The Magic of Four Zero Neutrino Yukawa Textures

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TALK PLAN

- Four zero textures and Motivation
- $\mu\tau$ symmetry, tribimaximal mixing and their effects
- $l_\alpha \to l_\beta \gamma$ and leptogenesis
- Phenomenology
- Deviations from RG running
- Conclusion

Four zero textures and Motivation

Type-I See-Saw

Lagrangian mass term

$$-\mathscr{L}^m = \bar{l}_L (M_\ell)_{ll'} l'_R + \bar{\nu}_{lL} (m_D)_{ll'} N_{l'R} + \frac{1}{2} \bar{N}^c_{lL} (M_R)_{ll'} N_{l'R} + h.c.$$

Neutrino mass matrix

$$-\mathscr{L}^m_\nu = \frac{1}{2} \begin{pmatrix} \bar{\nu}_L & \bar{N}^c_L \end{pmatrix}_l \begin{pmatrix} 0 & m_D \\ m_D^T & M_R \end{pmatrix}_{ll'} \begin{pmatrix} \nu^c_R \\ N_R \end{pmatrix}_{l'} + h.c.$$

for $O(M_R) \gg O(m_D) \Longrightarrow$

$$-\mathscr{L}^m_{\nu_L} = \frac{1}{2} \bar{\nu}_{lL} (M_\nu)_{ll'} \nu^c_{lR} + h.c.$$

$M_\nu \simeq -m_D M_R^{-1} m_D^T \quad (\mathrm{See - Saw\ formula})$.

Without loss of generality choose basis: Diagonal M_ℓ and M_R with real positive entries.

M_ν Diagonalisation:

$$U^\dagger M_\nu U^* = M_d^\nu = \mathrm{diag}(m_1,\ m_2,\ m_3)$$

m_i's Real Positive.

Relation of flavor basis to mass basis: $(\nu_L)_l = (U)_{li}\,\nu_{iL}$

Charged current interaction in mass basis:

$$\mathscr{L}_l^{cc} = (-g/\sqrt{2})\bar{l}_{iL}\gamma^\mu U_{ij}\nu_{Lj}W_\mu^-$$

U Parametrization:

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & -s_{13}e^{-i\delta_D} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_D} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_D} & -s_{23}c_{13} \\ -s_{12}s_{23} + c_{12}c_{23}s_{13}e^{i\delta_D} & c_{12}s_{23} + s_{12}c_{23}s_{13}e^{i\delta_D} & c_{23}c_{13} \end{pmatrix} \times \begin{pmatrix} e^{i\alpha_M} & 0 & 0 \\ 0 & e^{i\beta_M} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

= **PMNS** $\times$ Majorana Phase Matrix

$c_{ij} \equiv \cos\theta_{ij}, \quad s_{ij} \equiv \sin\theta_{ij}$

$\delta_D =$ Dirac phase

$\alpha_M, \beta_M =$ Majorana phases

In our chosen basis :

$$M_\ell = \begin{pmatrix} m_e & 0 & 0 \\ 0 & m_\mu & 0 \\ 0 & 0 & m_\tau \end{pmatrix} \qquad M_R = \begin{pmatrix} M_1 & 0 & 0 \\ 0 & M_2 & 0 \\ 0 & 0 & M_3 \end{pmatrix}$$

and in this basis

$$m_D = \begin{pmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{pmatrix}$$

with complex elements.

M_ν from See-Saw

$$M_\nu = -m_D M_R^{-1} m_D^T = -\begin{pmatrix} \frac{a_1^2}{M_1} + \frac{a_2^2}{M_2} + \frac{a_3^2}{M_3} & \frac{a_1b_1}{M_1} + \frac{a_2b_2}{M_2} + \frac{a_3b_3}{M_3} & \frac{a_1c_1}{M_1} + \frac{a_2c_2}{M_2} + \frac{a_3c_3}{M_3} \\ \frac{a_1b_1}{M_1} + \frac{a_2b_2}{M_2} + \frac{a_3b3}{M_3} & \frac{b_1^2}{M_1} + \frac{b_2^2}{M_2} + \frac{b_3^2}{M_3} & \frac{b_1c_1}{M_1} + \frac{b_2c_2}{M_2} + \frac{b_3c_3}{M_3} \\ \frac{a_1c_1}{M_1} + \frac{a_2c_2}{M_2} + \frac{a_3c_3}{M_3} & \frac{b_1c_1}{M_1} + \frac{b_2c_2}{M_2} + \frac{b_3c_3}{M_3} & \frac{c_1^2}{M_1} + \frac{c_2^2}{M_2} + \frac{c_3^2}{M_3} \end{pmatrix}$$

Experimental Facts:

- $|R| = \frac{\Delta m^2_{21}}{|\Delta m^2_{32}|} \simeq 3.2 \times 10^{-2}$
- $\theta_{23} \simeq \frac{\pi}{4}$, $\theta_{12} \simeq \sin^{-1}\frac{1}{\sqrt{3}}$, θ_{13} small
- No family of neutrino is decoupled

Additional inputs: $m_i \neq 0$ and all M_i's are large (like 10^9 Gev or more)

$$\det M_\nu \neq 0 \neq \det m_D$$

Texture zeros assumed in the Lagrangian, i.e. in m_D

Maximum number of zeros allowed in m_D with above constraints $= 4$

(G.C. Branco, D.E.-Costa, M.N. Rebelo and P. Roy, PRD 08)

All four zero textures have been classified. 72 allowed textures

One important property of these:

High Scale CP Violation completely determined by the Low Energy CP Violation

72 textures categorised into two

Category **A:** 54 Textures:

- 18 Textures, First row orthogonal to second row, $(M_\nu)_{12} = (M_\nu)_{21} = 0$
- 18 Textures, First row orthogonal to third row, $(M_\nu)_{13} = (M_\nu)_{31} = 0$
- 18 Textures, Second row orthogonal to third row, $(M_\nu)_{23} = (M_\nu)_{32} = 0$

Category B: Textures with two zeros in one row and one each in the rest

- 6 Textures, First row with two zeros
- 6 Textures, Second row with two zeros
- 6 Textures, Third row with two zeros

If the rows with one zero each are k, l then det cofactor $[(M_\nu)_{kl}] = 0$

In addition for each textures in B: $(M_\nu)_{ll'} \neq 0$

$\mu\tau$ symmetry, tribimaximal mixing and the consequences

Under $\mu\tau$ symmetry,

$$\nu_\mu \longleftrightarrow \nu_\tau,\ N_\mu \longleftrightarrow N_\tau : \quad m_D = \begin{pmatrix} a_1 & a_2 & a_2 \\ b_1 & b_2 & b_3 \\ b_1 & b_3 & b_2 \end{pmatrix}, \quad M_\nu = \begin{pmatrix} M_1 & 0 & 0 \\ 0 & M_2 & 0 \\ 0 & 0 & M_2 \end{pmatrix}.$$

Custodial $\mu\tau$ symmetry in M_ν:

$$M_\nu = \begin{pmatrix} A & B & B \\ B & C & D \\ B & D & C \end{pmatrix} \Longrightarrow \quad h = M_\nu M_\nu^\dagger = \begin{pmatrix} P & Q & Q \\ Q^* & R & S \\ Q^* & S & R \end{pmatrix}$$

where

$$\begin{aligned} P &= |A|^2 + 2|B|^2, \qquad Q = A^*B + B^*(C+D) \\ R &= |B|^2 + |C|^2 + |D|^2, \qquad S = |B|^2 + CD^* + DC^*. \end{aligned}$$

Apply to four zero textures.

Category A:

- $\mu\tau$ symmetry in Lagrangian is incompatible with only 4 zeros in m_D for 52 textures here
- In the remaining 2 textures $\mu\tau$ symmetry can be fitted

$$m_D^{(1)} = \begin{pmatrix} a_1 & a_2 & a_2 \\ 0 & b_2 & 0 \\ 0 & 0 & b_2 \end{pmatrix}, \qquad m_D^{(2)} = \begin{pmatrix} a_1 & a_2 & a_2 \\ 0 & 0 & b_2 \\ 0 & b_2 & 0 \end{pmatrix} \qquad M_R = \begin{pmatrix} M_1 & 0 & 0 \\ 0 & M_2 & 0 \\ 0 & 0 & M_2 \end{pmatrix}$$

See-Saw formula$\Longrightarrow$

$$M_\nu = -\begin{pmatrix} a_1^2/M_1 + 2a_2^2/M_2 & a_2 b_2/M_2 & a_2 b_2/M_2 \\ a_2 b_2/M_2 & b_2^2/M_2 & 0 \\ a_2 b_2/M_2 & 0 & b_2^2/M_2 \end{pmatrix},$$

for both.

Under parametrization

$$m_3 = -\frac{b_2^2}{M_2}, \qquad \sqrt{\frac{M_2}{M_1}} \times \frac{a_1}{b_2} = k_1 e^{i(\alpha+\alpha')} \qquad \frac{a_2}{b_2} = k_2 e^{i\alpha'}$$

M_ν after phase factor absorption $e^{i\alpha'}$ by ν_e :

$$M_\nu = m_3 \begin{pmatrix} k_1^2 e^{2i\alpha} + 2k_2^2 & k_2 & k_2 \\ k_2 & 1 & 0 \\ k_2 & 0 & 1 \end{pmatrix}.$$

3 real parameters $k_1,\ k_2,\ \alpha$

Further, tribimaximal mixing assumption, i.e. $\theta_{12} = \sin^{-1}(1/\sqrt{3}) \Longrightarrow$

$$A + B = C + D$$

$$\Longrightarrow \qquad \alpha = \pi/2 \qquad k_1 = (2k_2^2 + k_2 - 1)^{1/2}.$$

One real parameter.

Category B:

Again, 16 textures here with only four zeros in m_D are incompatible with $\mu\tau$ symmetry. Hence ruled out

In remaining two textures $\mu\tau$ Symmetry imposed without ambiguity

$$m_D = \begin{pmatrix} a_1 & 0 & 0 \\ b_1 & b_2 & 0 \\ b_1 & 0 & b_2 \end{pmatrix}, \qquad \begin{pmatrix} a_1 & 0 & 0 \\ b_1 & 0 & b_2 \\ b_1 & b_2 & 0 \end{pmatrix} \qquad M_R = \begin{pmatrix} M_1 & 0 & 0 \\ 0 & M_2 & 0 \\ 0 & 0 & M_2 \end{pmatrix}$$

See-Saw formula$\Longrightarrow$

$$M_\nu = -\begin{pmatrix} a_1^2/M_1 & a_1b_1/M_1 & a_1b_1/M_1 \\ a_1b_1/M_1 & b_1^2/M_1 + b_2^2/M_2 & b_1^2/M_1 \\ a_1b_1/M_1 & b_1^2/M_1 & b_1^2/M_1 + b_2^2/M_2 \end{pmatrix},$$

for both.

Under parametrization

$$m_3 = -\frac{b_2^2}{M_2}, \qquad \sqrt{\frac{M_2}{M_1}} \times \frac{a_1}{b_2} = l_1 e^{i\beta'} \qquad \sqrt{\frac{M_2}{M_1}} \times \frac{b_1}{b_2} = l_2 e^{i\beta}$$

Absorbing phase factor $e^{i\beta'}$ in ν_e:

$$M_\nu = m_3 \begin{pmatrix} l_1^2 & l_1 l_2 e^{i\beta} & l_1 l_2 e^{i\beta} \\ l_1 l_2 e^{i\beta} & l_2^2 e^{2i\beta} + 1 & l_2^2 e^{2i\beta} \\ l_1 l_2 e^{i\beta} & l_2^2 e^{2i\beta} & l_2^2 e^{2i\beta} + 1 \end{pmatrix}.$$

Again, 3 real parameters $l_1, \; l_2, \; \beta$.

Now, additional TBM assumption $\theta_{12} \; = \; \sin^{-1}(1/\sqrt{3}) \Rightarrow$

$$\beta = \cos^{-1}(l_1/4l_2),$$
$$l_2 = \frac{1}{2}(1 - l_1^2)^{1/2}.$$

One real parameter.

Phenomenology

Diagonlisation of $\mu\tau$ symmetric $h = M_\nu M_\nu^\dagger \Longrightarrow$

$$\tan 2\theta_{12} = \frac{2\sqrt{2}|Q|}{R+S-P} \quad \text{along with } \theta_{23} = \frac{\pi}{4},\ \theta_{13} = 0$$

Solar mass squared difference

$$\Delta m^2_{21} \ = \ m_2^2 - m_1^2 = \sqrt{(R+S-P)^2 + 8|Q|^2}$$

Atmospheric mass squared difference

$$\Delta m^2_{32} \ = \ m_3^2 - m_2^2 = \frac{R - 3S - P - \sqrt{(R+S-P)^2 + 8|Q|^2}}{2}$$

$$> 0 \qquad \text{(normal mass ordering)}$$

$$< 0 \qquad \text{(inverted mass ordering)}$$

Category **A** with $\mu\tau$ symmetry

$$\tan 2\theta_{12} = \frac{X_1}{X_2}$$
$$R = \frac{\Delta m^2_{21}}{\Delta m^2_{32}} = \frac{2\left(X_1^2 + X_2^2\right)^{1/2}}{X_3 - \left(X_1^2 + X_2^2\right)^{1/2}}$$

where,

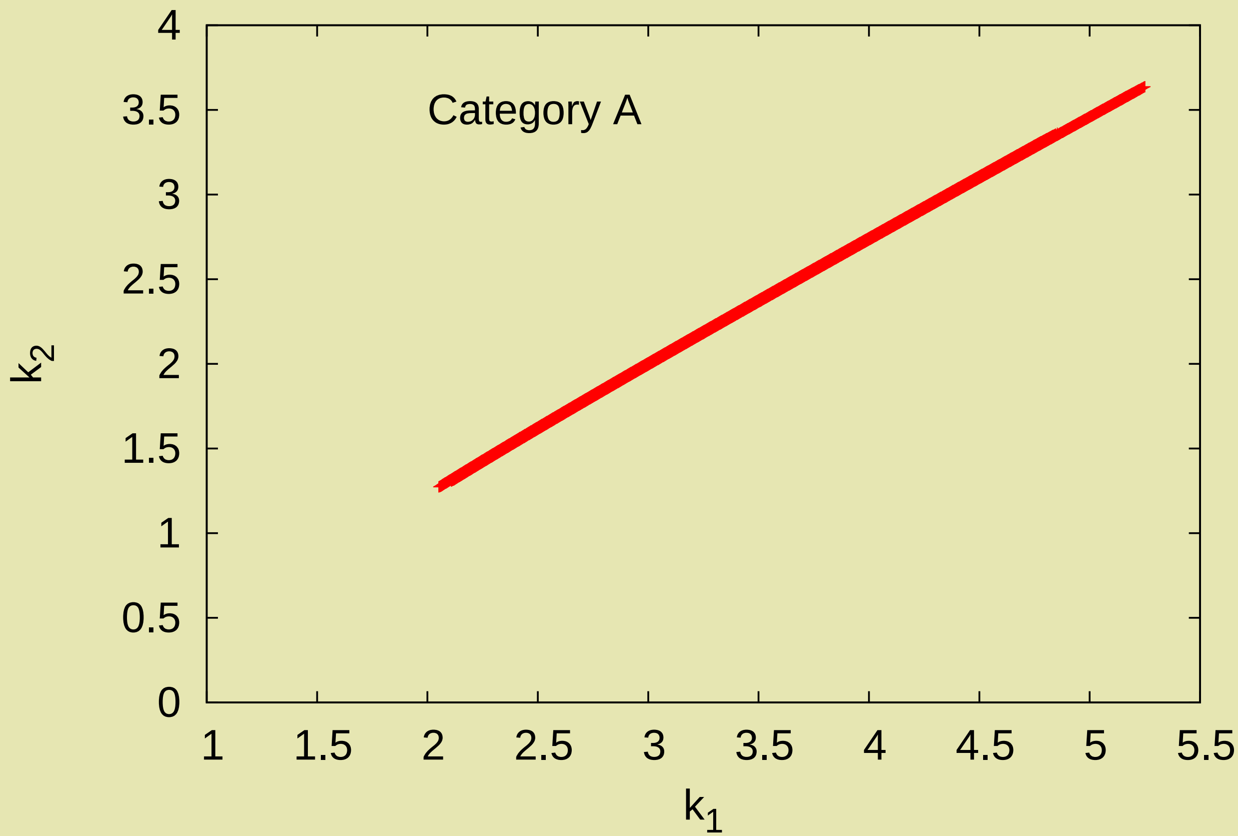
$$X_1 = 2\sqrt{2}k_2\sqrt{(2k_2^2+1)^2 + 2k_1^2(2k_2^2+1)\cos 2\alpha + k_1^4}$$
$$X_2 = 1 - 4k_2^4 - k_1^4 - 4k_1^2k_2^2\cos 2\alpha$$
$$X_3 = 1 - 4k_2^4 - k_1^4 - 4k_1^2k_2^2\cos 2\alpha - 4k_2^2.$$

TBM $\Longrightarrow$ $R = \frac{3(k_2-2)}{k_2+1}$

Only inverted ordered mass spectrum is allowed in this category: a result originally due to Merle, Rodejohann, PRD06

There is no common parameter space $(k_1,\ k_2,\ \alpha)$ for 1σ range of $R \times 10^2 = -(2.88 - 3.37)$ and $\theta_{12} = 33.15^\circ - 35.91^\circ$

3D allowed common parameter space $(k_1,\ k_2,\ \alpha)$: For $R \times 10^2 = -(2.46 - 3.99)(3\sigma)$ and $\theta_{12} = 30.66^\circ - 39.23^\circ(3\sigma)$ shown in projected $(k_1,\ k_2)$plane.


Category A
4
3.5
3
2.5
2
1.5
1
0.5
0
1
1.5
2
2.5
3
3.5
4
4.5
5
5.5
k1
k2

Restrictions on parameter space

- $89^\circ \leq \alpha \leq 90^\circ$
- $2.0 < k_1 < 5.3$
- $1.2 < k_2 < 3.7$
- Only inverted mass ordering allowed

TBM $\Longrightarrow$ $1.95 \leq k_2 \leq 1.97$ at the 3σ level

Category B:

$\mu\tau$ symmetry again $\Rightarrow$

$$\tan 2\theta_{12} = \frac{X_1}{X_2}$$

$$R = \frac{\Delta m_{21}^2}{\Delta m_{32}^2} = \frac{2\left(X_1^2 + X_2^2\right)^{1/2}}{X_3 - (X_1^2 + X_2^2)^{1/2}}$$

where,

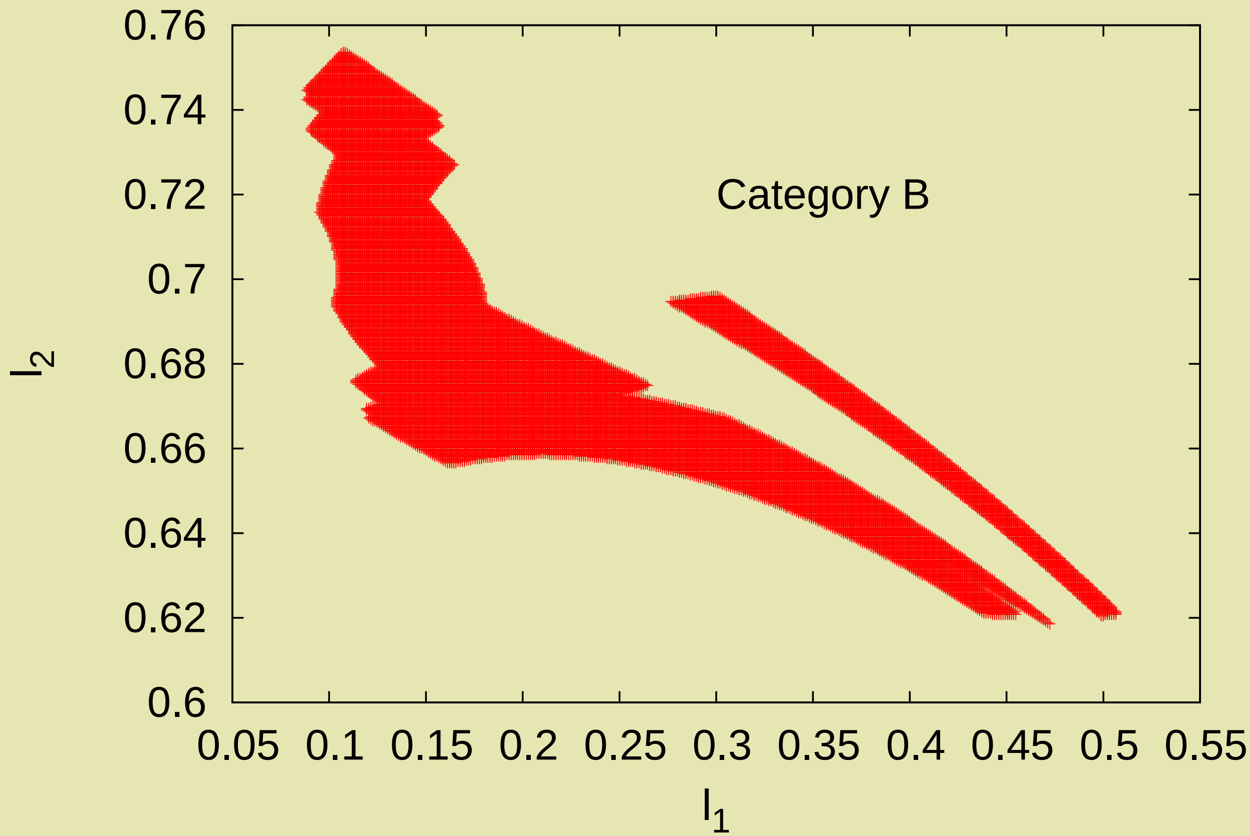
$$X_1 = 2\sqrt{2}k_2\sqrt{(2l_2^2 + l_1^2)^2 + 2(2l_2^2 + l_1^2)\cos 2\beta + 1}$$
$$X_2 = 4l_2^4 - l_1^4 + 4l_2^2 \cos 2\beta + 1$$
$$X_3 = 1 - (2l_2^2 + l_1^2)^2 - 4l_2^2 \cos 2\beta.$$

TBM $\Longrightarrow$

$$R = \frac{3l_1^2}{2 - 4l_1^2}$$

Only normal ordered mass spectrum is allowed in this category

3D allowed parameter space $(l_1,\ l_2,\ \beta)$: For $R \times 10^2 = 2.52 - 4.07(3\sigma)$ and $\theta_{12} = 30.66^\circ - 39.23^\circ$ (3σ) Shown in projected $(l_1,\ l_2)$ plane



Category B
0.76
0.74
0.72
0.7
0.68
0.66
0.64
0.62
0.6
l_2
0.05 0.1 0.15 0.2 0.25 0.3 0.35 0.4 0.45 0.5 0.55
l_1

Restrictions on parameter space

- $87^\circ \leq \beta \leq 90^\circ$
- $0.1 < l_1 < 0.55$
- $0.6 < l_2 < 0.76$
- Only normal ordered mass spectrum allowed

TBM $\Longrightarrow$ $0.11 \leq l_1 \leq 0.15$ at the 3σ level

Radiative lepton decay: $l_\alpha \to l_\beta \gamma$

$$l_\alpha \to l_\beta\gamma \qquad \alpha > \beta : l_1 = e, \quad l_2 = \mu, \quad l_3 = \tau$$

mSUGRA scenarios with universal scalar masses at high scale M_X(∼ 2×10^{16}**GeV**)⇒

$$\mathrm{BR}(l_\alpha \to l_\beta\gamma) \propto \mathrm{BR}(l_\alpha \to l_\beta\nu\bar{\nu}) \left|(m_D)_{\alpha i}(m_D)^*_{\beta i} \ln \frac{M_X}{M_i}\right|$$

M_i = mass of i-th heavy right chiral neutrino

Category **A**:

Allowed two textures→ $(m_\nu)_{23} = 0$⇒ BR($\tau \to \mu\gamma = 0$)

In both Category **A** and Category **B**:

$(m_\nu)_{13} = (m_\nu)_{12} \neq 0$⇒ BR($\tau \to e\gamma$)$\neq 0 \neq$BR($\mu \to e\gamma$)

$$\frac{\mathrm{BR}(\tau \to e\gamma)}{\mathrm{BR}(\mu \to e\gamma)} \simeq \frac{\mathrm{BR}(\tau \to e\nu_e\bar{\nu}_e)}{\mathrm{BR}(\mu \to e\nu_\mu\bar{\nu}_\mu)} \simeq 0.178.$$

Leptogenesis

Flavor dependent lepton asymmetry:

$$\epsilon_i^\alpha \equiv \frac{\Gamma(N_i \to \phi \bar{l}_\alpha) - \Gamma(N_i \to \phi^\dagger l_\alpha)}{\Sigma_\beta[\Gamma(N_i \to \phi \bar{l}_\beta) + \Gamma(N_i \to \phi^\dagger l_\beta)]}$$

$$\propto \sum_{j \neq i} \left[\mathcal{I}_{ij}^\alpha f\left(\frac{M_j^2}{M_i^2}\right) + \mathcal{J}_{ij}^\alpha \left(1 - \frac{M_j^2}{M_i^2}\right)^{-1} \right] + O(M_W^2/M_i^2)$$

$$\mathcal{I}_{ij}^\alpha = \mathrm{Im}(m_D^\dagger)_{i\alpha}(m_D)_{\alpha j}(m_D^\dagger m_D)_{ij} = -\mathcal{I}_{ji}^\alpha$$

$$\mathcal{J}_{ij}^\alpha = \mathrm{Im}(m_D^\dagger)_{i\alpha}(m_D)_{\alpha j}(m_D^\dagger m_D)_{ji} = -\mathcal{J}_{ji}^\alpha$$

$$f(x) = \sqrt{x}\left[\frac{2}{1-x} - \ln\frac{1+x}{x}\right] \quad (\mathrm{MSSM})$$

Flavor independent leptogenesis

$$\epsilon_i = \sum_\alpha \epsilon_i^\alpha \propto \sum_{j\neq i} \left[(m_D^\dagger m_D)_{ij}\right]^2 f\left(M_j^2/M_i^2\right)$$

For $M_1 \ll M_{2,3}$, $f(M_{2,3}^2/M_1^2) \neq -3M_1/M_{2,3}$

Effective mass for washout of a flavor asymmetry

$$\tilde{m}_1^\alpha = |(m_D)_{\alpha 1}|^2/M_1$$

$configuration$	I_{ij}^α	J_{ij}^α	$\widetilde{m_1}^e$	$\widetilde{m_1}^\mu$	$\widetilde{m_1}^\tau$
$\mathrm{m}_D^{(1)}$	$\mathrm{I}_{12}^e = \mathcal{I}_{13}^e \neq 0$, rest zero	0	$nonzero$	0	0
$\mathrm{m}_D^{(2)}$	$--do--$	0	$nonzero$	0	0
$\mathrm{m}_D^{(3)}$	$\mathrm{I}_{12}^\mu = \mathcal{I}_{13}^\tau \neq 0$, rest zero	0	$nonzero$	$nonzero$	$equals\ \widetilde{m_1}^\mu$
$\mathrm{m}_D^{(4)}$	$\mathrm{I}_{13}^\mu = \mathcal{I}_{12}^\tau \neq 0$, rest zero	0	$nonzero$	$nonzero$	$equals\ \widetilde{m_1}^\mu$

Deviations from RG

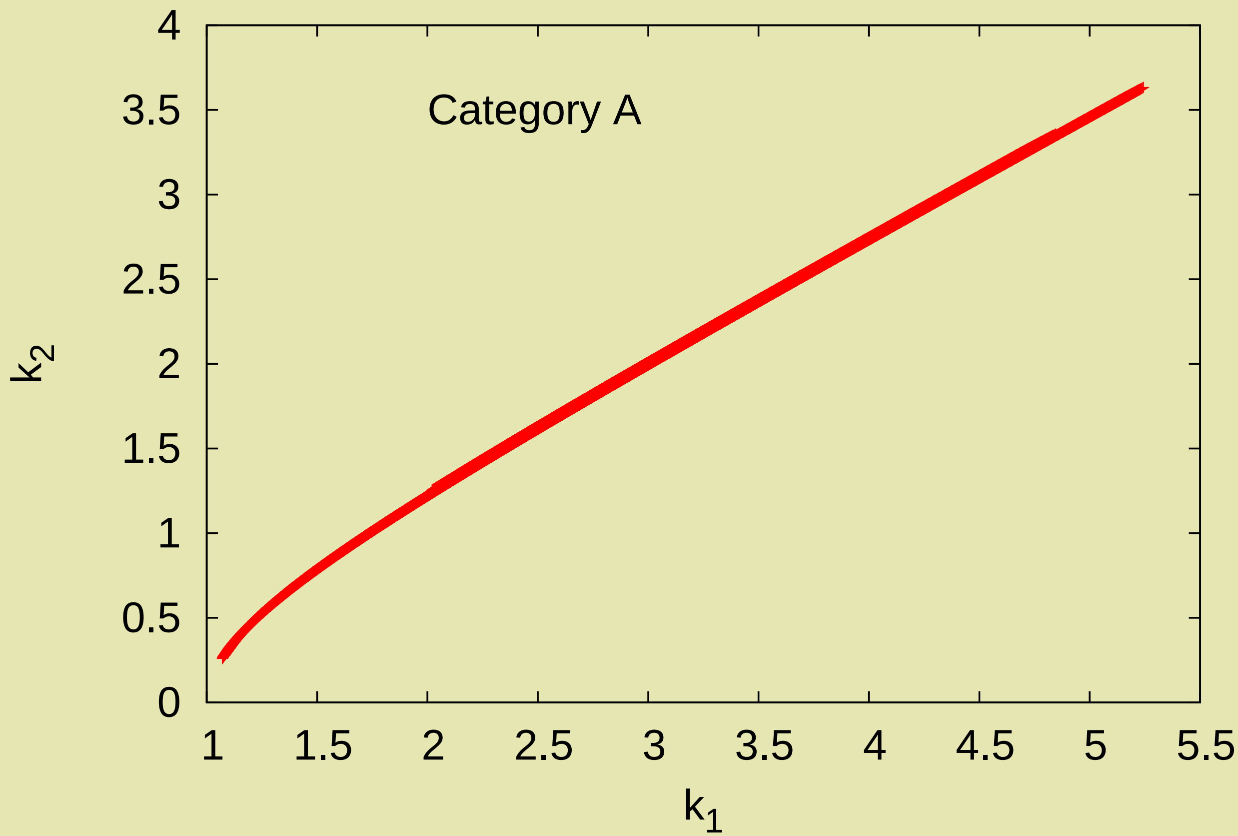
$\mu\tau$ symmetry imposed at $\Lambda \sim 10^{12}$ GeV
One loop RG running from Λ to $\lambda \sim 10^3$ GeV$\rightarrow$deviations
In MSSM using $m_\tau^2 \gg m_{e,\mu}^2$ deviation Δ_τ:

$$\Delta_\tau \simeq \frac{m_\tau^2}{8\pi^2 v^2}(\tan^2\beta + 1)\ln\left(\frac{\Lambda}{\lambda}\right),$$
$$v^2 = 2(v_u^2 + v_d^2), \quad \tan\beta = \frac{v_u}{v_d}.$$

Now

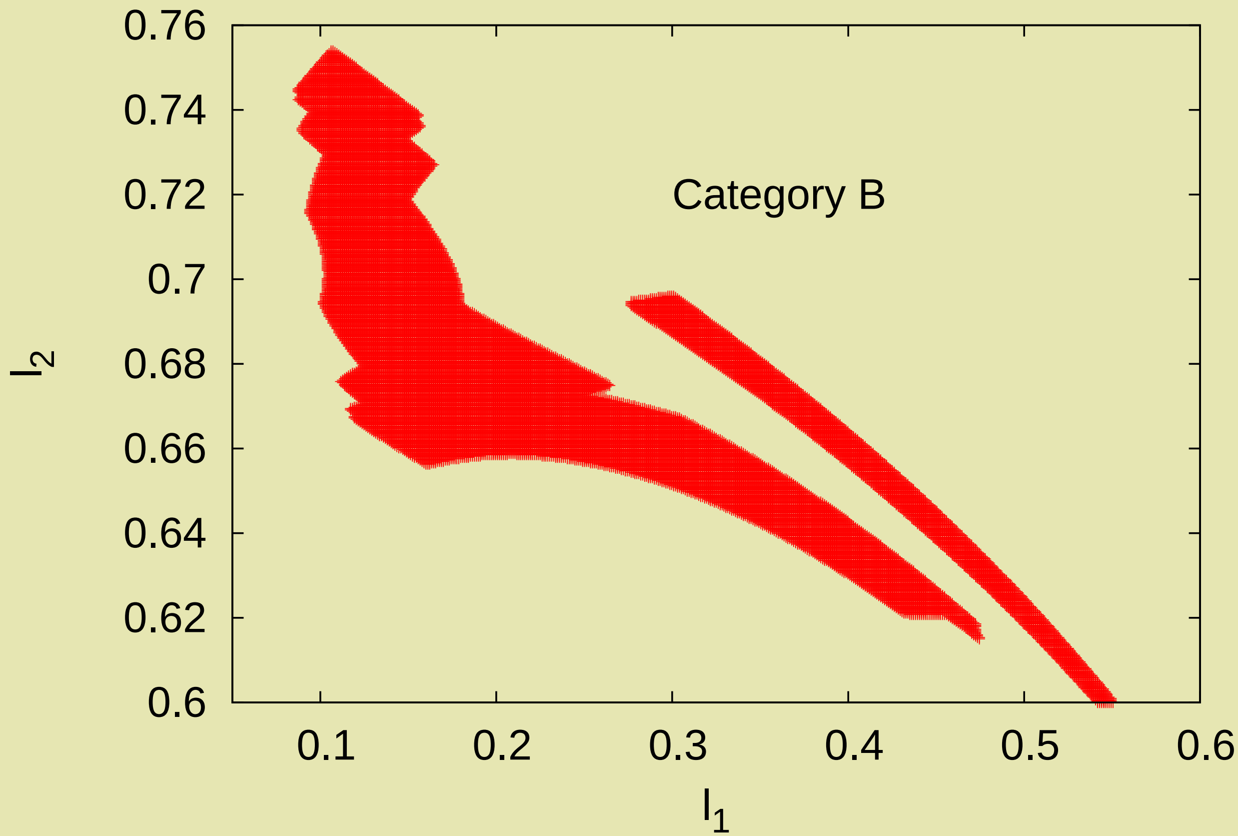
$$m_\nu^\lambda = \begin{pmatrix} A & B & B(1-\Delta_\tau) \\ B & C & D(1-\Delta_\tau) \\ B(1-\Delta_\tau) & D(1-\Delta_\tau) & C(1-2\Delta_\tau) \end{pmatrix}$$

Deviations affect the results in both categories but marginally. New parameter space are shown in figures.


Category A
k2
k1
0
0.5
1
1.5
2
2.5
3
3.5
4
1
1.5
2
2.5
3
3.5
4
4.5
5
5.5

Results in category A for 3σ variations of R and θ_{12}

- $0^\circ \leq \theta_{13}^{\lambda} \leq 2.7^\circ$
- $\theta_{23}^{\lambda} \leq 45^\circ$
- Inverted mass ordering retained


0.76
0.74
0.72
0.7
0.68
0.66
0.64
0.62
0.6
l_2
Category B
0.1
0.2
0.3
0.4
0.5
0.6
l_1

Results in category B for 3σ variations of R and θ_{12}

- $0^\circ \leq \theta_{13}^{\lambda} \leq 0.85^\circ$
- $\theta_{23}^{\lambda} \geq 45^\circ$
- Normal mass ordering retained

Conclusion

- Out of 126 four zero textures $\mu\tau$ symmetry compatible with FOUR textures only leading to only two forms of M_ν: one for category A, one for category B

- For these θ_{12} and R admit restricted regions in the parameter space and $M_\nu^{(A)}$ is in some tension with data

- Tri-bimaximal mixing farther highly restricts the parameters

- Small radiative deviations from $\mu\tau$ yield rather small θ_{13} and further restrict θ_{23}