Collective Flavor Oscillations For Supernova Neutrinos And r-Process Nucleosynthesis

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Plan of the Talk:

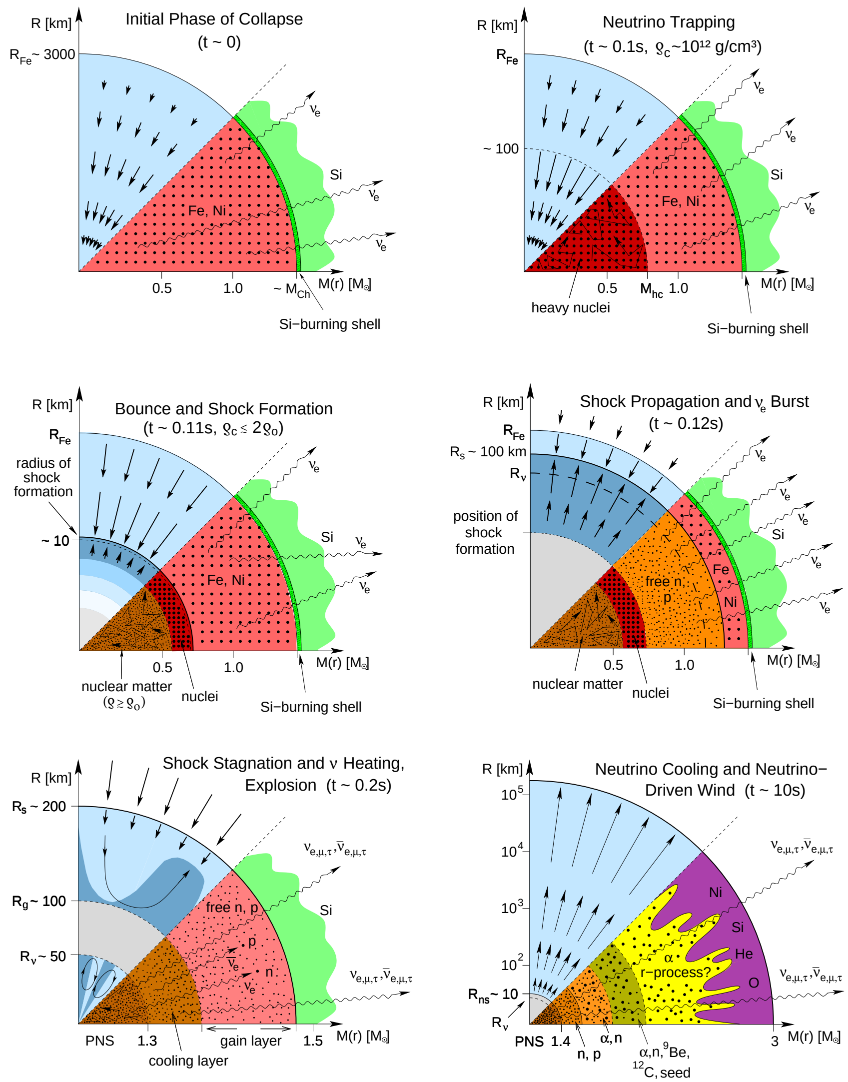
- Supernovae and Neutrinos
- Collective Neutrino Effects
- Luminosity Variation and Collective Effects
- r-Process Nucleosynthesis and Collective Effect
- Summary

- 1. Supernovae and Neutrinos

SN and Neutrinos

- Supernova explosions are accompanied by emission of a huge flux of neutrinos and antineutrinos
- These neutrinos play important role in shock revival and subsequent explosion
- Neutrinos cross SN mantle and envelope
- Neutrinos may get affected by earth matter
- Interaction with matter constituent particles inside SN are very crucial

Different stages of SN



Janka et al, Phys. Rep. 442(2007)38

- 2. Collective Effects

Collective Neutrino Flavor transition

- Neutrino-Neutrino interaction in SN core is not negligible.
- 'neutrino-neutrino interactions' can lead to oscillation of neutrinos and antineutrinos of different energies with same frequency $\Longrightarrow$Synchronized Oscillation
- After a few hundred kilometers these interaction effects become smaller and eventually end with
  - one or multiple swapping i.e, complete oscillation between ν_e and ν_x and between $\bar{\nu}_e$ and $\bar{\nu}_x$

    Dasgupta et al hep-ph/0904.3542
  - The critical energy of swapping depends on the initial flux of the emitted neutrino and antineutrino.
  - The effect more pronounced for IH than NH.

Hamiltonian

Hamiltonian of a neutrino/antineutrino in the ensemble of relativistic neutrinos and Antineutrinos.

$$H = H_{vacuum} + H_{MSW} + H_{\nu\nu}$$

For the density matrix in flavor basis

$$\rho = \sum |\psi_i\rangle\langle\psi_i| \ \ , \ \ \bar{\rho} = \sum |\bar{\psi}_i\rangle\langle\bar{\psi}_i|$$

Evolution equation is

$$\partial_t \rho = -i[H, \rho] \ \ , \ \ \partial_t \bar{\rho} = -i[H, \bar{\rho}]$$

2 Flavor system:

For a 2 flavor system $\nu_e(\bar{\nu}_e)$ and $\nu_x(\bar{\nu}_x)$

$$H_{vacuum} = E + \frac{m_1^2 + m_2^2}{4E} + \frac{\triangle m^2}{4E} \begin{pmatrix} -cos2\theta & sin2\theta \\ sin2\theta & cos2\theta \end{pmatrix} ,$$

where

$$\triangle m^2 = m_2^2 - m_1^2 \;\; ; \;\; \theta = MixingAngle$$

$$H_{MSW} = -\frac{G_F N_n}{\sqrt{2}} + \begin{pmatrix} \sqrt{2} G_F N_e & 0 \\ 0 & 0 \end{pmatrix} ,$$

N_e= Electron Number Density, N_n= Neutron Number Density.

2 Flavor $\nu\nu$ Interaction Term:

Contribution from $\nu\nu$ forward scattering to the leading order of G_F

$$H_{\nu\nu} = \sqrt{2} G_F \int d^3q (1 - cos\theta_{pq})(n\rho_q - \bar{n}\bar{\rho}_q)$$

Single Angle Approximation :
Flavor evolution is trajectory independent and neutrinos in any trajectory transform like ν's propagating in radial direction.

$$H_{\nu\nu} = \sqrt{2} G_F D(r) \int dq (n\rho_q - \bar{n}\bar{\rho}_q)$$

where Geometrical factor $D(r) = \frac{1}{2}\left(1 - \sqrt{1 - \left(\frac{R_\nu}{r}\right)^2}\right)^2$

Bloch vector and E.O.M:

For a 2×2 Hermitian Matrix M

$$M = \frac{1}{2}(1 + \mathbf{P}.\bar{\sigma})$$

$\sigma \rightarrow$ Pauli spin Matrices,
P $\rightarrow$ Bloch Vector corresponding to ρ

Fogli et al hep-ph/0707.1998

E.O.M for ν and $\bar{\nu}$ polarization vector P and $P^{'}$

$$\dot{\mathbf{P}} = \mathbf{P} \times (\omega \mathbf{B} - \lambda \hat{\mathbf{L}} - \mu \mathbf{D})$$

$$\dot{\mathbf{P}}^{'} = \mathbf{P}^{'} \times (-\omega \mathbf{B} - \lambda \hat{\mathbf{L}} - \mu \mathbf{D})$$

$$\omega = \frac{\Delta^2}{2E}$$

$$\lambda = \sqrt{2} G_F N_e$$

$$\mu = \sqrt{2} G_F (N_{\nu_e} + N_{\nu_x} + N_{\bar{\nu}_e} + N_{\bar{\nu}_x})$$

$$\mathbf{B} = (-sin2\theta, 0, cos2\theta)^T$$

$$\hat{\mathbf{L}} = (0, 0, 1)^T$$

$$\mathbf{D} = \frac{1}{(N_{\nu_e} + N_{\nu_x} + N_{\bar{\nu}_e} + N_{\bar{\nu}_x})} \int dE (n\mathbf{P} - \bar{n}\mathbf{P}^{'})$$

Note that collective effects change both ν and $\bar{\nu}$ fluxes whereas MSW effect changes only one

E.O.M:

N_α's represent the total effective number density of the αth species.

$$N_\alpha = \int dE \, n_\alpha, \qquad \begin{aligned} n &= n_{\nu_e} + n_{\nu_e} \\ \bar{n} &= n_{\bar{\nu}_e} + n_{\bar{\nu}_e} \end{aligned}$$

n_α's $\longrightarrow$ effective number per unit volume per unit energy.

The effective number density for the αth species per unit volume per unit energy is given by

$$n_\alpha(r, E) = \frac{D(r)}{2\pi R_\alpha^2} \frac{L_\alpha}{\langle E_\alpha \rangle} \Psi(E)_\alpha$$

E.O.M:

Consider a frame rotating with angular velocity -λ**L**.

The evolution equations are

$$\dot{\mathbf{P}} = \mathbf{P} \times (\omega \mathbf{B} - \mu \mathbf{D})$$

$$\dot{\mathbf{P}}^{'} = \mathbf{P}^{'} \times (-\omega \mathbf{B} - \mu \mathbf{D})$$

Nonlinear coupled equations $\longrightarrow$ Integro-Differential Equations.

Survival Probability :

$$P_{\nu_e \nu_e} = \frac{1}{2}(1 + P_z) \quad , \quad P_{\bar{\nu}_e \bar{\nu}_e} = \frac{1}{2}(1 + P_z^{'})$$

- 3. Luminosity Variation and Collective Effect

Initial Energy Distribution :

Effective number density for the αth species per unit energy

$$n_\alpha(r,E) = \frac{D(r)}{2\pi R_\alpha^2}\frac{L_\alpha}{\langle E_\alpha\rangle}\Psi(E)_\alpha$$

For Thermal neutrinos initial energy distribution is Fermi-Dirac

$$\Psi_\alpha^{FD}(E) \propto \frac{\beta_\alpha \quad (\beta_\alpha E)^2}{e^{\beta_\alpha E}+1} \qquad \beta_{\nu_e} = 0.315\ MeV^{-1};\ \beta_{\bar{\nu}_e} = 0.210\ MeV^{-1}$$

$$\beta_{\nu_x} = \beta_{\bar{\nu}_x} = 0.131 MeV^{-1}.$$

Pinched spectra for different simulations are parameterized as

$$\Psi_\alpha(E) = \frac{(1+\zeta_\alpha)^{1+\zeta_\alpha}}{\Gamma(1+\zeta_\alpha)}\left(\frac{E_\alpha}{\langle E_\alpha\rangle}\right)^{\zeta_\alpha}\frac{\exp\left(-(1+\zeta_\alpha)\frac{E_\alpha}{\langle E_\alpha\rangle}\right)}{\langle E_\alpha\rangle},$$

$\zeta_\alpha \rightarrow$ the pinching parameter

Different simulation models have different ζ_α and $\langle E_\alpha\rangle$

Initial Neutrino Flux:

The initial flux (ϕ = $\frac{L_\alpha}{\langle E_\alpha \rangle}$) is another important input parameter.

Total emitted SN energy (E_B = 3×10^{53} erg) puts a constraint on flavor luminosities.

$$L_{\nu_e} + L_{\bar{\nu}_e} + 4L_{\nu_x} = \frac{E_B}{T}$$

We took a constant profile and emission time T = 10 seconds. Thus the initial fluxes of different flavors are also constrained

$$\phi_{\nu_e} \langle E_{\nu_e} \rangle + \phi_{\bar{\nu}_e} \langle E_{\bar{\nu}_e} \rangle + 4\phi_{\nu_x} \langle E_{\nu_x} \rangle = 3 \times 10^{52} \mathrm{erg/sec}$$

Initial Neutrino Fluxes

If the ratio between the initial fluxes of different flavors are

$$\phi_{\nu_e} : \phi_{\bar{\nu}_e} : \phi_{\nu_x} = a : b : 1.00$$

then

$$\phi_{\nu_x}(a\langle E_{\nu_e}\rangle + b\langle E_{\bar{\nu}_e}\rangle + 4\langle E_{\nu_x}\rangle) = 3 \times 10^{52}\mathrm{erg/sec}$$

These parameters are initial relative fluxes

$$a = \frac{\phi_{\nu_e}}{\phi_{\nu_x}}; \quad b = \frac{\phi_{\bar{\nu}_e}}{\phi_{\nu_x}}$$

SN Neutrino Parameters

Other than FD there are mainly four representative models motivated by SN simulations.

One by Lawrence Livermore group (LL) and three by the Garching group (G1, G2, G3).

Fitting Parameters

Model	$\langle E_{\nu_e} \rangle$ (MeV)	$\langle E_{\bar{\nu}_e} \rangle$ (MeV)	$\langle E_{\nu_x} \rangle$ (MeV)	$\zeta_{\bar{\nu}_e}$	$\zeta_{\bar{\nu}_x}$	$\Phi_{\bar{\nu}_e}/\Phi_{\bar{\nu}_x}(a)$	$\Phi_{\bar{\nu}}/\Phi_{\bar{\nu}_x}(b)$
LL	12	15	24	3	4	2.00	1.60
G1	12	15	18	3	4	0.80	0.50
G2	12	15	15	3	4	0.50	0.50
G3	12	15	18	3	3	0.85	0.75

Luminosity Variation

Phenomenologically Important Region:

$$\frac{1}{2} \leq \frac{L_{\nu_e}}{L_{\nu_x}} \leq 2 \;\; ; \;\; \frac{1}{2} \leq \frac{L_{\bar{\nu}_e}}{L_{\nu_x}} \leq 2$$

Lunardini and Smirnov (2003)

Therefore

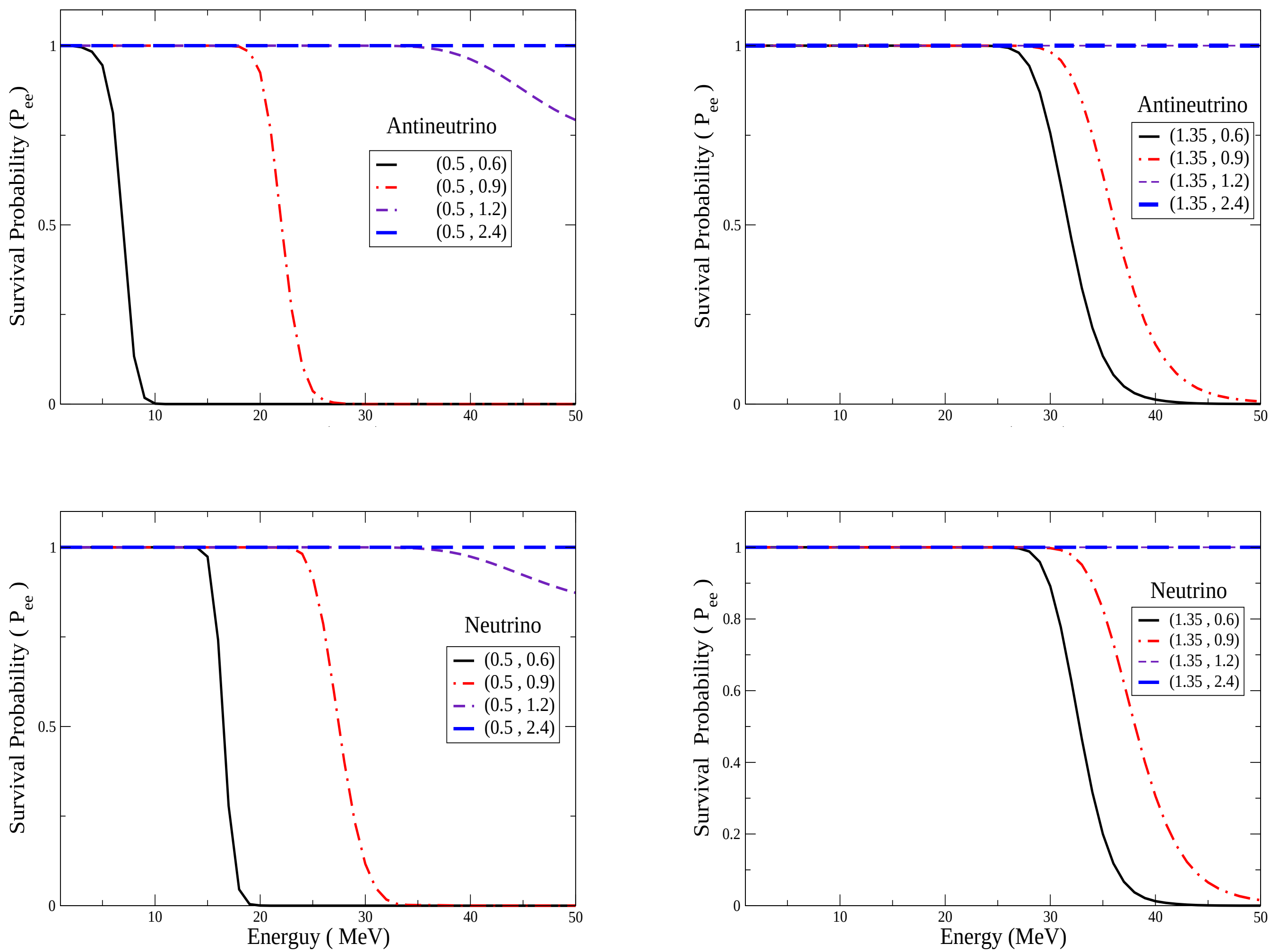
$$\frac{1}{2}\frac{\langle E_{\nu_x}\rangle}{\langle E_{\nu_e}\rangle} \leq a \leq 2\frac{\langle E_{\nu_x}\rangle}{\langle E_{\nu_e}\rangle} \;\; ; \;\; \frac{1}{2}\frac{\langle E_{\nu_x}\rangle}{\langle E_{\bar{\nu}_e}\rangle} \leq b \leq 2\frac{\langle E_{\nu_x}\rangle}{\langle E_{\bar{\nu}_e}\rangle}$$

We varied a and b in the above mentioned region.

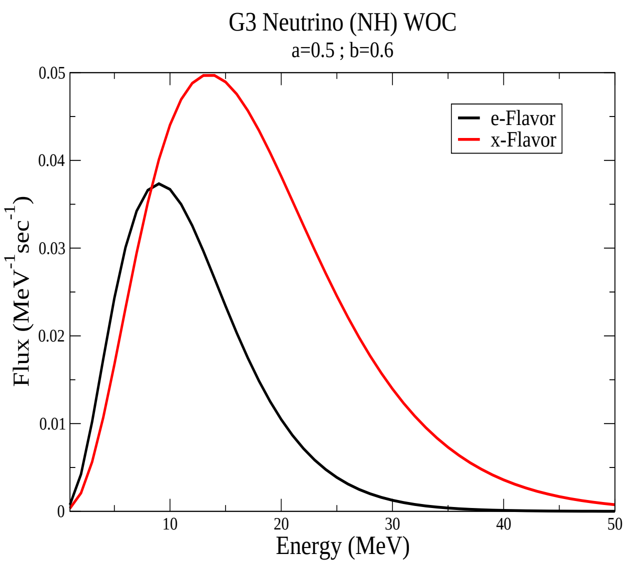
Fogli et al hep-ph/0907.5115

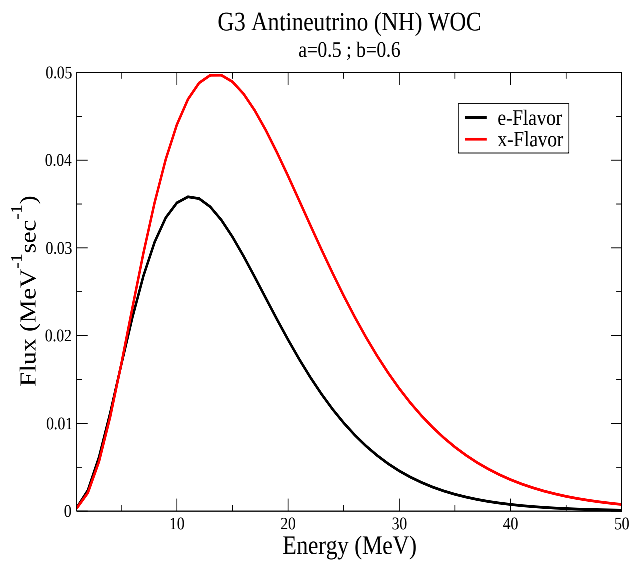
For Normal Hierarchy

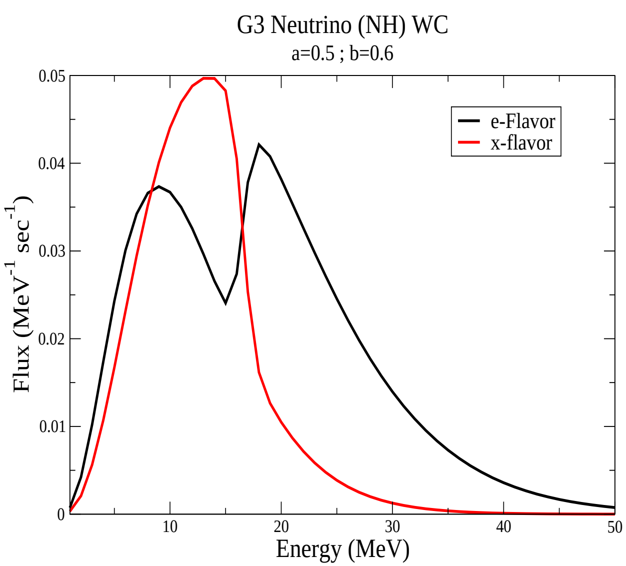
- Effective two flavour formalism: $\Delta m^2 = 3 \times 10^{-3}$, $\theta_{eff} = 10^{-5}$
- Survival prob. for ν and $\bar{\nu}$ for different luminosity combinations

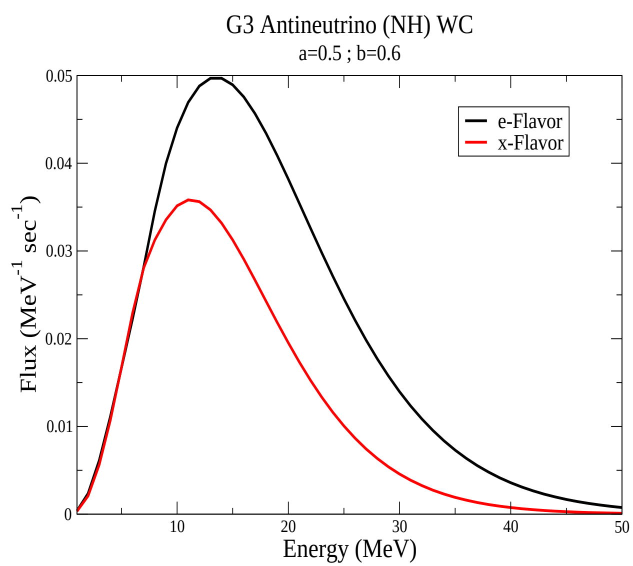


Flux (Normal Hierarchy)

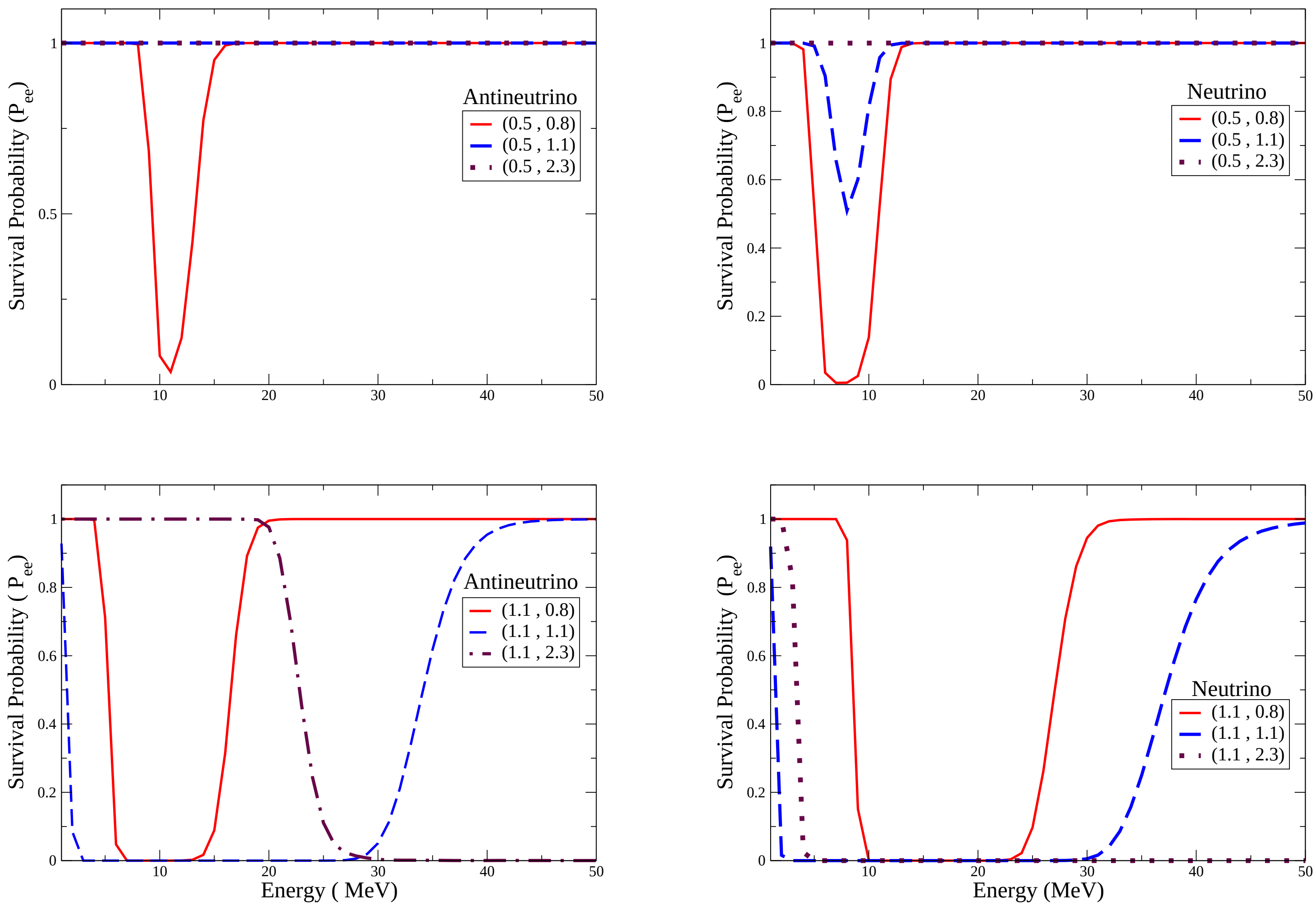




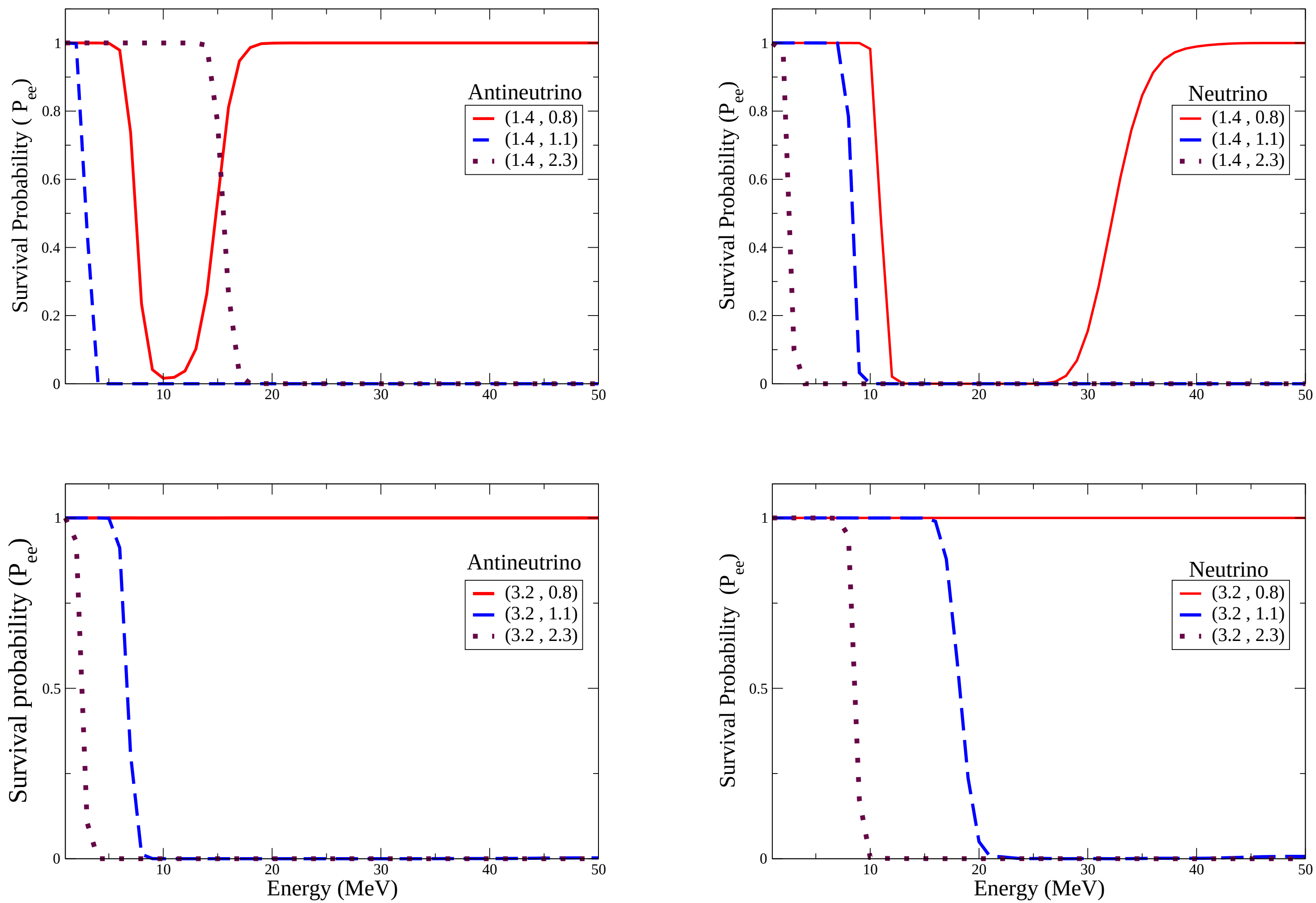




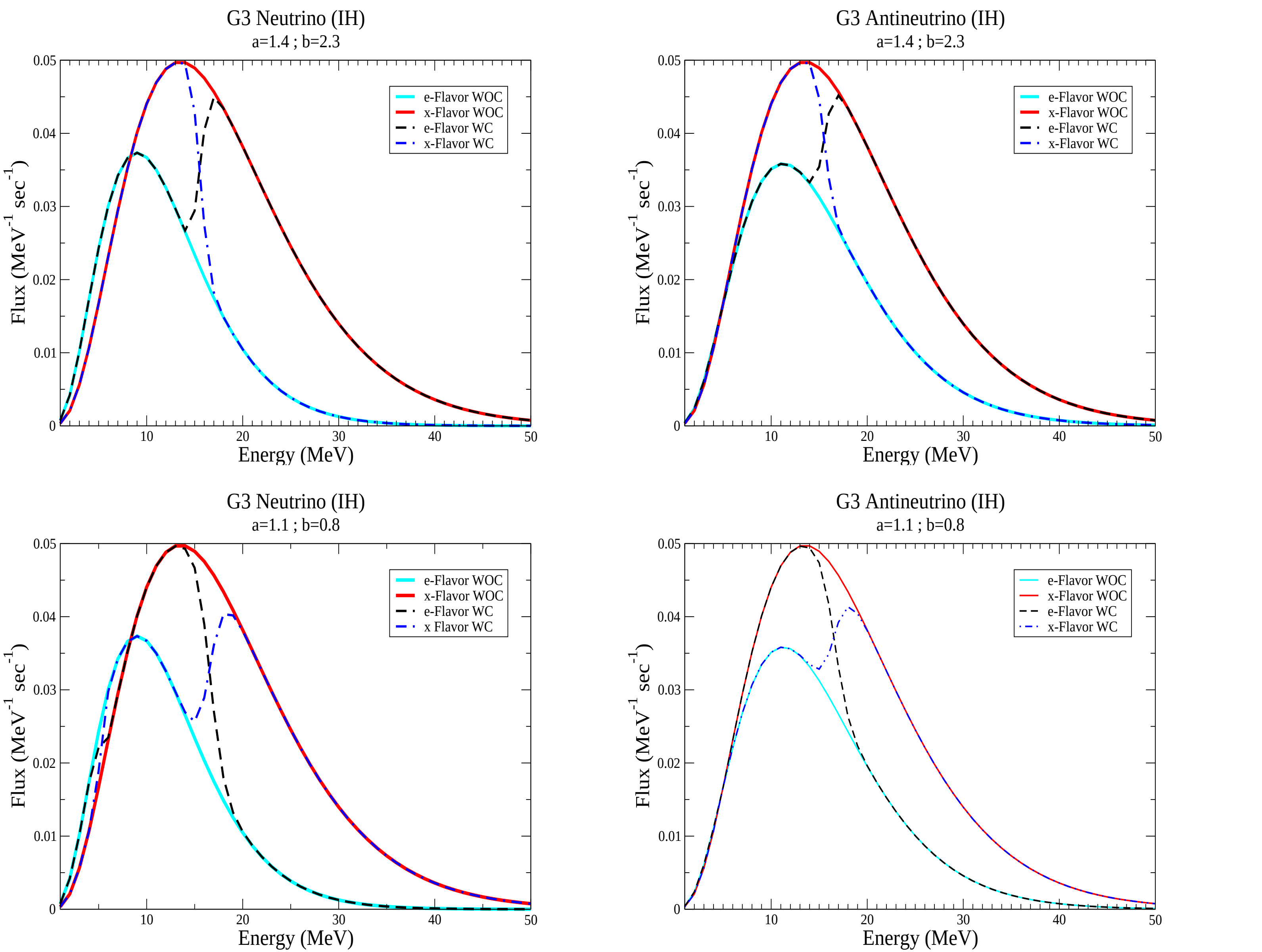
For Inverted Hierarchy



For Inverted Hierarchy



Flux (Inverted Hierarchy)



- 4. r-Process Nucleosynthesis and Collective Effects

r-Process Nucleosynthesis

- Heavy n-rich nuclei beyond Fe synthesised by n-capture

  Recent review: Qian,Prog Part Nucl Phys 50(2003), Arnould *et al* Phys Rep 450 (2007)

- Rapid(r-) Process: $t_{n,\gamma} << t_\beta$
- Possible site $\rightarrow$ "ν-driven wind" and the "hot bubble" in SN
- Heating by ν-driven wind coming from ν-sphere

$$\nu_e + n \rightarrow e^- + p \quad ; \quad \bar{\nu}_e + p \rightarrow e^+ + n$$

- Important quantity is Electron Fraction (Y_e)

$$Y_e = \frac{\text{no.of electrons}}{\text{no. of baryons}}$$

Its evolution needs to be studied

Pastor and Raffelt (2002); Balantekin and Yuksel (2005)

The variation of Y_e

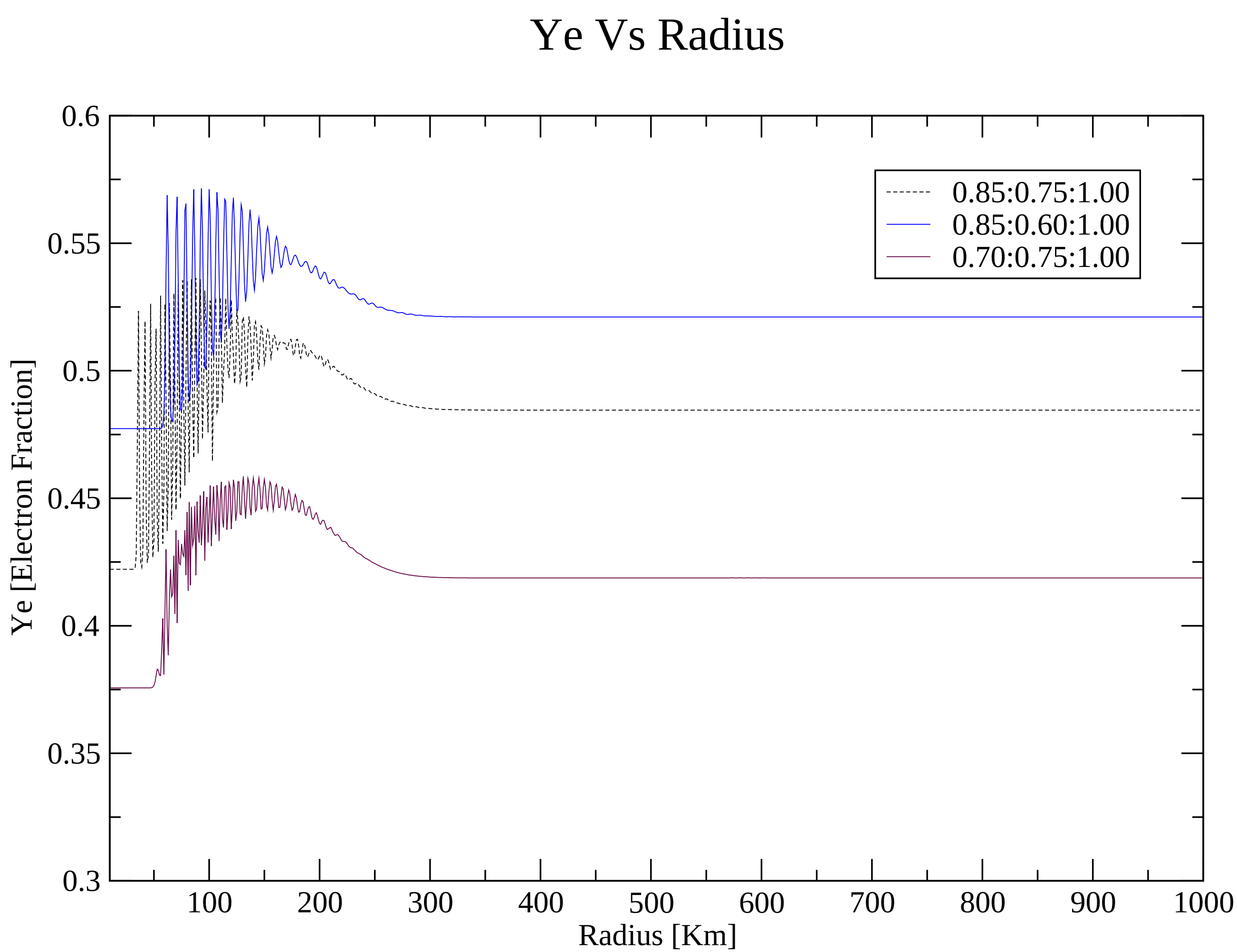
- To be n-rich minimal condition Y_e <0.5
- Better n-rich condition Y_e < 0.45 or 0.40
- Limiting Cases
  - No ν-ν interaction, no matter effect, for FD energy dist.

$$Y_e = \frac{1}{1 + T_{\bar{\nu}_e}/T_{\nu_e}}$$

    Y_e=0.4 for our choice of temperatures
  - Limit of complete conversion of ν and $\bar{\nu}$ gives Y_e=0.5
- We have varied the luminosities to study the effect of multiple spectral splits on Y_e

Y_e with Collective Effect for IH

Y_e initially has a fast oscillation due to bipolar collective effects



Comments

- Note that for SN MSW resonance occurs at very large value of 'r' and hence not included
- If matter has also alpha particles (strongly bound, no interaction with ν or $\bar{\nu}$) then

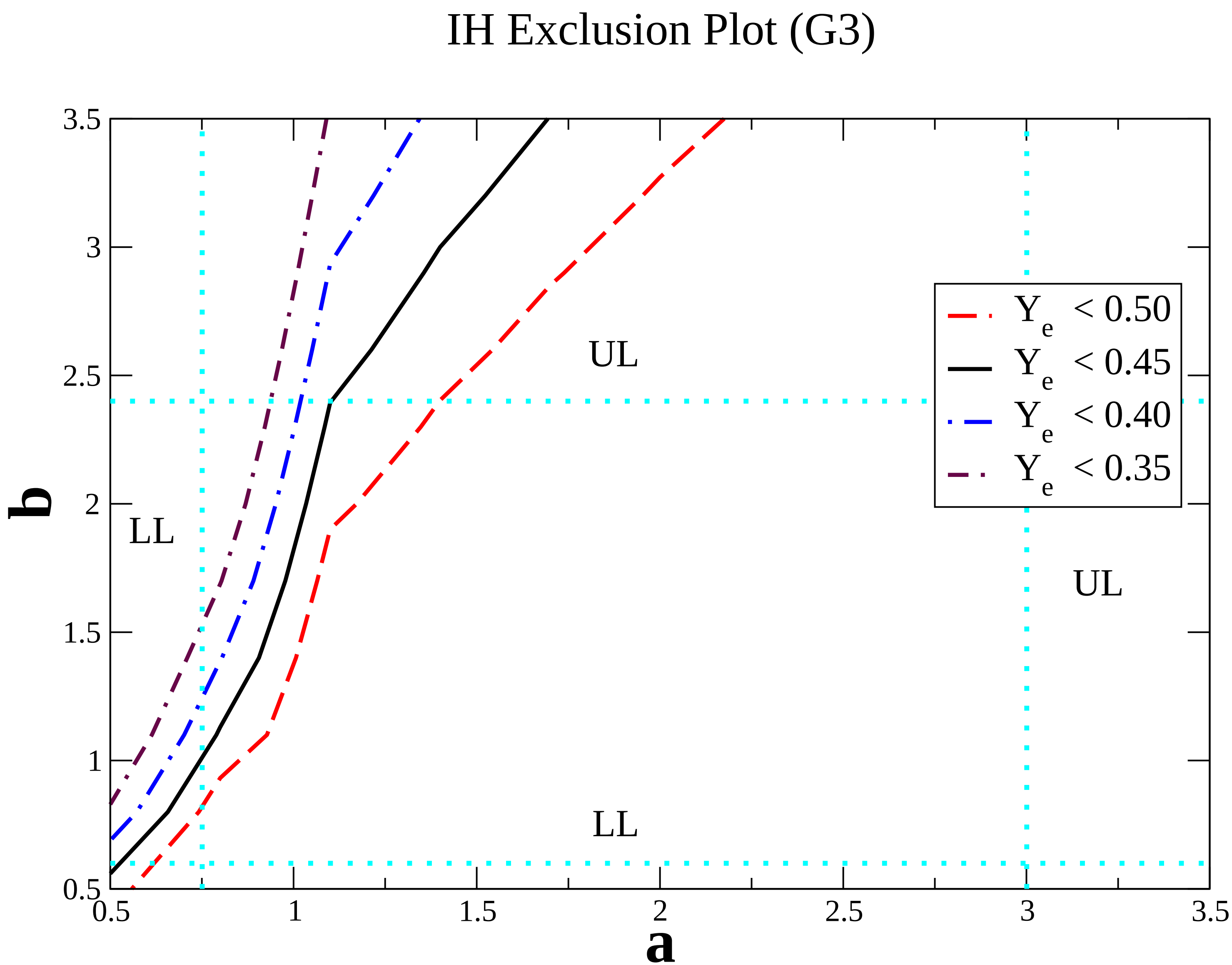
Balantekin and Yuksel, 2005

$$Y_e = Y_e^{(0)} + (1/2 - Y_e^0) X_\alpha$$

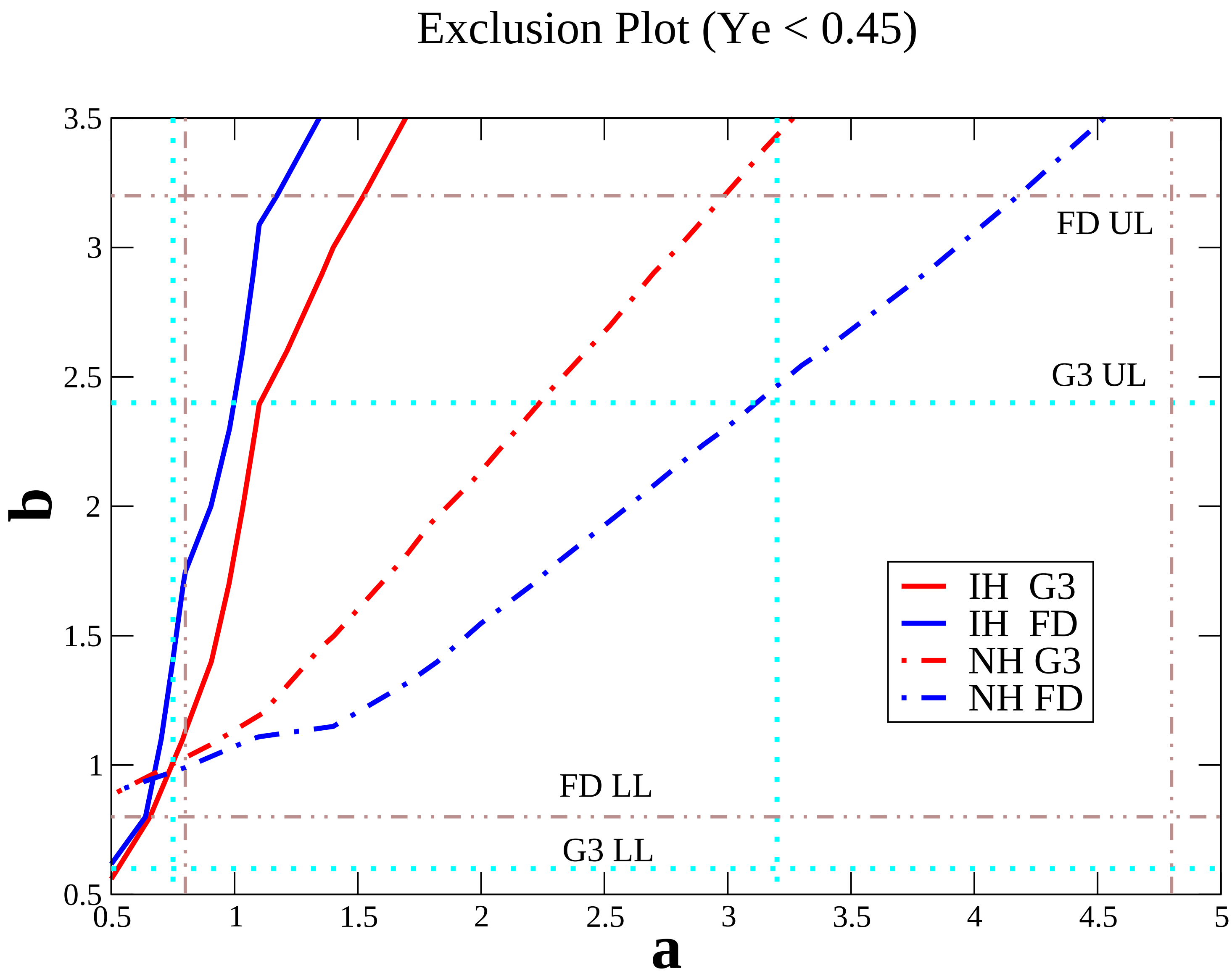
where $Y^{(0)}$ is electron fraction without α and X_α is the alpha fraction

- Of course high entropy and other considerations needed to make r-process viable

Exclusion Plot



Exclusion Plot



Summary

- The neutrino flux coming out of core collapse SN depend strongly on the collective effects produced due to $\nu - \nu$ interaction
- The collective effects depend sensitively on the spectra as well as the hierarchy
- Effects of bipolar collective oscillation seen in Y_e
- Allowed regions in the luminosities to give n-rich matter are plotted
- Effect of spectral splits on Y_e studied- NH gives more allowed region than IH

THANK YOU