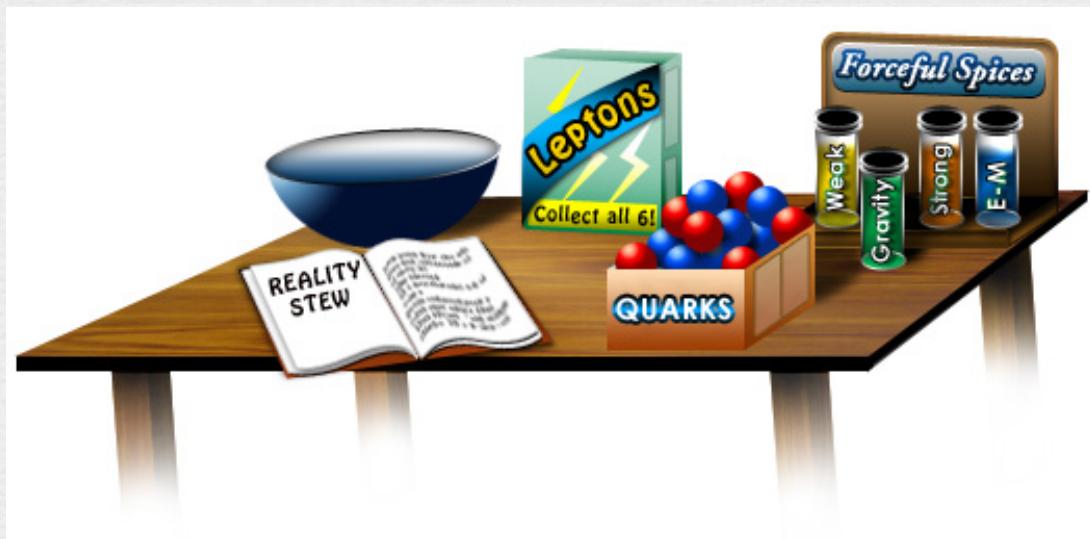


# Flavor Physics and Lattice QCD



*Damir Becirevic, LPT Orsay*

*Weak Interactions and Neutrinos - WIN'09  
Relais San Clemente, 14-19 September 2009*



# What are we after?

Sensitivity to New Physics  $\Lambda_{\text{NP}} \gg E_{\text{exp}}$

## QUESTIONS:

### ① Standard Model Flavor Puzzle

Why is there a hierarchy of small parameters?

### ② New Physics Flavor Puzzle

If the new physics is at about 1 TeV, why FCNC are so small?

### ③ What about the baryon asymmetry?

Not enough CP for baryogenesis ; flavor matters in leptogenesis

Strong CP-problem :  $\tilde{\theta} \lesssim 10^{-10}$  Vs  $\delta_{\text{CKM}} \sim 1$

# SM Flavor Physics Issues

Interaction occurring between flavor generations

- ✗ Yukawa interactions i.e. no self-interactions  
(unlike gauge and Higgs sectors)
- ✗ Flavor physics parameters :  
fermion masses and mixing parameters (angles and phases)

**Extract them by comparing theory  
expressions to experimental data**

Non-perturbative QCD ?

# Lattice QCD

We can:

- ✓ calculate hadronic properties from first principles - non-perturbative QCD
- ✓ reach an arbitrary accuracy (in principle)

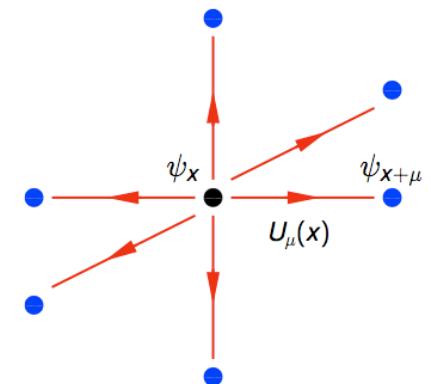
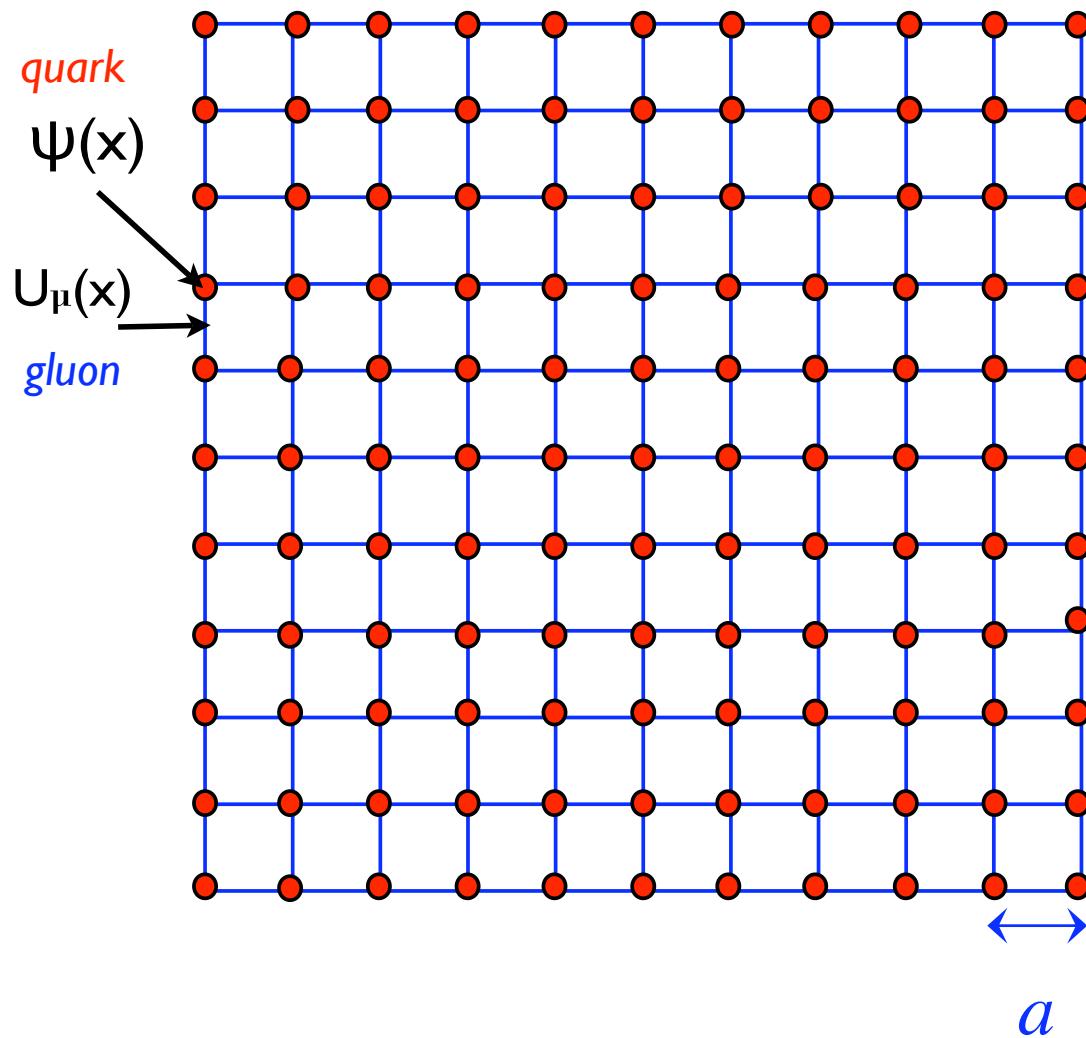
Not now but maybe...

- ? compute the strong phase (accurately)
- ? control the systematics to arbitrary accuracy

✓ hadronic matrix elements: hadron leptonic,  
semileptonic, radiative, some rare decays  
[hadron  $\rightarrow$  vacuum, hadron  $\rightarrow$  hadron]

✗ non-leptonic decays [hadron  $\rightarrow$  ( $n \geq 2$ ) hadrons]

# Lattice QCD

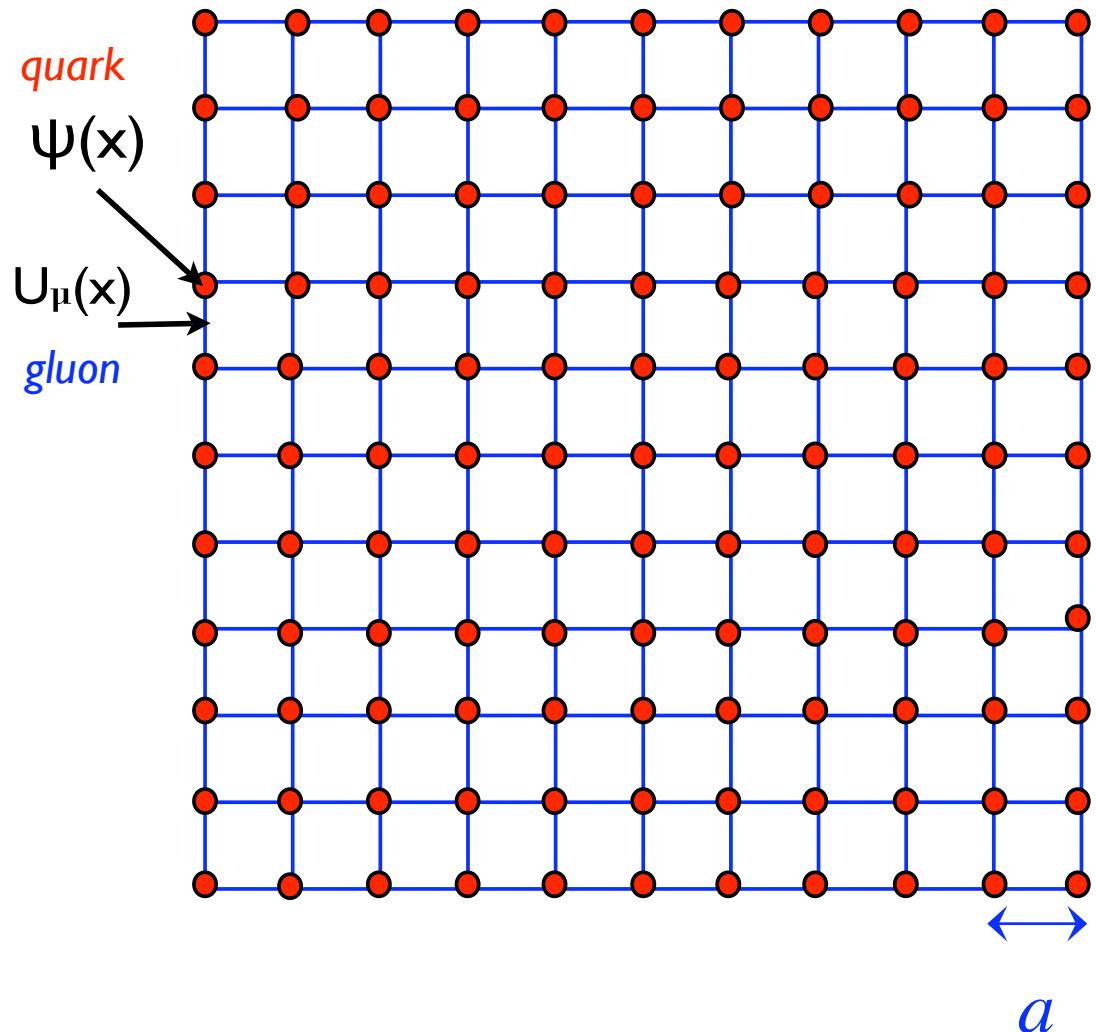


IR cutoff

Euclidean space-time

UV cutoff

# Lattice QCD



Clean (the only?) way to define QFT

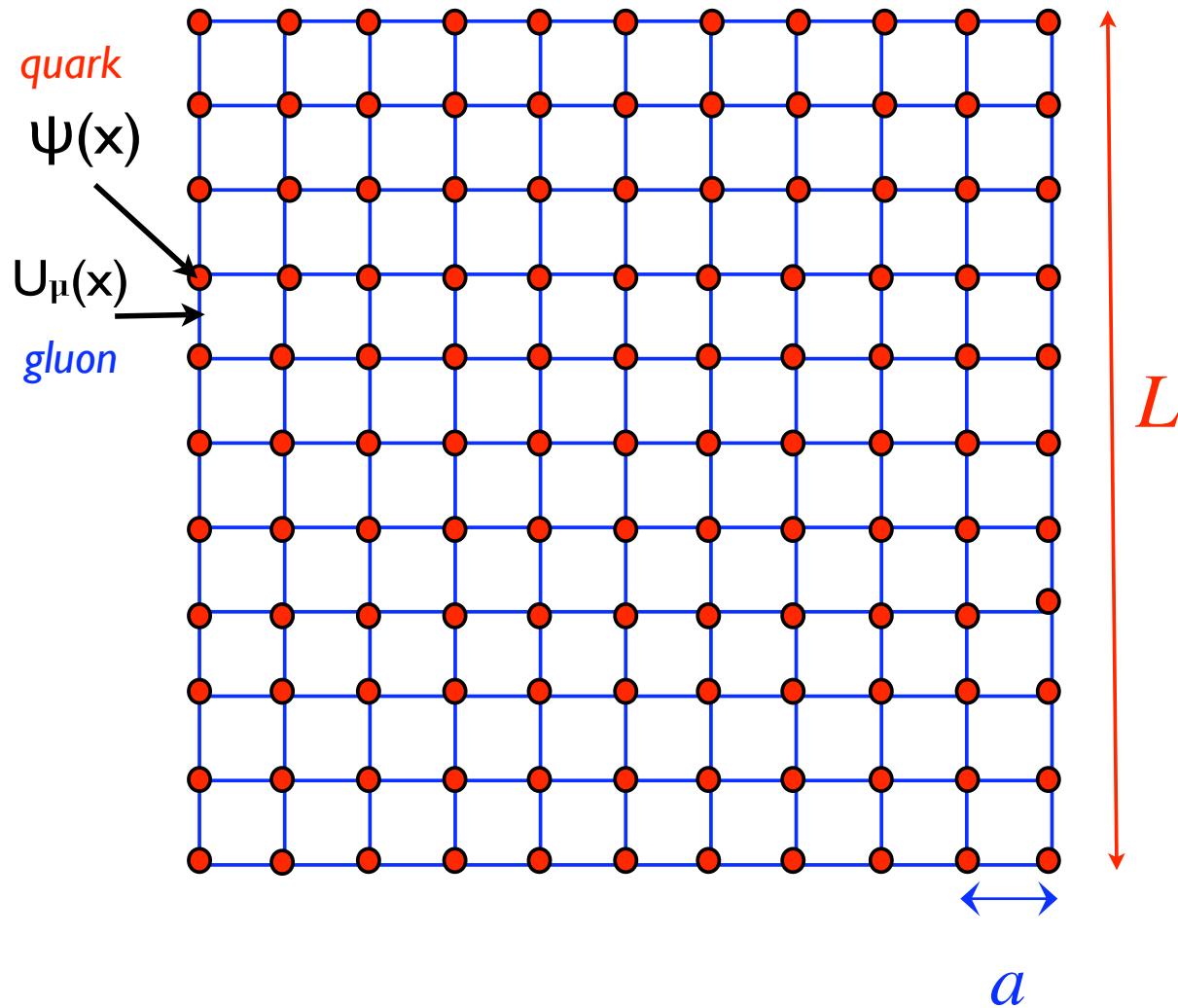
$$\mathcal{L}_{\text{YM}} \rightarrow \mathcal{L}_{LYM}$$

$$\mathcal{L}_{\text{Dirac}} \rightarrow \mathcal{L}_{\text{naive}}$$

SIC : doublers  $2^4 = 16$   
when  $a \rightarrow 0$

UV cutoff

# Lattice QCD



Get rid of the doublers :

- \* add a D'Alambert term to decouple doublers [WILSON] (breaks chiral symmetry)
  - Overlap to restore a peculiar form of chiral symmetry [expensive]
  - Domain Wall in 5th dim. to separate chiralities in 4D [need large 5th Dim, residual mass very small]
  - twisted (maximally)
- $\bar{\psi}(x)[D_\mu\gamma_\mu + i\mu\gamma_5\tau_3]\psi(x)$  to get closer to chiral limit
- \* distribute spinor components over a lattice cell :  $16 \rightarrow 4$  [staggered quarks, U(1)]

# QCD on the lattice

$$\langle \mathcal{O} \rangle = \frac{1}{\mathcal{Z}} \prod_{x,\mu} dU_\mu(x) \mathcal{O}(U) \det M(U) e^{-S_g(U)}$$

## Unquenched - extra problems

Manipulating large determinants is computationally expensive. A “bosonisation” trick is commonly used: the pseudofermion field.

For two flavours of fermions with a  $\gamma_5$ -hermian lattice representation of the Dirac matrix, we have

$$\det M^2 = \det M^\dagger M = \frac{1}{\det [M^\dagger M]^{-1}}$$

and this is represented as a gaussian integral;

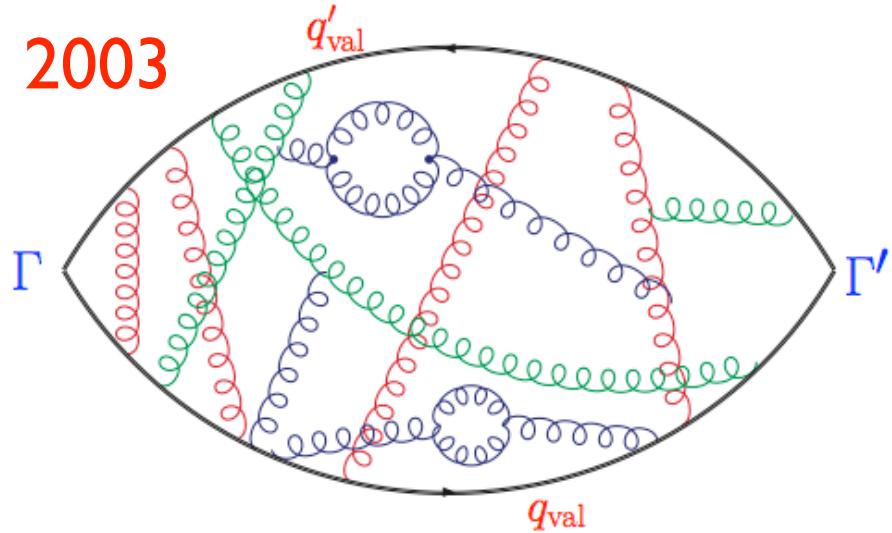
$$\frac{1}{\det [M^\dagger M]^{-1}} = \int \mathcal{D}\phi \mathcal{D}\phi^* e^{-\phi^* [M^\dagger M]^{-1} \phi}$$

The pseudofermions inherit the spin structure and colour charge.

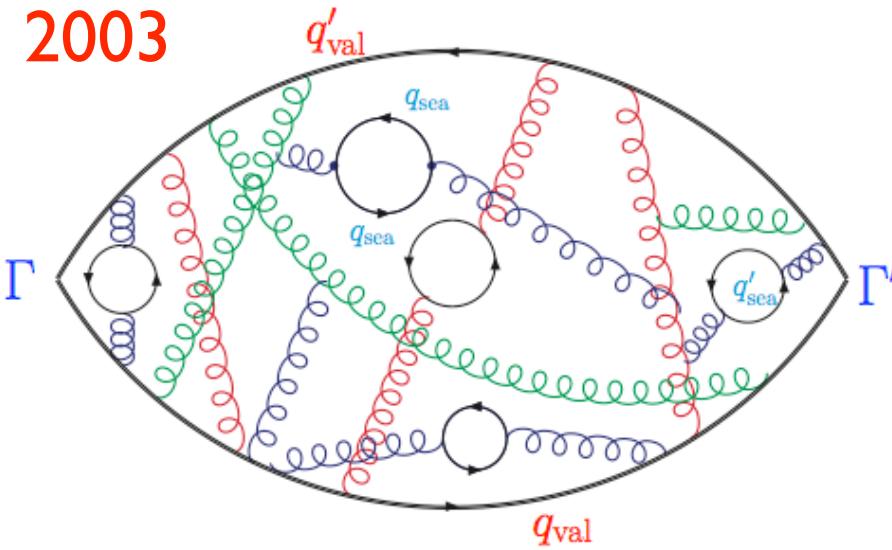
If the staggered pions were degenerate, taking the 4th root would be fine

Otherwise -- non-localities shadow of doubt in spite of correct phenomenology so far...

< 2003



> 2003



$$m_q^{\text{sea}} \rightarrow \infty$$

Quenched QCD

Partially Quenched  
QCD

$$m_q^{\text{val}} \neq m_q^{\text{sea}}$$

UNQuenched QCD

$$m_q^{\text{val}} = m_q^{\text{sea}}$$

$$\mathcal{L}_{\text{LQCD}}^{\text{val}} \neq \mathcal{L}_{\text{LQCD}}^{\text{sea}}$$

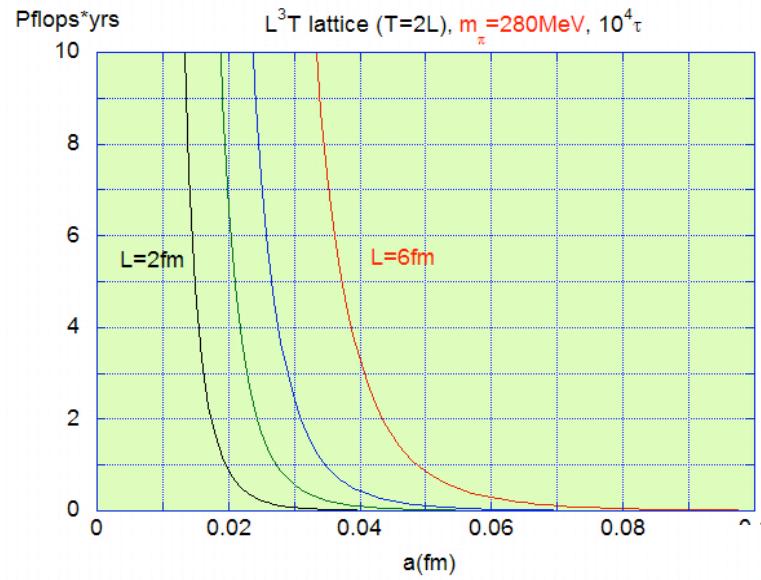
$$\mathcal{L}_{\text{LQCD}}^{\text{val}} = \mathcal{L}_{\text{LQCD}}^{\text{sea}}$$

# Lattice QCD

## Main worries:

- ✓ reduce statistical errors
- ✓? control chiral extrapolation while keeping the finite volume effects under control
- ✓? control the discretization effects
  - matching and renormalization
- ✓? inclusion of 2,3,4 sea quark flavors
- ✗ unstable resonances (signal extraction...)

2+1 Wilson-clover calculation off the physical point



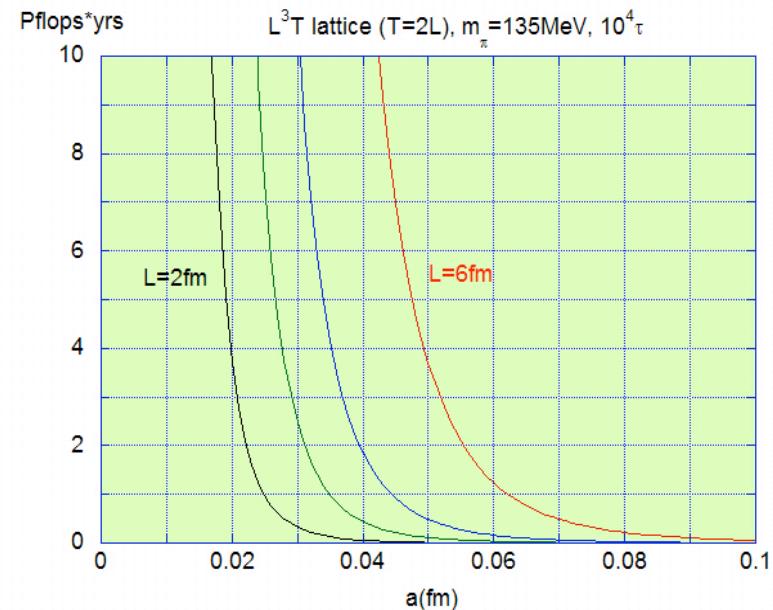
Need Better  
Algorithms!

Improve HMC

Need Better  
Machines!

Teraflop era  
PetaQCD [on CELL...]

2+1 Wilson-clover Calculation on the physical point



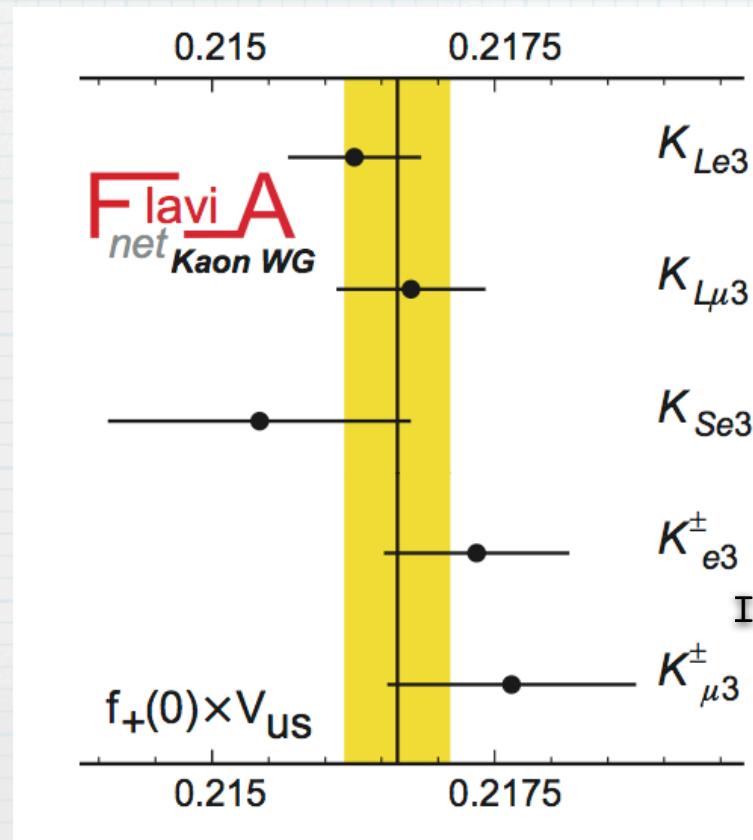
# Kaon physics: semileptonics

Tree level decay

$$K \rightarrow \pi \ell \nu$$

$$\left( |V_{ud}|^2 + |V_{us}|^2 + \cancel{|V_{ub}|^2} \right) \stackrel{?}{=} 1$$

$|V_{ud}| = 0.97425(22)$  from Hardy & Towner 08



$$|V_{ud}|^2 + |V_{us}^{Kl3}|^2 = 0.9995(7)$$

Impressive check of the CKM unitarity

$\Leftrightarrow$  universality of  $G_\mu$

# Kaon physics: semileptonics

$$K \rightarrow \pi \ell \nu$$

hadronic uncertainty!

$$\langle \pi | \bar{s} \gamma_\mu u | K \rangle \rightarrow f_{0,+}(q^2)$$

$$f_0(0) = f_+(0)$$

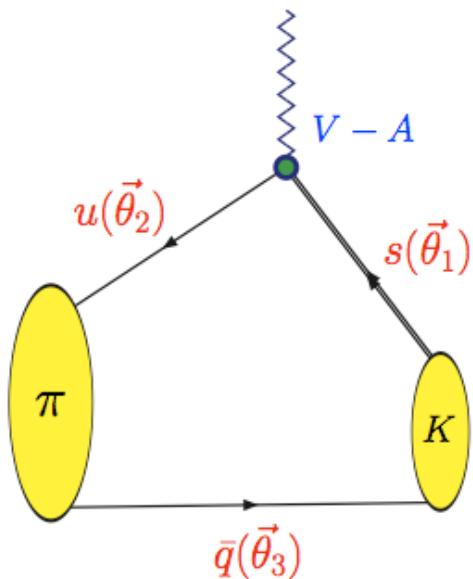
$$f_+(0) = 1 + f_2 + f_4 + \mathcal{O}(p^8)$$

CVC

-0.023

Ademollo-Gatto

$\propto (m_s - m_u)^2$   
lattice QCD



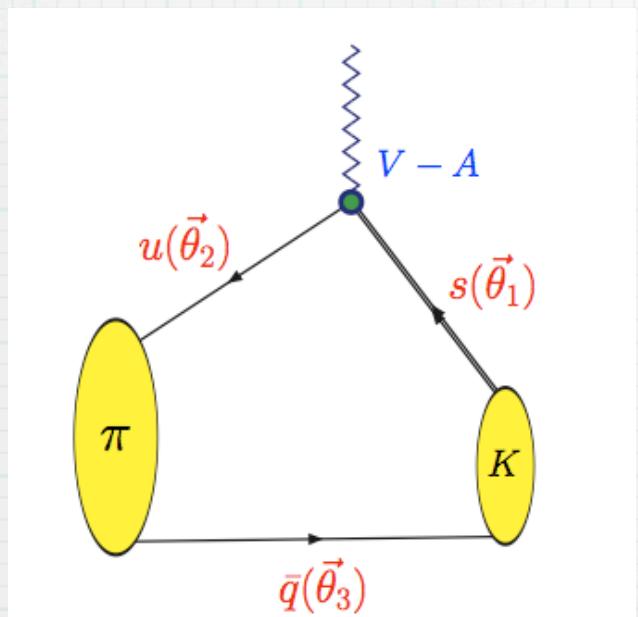
$$\frac{\langle \pi | \bar{s} \gamma_0 u | K \rangle \langle K | \bar{u} \gamma_0 s | \pi \rangle}{\langle K | \bar{s} \gamma_0 s | K \rangle \langle \pi | \bar{u} \gamma_0 u | \pi \rangle} = \frac{(m_K + m_\pi)^2}{4m_K m_\pi} \left( f_0(q_{\max}^2) \right)^2$$

Nf=2+1 by RBC/UKQCD 0710.5136

- use twisted boundary condition to go to  $q^2=0$
- extrapolation to physical pion and kaon by using ChPT
- $m_\pi \geq 330 \text{ MeV}$ ,  $Lm_\pi \geq 4$
- $a=0.11 \text{ fm}$

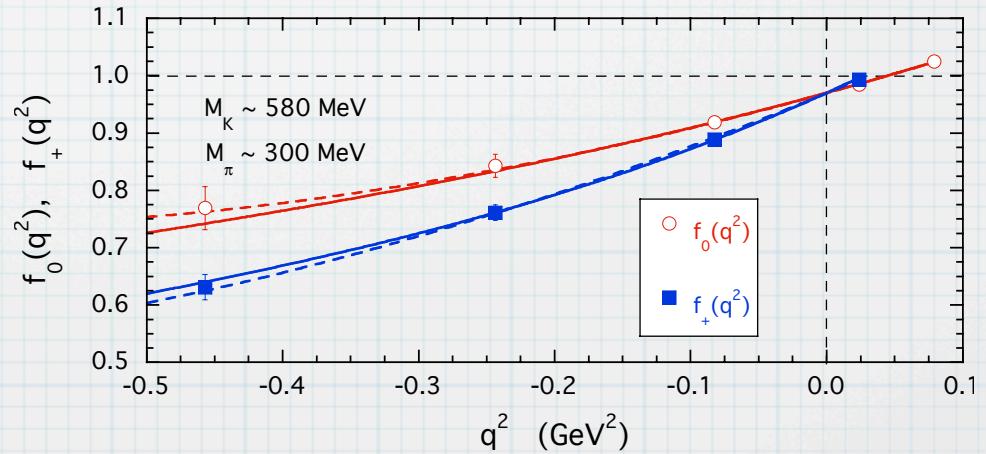
# Kaon physics: semileptonics

$$K \rightarrow \pi \ell \nu$$



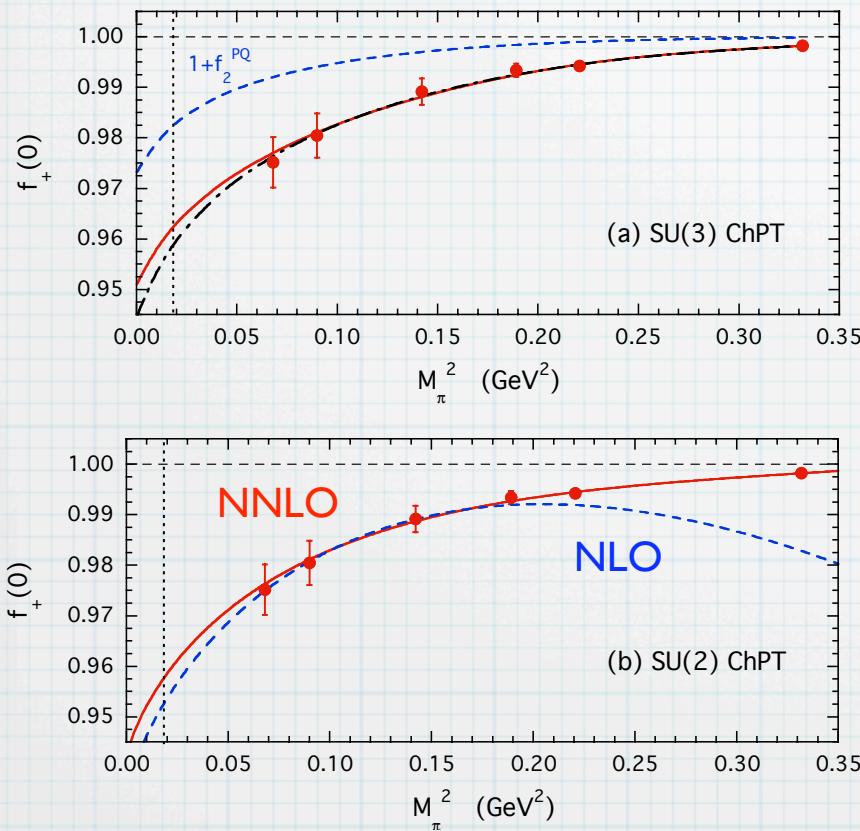
Nf=2 by **ETMC 0906.4728**

- use twisted boundary conditions to smoothly reach  $q^2=0$
- extrapolation to physical pion and kaon by using ChPT
- $m_\pi \geq 260 \text{ MeV}, Lm_\pi \geq 3.7$
- 2 lattice spacings



# Kaon physics: semileptonics

$K \rightarrow \pi \ell \nu$



Nf=2 by ETMC 0906.4728

- use twisted boundary condition to go to  $q^2=0$
- extrapolation to physical pion and kaon by using ChPT
- $m_\pi \geq 260 \text{ MeV}$ ,  $Lm_\pi \geq 3.7$
- 2 lattice spacings

$$f_+(0) = 0.9644(33)(37)$$

RBC/UKQCD Nf=2+I

$$f_+(0) = 0.9560(57)(62)$$

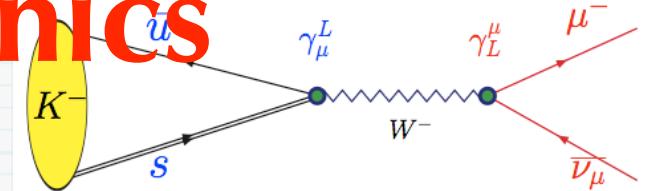
ETMC Nf=2

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$$f_+(0) = 0.962(3)(4)$$

V.Lubicz @ Lattice'09

# Kaon physics: leptonic

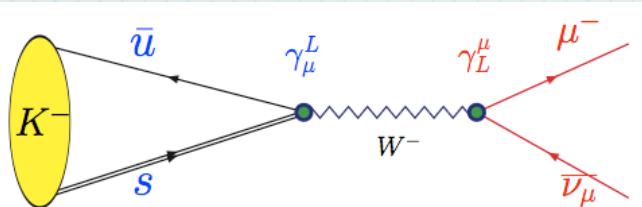


	$N_f$	$a$ [fm]	$m_{\text{PS}}$ [MeV]	$\text{N}^n\text{LO}$	$m_{\text{PS}} L$	
ETMC	2 twisted m.	0.08, 0.07, 0.05 → 0	250–600	2	compl. SU(2)	$\geq 3.0$
JLQCD	2 overlap	0.12	290–750	2	" $\xi$ " SU(2)	$\geq 2.8$
	2+1 overlap	0.10	320–800	2	" $\xi$ " SU(2),SU(3)	$\geq 2.8$
PACS-CS	2+1 iWilson	0.09	160–410 reweight → $m_\pi$	1	compl. SU(2)	$\geq 2.3$
MILC	2+1 staggered	0.09, 0.06 → 0	180–380 180–540	1 2 3,4	r $\Sigma\chi$ PT SU(3) compl. SU(3) analytic	$\geq 4.0$
Aubin et al.	2+1 stagg/DWF	0.12, 0.09 → 0	240–500	1 $\geq 3$	SU(3) MA $\chi$ PT analytic	$\geq 4.0$
RBC/UKQCD	2+1 DWF	0.11, 0.09 → 0	290–420	1,2	compl. SU(2)	$\geq 4$

# Kaon physics: leptomics

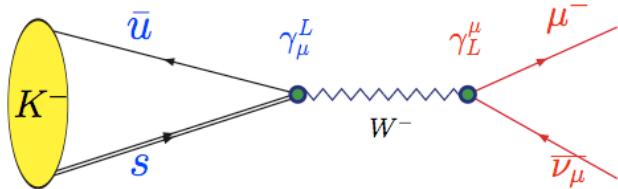
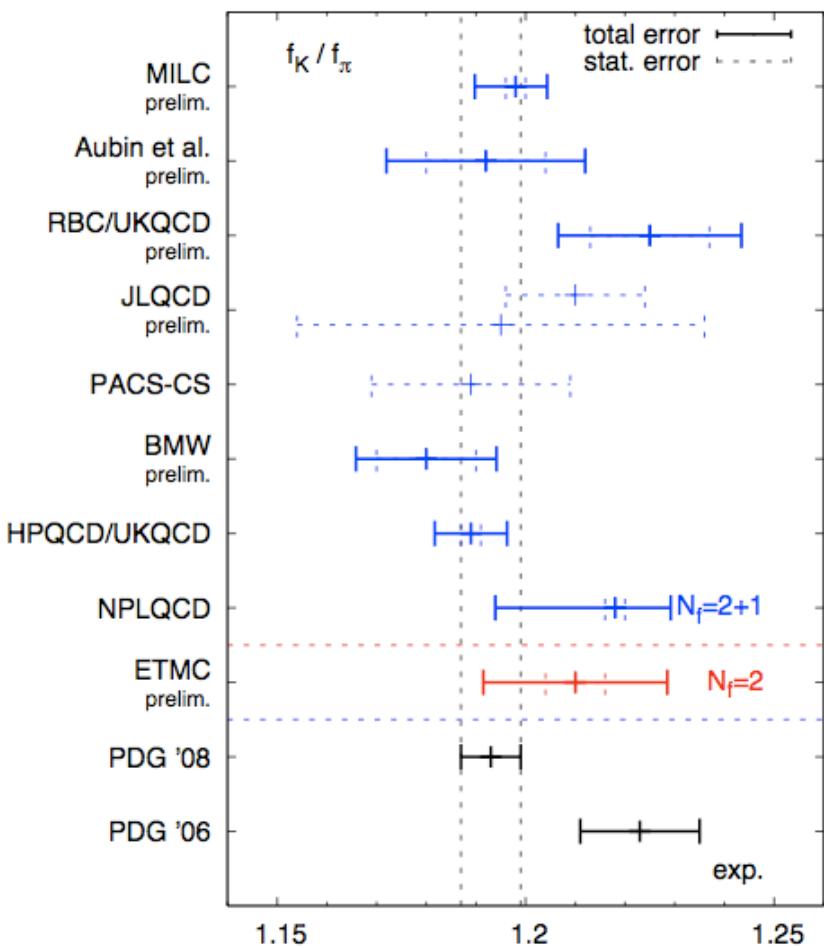
	$f_\pi$	$f_K$	$f_{\text{SU}(2)}$	$f_0 \text{ SU}(3)$
ETMC	input	158.1(0.8)(2.0)(1.1)		
JLQCD (2)	119.6(3.0)(1.0)( $^{+6.4}_{-0.0}$ )		121.57(70)	
JLQCD (2+1)	121.2(5.6)	149.6(6.0)	111.7(3.5)(1.0)( $^{+6.0}_{-0.0}$ )	
JLQCD (2+1)	input	157.2(6.4)	112.5(9.2)	73(19)
RBC/UKQCD	122.2(3.4)(7.3)	149.7(3.8)(2.0)	121(14)	79(20)
PACS-CS	134.0(4.3)	159.4(3.1)	113.0(3.8)(6.8)	93.5(7.3) ['08]
MILC	128.0(.3)(2.9)	153.8(0.3)(3.9)	126.4(4.7)	118.5(9.0)
Aubin et al.	131.1(1.3)(2.2)	156.3(1.3)(2.0)	122.8(.3)(.5)	111.0(2.0)(4.1)
PDG '08	130.4(.04)(.2)	155.5(.2)(.8)(.2)	blue marks prelim. results	

NEW RESULTS : summary by Scholz @ Lattice'09



Convergence of ChPT with Nf=3 is still unclear :  
need checks with lower/unphysical strange quark

# Kaon physics: leptronics



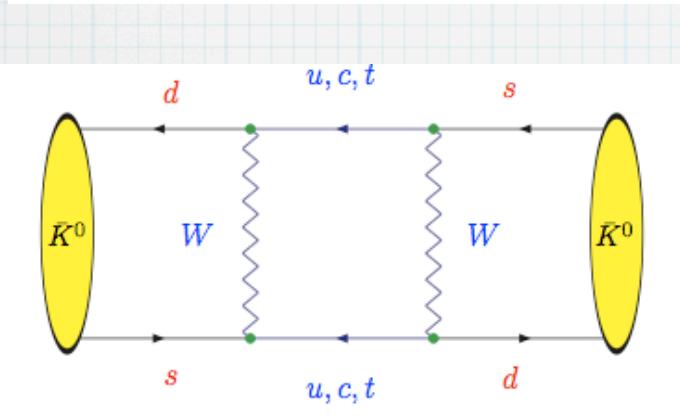
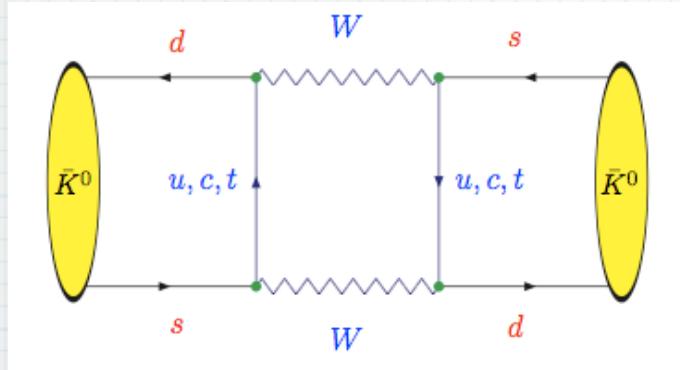
New Average @ Lattice'09

$$f_K/f_\pi = 1.196(1)(10)$$

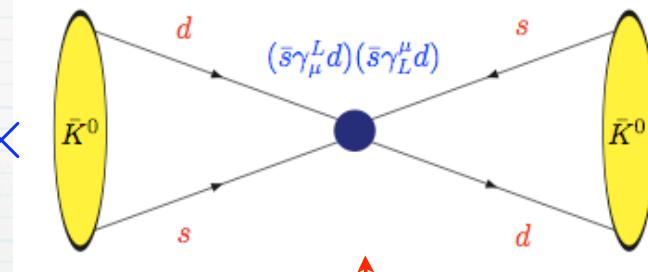
UT +  $|V_{ud}|$  + experiment

$$f_K/f_\pi = 1.193(6)$$

# Kaon physics: $B_K$



$$= C_{\text{SM}}^{\text{RGI}} \times$$



$$\langle \bar{K}^0 | (\bar{s}\gamma_\mu^L d)(\bar{s}\gamma^\mu_L d) | K^0 \rangle = \frac{8}{3} m_K^2 f_K^2 B_K$$

RGI from NLO QCD pert.th  
(NDR-MS,RI/MOM,SF)

LATTICE QCD  
 $B_K$

In 10 years the central value for  $B_K$  changed from 0.90 to 0.72 and the error from about 20% to about 5%

# Kaon physics: $B_K$

## 3 NEW RESULTS :

(1) Aubin et al, 0905.3947

`mixed' action Stagg/DWF with  $N_f=2+1$ ;  $a=0.12, 0.09$  fm; NP-Renorm

$$\hat{B}_K = 0.724(8)(28)$$

(2) ETMC, to appear

OS/tmQCD with  $N_f=2$ ;  $a=0.10, 0.09, 0.07$  fm; NP-Renormalization

$$\hat{B}_K = 0.730(30)(30)$$

(3) RBC/UKQCD, updating on 0710.5136

DWF with  $N_f=2+1$ ;  $a=0.11, 0.08$  fm; NP-Renormalization

$$\hat{B}_K = 0.738(8)(25)$$

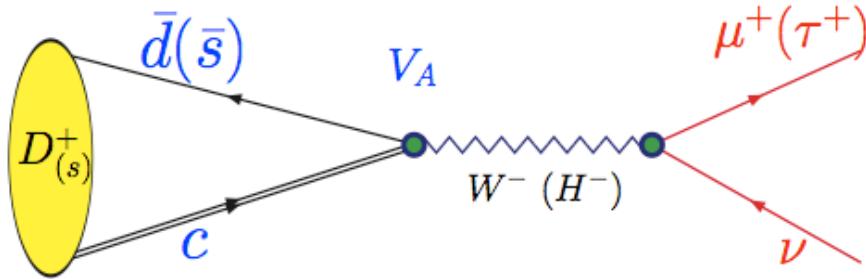
avg :  $\hat{B}_K = 0.731(7)(35)$

Note that this error is  
no more dominant :

$$\varepsilon_K^{\text{exp}} \propto A^2 \bar{\eta} \hat{B}_K$$

It is comparable to  
that of  $A = V_{cb}/\lambda^2$

# Charm physics: $f_D$ , $f_{D_s}$



$$\langle 0 | \bar{q} \gamma_\mu \gamma_5 c | D_q(p) \rangle = i p_\mu f_{D_q}$$

$$\Gamma(D_q \rightarrow \ell \nu) = \frac{G_F^2}{8\pi} f_{D_q}^2 |V_{cq}|^2 m_\ell^2 m_{D_q} \left(1 - \frac{m_\ell^2}{m_{D_q}^2}\right)^2 R_q^2$$

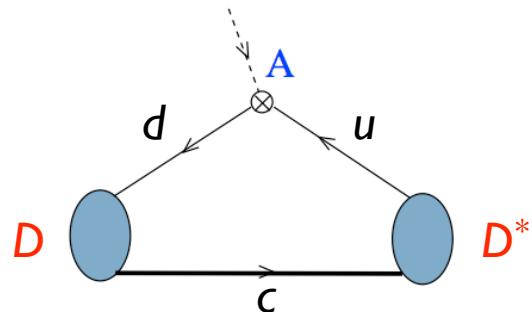
$$R_q = 1 + \frac{m_{D_q}^2}{M_H^2} \frac{m_c - m_q \tan^2 \beta}{m_c + m_q}$$

$|V_{cs}|$  and  $|V_{cd}|$  only known from the CKM unitarity :

CLEO-c from leptonic decays extracted

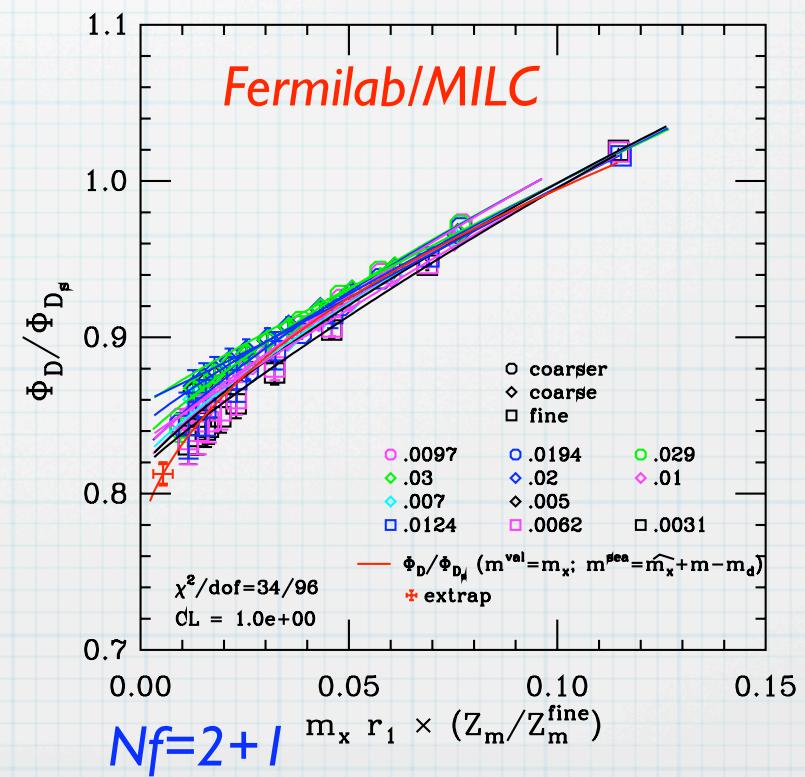
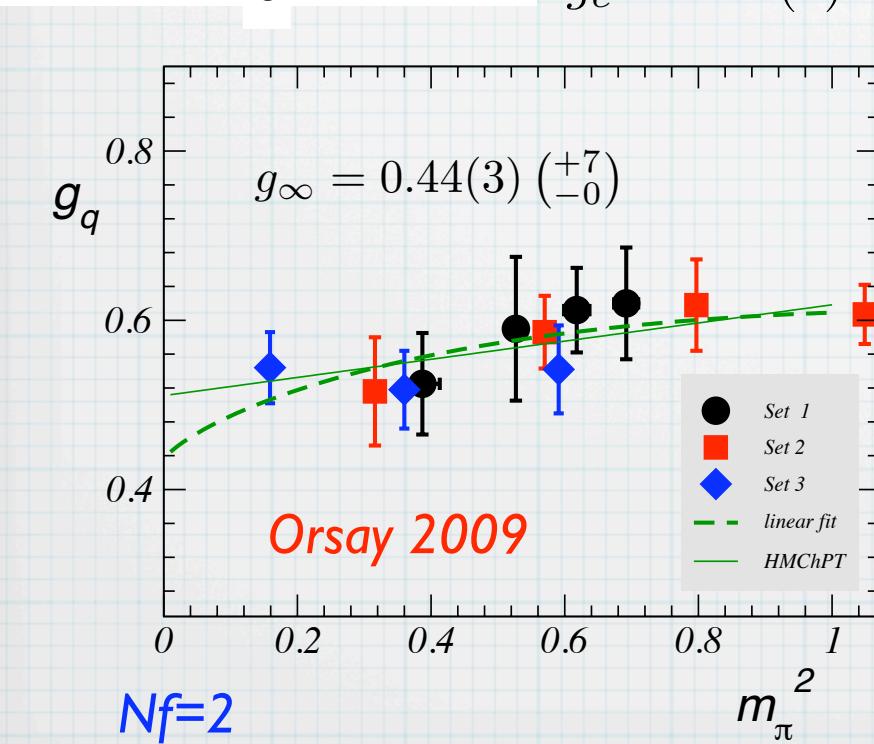
$$|V_{cs}| = (1.08 \pm 0.05) |V_{cs}|^{\text{UTA}} \text{ and } |V_{cd}| = (1.00 \pm 0.05) |V_{cd}|^{\text{UTA}}$$

# Charm physics: $f_D$ , $f_{D_s}$



$\Phi_{D_q} = f_{D_q} \sqrt{m_{D_q}}$

Use of HMChPT for the charm ( $g=?$ , static value < charmed )  
StagHMChPT... for Fermilab/MILC and HPQCD/MILC



# 2009 - Epilogue (?)

Experimental average (CLEO-c!)

$$f_{D_s} = (277 \pm 9) \text{ MeV [2007]} \rightarrow (271 \pm 8) \text{ MeV [2008]} \rightarrow (263 \pm 9) \text{ MeV [2009]}$$

CLEO-c in PRD79:052001, 2009

WHY?  $D \rightarrow \mu\nu_\mu$  stable,  $D \rightarrow \tau\nu_\tau$  changed a lot!

$$f_{D_s}^{(\tau)} = (285 \pm 15) \text{ MeV [2007]} \rightarrow (272 \pm 13) \text{ MeV [2008]} \rightarrow (260 \pm 8) \text{ MeV [2009]}$$

---

Lattice QCD -  $f_{D_s}$

Staggered

$N_f = 2 + 1$

Fermilab/MILC

$249 \pm 11 \text{ MeV [0904.1895]}$

$241 \pm 3 \text{ MeV [0706.1726]}$

HPQCD/MILC

tmLQCD

$N_f = 2$

ETMC

$244 \pm 8 \text{ MeV [0904.0954]}$

Wilson-impr.

$N_f = 2$

DB,B.Haas 2009

$242 \pm 19 \text{ MeV}$

$$f_{D_s}^{\text{quench}} = 252 \pm 9 \text{ MeV [PLB560, 59(2003)]}$$

Alpha

**NO MORE PUZZLE (?)**

# B-physics

$V_{cb}$ :

- ✓ HFAG (winter 2009) :  $|V_{cb}|^{\text{incl.}} = 41.5(5)(7) \times 10^{-3}$
- ✓ Fermilab/MILC (2008) :  $|V_{cb}|^{\text{excl.}} = 38.6(1.2) \times 10^{-3}$
- ✓ any improvement on inclusive side?
- ✓ on exclusive end, revise the strategy which relies on HQET (relations among slope and curvature...) There is a full QCD method which can be used to compute both form factors without recourse to HQET

$V_{ub}$ ::

- ? tension still present  $|V_{ub}|^{\text{excl.}} = 3.42(37) \times 10^{-3}$
- ? inclusive depend on the ``scheme''/strategy and  
 $\text{cv} > 4 \times 10^{-3}$

# B-physics:Step Scaling Method

Method proposed and tested in Quenched Approximation to obtain accurate values for D, B decay constants and form factors in QCD (no recourse to HQET)

Rome-2 : N.Tantalo et al 2002, 2007, 2008

- Compute an  $F(m_b, m_q, L)$
- Compute an  $F(m_b, m_q, 2L)$  and define

$$\sigma(m_b, m_q, L) = \frac{F(m_b, m_q, 2L)}{F(m_b, m_q, L)}$$

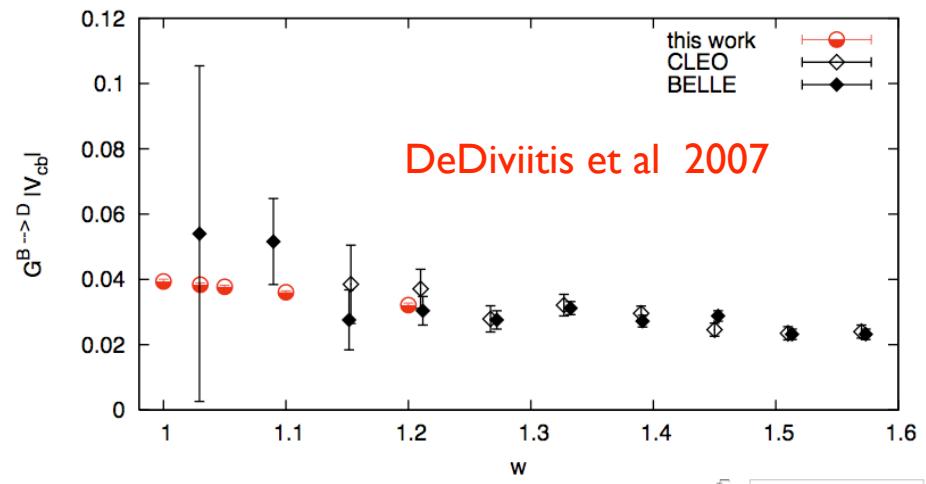
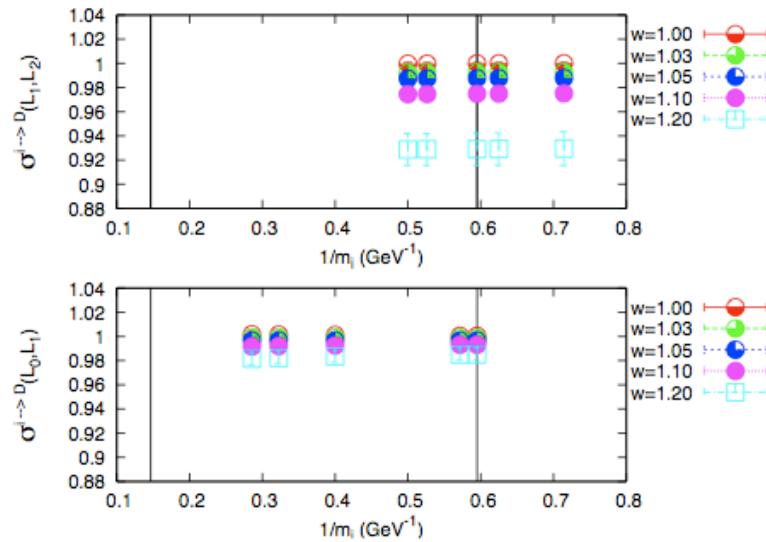
- Compute an  $F(m_b, m_q, 4L)$  and check how fast  $\sigma(m_b, m_q, 2nL)$  converges to 1
- Indeed for

$$F(m_b, m_q, L) = F_0(m_q, L) \left[ 1 + \frac{F_1(m_q, L)}{m_b} \right] \implies$$

$$\sigma(m_b, m_q, L) = \frac{F_0(m_q, 2L)}{F_0(m_q, L)} \left[ 1 + \frac{F_1(m_q, 2L) - F_1(m_q, L)}{m_b} \right]$$

# B-physics:Step Scaling Method

$$\sigma^{i \rightarrow D}(w; L_0, L_1) = \frac{G^{i \rightarrow D}(w; L_1)}{G^{i \rightarrow D}(w; L_0)}$$



Unquenching is hard but feasible! Underway!  
These results will be available and one then should use standard QCD expression for the decay spectra (no HQET).

Extending this strategy to  $B \rightarrow \pi \ell \nu$  (?)

# B-physics

$f_{B(s)}$  and  $f_{+,0}^{B/D}(q^2)$  simulations made with  $N_f = 2 + 1$  but only by using Staggered light quark = MILC configurations

$f_B$

Fermilab/MILC 0904.1895

$195 \pm 11$  MeV

HPQCD/MILC 0507.015  
 $216 \pm 22 \pm 7$  MeV

$f_{B_s}$

Fermilab/MILC 0904.1895

$243 \pm 11$  MeV

HPQCD/MILC 0507.015  
 $1.20(3)(1) \times f_B$

$f_D$

Fermilab/MILC 0904.1895

$207 \pm 11$  MeV

HPQCD/MILC 0706.1726  
 $208 \pm 4$  MeV

Chiral extrapolations guided by HMChPT

Range of validity compromised by the nearness  
of orbital excitations

$$\Delta_{S_s} \equiv m_{D_{0s}^*} - m_{D_s} = m_{D_{1s}} - m_{D_s^*} = 350 \text{ MeV}$$

$$\Delta_{S_{s,q}} < \Lambda_\chi, m_\eta, m_K$$

Work only with very light pions (cf.hep-ph/  
0612224)

ETMC 0904.0954

$197 \pm 9$  MeV

DB & B.Haas to appear

$205 \pm 18$  MeV

# B-physics

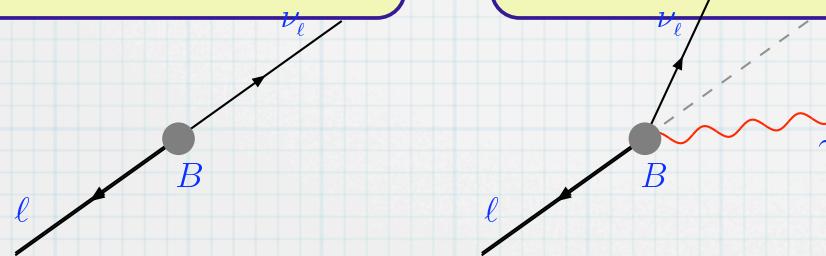
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$$f_B$$

Fermilab/MILC 0904.1895

$$195 \pm 11 \text{ MeV}$$

HPQCD/MILC 0507.015  
 $216 \pm 22 \pm 7 \text{ MeV}$



Be VERY afraid of soft photon radiation : SD may

be huge and compromise the study of  $B \rightarrow \mu\nu_\mu$   
c.f. 0907.1845 !!!

$$f_{B_s}$$

Fermilab/MILC 0904.1895

$$243 \pm 11 \text{ MeV}$$

HPQCD/MILC 0507.015  
 $1.20(3)(1) \times f_B$

$$f_D$$

Fermilab/MILC 0904.1895

$$207 \pm 11 \text{ MeV}$$

HPQCD/MILC 0706.1726  
 $208 \pm 4 \text{ MeV}$

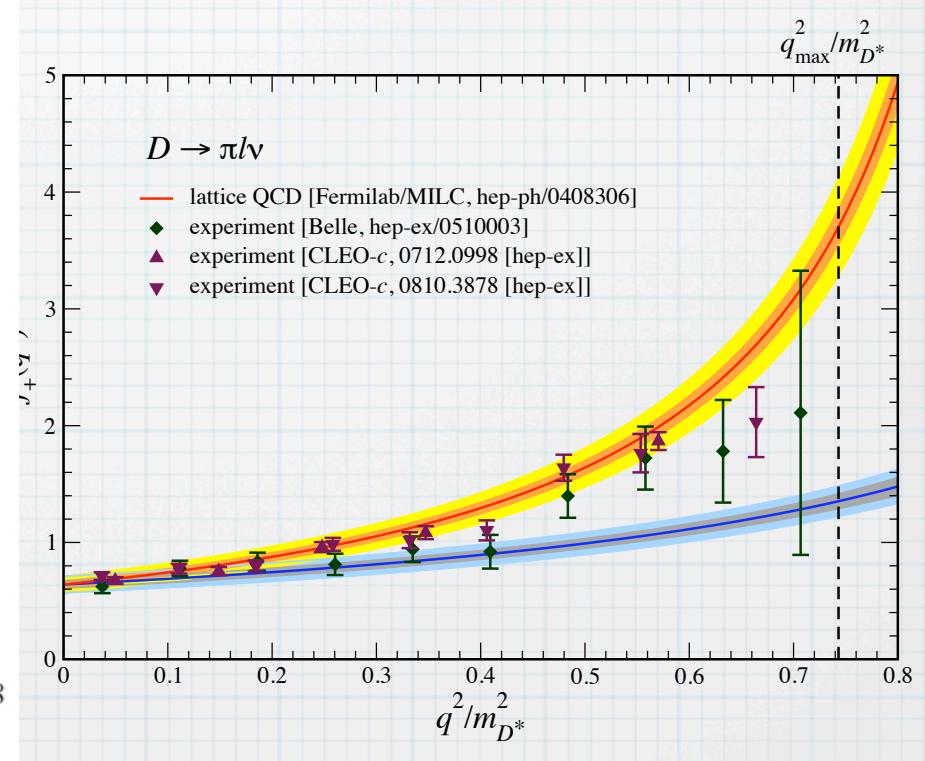
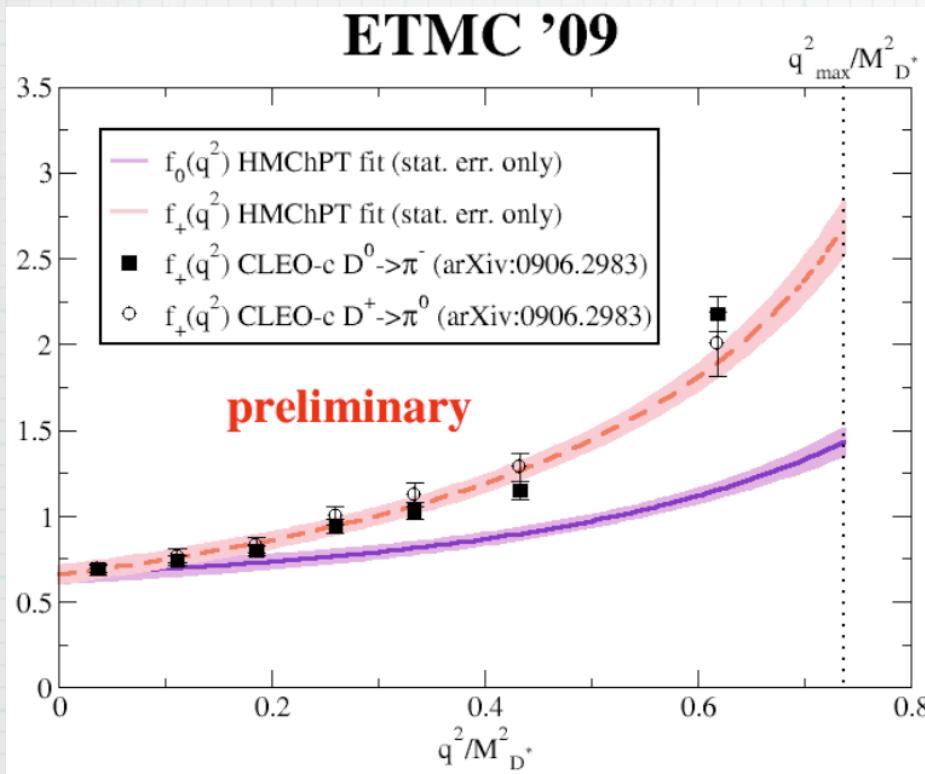
ETMC 0904.0954

$$197 \pm 9 \text{ MeV}$$

DB & B.Haas to appear

$$205 \pm 18 \text{ MeV}$$

# Heavy-to-light decay form factors



# New physics flavor problem

New physics effects through higher dim. operators

e.g.

$$\begin{aligned} & \frac{C_{sd}}{\Lambda_{\text{NP}}^2} (\bar{s}\gamma_L^L d)(\bar{s}\gamma_L^\mu d) + \frac{C_{cu}}{\Lambda_{\text{NP}}^2} (\bar{c}\gamma_L^L u)(\bar{c}\gamma_L^\mu u) \\ & + \frac{C_{bd}}{\Lambda_{\text{NP}}^2} (\bar{b}\gamma_L^L d)(\bar{b}\gamma_L^\mu d) + \frac{C_{bs}}{\Lambda_{\text{NP}}^2} (\bar{b}\gamma_L^L s)(\bar{b}\gamma_L^\mu s) \end{aligned}$$

if NP had a generic flavor structure then all  $C_{xy} \sim 1$

Meson mixing:

$$\frac{\Delta M_B}{M_B} \approx C_{bd} \left( \frac{f_B}{\Lambda_{\text{NP}}} \right)^2$$

# New physics flavor problem

if NP had a generic flavor structure then all  $C_{xy} \sim 1$

As a result

$$\Lambda_{\text{NP}} \gtrsim \begin{cases} 1 \times 10^4 \text{ TeV} & \varepsilon_K \\ 9 \times 10^2 \text{ TeV} & \Delta M_D \\ 4 \times 10^2 \text{ TeV} & \Delta M_B \\ 7 \times 10^1 \text{ TeV} & \Delta M_{B_s} \end{cases}$$

# New physics flavor problem

For  $\Lambda_{\text{NP}} = 1 \text{ TeV}$

$$\text{Im}[C_{sd}] \lesssim 6 \times 10^{-9}$$

$$\text{Im}[C_{cu}] \lesssim 1 \times 10^{-6}$$

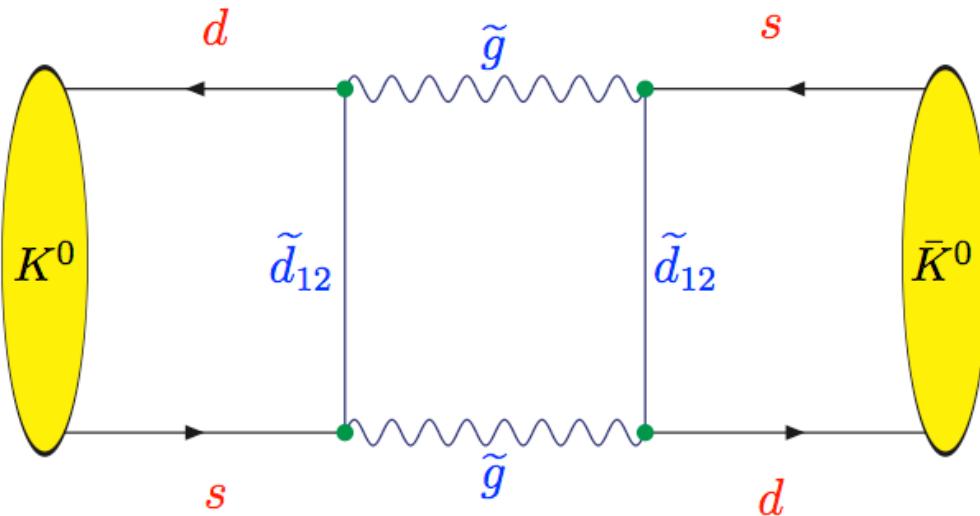
$$\text{Im}[C_{bd}] \lesssim 2 \times 10^{-6}$$

$$\text{Im}[C_{bs}] \lesssim 2 \times 10^{-4}$$

The flavor structure of NP is not generic! Why?

# SUSY flavor problem

For



$$\begin{aligned}\frac{\Delta M_K}{M_K} &= 7.0 \times 10^{-15} \\ &= \eta_K g(m_{\tilde{g}}^2/m_{\tilde{d}}^2) \frac{\alpha_s^2}{108} \frac{B_K f_K}{m_{\tilde{d}}^2} \frac{(\Delta m_{\tilde{d}}^2)^2}{m_{\tilde{d}}^4} (K_{21}^d K_{11}^d)^2 \\ \implies &\frac{\text{TeV}}{m_{\tilde{q}}} \times \frac{\Delta m_{\tilde{q}}^2}{m_{\tilde{q}}^2} \times \sin \theta_d \leq 0.01\end{aligned}$$

# SUSY flavor problem

Beyond MFV - extra operators LR, RL, RR

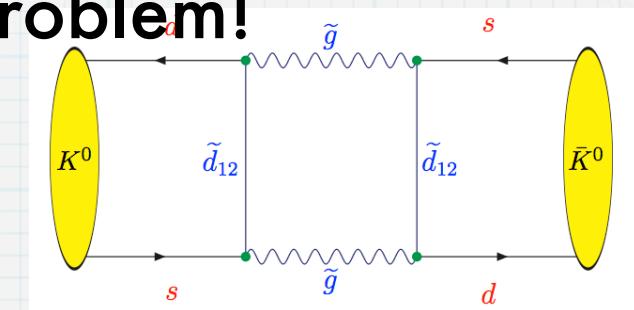
- matrix elements LQCD + 2loop ADM
- many more parameters

**you do not dodge the flavor problem!**

$$\frac{\Delta M_K}{M_K} = 7.0 \times 10^{-15}$$

$$= \eta_K g(m_{\tilde{g}}^2/m_{\tilde{d}}^2) \frac{\alpha_s^2}{108} \frac{B_K f_K}{m_{\tilde{d}}^2} \frac{(\Delta m_{\tilde{d}}^2)^2}{m_{\tilde{d}}^4} (K_{21}^d K_{11}^d)^2$$

$$\implies \frac{\text{TeV}}{m_{\tilde{q}}} \times \frac{\Delta m_{\tilde{q}}^2}{m_{\tilde{q}}^2} \times \sin \theta_d \leq 0.01$$



# SUSY flavor problem

$$\frac{\Delta M_K}{M_K} = 7.0 \times 10^{-15} \implies \frac{\text{TeV}}{m_{\tilde{q}}} \times \frac{\Delta m_{\tilde{q}}^2}{m_{\tilde{q}}^2} \times \sin \theta_d \leq 0.01$$
$$\frac{\Delta M_D}{M_D} \lesssim 2.0 \times 10^{-14} \implies \frac{\text{TeV}}{m_{\tilde{q}}} \times \frac{\Delta m_{\tilde{q}}^2}{m_{\tilde{q}}^2} \times \sin \theta_u \leq 0.10$$

**Heavy**  $m_{\tilde{q}} \gg 1 \text{ TeV}$

**Degeneracy**  $(\Delta m_{\tilde{q}})^2 \ll m_{\tilde{q}}^2$

**Alignment**  $\sin \theta_d \ll 1$

[split SUSY]

[gauge mediation]

[horizontal symm]