Review on rare kaon decays

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Outline

- SM-Flavour physics tests - CKM determination
- Rare Kaon decays
- $K^+ \longrightarrow \pi^+ \nu \bar{\nu}$
- Chiral Perturbation theory
- $K_L \longrightarrow \pi^0 ee$ and related channels $K \longrightarrow \pi\gamma\gamma$ and $K \longrightarrow \pi ee$
- $K^+ \longrightarrow \pi\pi\gamma$ and CP violation
- Conclusions

Standard Model FCNC

- SM with 3 families $\Longrightarrow$ weak int. with an unit. mat. V_{ij}: 3 angles and 1 phase (CPV)

$$\overbrace{V_{ud}, V_{cb}, V_{td}}^{\text{Wolfenstein}} \Longrightarrow \lambda, A\lambda^2, A\lambda^3(1-\rho-i\eta)$$

- FCNC only at 1-loop

$$\mathcal{H}^{SM}_{\Delta F=2} \sim \frac{G_F^2 M_W^2}{16\pi^2}\left[\frac{(V_{td}^* m_t^2 V_{tb})^2}{v^4}(\bar{d}_L\gamma^\mu b_L)^2 + \frac{(V_{td}^* m_t^2 V_{ts})^2}{v^4}(\bar{d}_L\gamma^\mu s_L)^2\right] + \text{charm}$$

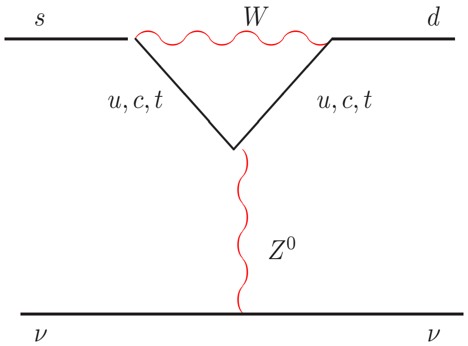
- The area of all possible CKM-unitarity triangles is an invariant:

$$|J_{CP}| \overset{Wolfenstein}{\simeq} A^2\lambda^6\eta$$

- As we shall see $B(K_L \to \pi^0\nu\overline{\nu})$ will measure this area

$K \to \pi\nu\overline{\nu}$

$$A(s \to d\nu\overline{\nu})_{\rm SM} \sim \overline{s}_L\gamma_\mu d_L \quad \overline{\nu}_L\gamma^\mu\nu_L \ \times \left[\sum_{q=c,t} V^*_{qs}V_{qd}\, m_q^2\right]$$



$$\sim \qquad \left[A^2\lambda^5\,(1-\rho-i\eta)m_t^2 + \lambda m_c^2\right]$$

SM: $\underbrace{V-A\otimes V-A}_{\Downarrow}$ Littenberg

$$\Gamma(K_L \to \pi^0\nu\overline{\nu}) \quad \left\{ \begin{array}{l} \text{CP violating} \\ \Rightarrow \ J = A^2\lambda^6\eta \\ \text{Only } top \end{array} \right.$$

Theory versus expts.

- K^+: $t >$ charm $\overset{\rm NLO-QCD}{\Longrightarrow}$ $K^+ \ \mathcal{O}(5\%)$, $K_L \ \mathcal{O}(1\%)$
- BR$(K \to \pi\nu\overline{\nu})_{\rm TH}$ $\quad K^+ \quad (0.8 \pm 0.1)\cdot 10^{-10} \quad K_L : (3.0 \pm 0.6)\cdot 10^{-11}$
- $B(K^+) = (1.5^{+1.3}_{-0.9})\cdot 10^{-10}$ BNL-E787/E949 P326 at CERN

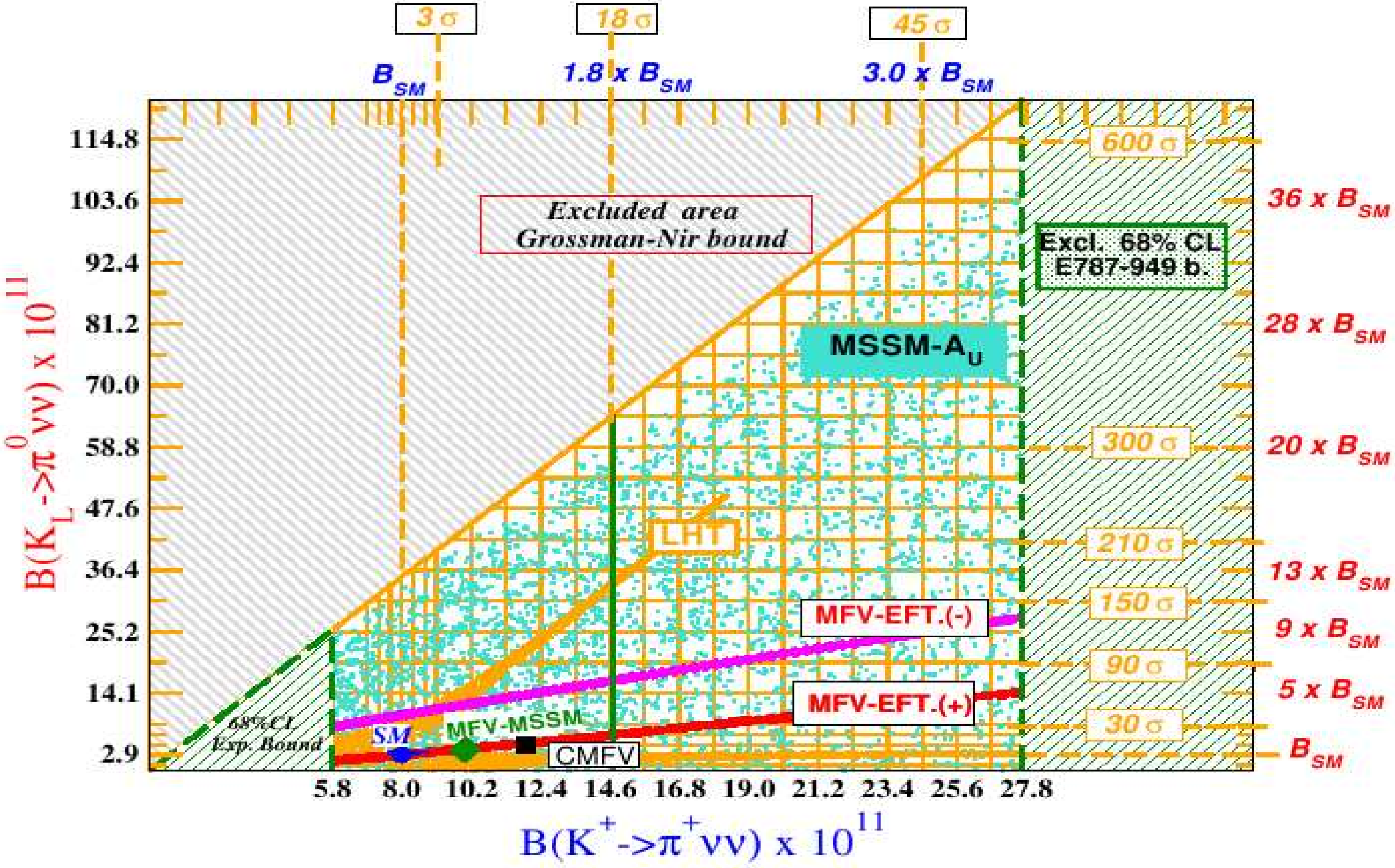
  $B(K_L) \leq 2.1 \times 10^{-7}$ at $90\% C.L.$ E391 at KEK 10% data

- K_L Model-independent bound, based on $SU(2)$ properties dim-6 operators for $\overline{s}d\overline{\nu}\nu$

  Grossman-Nir

$$B(K_L) \ \leq \frac{\tau_L}{\tau_+} \times B(K^{\pm})_{\rm E949} \leq 1.4 \times 10^{-9} \quad \text{at } 90\% C.L.$$

New Physics searches at NA62 100 $K^+ \to \pi^+ \nu\bar{\nu}$ Mescia



Chiral Perturbation Theory

χPT effective field theory based on the two assumptions

- π's are the Goldstone boson of $SU(3)_L \otimes SU(3)_R \to SU(3)_V$
- *(chiral) power counting* i.e. the theory has a small expansion parameter: $p^2/\ \Lambda^2_{\chi SB}$: $\Lambda_{\chi SB} \sim 4\pi F_\pi \sim 1.2$ GeV

$$\mathcal{L}_{\Delta S=0} = \mathcal{L}^2_{\Delta S=0} + \mathcal{L}^4_{\Delta S=0} + \cdots = \frac{F_\pi{}^2}{4} \overbrace{\langle D_\mu U D^\mu U^\dagger + \chi U^\dagger + U \chi^\dagger \rangle}^{\pi\to l\nu,\ \pi\pi\to\pi\pi,\ K\to\pi..} + \overbrace{\sum_i L_i O_i}^{K\to\pi..} + \cdots$$

Fantastic chiral prediction $A_{\pi\pi} \sim (s - m_\pi^2)/F_\pi{}^2$ Weinberg, Colangelo *et al*

$$\mathcal{L}_{\Delta S=1} = \mathcal{L}^2_{\Delta S=1} + \mathcal{L}^4_{\Delta S=1} + \cdots = G_8 F^4 \underbrace{\langle \lambda_6 D_\mu U^\dagger D^\mu U \rangle}_{K\to 2\pi/3\pi} + \underbrace{G_8 F^2 \sum_i N_i W_i}_{K^+\to\pi^+\gamma\gamma,\ K\to\pi l^+ l^-} + \cdots$$

Vector Meson Dominance in the strong sector

Ecker,Gasser,de Rafael,Pich

i	$L_i^r(M_\rho)$	V	A	Total	Total[c)]
1	0.4 ± 0.3	0.6	0	0.6	0.9
2	1.4 ± 0.3	1.2	0	1.2	1.8
3	-3.5 ± 1.1	-3.6	0	-3.0	-4.9
4	-0.3 ± 0.5	0	0	0.0	0.0
5	1.4 ± 0.5	0	0	1.4	1.4
6	-0.2 ± 0.3	0	0	0.0	0.0
7	-0.4 ± 0.2	0	0	-0.3	-0.3
8	0.9 ± 0.3	0	0	0.9	0.9
9	6.9 ± 0.7	6.9[a)]	0	6.9	7.3
10	-5.5 ± 0.7	-10.0	4.0	-6.0	-5.5

[c)] uses QCD "inspired" relations

$$F_V = 2G_V = \sqrt{2} f_\pi, \quad F_A = f_\pi, \quad M_A = \sqrt{2} M_V$$

$$L_1^V = \frac{L_2^V}{2} = -\frac{L_3^V}{6} = \frac{G_V^2}{8M_V^2}, \qquad L_9^V = \frac{F_V G_V}{2M_V^2}, \qquad L_{10}^{V+A} = -\frac{F_V^2}{4M_V^2} + \frac{F_A^2}{4M_A^2}$$

Theoretical motivations of success of VMD

- VMD improves the matching with QCD Ecker,Gasser,Leutwyler,Pich,de Rafael

  Donoghue, Ramirez, Valencia

$$F_V^{\pi^\pm}(t) \approx \frac{M_\rho^2}{M_\rho^2 - t},$$

$$L_1^V = L_2^V/2 = -L_3^V/6 = L_9^V/8 = -L_{10}^{V+A}/6 = f_\pi^2/(16 M_V^2).$$

- Picture consolidated by in the chiral quark model $\chi\mathrm{QM}$ Georgi-Manohar;Espriu,de Rafael,Taron
  This is a dynamical interpretation (in CHPT) of the success of the non-relativstic quark model. In this model

$$L_i^{\chi\mathrm{QM}} \sim L_i^{VMD}$$

- Also Large N

- Strong consolidated picture: Must work also in the WEAK sector in some way: **WHICH WAY?**

- Many CT's (37) Ecker,Kambor, Wyler; G.D. Portoles

$$\mathcal{L}^{(4)}_{|\Delta S|=1} = G_8 F^2 \sum_{i=1}^{37} N_i W_i$$

- There are **tests**
- $K^+ \longrightarrow \pi^+\pi^0\gamma$ and $K_S \longrightarrow \pi^+\pi^-\gamma$ same CT combination
- $K \longrightarrow \pi l^+ l^-$, $K^+ \longrightarrow \pi^+\pi^0\gamma$, $K^+ \longrightarrow \pi^+\gamma\gamma$ (observed or close to observation) and others probe the the same CT's $\Longrightarrow$ **CHPT tests**
- Low energy constants of the Electroweak Lagrangian G_8, N_i are integrals of appropriate QCD Greens Functions Knecht,Peris, de Rafael
- We need desperately some dynamical info and also some theory prejudice
- VMD must work and it is easy testable: form factor

Counterterm combination	Processes	VMD weak coupling
$N_{14}^r - N_{15}^r$	$K^+ \to \pi^+\gamma^*$ $K^+ \to \pi^+\pi^0\gamma^*$	$-0.020\,\eta_V + 0.004\,\eta_A$
$2N_{14}^r + N_{15}^r$	$K_S \to \pi^0\gamma^*$	$0.08\,\eta_V$
$N_{14} - N_{15} - 2N_{18}$	$K^+ \to \pi^+\gamma\gamma$ $K^+ \to \pi^+\pi^0\gamma\gamma$	$-0.01\,\eta_A$
$N_{14} - N_{15} - N_{16} - N_{17}$	$K^+ \to \pi^+\pi^0\gamma$ $K_S \to \pi^+\pi^-\gamma$	$-0.010\,\eta_A$
$N_{14}^r - N_{15}^r - 3(N_{16}^r - N_{17})$	$K_L \to \pi^+\pi^-\gamma^*$	$-0.004\,\eta_V + 0.018\,\eta_A$
$N_{14}^r - N_{15}^r - 3(N_{16}^r + N_{17})$	$K_S \to \pi^+\pi^-\gamma^*$	$0.05\,\eta_V - 0.04\,\eta_A$
$N_{14}^r + 2N_{15}^r - 3(N_{16}^r - N_{17})$	$K^+ \to \pi^+\pi^0\gamma^*$	$0.12\,\eta_V + 0.01\,\eta_A$
$N_{29} + N_{31}$	$K_L \to \pi^+\pi^-\gamma$	$0.005\,\eta_V + 0.003\,\eta_A$
$3N_{29} - N_{30}$	$K^+ \to \pi^+\pi^0\gamma$	$-0.005\,\eta_V - 0.003\,\eta_A$

Observation hidden by other effects: different analysis maybe useful (Kaon charge radius)
NA48 has a good chance

$K_L(p) \to \pi^0(p_3)\gamma(q_1)\gamma(q_2)$

$$\text{Lorentz} + \text{gauge invariance} \quad \Rightarrow \quad M \sim \begin{array}{cc} A(y,z) & B(y,z) \\ \gamma\gamma & \gamma\gamma \\ J=0 & \text{D}-\text{wave too} \\ F^{\mu\nu}F_{\mu\nu} & F^{\mu\nu}F_{\mu\lambda}\partial_\nu K_L \partial^\lambda \pi^0 \end{array}$$

$y = p\cdot(q_1 - q_2)/m_K^2, \quad z = (q_1+q_2)^2/m_K^2$

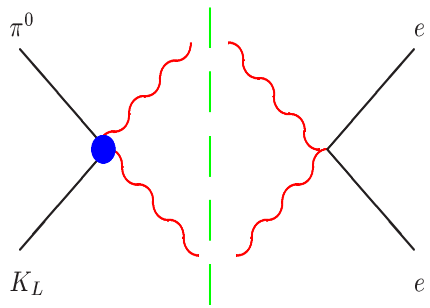
$r_\pi = m_\pi/m_K$

- $\frac{d^2\Gamma}{dydz} \sim z^2|A+B|^2 + \left(y^2 - \left(\frac{(1+r_\pi^2-z)^2}{4} - r_\pi^2\right)\right)^2 |B|^2$ $\qquad S, B$
- Different gauge structure $\Rightarrow$ B$\neq$0 at $z \to 0$ (collinear photons).

Crucial role in $K_L \to \pi^0 e^+ e^-$

A suppressed by m_e/m_K

B is not



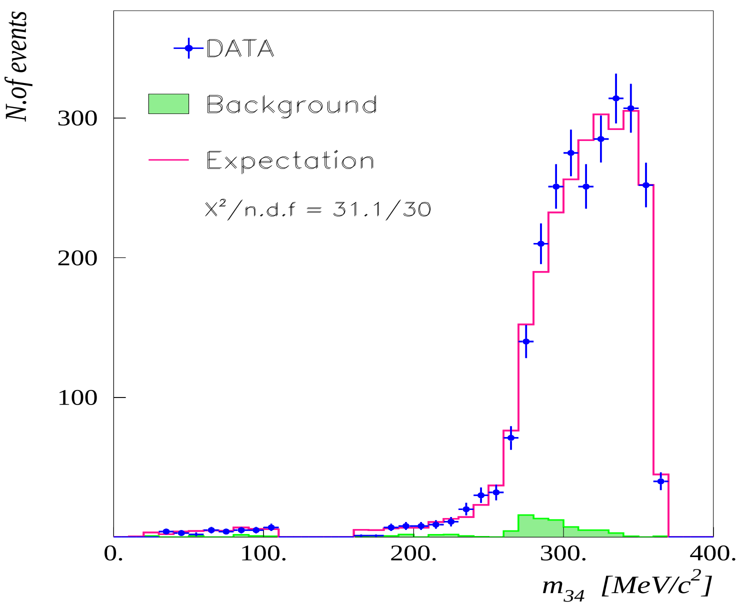
Morozumi et al, Flynn Randall

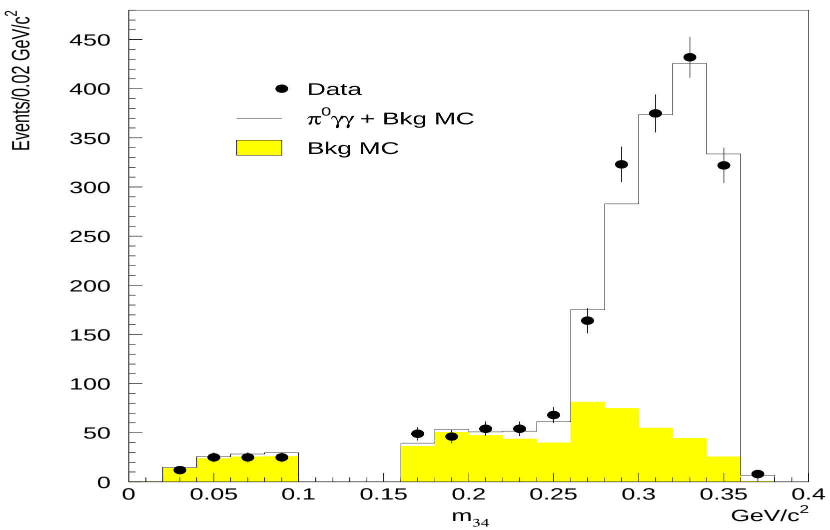
Sehgal Heiliger, Ecker et al., Donoghue et al.

NA48 and **KTeV** : 1 parameter fit a_V with unitarity corrections

NA48 $B(K_L \to \pi^0\gamma\gamma) = (1.36 \pm 0.05) \cdot 10^{-6}$ $a_V = -0.46 \pm 0.05$

KTeV $B = (1.29 \pm 0.03 \pm 0.05) \times 10^{-6}$ $a_V = -0.31 \pm 0.05 \pm 0.07$





No evts. at low $m_{\gamma\gamma} \Rightarrow$ B$(K_L \to \pi^0 e^+ e^-) < 5 \cdot 10^{-13}$

$K^+ \to \pi^+\gamma\gamma$

$$\gamma\gamma \quad \text{in} \quad \overbrace{\begin{matrix} F_{\mu\nu}F^{\mu\nu} & F\tilde{F} \end{matrix}}^{J=0} \quad J=2$$

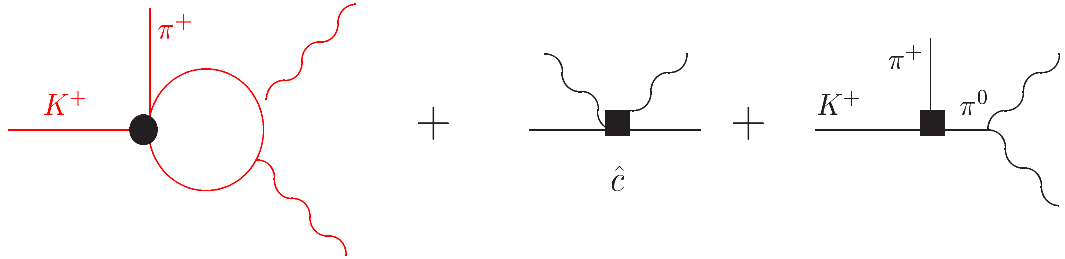
$$\begin{matrix} P=+1 & P=-1 & & \\ A & C & B & + \ \ldots \end{matrix}$$

Lorentz + gauge invariance

$$\frac{d^2\Gamma}{dydz} \sim \left[z^2(|A+B|^2+|C|^2) + \left(y^2 - \left(\frac{(1+r_\pi^2-z)^2}{4} - r_\pi^2\right)\right)^2 |B|^2\right]$$

$K^+ \to \pi^+\gamma\gamma$

- $O(p^4)$



Ecker, Pich, de Rafael

In factorization $\hat{c} = \frac{128\pi^2}{3}[3(L_9 + L_{10}) + N_{14} - N_{15} - 2N_{18})] = 2.3(1 - 2k_f)$

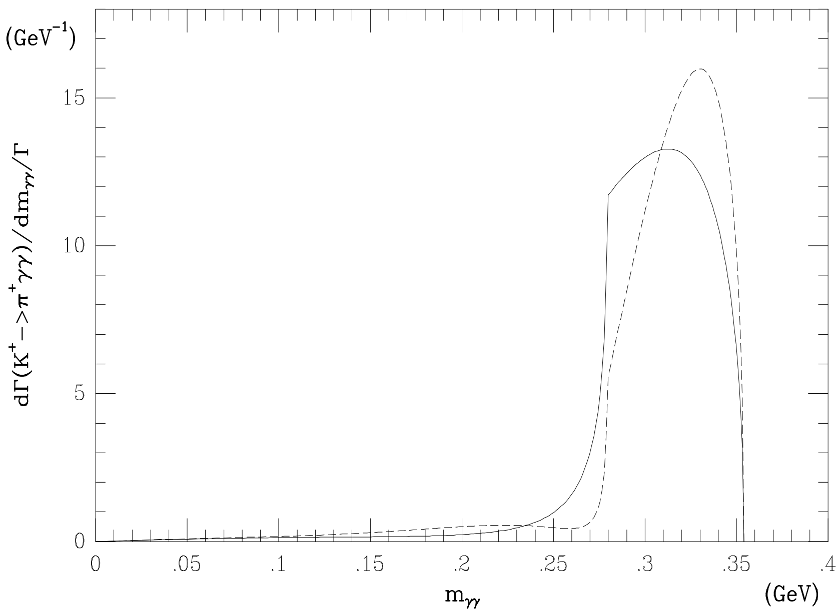
spin-1 contributions (axials) to $\hat{c}$

- $O(p^6)$ G.D., Portoles 96

Unitarity corrections: 30%-40%

a_{V^+} negligible

$K^+ \to \pi^+\gamma\gamma$



	$\hat{c}$
——	0
– – –	-2.3

BNL 787 (96) got 31 events
i) confirm $O(p^6)$
ii) Br$\sim (6 \pm 1.6) \cdot 10^{-7}$
iii) $\hat{c} = 1.8 \pm 0.6$

E949 no events at low $m_{\gamma\gamma}$:

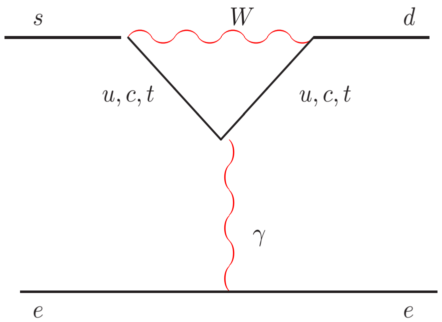
NA48 published $K^+ \to \pi^+ e^+ e^- \gamma$ $\hat{c} = 0.90 \pm 0.45$

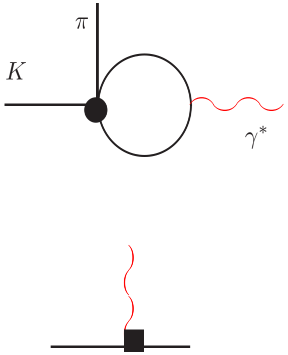
NA48 preliminary $B = (1.07 \pm 0.04 \pm 0.08) \cdot 10^{-6}$ assuming $\hat{c} = 2$

$K^{\pm}(K_S) \to \pi^{\pm}(\pi^0)\ell^+\ell^-$

- short distance << long distance

LD described by form factor W





$$W^i = G_F m_K^2 (a_i + b_i z) + W^i_{\pi\pi}(z)$$

$$i = \pm, S$$

$$a_i, b_i \sim O(1), \qquad z = \frac{q^2}{m_K^2}$$

- Observables $\Gamma(K^+ \to \pi^+ e^+ e^-)$, $\Gamma(K^+ \to \pi^+ \mu\overline{\mu})$, slopes
- a_i $\quad O(p^4)$ $\qquad a_+ \sim N_{14} - N_{15}$, $\qquad a_S \sim 2N_{14} + N_{15}$ — Ecker, Pich, de Rafael
- b_i $\quad O(p^6)$ — G.D., Ecker, Isidori, Portoles
- $a_+,\ b_+$ in general not related to $a_S,\ b_S$

- Expt. E865

  $K^+ \to \pi^+ e^+ e^- : \quad a_+ = -0.586 \pm 0.010 \quad b_+ = -0.655 \pm 0.044$

  confirmed by NA48/2 (1.4 σ's away) also in $K^+ \to \pi^+ \mu \overline{\mu}$

  Problems: a_i b_i same phenomenological size; p^4 p^6 different theoretical order

  Explained by interplay of the large $\pi\pi$ rescattering effect and VMD. $\pi\pi$ rescattering is not present in K_S-decays. Then we can just parameterize

$$\mathrm{Br}(K_S \to \pi^0 e^+ e^-) = 4.6 \times 10^{-9} a_S^2$$

  not predicted but dynamically interesting: $a_S \sim \mathcal{O}(1)$

$K_S \to \pi^0 l^+ l^-$ at NA48/1 Collaboration at CERN

- $K_S \to \pi^0 e^+ e^-$ 7 evts observed (with 0.15 expected bkg evts)

$$B(K_S \to \pi^0 e^+ e^-)_{m_{ee}>165 \text{ MeV}} = \left(3.0^{+1.5}_{-1.2} \pm 0.2\right) \times 10^{-9}$$

$$|a_S| = 1.08^{+0.26}_{-0.21}$$

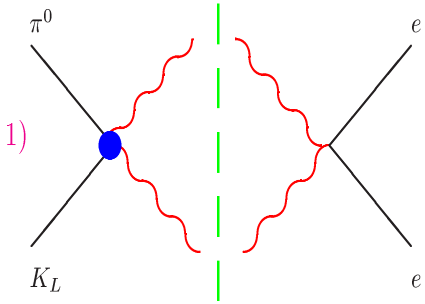
- $K_S \to \pi^0 \mu^+ \mu^-$ 6 events observed

$$B(K_S \to \pi^0 \mu^+ \mu^-) = \left(2.9^{+1.5}_{-1.2} \pm 0.2\right) \times 10^{-9}$$

$$|a_S|_{\mu\mu} = 1.54^{+0.40}_{-0.32} \pm 0.06$$

$K_L \to \pi^0 e^+ e^-$: summary

$\mathrm{Br}(K_L \to \pi^0 e^+ e^-) \leq 2.8 \cdot 10^{-10}$ KTeV

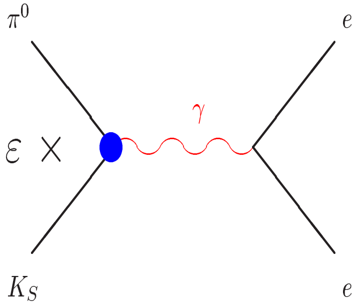


CP conserving NA48

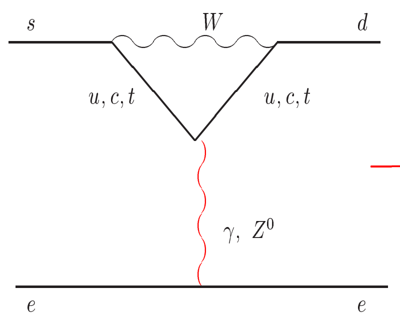
$\mathrm{Br}(K_L \to \pi^0 e^+ e^-) < 3 \cdot 10^{-12}$

V-A$\otimes$ V-A $\Rightarrow \langle \pi^0 e^+ e^- | (\bar{s}d)_{V-A} (\bar{e}e)_{V-A} | K_L \rangle$ violates CP

2) ε $\times$



$+$ 3)



$\longrightarrow$ Im λ_t

$\lambda_t = V_{td}V_{ts}^*$

$\uparrow$ $\mathrm{B}(K_S \to \pi^0 e^+ e^-) = 4.6 a_S^2 \times 10^{-9}$

Possible large interference: $a_S < -0.5$ or $a_S > 1$; short distance probe even for a_S large

$$|2) + 3)|^2 = \left[15.3\ a_S^2 - 6.8 \frac{Im\lambda_t}{10^{-4}}\ a_S + 2.8 \left(\frac{Im\lambda_t}{10^{-4}} \right)^2 \right] \cdot 10^{-12}$$

$$[17.7 \pm \quad 9.5 + \quad 4.7] \quad \cdot 10^{-12}$$

$$K(p_K) \to \pi(p_1)\pi(p_2)\gamma(q)$$

- Lorentz + gauge invariance $\Rightarrow$ Electric (E) and Magnetic(M) amplitude

$$A(K \to \pi\pi\gamma) = F^{\mu\nu}\left[E\partial_\mu K\partial_\nu\pi + M\varepsilon_{\mu\nu\rho\sigma}\partial^\rho K\partial^\sigma\pi\right]$$

- Unpolarizated photons

$$\frac{d^2\Gamma}{dz_1 dz_2} \sim |E|^2 + |M|^2$$

$$|E^2| = |E_{IB}|^2 + 2Re(E^*_{IB}E_D) + |E_D|^2$$

$$\downarrow$$

Low Theorem $\Rightarrow E_{IB} \sim \frac{1}{E^*_\gamma} + \text{c}$ $\qquad E_D$, M chiral tests

We need FIGHT DE/IB$\sim 10^{-3}$

	IB	DE_{exp}	
$K_S \to \pi^+\pi^-\gamma$	10^{-3}	$< 9 \cdot 10^{-5}$	$E1$
$K^+ \to \pi^+\pi^0\gamma$	10^{-4} $(\Delta I = \frac{3}{2})$	$(0.44 \pm 0.07)10^{-5}$ PDG	$M1$,$E1$
$K_L \to \pi^+\pi^-\gamma$	10^{-5} (CPV)	$(2.92 \pm 0.07)10^{-5}$ KTeVnew	$M1$, VMD

CPV is only from IB K_L (also measured in $K_L \to \pi^+\pi^-e^+e^-$)

BUT IB suppressed in K^+ and K_L.

$K_L \to \pi^+\pi^-\gamma$

M1 transtions clearly measured KTeV (00) with large slope

form factor measured

$$\mathcal{F} = 1 + \frac{a}{1 - \frac{m_k^2}{m_\rho^2} + \frac{2m_K E_\gamma^*}{m_\rho^2}}$$

E_γ^* photon energy

KTeV:

- a$=-1.243 \pm 0.057$
-

	linear slope	quadratic slope	$\mathcal{F}$
χ^2/DOF	$43.2/27$	$37.6/26$	$38.8/27$

$\Rightarrow$ Large VMD: $\rho-$pole

$$\text{Theory} \quad M1 \sim \quad \overbrace{a_2 + 2a_4}^{p^4} \quad + h.o.$$

$$\Downarrow$$

Large VMD in the a_i. Not automatic in all spin-1 formulations

[G.D. Portoles, G.D. Gao]

We should find a consistent treatment with M1 in $K^+ \to \pi^+ \pi^0 \gamma$

$K^+ \to \pi^+\pi^0\gamma$

$$A(K \to \pi\pi\gamma) = F^{\mu\nu}\left[E\partial_\mu K \partial_\nu \pi + M\varepsilon_{\mu\nu\rho\sigma}\partial^\rho K \partial^\sigma \pi\right]$$

$E1$ and $M1$ are measured with Dalitz plot

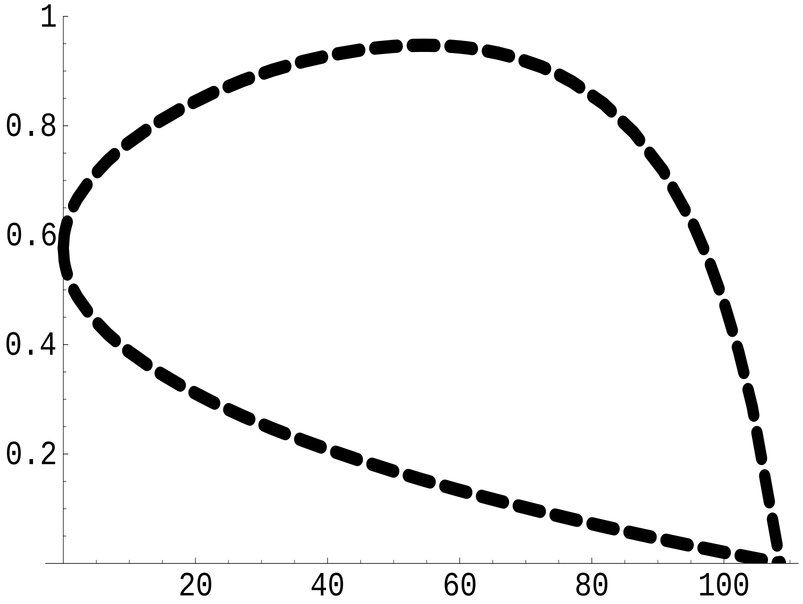
$$\frac{\partial^2\Gamma}{\partial T_c^* \partial W^2} = \frac{\partial^2\Gamma_{IB}}{\partial T_c^* \partial W^2}\left[1 + \frac{m_{\pi^+}^2}{m_K} 2Re\left(\frac{E1}{eA}\right) W^2 + \frac{m_{\pi^+}^4}{m_K^2}\left(\left|\frac{E1}{eA}\right|^2 + \left|\frac{M1}{eA}\right|^2\right) W^4\right]$$

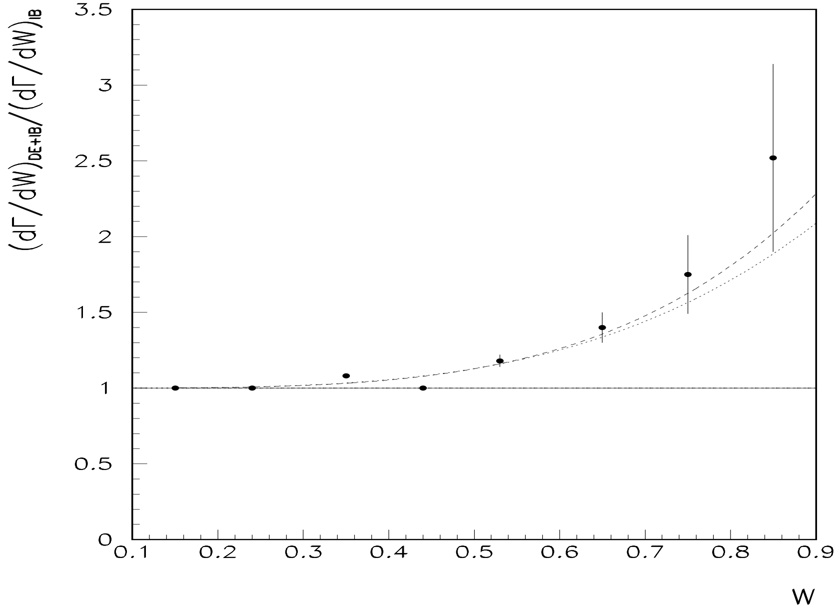
$$W^2 = (q \cdot p_K)(q \cdot p_+)/(m_\pi^2 m_K^2)$$

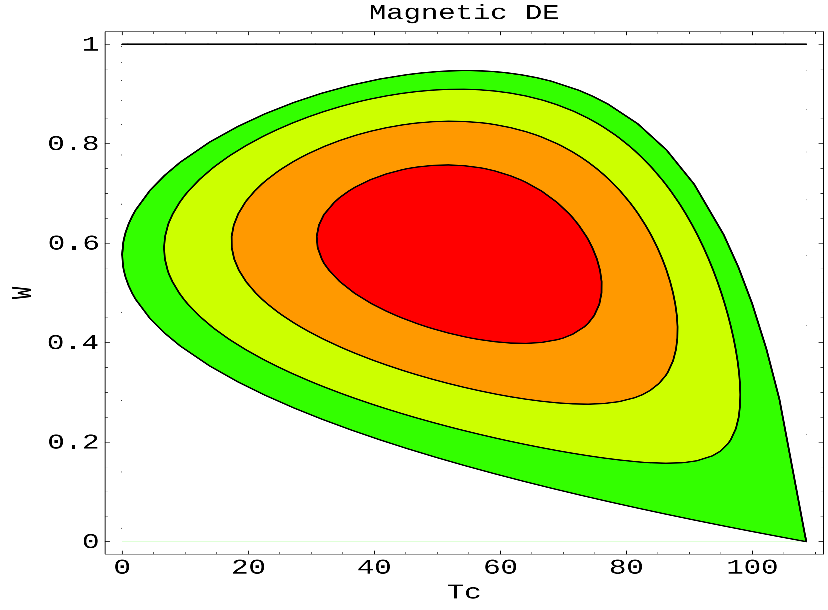
$$A = A(K^+ \to \pi^+\pi^0)$$

$K^+ \to \pi^+\pi^0\gamma$ $W - T_c$ Dalitz plot

Integrating over T_c deviations from IB measured





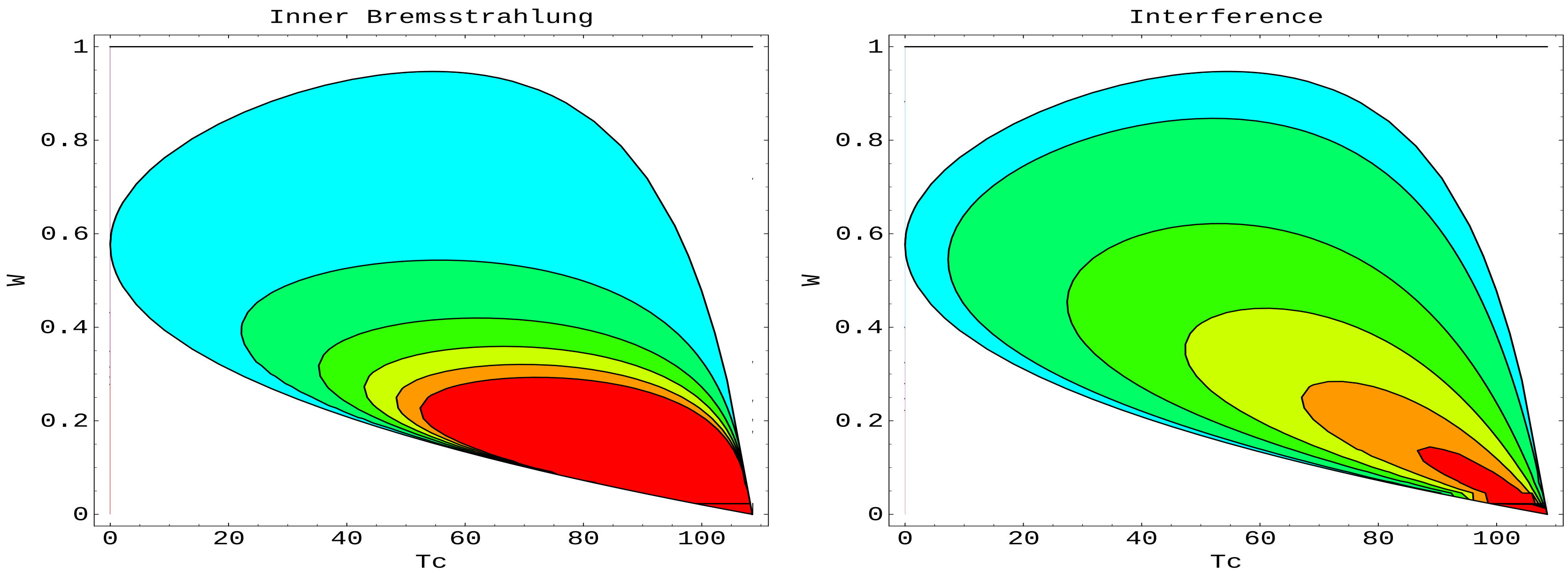


- M1 $\sim$ (-2 + $3a_2 - 6a_3$) -2.5

  $\uparrow$ Wess Zumino $\quad$ $\uparrow$ p^4 CT's (VMD) $\quad$ $\Downarrow$ a_i small?

Cheng
Bijnens, Ecker, Pich; G.D.,Gao

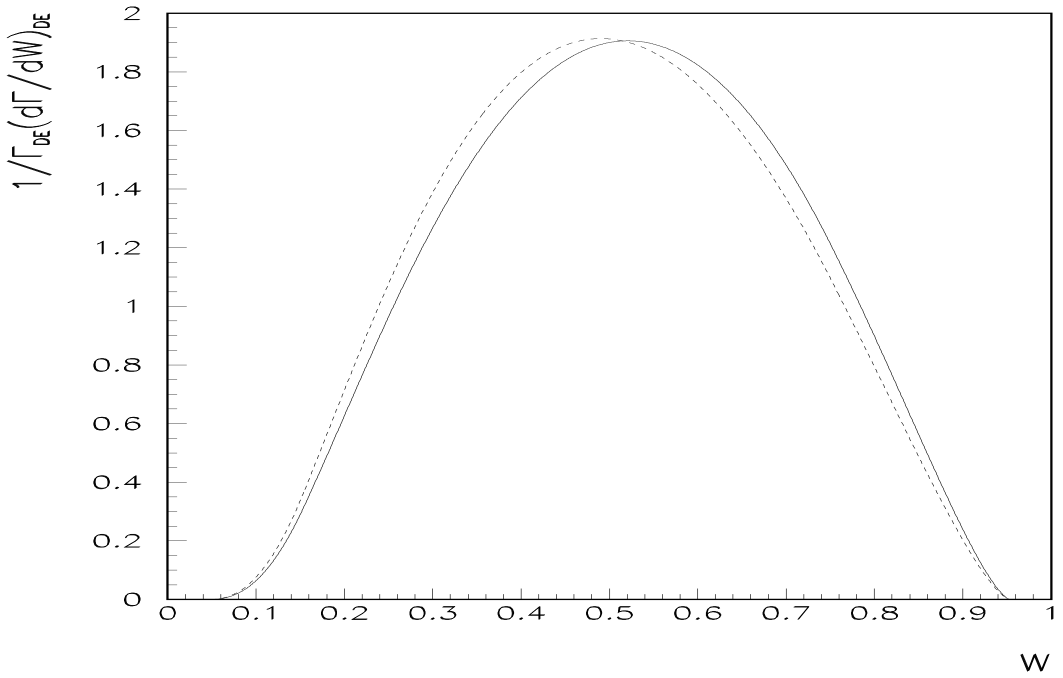
$K^+ \to \pi^+\pi^0\gamma$



- E787 has measured M1 and Re$\left(\frac{E1}{E_{IB}}\right) \sim (-0.4 \pm 1.6)\%$

$$\downarrow$$

- E1 dominated by CT $\Rightarrow$ E787 constrains models $(k_f < 1)$



- $\frac{d\Gamma}{dW}$ may be crucial to study well the form factor $W^2 = (q \cdot p_K)(q \cdot p_+)/(m_\pi^2 m_K^2)$

- to establish VMD

CP asymmetry $K^{\pm} \to \pi^{\pm}\pi^{0}\gamma$

- Bettter to measure asymm. in the slope than in the rate: in $\frac{\partial^2 \Gamma^{\pm}}{\partial T_c^* \partial W^2}$ we can select a favourable kin. region (large W^2)
- For the calculation useful the compensating channel $K^{\pm} \to \pi^{\pm}\pi^{0}$
- This asymm., Ω, in extensions of SM $\sim \mathcal{O}(10^{-4})$ Colangelo et al.
- SM $\leq \mathcal{O}(10^{-5})$ Paver et al.
- Assuming the expts. are almost seeing the CP conserving $E1$ Statistics seems tough but previous limit (Smith eta l. 76) weak
- Similar analysis for CPV in K_L: but time interf. required

Conclusions

- $K \to \pi\nu\nu$
- Hard work for other channels
- phenomenology+ theory very important
- important to see $K_L \to \pi^0 e^+ e^-$
- CHPT useful to uncover New Physics

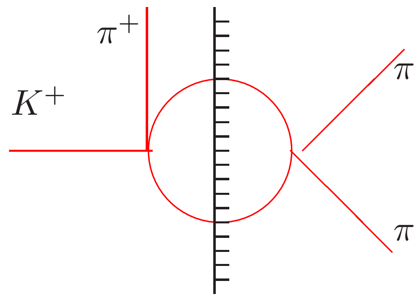
$K \to 3\pi$: slope asymm. $\Delta g/2g = (g_+ - g_-)/(g_+ + g_-)$

- Isospin decomposition, rescattering properties

$$\Downarrow$$

- $$A(K^+ \to \pi^+\pi^+\pi^-) \quad = \quad a e^{i\alpha_0} + b e^{i\beta_0} Y + \mathcal{O}(Y^2)$$

Final State Interaction



Zeldovich,Grinstein et al
Isidori,Maiani,Pugliese

Compared to $K \to \pi\pi$

- two $\Delta I = 1/2$ transitions (a, b)
- final state small ($\alpha_0, \beta_0 \sim 0.1$)

- CPT relates
$$\begin{aligned} A(K^+ \to \pi^+\pi^+\pi^-) &= a e^{i\alpha_0} + b e^{i\beta_0} Y \\ A(K^- \to \pi^-\pi^-\pi^+) &= a^* e^{i\alpha_0} + b^* e^{i\beta_0} Y \end{aligned}$$

- The asymmetry
$$\frac{g_+ - g_-}{g_+ + g_-} = \left[\frac{\Im b}{\Re b} - \frac{\Im a}{\Re a}\right] \sin(\alpha_0 - \beta_0) \, ,$$
can be evaluated in CHPT

- Only one operator at $\mathcal{O}(p^2)$:
$$G_8 e^{i\phi} \langle \lambda_6 \partial_\mu U \partial^\mu U^\dagger \rangle$$
ϕ $\Delta I = 1/2$ electroweak phase

- Then if we stop at $\mathcal{O}(p^2)$
$$\frac{\Im b}{\Re b} = \frac{\Im a}{\Re a} \Rightarrow \Delta g / 2g = 0$$

- However $\mathcal{O}(p^4)$ is necessary in order to reproduce the phenomenological values $\frac{\Delta a}{a} \sim \frac{\Delta b}{b} \sim 30\%$

$$\Downarrow$$

- splitting $a = a^{(2)} + a^{(4)}$ and $b = b^{(2)} + b^{(4)}$

$$\frac{\Delta g}{2g} = \frac{\Im A^0}{\Re A^0}\,(\alpha_0 - \beta_0)\left(\frac{\Re b^{(4)}}{\Re b^{(2)}} - \frac{\Im b^{(4)}}{\Im b^{(2)}} + \frac{\Im a^{(4)}}{\Im a^{(2)}} - \frac{\Re a^{(4)}}{\Re a^{(2)}}\right)$$

G.D.,Isidori,Paver

- Since
  - $|\frac{\Im A^0}{\Re A^0}| \sim 22\epsilon' \sim 10^{-4}$ $(\alpha_0 - \beta_0) \sim 0.1$
  - to maximize Δg, we take $\mathcal{O}(p^4) \sim \mathcal{O}(p^2)$

- $\Delta g/2g \leq 10^{-5}$ NA48 $(1.5 \pm 2.2) \cdot 10^{-4}$

New Physics

- New Physics: crucial to have large $\Delta g/2g$ is to find an operator which affects $K \to 3\pi$ but not $K \to 2\pi$, limited by the experimental size of ϵ'
- In fact Masiero- Murayama explanation of a possible large size of ϵ' does the job!

- They suggest that susy (s-)particles, introduce new flavour structures affecting only the $\Delta S = 1$ and not $\Delta S = 2$ interactions

$$\mathcal{H}_{\rm mag} = C_g^+ Q_g^+ + C_g^- Q_g^- + {\rm h.c.}$$

$$Q_g^{\pm} = \frac{g}{16\pi^2} \left(\bar{s}_L \sigma^{\mu\nu} t^a G^a_{\mu\nu} d_R \pm \bar{s}_R \sigma^{\mu\nu} t^a G^a_{\mu\nu} d_L \right)$$

- Q_g^+ is affects only $K \to 3\pi$; Q_g^- only $K \to 2\pi$ G.D,Isidori,Martinelli

- As a result by tuning properly $C_g^{\pm}$ we can generate large $\Delta g/2g$ ($\leq 10^{-4}$)

- The large slope for $K^+ \to \pi^+ e^+ e^-$ calls for large VMD
- $K^+ \to \pi^+ e^+ e^-$ receives substantial $\pi\pi$-loop, contrary to $K_S \to \pi^0 e^+ e^-$ (~ 0),
- if we split

$$\left(\frac{a_i^{\rm VMD}}{1 - z m_K^2/m_V^2} + a_i^{\rm nVMD} \right) \approx \left[\left(a_i^{\rm VMD} + a_i^{\rm nVMD} \right) + a_i^{\rm VMD} \frac{m_K^2}{m_V^2} z \right]$$

Then we can determine both terms from expt.

$$a_+^{\rm VMD} = \frac{m_V^2}{m_K^2} b_+^{\rm exp} = -1.6 \pm 0.1 \; , \qquad a_+^{\rm nVMD} = a_+^{\rm exp} - a_+^{\rm VMD} = 1.0 \pm 0.1$$

- Also we can hope $a_i^{\rm VMD}$ obey a short distance relation since i) VMD is a larger scale and ii) NOT affected by $\pi\pi$-loop
- The only operator at short distances is $Q_7 = \overline{s}\gamma^\mu(1-\gamma_5)d\,\overline{\ell}\gamma_\mu\ell$,

$$\mathcal{H}_{eff}^{|\Delta S|=1} = \frac{G_F}{\sqrt{2}} V_{us}^* V_{ud} \Big[\sum_{i=1}^{6,7V} (z_i(\mu) + \tau y_i(\mu)) Q_i(\mu) \; + \; \tau y_{7A}(M_W) Q_{7A}(M_W) \Big]$$

$\tau = -V_{ts}^* V_{td}/V_{us}^* V_{ud}$. The Wilson coefficients $z_7(\mu)$ and $\tau y_7(\mu)$ determine the CPC CPV amplitudes and their relative sign. The isospin structure of Q_{7V} leads

$$(a_S)_{\langle Q_{7V}\rangle} = -(a_+)_{\langle Q_{7V}\rangle}$$

- If this relation is obeyed by the full VMD amplitude

$$(a_S^{\rm VMD})_{\langle Q_{7V}\rangle} = -a_+^{\rm VMD} = 1.6 \pm 0.1$$

in good agreement with NA48 $(|a_S| = 1.08^{+0.26}_{-0.21})$

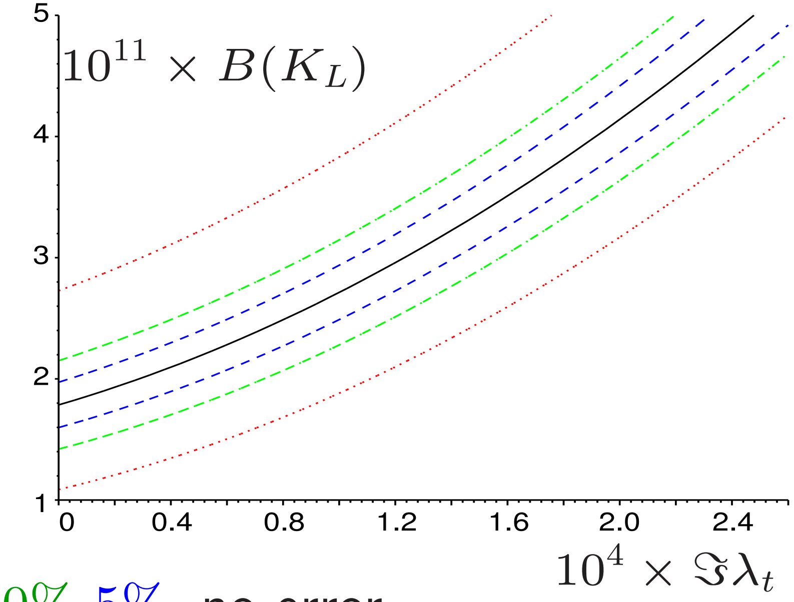
- Having i) separated the contribution better suited to comparison with s.d. (VMD) and ii) realized that this dominates Theoret. and Phenom.(NA48) a_S

- we believe the positive interference of s.d.

$$B(K_L \to \pi^0 e^+ e^-)_{\rm SM} = (3.1^{+1.2}_{-0.9}) \times 10^{-11} \quad a_S$$

KTeV $B(K_L \to \pi^0 e^+ e^-)$

$< 2.8 \times 10^{-10}$ at $90\,\%$C.L.



Present error on $a_S = 1.08$, 10% 5%, no error

K-physics bound: $-1.2 \times 10^{-3} < \Im\lambda_t < 1.0 \times 10^{-3}$ at $90\,\%$C.L.