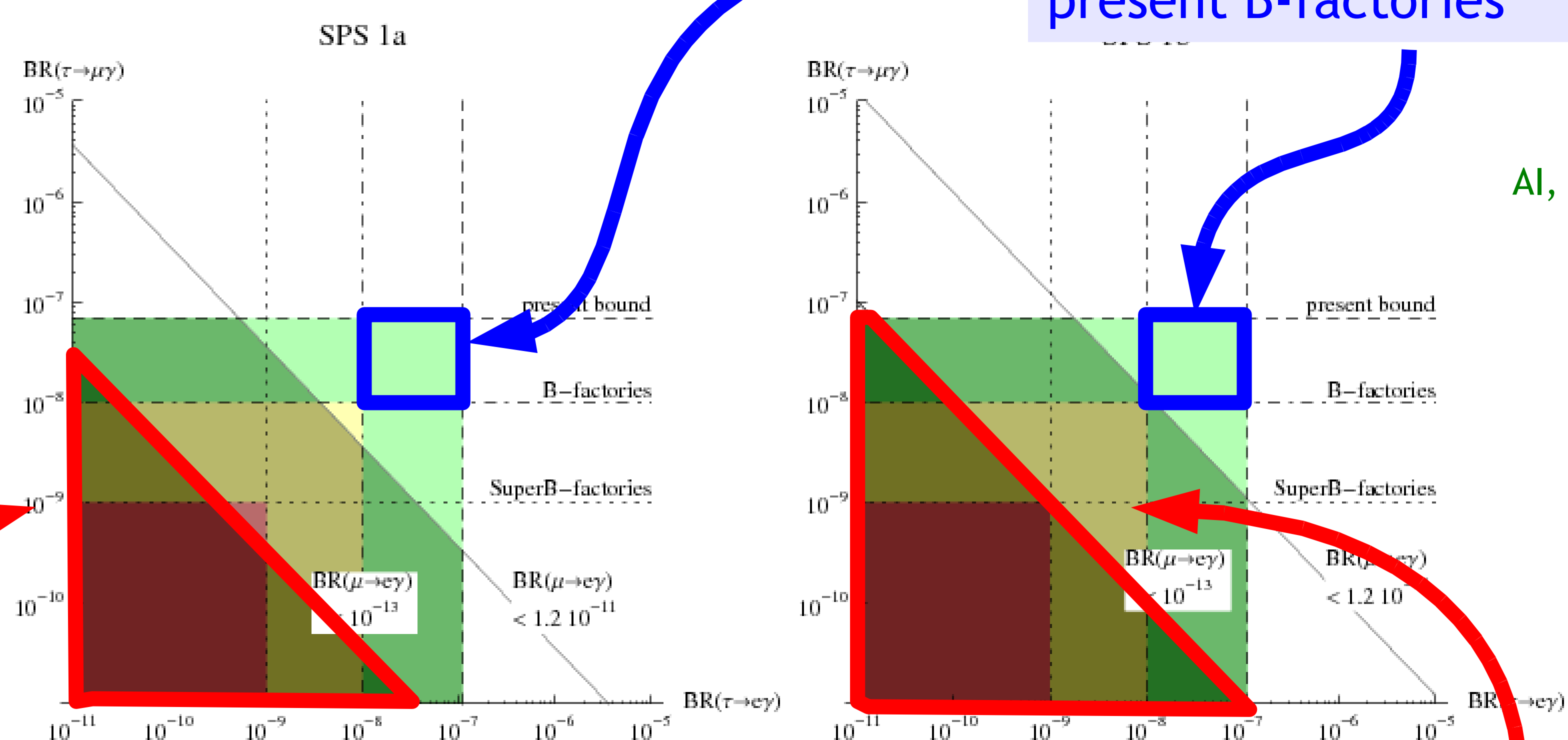
AI, Simonetto



Region of the parameter space allowed by searches of rare tau decays + present bound on $\mu \to e\gamma$ ($BR < 1.2 \times 10^{-11}$)

Region where *both* τ→μγ and τ→eγ could be observed at present B-factories

Al, Simonetto



Region of the parameter space allowed by searches of rare tau decays + projected bound on $\mu \to e\gamma$ ($BR<10^{-13}$)

Prediction II.

Assume again the worst case for the detection of $l_i \to l_j \gamma$

- $(\mathbf{m}_L^2)_{ij}$, $(\mathbf{m}_e^2)_{ij}$, $\mathbf{A}_{eij}$, $i \neq j$ vanish at high energies (no LFV in the soft terms at tree level)

AND

- $(Y_\nu^\dagger Y_\nu)$ diagonal

The back of the envelope calculation gives BR($l_i \to l_j \gamma$)=0

$$\mathrm{BR}(\ell_j \to \ell_i \gamma) \simeq \frac{\alpha^3}{G_F^2} \frac{|(\mathbf{m}_L^2)_{ij}|^2}{m_S^8} \tan^2\beta \, \mathrm{BR}(\ell_j \to \ell_i \nu_j \bar{\nu}_i)$$

$$(\mathbf{m}_L^2)_{ij} \simeq \mathbf{m}_L^2(\Lambda)_{ij} - \frac{1}{8\pi^2}(3m_0^2 + |A_0|^2)(\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu)_{ij} \log\left(\frac{\Lambda}{M_{\mathrm{maj}}}\right)$$

Prediction II.

Assume again the worst case for the detection of $l_i \to l_j \gamma$

- $(\mathbf{m}_L^2)_{ij}$, $(\mathbf{m}_e^2)_{ij}$, $\mathbf{A}_{eij}$, $i \neq j$ vanish at high energies (no LFV in the soft terms at tree level)

AND

- $(Y_\nu^\dagger Y_\nu)$ diagonal

The back of the envelope calculation gives $\mathrm{BR}(l_i \to l_j \gamma)=0$

$$\mathrm{BR}(\ell_j \to \ell_i \gamma) \simeq \frac{\alpha^3}{G_F^2} \frac{|(\mathbf{m}_L^2)_{ij}|^2}{m_S^8} \tan^2\beta \, \mathrm{BR}(\ell_j \to \ell_i \nu_j \bar{\nu}_i)$$

$$(\mathbf{m}_L^2)_{ij} \simeq \mathbf{m}_L^2(\Lambda)_{ij} - \frac{1}{8\pi^2}(3m_0^2 + |A_0|^2)(\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu)_{ij} \log\left(\frac{\Lambda}{M_{\mathrm{maj}}}\right)$$

All the right-handed neutrinos decouple at the same scale M_{maj}

Strictly speaking $(\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu)_{ij} \log\left(\frac{\Lambda}{M_{\mathrm{maj}}}\right) \longrightarrow \sum_k \mathbf{Y}_{\nu ki}^* \log\left(\frac{\Lambda}{M_k}\right) \mathbf{Y}_{\nu kj}$

Which is necessarily different from zero (unless cancellations take place)

Assume:

- No cancellations
- hierarchical neutrino Yukawa eigenvalues: $y_1 << y_2 << y_3$
  (as in the rest of known Yukawa matrices)
- Cut-off scale $\Lambda > M_3$

$$\mathrm{BR}(\mu \to e\gamma) \gtrsim \frac{\alpha^3}{G_F^2}\left(\frac{3m_0^2+|A_0|^2}{8\pi^2 m_S^4}\right)^2 y_1^4 \log^2\frac{M_2}{M_1}\tan^2\beta\ ,$$

$$\mathrm{BR}(\tau \to e\gamma) \gtrsim \frac{\alpha^3}{G_F^2}\left(\frac{3m_0^2+|A_0|^2}{8\pi^2 m_S^4}\right)^2 y_1^4 \left(2\log\frac{M_3}{M_2}+\log\frac{M_2}{M_1}\right)^2 \tan^2\beta\, \mathrm{BR}(\tau \to e\nu_\tau\bar{\nu}_\mu)\ ,$$

$$\mathrm{BR}(\tau \to \mu\gamma) \gtrsim \frac{\alpha^3}{G_F^2}\left(\frac{3m_0^2+|A_0|^2}{8\pi^2 m_S^4}\right)^2 y_2^4 \log^2\frac{M_3}{M_2}\tan^2\beta\, \mathrm{BR}(\tau \to \mu\nu_\tau\bar{\nu}_e)\ ,$$

$$|d_e| \gtrsim e\frac{\alpha}{\pi}\frac{m_e}{m_S^2}\left(\frac{1}{16\pi^2}\right)^2 y_1^4 \left|2\sqrt{2}\sin\theta_{13}\sin\delta + 6\frac{m_1}{m_2}\sin(\phi'-\phi)\right| \log\frac{M_2}{M_1}\log\frac{M_3}{M_2}$$

Assume:

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- hierarchical neutrino Yukawa eigenvalues: $y_1 << y_2 << y_3$

  (as in the rest of known Yukawa matrices)
- Cut-off scale $\Lambda > M_3$

$$\mathrm{BR}(\mu \to e\gamma) \gtrsim 1.2 \times 10^{-11} \left(\frac{y_1}{4 \times 10^{-2}}\right)^4 \left(\frac{m_S}{200\,\mathrm{GeV}}\right)^{-4} \left(\frac{\tan\beta}{10}\right)^2 ,$$

$$\mathrm{BR}(\tau \to \mu\gamma) \gtrsim 4.5 \times 10^{-8} \left(\frac{y_2}{0.5}\right)^4 \left(\frac{m_S}{200\,\mathrm{GeV}}\right)^{-4} \left(\frac{\tan\beta}{10}\right)^2 ,$$

For θ_{13}=0.2, and $\mathcal{O}(1)$ phases

$$|d_e| \gtrsim 10^{-27} \left(\frac{y_1}{2}\right)^4 \mathrm{e\,cm} \left(\frac{m_S}{200\mathrm{GeV}}\right)^{-2}$$

Where the smallest Yukawa coupling is related to the lightest right-handed neutrino mass through: $M_1 \lesssim \frac{y_1^2 \langle H_u^0 \rangle^2}{\sqrt{\Delta m^2_{\mathrm{sol}}}}$

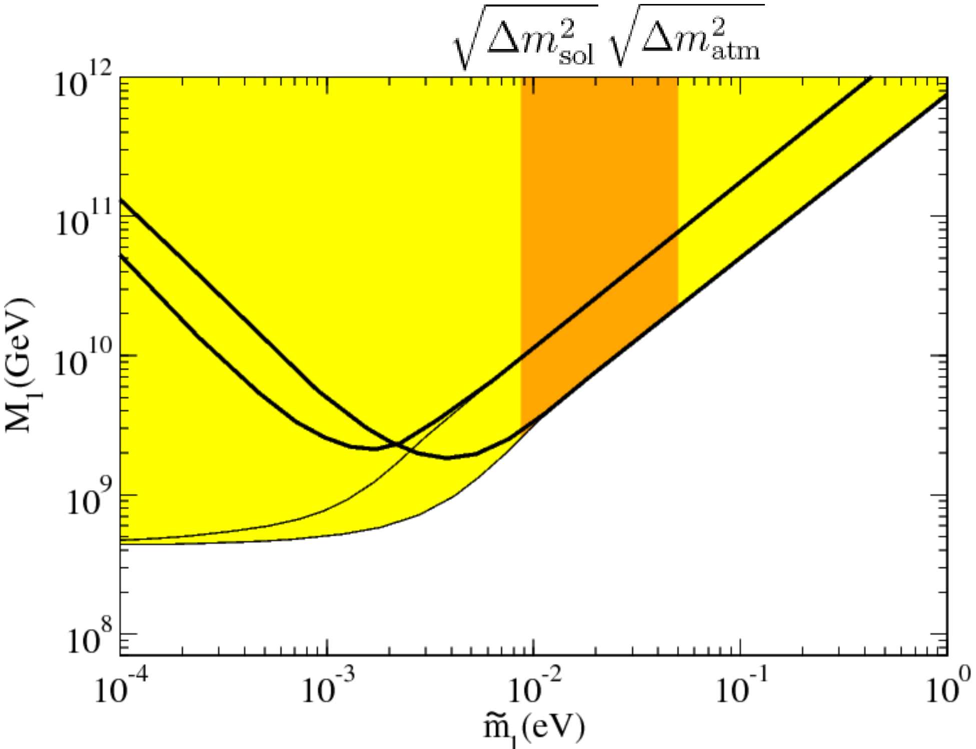
$$\mathrm{BR}(\mu \to e\gamma) \gtrsim 1.2 \times 10^{-11} \left(\frac{M_1}{5 \times 10^{12}\mathrm{GeV}}\right)^2 \left(\frac{m_S}{200\,\mathrm{GeV}}\right)^{-4} \left(\frac{\tan\beta}{10}\right)^2$$

AI, Simonetto

Assume:

- No cancellations
- hierarchical neutrino Yukawa eigenvalues: $y_1 << y_2 << y_3$
  (as in the rest of known Yukawa matrices)
- Leptogenesis as the origin of the BAU

Leptogenesis requires $M_1 > 10^9$ GeV $\Rightarrow$ gravitino mass >5 GeV, to avoid overclosure $\Rightarrow \Lambda > 10^{14\text{-}16}$ GeV. Normally, at least one right-handed neutrino will be coupled and induce LFV.



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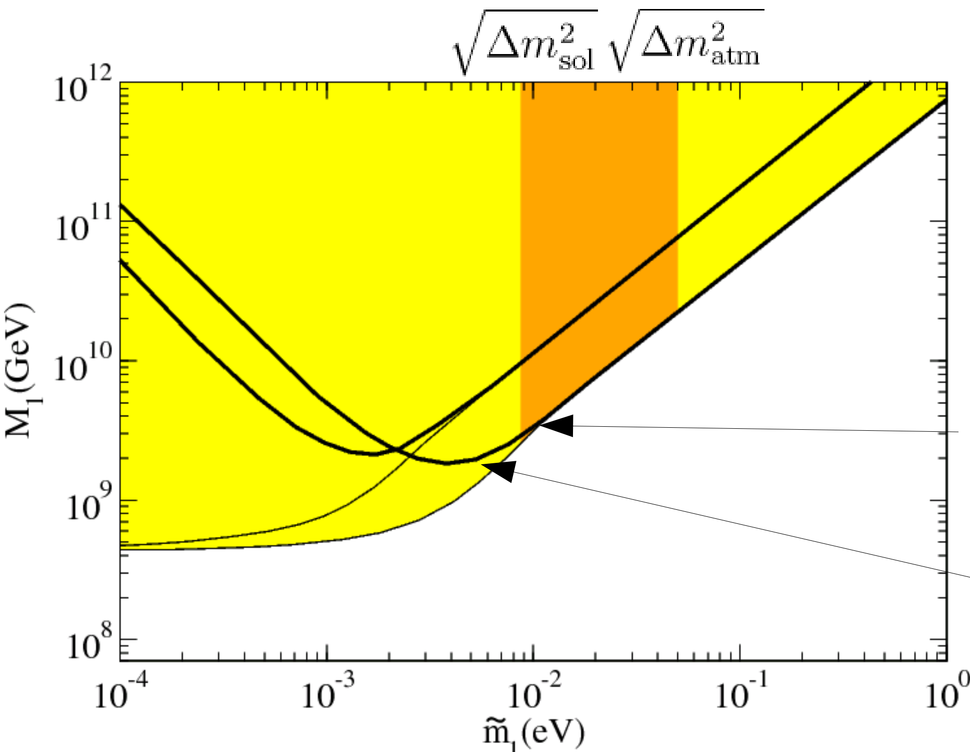


PRISM/PRIME at JPARC aims to a sensitivity to μTi-e Ti at the level of 10^{-18} (equivalent to ~10^{-16} in BR($\mu \to e\gamma$)).
Part of the parameter space can be covered

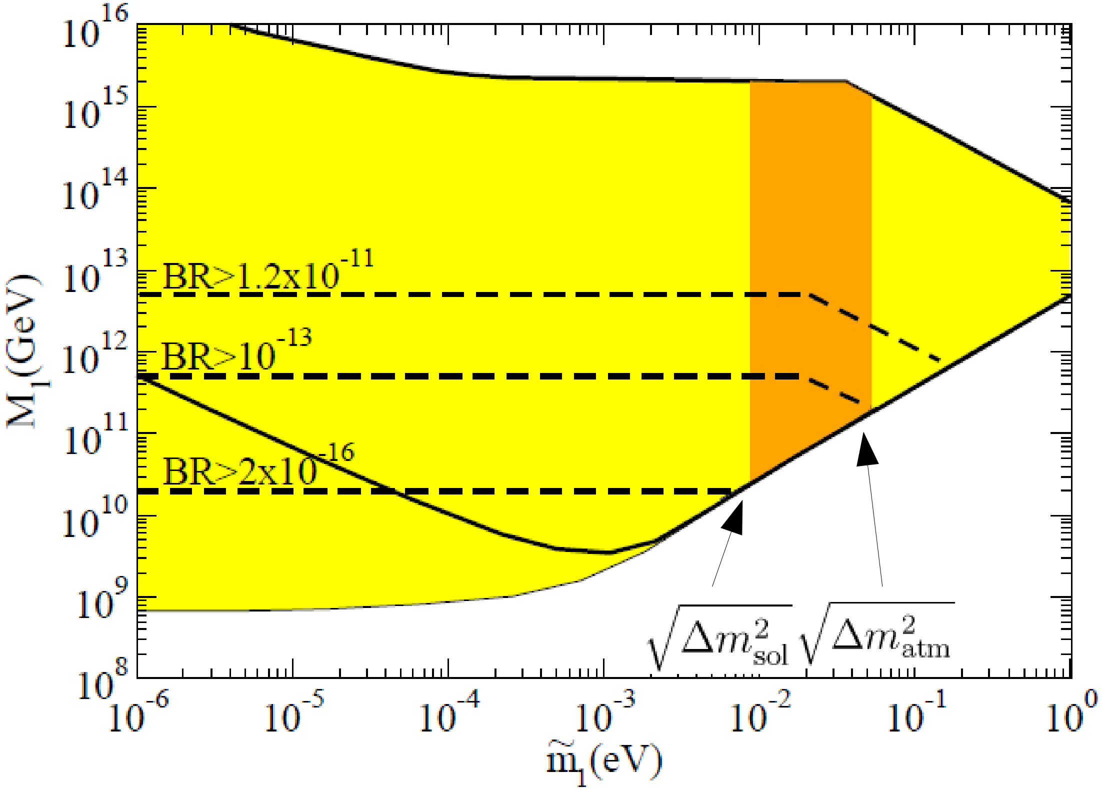
$$\mathrm{BR}(\mu \to e\gamma) \gtrsim 5 \times 10^{-18} \left(\frac{m_S}{200\ \mathrm{GeV}}\right)^{-4} \left(\frac{\tan\beta}{10}\right)^2$$

$$\mathrm{BR}(\mu \to e\gamma) \gtrsim 5 \times 10^{-19} \left(\frac{m_S}{200\ \mathrm{GeV}}\right)^{-4} \left(\frac{\tan\beta}{10}\right)^2$$

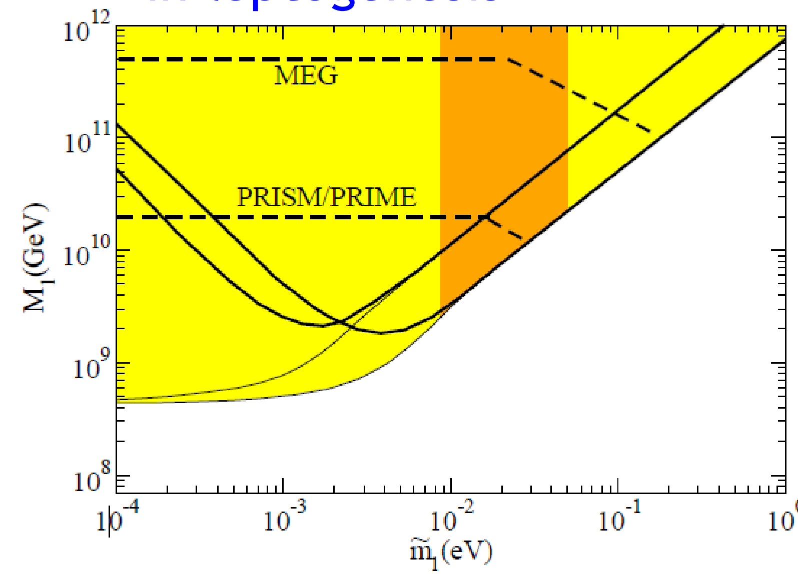
Probing SUSY leptogenesis with $\mu \to e\gamma$

Assumptions:

- No cancellations
- hierarchical neutrino Yukawa eigenvalues: $y_1 << y_2 << y_3$



Including flavour effects in leptogenesis



Type II see-saw model

There is a tight connection between the flavour structure of the complete theory and the effective theory

$$W = \frac{1}{\sqrt{2}}\mathbf{Y}_{\Delta ij} L_i \Delta L_j + \frac{1}{\sqrt{2}}\lambda H_u \bar{\Delta} H_u + M_\Delta \bar{\Delta}\Delta$$

$$\downarrow$$

$$\mathcal{M}_{\nu ij} = \frac{\lambda_2 \langle H_u^0 \rangle^2}{M_\Delta}\mathbf{Y}_{\Delta ij}$$

If this is the only source of LFV, the low energy flavour structure of the slepton mass matrix is dictated by the flavour structure of the neutrino mass matrix Rossi

$$\delta \mathbf{m}^2_{Lij} \propto m_0^2 (\mathbf{Y}_\Delta^\dagger \mathbf{Y}_\Delta)_{ij} \log \frac{\Lambda}{M_\Delta} = m_0^2 \left(\frac{M_\Delta}{\lambda_2 \langle H_u^0 \rangle^2}\right)^2 (\mathcal{M}_\nu^\dagger \mathcal{M}_\nu)_{ij} \log \frac{\Lambda}{M_\Delta}$$

Correlations!!

$$\frac{BR(\tau \to \mu\gamma)}{BR(\mu \to e\gamma)} \sim 10^3$$

$$\frac{BR(\tau \to e\gamma)}{BR(\mu \to e\gamma)} \sim 10^{-1}$$

Conclusions

- Lepton flavour violation and electric dipole moments are very powerful tools to probe physics beyond the Standard Model.
- Huge experimental effort ongoing to constrain (hopefully discover) leptonic rare decays or EDMs. On the theory side, there is also an intense activity computing predictions in particular scenarios.
- Within a particular model (MSSM, RS, LHT...), correlations between processes violating the same flavours can be derived. Tests!
- Correlations between processes violating different flavours are more difficult to derive, but possible under certain assumptions:

  - No cancellations: $\mathrm{BR}(\mu\to e\gamma) \gtrsim C\,\mathrm{BR}(\tau\to\mu\gamma)\mathrm{BR}(\tau\to e\gamma)$
  - SUSY+hierarchical Yukawas+thermal leptogenesis
    $\mathrm{BR}(\mu\to e\gamma) \gtrsim 5\times10^{-18}$ for typical SUSY parameters
  - SUSY+type II see-saw: $\mathrm{BR}(\tau\to\mu\gamma)/\mathrm{BR}(\mu\to e\gamma)\sim10^{3}$